Magnetostatic spin solitons in ferromagnetic nanotubes

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(Received 15 October 2003; revised manuscript received 23 April 2004; published 18 October 2004)

We study the linear and nonlinear evolution of a magnetostatic spin wave (MSW) in a charge free, isotropic ferromagnetic hollow nanotube. By analyzing the dispersion relation we observe that elliptically polarized forms of wave can propagate through the ferromagnetic nanotube. Using the multiple scale analysis we find that the dynamics of magnetization of the medium is governed by the cubic nonlinear Schrödinger equation. The stability of the continuous wave, related to the propagation of either bright or dark (MS) solitons in the nanotube, is governed by the direction of the external magnetic field relative to the magnetized nanotube.

DOI: 10.1103/PhysRevB.70.134413 PACS number(s): 61.46.+w, 41.20.Jb, 05.45.Yv

I. INTRODUCTION

The study of interaction of electromagnetic (EM) field in ordered magnetic media has become an emerging and growing field of research. In this context, ferromagnetic medium has assumed lot of importance in the field of magneto-optical recording for higher storage and faster reading of information.[1] The ferromagnetic medium with different magnetic interactions were identified as an interesting class of nonlinear medium exhibiting soliton excitations of their magnetization in the classical continuum limit.[2–4] This soliton excitation of magnetization is mainly due to the nonlinearity present in the ferromagnetic medium. Several studies on the propagation of EM wave (EMW) in a ferromagnetic medium were carried out in the recent past taking into account the nonlinear nature of the medium.[5,6] The authors of the present paper have separately investigated the nonlinear modulation of quasi-monochromatic EMW in a ferromagnetic medium with different magnetic interactions.[7–10] The results show that when the EMW propagates in a charge free isotropic or anisotropic ferromagnetic medium, the plane EMW is modulated in the form of solitons, while the magnetization of the medium exhibits soliton excitations.[7–10] This results in the dispersionless propagation of EMW in a ferromagnetic medium. The propagation of magnetostatic spin wave (MSW) has been also studied extensively, especially in ferromagnetic thin films.[6] It has been shown that magnetostatic backward volume waves (MSBVW) can propagate in the case when the direction of propagation of EMW is parallel to the constant applied field.[6] The nonlinear Schrödinger (NLS) equation which describes the propagation of MSW solitons in thin magnetic films has been first derived as early as 1982.[11] Further, bright and dark solitons have been observed experimentally a few years later.[12,13] In a recent paper one of the authors, Leblond, has given a rigorous derivation of the NLS equation which describes the propagation of the MSBVW solitons, using for the first time a multiscale approach which takes into account the wave guide properties.[14] Further, the propagation of EM soliton in isotropic and anisotropic antiferromagnetic media was studied by Daniel and Veerakumar.[15] The above studies were restricted to the bulk ferro/antiferromagnetic materials and ferromagnetic thin films.

In a different context many innovative magnetic materials are now synthesized experimentally in order to fulfill the demands of the data storage industry. Important among them are the nanoscale ferromagnetic tubes and wires which are synthesized using nanofabrication techniques and whose properties can be tailored as per our needs by changing the size, the shape or composition of the nanostructures.[16] In this case the size of the nanomagnets becomes comparable to the magnetic length scales such as exchange length or domain wall width. Therefore it will be interesting to understand the effect of interaction of EM field with the magnetization of the nanoferromagnetic tubes. The linear propagation of MSW in magnetic nanowires has been studied recently.[17] In the present paper we investigate the interaction of spatially varying EM field with the magnetization of a charge free isotropic ferromagnetic hollow nanotube and the nature of excitation of the later. The linear propagation is studied in Sec. II: first we formulate the model and derive the dynamical equations to be solved. Then, using a cylindrical model for the nanotube and linearizing the system of coupled equations, the analysis of the dispersion relation is carried out. Section III deals with the nonlinear problem of interaction of EM field with the magnetization of the ferromagnetic nanotube without free charges, and the results are concluded in Sec. IV.

II. LINEAR WAVES

A. Equations for an ultrathin ferromagnetic film

We consider an isotropic charge free ferromagnetic film in the form of a cylindrical hollow tube with the surface thickness very less compared to other dimensions. Practically such cylindrical tubes are realized in the form of ferromagnetic nanotubes which are experimentally synthesized.[16] The hollow cylindrical ferromagnetic tube is locally equivalent to a planar film as shown in Fig. 1, provided the thickness a of

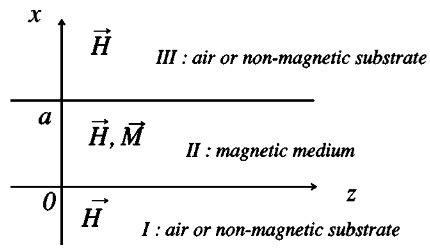

FIG. 1. Thin film representation of the ferromagnetic nanotube.

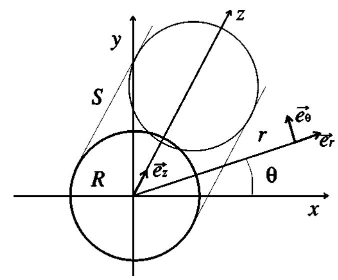

FIG. 2. Cylindrical model of the ferromagnetic nanotube.

the film is constant and very small in relation with the radius of curvature. In this case, the magnetic medium fills the region of space between $x=0$ and $x=a$, denoted by II. The regions above and below the magnetic medium are denoted as III and I, respectively. The magnetization dynamics in the isotropic charge free ferromagnetic film in a continuum limit is governed by the Landau-Lifshitz equation[18]

$$\partial_t \vec{M} = \vec{M} \wedge [\vec{H}^i + J\Delta \vec{M}], \tag{1}$$

where $\vec{M}=(M_x, M_y, M_z)$ is the magnetization of the medium, $\vec{H}^i=(H_x^i, H_y^i, H_z^i)$ is the magnetic field inside the film, J is the exchange integral and $\Delta=\partial_x^2+\partial_y^2+\partial_z^2$ is the Laplacian operator. The magnetization $\vec{M}$ is zero outside the medium. The term $\vec{H}^i$ accounts for both demagnetizing and dipolar fields. Thus the Landau-Lifshitz equation (1) gives account for the dipolar and exchange interactions.[6] The length of nanotubes, at most in the micrometer range, will be very small with regard to the considered wavelengths, rarely below the millimeter range. Therefore retardation can be neglected, and the components of the external magnetic field satisfy the magnetostatic Maxwell equations[19]

$$\vec{\nabla} \wedge \vec{H} = 0, \tag{2}$$

$$\vec{\nabla} \cdot (\vec{H} + \vec{M}) = 0. \tag{3}$$

The surface of the hollow cylinder can be considered as an ultrathin ferromagnetic film, of thickness a. Notice that a is well below the exchange length, and even smaller with regard to the thickness of the usual ferromagnetic thin films. Thus usual results on MSW in magnetic films do not apply here. Therefore we study the above equations in the limit $a \to 0$. In this limit it is reasonable to assume that $\vec{H}$ and $\vec{M}$ depend only on x. It follows from Eqs. (2) and (3) that H_y and H_z are uniform in regions I, II, and III, and so are $H_x + M_x$ in region II and H_x in regions I and III. Further, the boundary conditions state that these field components are continuous at the surfaces $x=0$ and $x=a$. Thus we obtain the following conditions specific to our problem:

$$H_{\mathrm{I},\beta} = H_{\mathrm{II},\beta} = H_{\mathrm{III},\beta}, \tag{4}$$

where $\beta=y,z$ and

$$H_{\mathrm{II},x} = H_{\mathrm{I},x} - M_x, \tag{5}$$

$$H_{\mathrm{III},x} = H_{\mathrm{I},x}. \tag{6}$$

The subscripts I, II, and III in Eqs. (4)–(6) represent the respective region.

B. Cylindrical model and linearized equations

Since the ultrathin ferromagnetic film is rolled to form a cylindrical hollow tube, the magnetization exists only on the surface S of the cylinder as illustrated in Fig. 2. We denote by z the axis of the cylinder and by $\vec{e}_r$, $\vec{e}_\theta$, and $\vec{e}_z$ the unit vectors along the cylindrical coordinates (r, θ, z). The cylindrical components of the magnetization and magnetic field are written as (M_r, M_θ, M_z) and (H_r, H_θ, H_z), respectively. The field $\vec{H}$ satisfy the magnetostatic equations (2) and (3) everywhere. Outside the magnetic medium, Eq. (3) reduces to

$$\vec{\nabla} \cdot \vec{H} = 0. \tag{7}$$

The surface S of the film is obtained at the limit $r \to R$. According to the results of the preceding section, the magnetic field inside the ferromagnetic surface $\vec{H}^i$ is related to the exterior field $\vec{H}$ and the magnetization $\vec{M}$ by

$$\vec{H}^i = \begin{pmatrix} H_r - M_r \\ H_\theta \\ H_z \end{pmatrix}. \tag{8}$$

In order to derive the dispersion relation we expand the magnetization of the medium and the magnetic field as

$$\vec{M} = \vec{M}_0 + \varepsilon \vec{m}, \tag{9}$$

$$\vec{H} = \vec{H}_0 + \varepsilon \vec{h}, \tag{10}$$

where $\vec{M}_0$ and $\vec{H}_0$ represent the uniform saturation magnetization and the applied static magnetic field, respectively, and ε is a small perturbation parameter. $\vec{m}$ and $\vec{h}$ are the wave magnetization and magnetic field, respectively. Using the expansions (9) and (10) in Eq. (1) and neglecting the quadratic terms we obtain the following equation at the order ε^0:

$$\vec{M}_0 \wedge \vec{H}_0^i = 0. \tag{11}$$

At order ε, we get

$$\partial_t \vec{m} = \vec{M}_0 \wedge (\vec{h}^i + J\Delta \vec{m}) + \vec{m} \wedge \vec{H}_0^i. \tag{12}$$

For sake of simplicity, we assume that the magnetization $\vec{M}_0$ has the symmetry of the cylinder. Since it is uniform, it is parallel to the axis of the cylinder (z axis), and so is $\vec{H}_0$. Then the uniform magnetization and magnetic field are

$$\vec{M}_0 = M\vec{e}_z, \tag{13}$$

$$\vec{H}_0 = \alpha M\vec{e}_z. \tag{14}$$

The parameter $\alpha = H_0/M_0$ measures the strength of the external field. In bulk materials, the steady state is stable when $\alpha > 0$ only. In the smallest nanotubes, the reversal of the magnetization which should occur for negative α can be prohibited by exchange effects.[16] The diameter of the tubes can indeed be less than the exchange length. Then steady states with negative values of α may become stable. Substituting Eqs. (9) and (10) into Eq. (8), we find that the magnetic field on the surface of the tube can be written as

$$\vec{h}^i = \vec{h} - m_r\vec{e}_r. \tag{15}$$

Using Eqs. (13)–(15), Eq. (12) can be rewritten as

$$\partial_t\vec{m} = M\vec{e}_z \wedge (\vec{h} - m_r\vec{e}_r - \alpha\vec{m} + J\Delta\vec{m}). \tag{16}$$

Here the term $-m_r\vec{e}_r$ is the demagnetizing field. It is shown below that the relative magnitude of the demagnetizing field and the exchange integral strongly influences the polarization of the MSW.

C. Dispersion relation

Let us consider a linear wave propagating along the axis of the cylinder. Therefore we look for a solution in the form

$$\vec{m} = \vec{A}(\theta)e^{i(kz-\omega t)}, \tag{17}$$

$$\vec{h} = \vec{C}(r,\theta)e^{i(kz-\omega t)}, \tag{18}$$

where $\vec{A}$ and $\vec{C}$ are the amplitudes of the wave magnetization and magnetic field, respectively. $\vec{A}$ depends only on θ since the magnetization exists in the surface of the cylinder. Substitution of Eqs. (17) and (18) reduces Eqs. (2), (7), and (16) to

$$(\vec{\nabla}_\perp + ik\vec{e}_z) \wedge \vec{C} = 0, \tag{19}$$

$$(\vec{\nabla}_\perp + ik\vec{e}_z) \cdot \vec{C} = 0, \tag{20}$$

$$-i\omega\vec{A} = M\vec{e}_z \wedge [\vec{C} - A_r\vec{e}_r - \alpha\vec{A} + J(\vec{\nabla}_\perp^2 - k^2)\vec{A}], \tag{21}$$

where $\vec{\nabla}_\perp = \vec{e}_r\partial_r + (1/r)\vec{e}_\theta\partial_\theta$. Equations (19) and (20) are satisfied by $\vec{C}=0$ (it is easy to show that this condition is necessary if $k=0$, but other solutions seem to be possible for the case of interest of a nonzero k). Further, due to the symmetry of the cylinder, we assume that $\vec{A}$ is constant with regard to θ. Thus the components A_r, A_θ, and A_z of $\vec{A}$ are uniform. The Laplacian operator can be expressed as

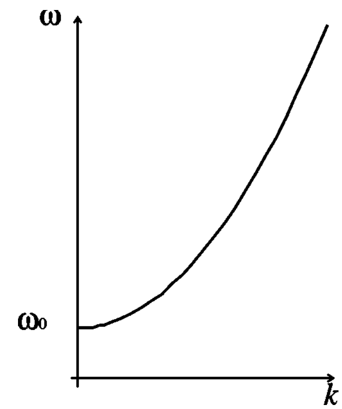

FIG. 3. The dispersion relation for MSW in hollow nanotubes.

$$\vec{\nabla}_\perp^2\vec{A} = -\frac{A_r}{r^2}\vec{e}_r - \frac{A_\theta}{r^2}\vec{e}_\theta, \tag{22}$$

which reduces Eq. (21) to

$$-i\Omega\vec{A} = \vec{e}_z \wedge \begin{pmatrix} -(1+\alpha)A_r - KA_r \\ -\alpha A_\theta - KA_\theta \\ -\alpha A_z - Jk^2A_z \end{pmatrix}. \tag{23}$$

We have set

$$K = J\left(k^2 + \frac{1}{R^2}\right), \tag{24}$$

$$\Omega = \frac{\omega}{M}. \tag{25}$$

In Eq. (23) we have used the limit $r \to R$ as we analyze the dynamics of magnetization on the surface of the ferromagnetic nanotube. From Eq. (23) we find the dispersion relation

$$\frac{\omega^2}{M^2} = \left[\alpha + J\left(k^2 + \frac{1}{R^2}\right)\right]\left[1 + \alpha + J\left(k^2 + \frac{1}{R^2}\right)\right], \tag{26}$$

which is plotted in Fig. 3. Due to the geometry, the wave guiding properties of the hollow tube differ from that of the thin films, thus the dispersion relation (26) coincides neither with that of MSW in thin films nor in the bulk medium. Notice that the curve $\omega = \omega(k)$ does not cross the origin, but presents some minimal frequency

$$\omega_0 = M\sqrt{\left(\alpha + \frac{J}{R^2}\right)\left(1 + \alpha + \frac{J}{R^2}\right)}. \tag{27}$$

The minimal frequency ω_0 also exists in the bulk, its value is $\omega_0 = M\sqrt{\alpha(1+\alpha)}$, which corresponds to the ferromagnetic resonance. The latter is indeed often considered at microwave frequencies, in a situation where the effect of the exchange on the dispersion is negligible. In the nanotubes, ω_0 is shifted up due to the combined effect of the exchange interaction and the tube curvature. From Eq. (23), we obtain the amplitude of the linear spin wave as

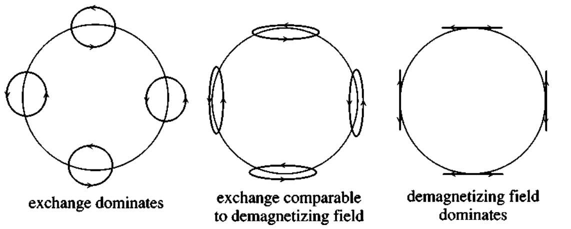

FIG. 4. The possible polarizations of a spin wave in a nanotube.

$$\vec{A} = \begin{pmatrix} -(\alpha + K) \\ i\Omega \\ 0 \end{pmatrix}. \tag{28}$$

If $(\alpha+K) \gg 1$, i.e., if the demagnetizing field is negligible compared to the external and exchange field, then Eq. (28) refers to a circularly polarized wave. In this case, the waves are clearly yielded by the precession of the magnetization about the axis of the cylinder. On the other hand, if the demagnetizing field is not negligible, then the precession is slightly distorted, giving rise to an elliptically polarized wave. On the contrary, if the demagnetizing field is dominant, $(\alpha+K) \ll 1$, the precession is not possible at all, and the polarization vector $\vec{A}$ becomes proportional to (0, 1, 0), which represent a wave linearly polarized in the tangential direction. In the same time the frequency goes to zero. Thus, in conclusion, the linear analysis shows the possibility of existence of an elliptically polarized wave. The elliptical polarization is close to circular when the exchange or the external field dominates, and becomes close to linear when the demagnetizing field dominates, as illustrated in Fig. 4.

III. NONLINEAR WAVES AND SOLITONS

A. Multiscale formalism

The nonlinear modulation of the linear wave found in the preceding section is now investigated using the multiple scale analysis. For this we expand the magnetization of the medium and the magnetic field in a series of harmonics of a fundamental phase $\varphi = kz - \omega t$ as

$$\vec{M} = \sum_{n \geq 0, |p| \leq n} \varepsilon^n \vec{M}_n^p e^{ip\varphi}, \tag{29}$$

$$\vec{H} = \sum_{n \geq 0, |p| \leq n} \varepsilon^n \vec{H}_n^p e^{ip\varphi}, \tag{30}$$

where again ε is the small perturbation parameter, that will give account of the smallness of the signal amplitude and of the spectral width. The profiles $\vec{M}_n^p$ and $\vec{H}_n^p$ are functions of the slow variables

$$\zeta = \varepsilon(z - Vt) \quad \text{and} \quad \tau = \varepsilon^2 t. \tag{31}$$

From Eqs. (2) and (7) we find that $\vec{H}$ is uniform. We still assume that the wave has the symmetry of the cylinder, then the cylindrical components of the magnetization are independent of θ.

We substitute the expansions (29) and (30) for $\vec{M}$ and $\vec{H}$ and the stretched variables (31) into Eq. (1), collect the terms proportional to different powers of ε and try to solve the resulting equations. At order ε^0, we find again that the magnetization density is collinear to the magnetic field. Equations (13) and (14) are still valid (with $\vec{M}_0^0 = \vec{M}_0$ and $\vec{H}_0^0 = \vec{H}_0$). Here M can take both signs. The positive z direction is defined by the propagation direction of the waves. At order ε, we are interested only in $p = \pm 1$. For $p=1$, we get

$$-i\Omega \vec{M}_1^1 = \vec{e}_z \wedge \begin{pmatrix} -(1+\alpha)M_{1,r}^1 - KM_{1,r}^1 \\ -\alpha M_{1,\theta}^1 - KM_{1,\theta}^1 \\ -\alpha M_{1,z}^1 - Jk^2 M_{1,z}^1 \end{pmatrix}. \tag{32}$$

Recall that K is defined by Eq. (24). From Eq. (32) we retrieve again the dispersion relation (26). The corresponding polarization vector is written as

$$\vec{M}_1^1 = \begin{pmatrix} -(\alpha + K) \\ i\Omega \\ 0 \end{pmatrix} g \quad \text{with } g = g(\zeta, \tau). \tag{33}$$

At order ε^2 we obtain

$$-ip\omega \vec{M}_2^p - V\partial_\zeta \vec{M}_1^p = M\vec{e}_z \wedge \vec{U}_2^p + \vec{X}_2^p, \tag{34}$$

where

$$\vec{U}_2^p = \begin{pmatrix} -(1+\alpha+K_p)M_{2,r}^p \\ -(\alpha+K_p)M_{2,\theta}^p \\ -(\alpha+Jp^2k^2)M_{2,z}^p \end{pmatrix} + 2ipkJ\partial_\zeta \vec{M}_1^p, \tag{35}$$

with

$$K_p = J\left(p^2k^2 + \frac{1}{R^2}\right) \tag{36}$$

and

$$\vec{X}_2^p = \sum_{q+s=p} \vec{M}_1^q \wedge \begin{pmatrix} -(1+K_s)M_{1,r}^s \\ -K_s M_{1,\theta}^s \\ 0 \end{pmatrix}. \tag{37}$$

The nonlinear term $\vec{X}_2^p$ can be nonzero for $p=0$ or ± 2 only. For $p=0$, using Eq. (33) we obtain after simplification $\vec{X}_2^0 = \vec{0}$. Since the solution of the profile equation (34) is unique for $p \neq \pm 1$, we find that

$$\vec{M}_2^0 = \vec{0}. \tag{38}$$

For $p=2$, still at order ε^2, we get

$$\vec{X}_2^2 = -i\Omega(\alpha+K)g^2\vec{e}_z, \tag{39}$$

which implies a nonzero $\vec{M}_2^2$, as

$$\vec{M}_2^2 = \frac{\alpha + K}{2M} g^2 \vec{e}_z. \tag{40}$$

For $p=1$, Eq. (34) yields $M_{2,z}^1 = 0$ and

$$-i\Omega M_{2,r}^{1}+\frac{V}{M}(\alpha+K)\partial_{\zeta}g=(\alpha+K)M_{2,\theta}^{1}+2k\Omega J\partial_{\zeta}g, \tag{41}$$

$$-i\Omega M_{2,\theta}^{1}-i\Omega\frac{V}{M}\partial_{\zeta}g=-(1+\alpha+K)M_{2,r}^{1}-2ik(\alpha+K)J\partial_{\zeta}g. \tag{42}$$

Combing Eq. (42) and Eq. (41) and simplifying the resulting equation using the dispersion relation (26), we obtain the wave velocity

$$\frac{V}{M}=\frac{kJ}{\Omega}(2\alpha+2K+1). \tag{43}$$

It is observed that Eq. (43) coincides with the usual expression for the group velocity

$$\frac{V}{M}=\frac{1}{M}\frac{d\omega}{dk}=\frac{d\Omega}{dk}. \tag{44}$$

Another combination of Eqs. (41) and (42) yields the relation

$$i\Omega M_{2,r}^{1}+(\alpha+K)M_{2,\theta}^{1}=\frac{-kJ(\alpha+K)}{\Omega}\partial_{\zeta}g, \tag{45}$$

which will be useful below.

The nonlinear evolution equation is found as usual at order ε^3, as a solvability condition for the fundamental frequency $p=1$. The equation satisfied by this component at this order is

$$-i\omega\vec{M}_{3}^{1}-V\partial_{\zeta}\vec{M}_{2}^{1}+\partial_{\tau}M_{1}^{1}=M\vec{e}_{z}\wedge\vec{U}_{3}^{1}+\vec{X}_{3}^{1}, \tag{46}$$

with

$$\vec{U}_{3}^{1}=\begin{pmatrix}-(1+K)M_{3,r}^{1}\\ -KM_{3,\theta}^{1}\\ 0\end{pmatrix}+2iJk\partial_{\zeta}\vec{M}_{2}^{1}+J\partial_{\zeta}^{2}\vec{M}_{1}^{1}-\alpha\vec{M}_{3}^{1} \tag{47}$$

and

$$\vec{X}_{3}^{1}=\sum_{q}(\vec{X}_{3,a}^{1,q}+\vec{X}_{3,b}^{1,q}). \tag{48}$$

The nonlinear terms $\vec{X}_{3,a}^{1,q}$ and $\vec{X}_{3,b}^{1,q}$ are defined as

$$\vec{X}_{3,a}^{1,q}=\vec{M}_{1}^{q}\wedge\left[\begin{pmatrix}-(1+K_{s})M_{2,r}^{s}\\ -K_{s}M_{2,\theta}^{s}\\ 0\end{pmatrix}+2iJs\partial_{\zeta}\vec{M}_{1}^{s}\right] \tag{49}$$

and

$$\vec{X}_{3,b}^{1,q}=\vec{M}_{2}^{q}\wedge\begin{bmatrix}-(1+K_{s})M_{1,r}^{s}\\ -K_{s}M_{1,\theta}^{s}\\ 0\end{bmatrix}, \tag{50}$$

where $s=1-q$. It is clear that $\vec{X}_{3,a}^{1,q}$ can be nonzero only for $q=\pm 1$ and $\vec{X}_{3,b}^{1,q}$ for $s=\pm 1$ only. Since $\vec{M}_{2}^{0}=\vec{0}$, two terms vanish and $\vec{X}_{3}^{1}$ reduces to $\vec{X}_{3,a}^{1,-1}+\vec{X}_{3,b}^{1,2}$. Using expression (40) for $\vec{M}_{2}^{2}$, we deduce

$$\vec{X}_{3}^{1}=\frac{\alpha+K}{2M}g|g|^{2}\begin{pmatrix}-i\Omega(K-4k^{2}J)\\ (\alpha+K)(1+K-4k^{2}J)\\ 0\end{pmatrix}. \tag{51}$$

The r and θ components of Eq. (46), can be combined to obtain a new equation, in which the terms involving $M_{3,\theta}^{1}$ vanish directly. The terms with $M_{3,r}^{1}$ vanish after using the dispersion relation (26). The term involving $\vec{M}_{2}^{1}$ in the new combined equation is

$$T=\frac{V}{M}\partial_{\zeta}[i\Omega M_{2,r}^{1}-(\alpha+K)M_{2,\theta}^{1}] -2iJk\partial_{\zeta}[i\Omega M_{2,\theta}^{1}+(\alpha+K)M_{2,r}^{1}]. \tag{52}$$

Using the expression (43) of the group velocity V, it reduces to

$$T=\frac{-k^{2}J^{2}}{\Omega^{2}}(\alpha+K)\partial_{\zeta}^{2}g. \tag{53}$$

Substituting Eqs. (53) and (33), the equation obtained by combining the r and θ components of Eq. (46), reduces to the integrable cubic NLS equation

$$iG\partial_{\tau}g+B\partial_{\zeta}^{2}g+Cg|g|^{2}=0, \tag{54}$$

where

$$G=\frac{2\Omega(\alpha+K)}{M}, \tag{55}$$

$$B=\frac{-k^{2}J^{2}}{\Omega^{2}}(\alpha+K)+J[\Omega^{2}+(\alpha+K)^{2}], \tag{56}$$

$$C=\frac{1}{M}[i\Omega X_{3,r}^{1}-(\alpha+K)X_{3,\theta}^{1}]. \tag{57}$$

Substituting the expression (51) of $\vec{X}_{3}^{1}$, the nonlinear coefficient C reduces to

$$C=\frac{-(\alpha+K)^{2}}{2M^{2}}(\alpha+4k^{2}J) \tag{58}$$

and using Eq. (26), we reduce the expression (56) of the dispersion coefficient B to

$$B=J[\alpha+K]\left[2(\alpha+K)+1-\frac{k^{2}J}{\Omega^{2}}\right]. \tag{59}$$

Taking the derivative of the group velocity V given by (43), we check that the usual formula

$$\frac{B}{G}=\frac{1}{2}\frac{d^{2}\omega}{dk^{2}} \tag{60}$$

is satisfied.

B. Bright and dark solitons

Equation (54) is the well-known completely integrable cubic NLS equation which has been solved for N-bright and dark soliton solutions using inverse scattering transform method.[20] In Eq. (54) the dark and bright soliton solutions are characterized by the sign of the product of coefficient of nonlinear and dispersion terms. If $BC>0$, the bright soliton solution exists. The single bright soliton of Eq. (54) is given by

$$g(\zeta,\tau) = 2\eta \operatorname{sech} 2\eta\left(\sqrt{\frac{C}{2B}}(\zeta-\zeta_0) + \frac{2\lambda C}{G}\tau\right) \times \exp\left(-i\left[2\lambda\sqrt{\frac{C}{2B}}\zeta + 4\frac{C}{2G}(\lambda^2-\eta^2)t\right]\right) \tag{61}$$

where η is the amplitude, λ and ζ_0 are all free real parameters. If $BC<0$, the formation of dark soliton is possible. The single dark soliton has the form[21]

$$g(\zeta,\tau) = \eta e^{i(k\zeta-\omega\tau)}\frac{1+be^{p\zeta-\Omega\tau}}{1+e^{p\zeta-\Omega\tau}}, \tag{62}$$

where η is again the amplitude,

$$\Omega = \frac{Bp}{G}\left(2k + \sqrt{\frac{-2C\eta^2}{B}-p^2}\right), \tag{63}$$

$$\omega = \frac{B}{G}\left(k^2 - \frac{C\eta^2}{B}\right), \tag{64}$$

$$b = \frac{p^2 + i\left(\frac{G\Omega}{B}-2kp\right)}{-p^2 + i\left(\frac{G\Omega}{B}-2kp\right)}, \tag{65}$$

and p, k are arbitrary constants with $p^2 \leqslant -2C\eta^2/B$. Substituting Eqs. (61) and (62) into Eq. (33) yields the expression of the wave magnetization amplitude $\vec{M}_1^1$ for either the bright or dark soliton. It is the lowest order of the expansion. A first correction to this approximate amplitude is the second harmonic term obtained by substituting Eqs. (61) and (62) into the expression (40) of $\vec{M}_2^2$. The full wave magnetization density is then

$$\vec{M}_w \simeq \varepsilon\vec{M}_1^1 e^{i\varphi} + \varepsilon^2\vec{M}_2^2 e^{2i\varphi} + \text{c.c.}, \tag{66}$$

where c.c. stands for complex conjugate.

Moreover, if $BC>0$, the so-called modulational instability of Benjamin-Feir type can occur:[22] an incident continuous wave, especially when it is periodically modulated, is destroyed by this instability and transformed into a train of solitons. On contrary, if $BC<0$, the continuous and periodically modulated waves are stable. Let us determine this sign. For positive values of the field constant α, the BC product is always negative: only dark solitons can be formed, and the continuous wave is stable. Indeed, from the definition (24) of K, we have $K>Jk^2$ and since $\Omega^2(2\alpha+2K+1)>K$, we find that B is always positive. On the other hand, expression (58) shows that the coefficient C is always negative when $\alpha>0$ (recall that $J>0$, and hence $K>0$). Solitons can thus be formed only when $\alpha<0$, i.e., when the direction of the magnetization is opposite to that of the external field. This condition may be achieved by changing the direction of the applied magnetic field. While in bulk this does not lead to a stable ground state, it can be stable in nanotubes. This is possible if the tube is very narrow, so that the exchange effect due to the curvature of the tube is strong enough to prevent the magnetization reversal.[16] Let us precise the sign of the product BC in the corresponding limit $R\to 0$. Then $K\to+\infty$ and $B\sim 2JK^2$ is positive. The nonlinear constant is

$$C \sim -K^2M^2(\alpha+4k^2J). \tag{67}$$

Thus BC is positive and solitons can be formed if the wavelength is long enough, depending on the external field strength,

$$k^2 < \frac{-\alpha}{4J}. \tag{68}$$

IV. CONCLUSIONS

In this paper we studied the linear and nonlinear interaction of an EM field with a charge free, isotropic ferromagnetic hollow nanotube, in the presence of an applied constant field, by assuming the surface thickness to be very small compared to other dimensions with the magnetization concentrated only on the surface of the nanotube. The analysis of the dispersion relation shows that when the demagnetizing field is negligible compared to the exchange field, the precession of the magnetization vector leads to circularly polarized wave while elliptically polarized waves are formed when the demagnetizing field is not negligible. The polarization goes close to linear when it dominates. Further, the multiple scale analysis shows that the magnetization on the surface of the ferromagnetic nanotube at the lowest order of expansion is governed by the completely integrable NLS equation. For the nanotubes of the largest diameter, the direction of the magnetization is always the same as the direction of the external field. In this case, no bright soliton can be formed. The propagation of a continuous wave is stable, and could support dark solitons. For the smallest tubes, magnetization reversal can be prevented when the direction of the external magnetic field is changed. In this case, under a specific condition on the wavelength and the field strength, solitons can be formed, while the continuous wave is modulationally unstable.

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