Dynamic localization and quasi-Bloch oscillations in general periodic ac-dc electric fields

J Wan,[1] C. Martijn de Sterke,[2] and M. M. Dignam[1]
[1]*Department of Physics, Queen's University, Kingston, Ontario, K7L 3N6, Canada*
[2]*Centre for Ultrahigh-bandwidth Devices for Optical Systems (CUDOS), and School of Physics, University of Sydney, Sydney, New South Wales 2006, Australia*

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We examine the conditions for dynamic localization in general applied periodic ac-dc electric fields. We find that in the presence of both ac and dc components to the field, in addition to traditional dynamic localization, a different type of dynamic localization can occur irrespective of the shape and amplitude of the ac part of the field. These "quasi-Bloch oscillations" take place if the ratio of the Bloch frequency to the ac frequency is a noninteger rational number. Quasi-Bloch oscillations occur only within the tight-binding approximation, but are not restricted to the nearest-neighbor tight-binding limit. This type of dynamic localization provides a more experimentally accessible way to observe dynamic localization in an electronic system than conventional dynamic localization.

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I. INTRODUCTION

Since the development of semiconductor superlattices in recent decades, the study of electrons in a periodic potential in the presence of an electric field has attracted increasing attention. The history of these investigations can be traced back to early in the last century when Bloch discussed the electron behavior in a dc electric field[1] and Zener predicted that an electronic wave packet in a periodic potential would exhibit periodic oscillations under the application of a dc field, the so-called Bloch oscillations (BOs).[2] The frequency of these periodic oscillations is the Bloch frequency $\omega_B = edF_o/\hbar$, where F_o is the amplitude of the dc field and d is the period of the potential. In bulk materials, however, BOs are almost impossible to observe because the electron is scattered before its first return to its initial state. BOs were not experimentally observed[3] until the development of semiconductor superlattices, which have Brillouin zones small enough to allow for a few complete transversals of the zone before scattering.

Dynamic localization (DL) is similar to BOs but the periodic return of the electron to its initial state occurs with the application of a periodic, purely ac field. Unlike BOs, which occur for any dc field amplitude, DL only occurs if the ac field has a particular shape and an amplitude that is one of a discrete set E_n. This phenomenon was initially addressed by a number of authors.[4–6] They found that in a sinusoidal ac field of period τ with the amplitude E_n, an electron periodically returns to its initial quantum state with period τ if the field amplitude satisfies the condition $J_0(edE_n\tau/h)=0$,[4–6] where e and h are the modulus of the electron charge and Planck's constant, respectively. However, DL in a continuous field, such as a sinusoidal field, occurs only in a one-band, nearest-neighbor tight-binding (NNTB) model and disappears beyond the NNTB approximation.[7–10] We refer to this form of DL as *approximate dynamic localization* (ADL). We recently showed that for most symmetric ac fields, it is guaranteed that field amplitudes exist for which ADL occurs.[9]

A number of authors[10–12] have shown that within a one-band model, DL can occur for periodic square-wave ac fields with specific amplitudes for any arbitrary band dispersion. We refer to this type of DL, which occurs for arbitrary band structures beyond the NNTB approximation, as *exact dynamic localization* (EDL). More recently, Dignam and de Sterke derived a set of conditions for which a general ac electric field can yield EDL.[10] The theory was developed directly from the solution of the Schrödinger equation in a general electric field. They noted that *EDL can only occur for fields with discontinuities that occur whenever the electric field changes sign*. In that letter they also showed how to construct a general discontinuous field leading to EDL. Although the requirement for discontinuities to obtain EDL makes it impossible to obtain these fields *exactly* in experiments on an *electronic* system, it has been shown that electric fields with somewhat smoothed discontinuities can still lead to good dynamic localization.[9]

Dynamic localization is often discussed within the framework of Floquet theory when the applied electric field is periodic. In this approach, Floquet-Bloch states, eigenstates of the discrete spatial translation operator (translation by the period d, of the lattice) and the discrete temporal translation operator (translation in time by the period τ, of the field), are constructed. The eigenvalues of the temporal translation define the *quasienergy band* of the system in the presence of the ac field. In this approach, DL is considered to have occurred if the quasienergy miniband reduces to a single level, i.e., the quasienergy miniband collapses.[5,13]

One can also achieve DL in a *general* periodic electric field in which there is a dc component in addition to the purely ac part of the field. Floquet-Bloch theory[14] can also be applied in this situation, provided we consider the period of the field T to be not simply equal to the ac period $\tau=2\pi/\omega$, but rather, to be an integer multiple of this period τ and also an integer multiple of the Bloch period $\tau_B=2\pi/\omega_B$, associated with the dc component of the field. This indicates that for DL to occur in a combined ac-dc field, the ratio of the Bloch frequency ω_B to the ac frequency ω must be a rational number, which can be expressed as $\omega_B/\omega=Q/N$ (where N and Q are positive integers with no common factor). A number of authors have studied this system for sinusoidal[15] and square-wave fields[11] using Floquet-Bloch theory. However, the conditions for DL have not been examined for *general* combined ac-dc fields.

There are a number of potential systems in which DL could be seen experimentally. These include essentially any systems where BOs have been observed: optically-excited semiconductor superlattices,[16–18] atoms in periodic optical traps (where both BOs[19,20] and DL[21] have been observed), and light propagation in coupled optical waveguides.[22,23] It is easy to show that in all these systems a one-band model is valid as long as care is taken in the choice of the lattice potential, the "electric fields" are not too high, and the time over which experiments are conducted is much less than the Zener tunneling time.[10,24,25] In addition, in all systems, the experiments can be performed such that carrier-carrier interaction effects and dephasing have a minimal impact on the qualitative dynamic behavior over the first few periods of the ac field. In the atomic and the optical systems, the carrier-carrier interactions are intrinsically small or nonexistent. In superlattices, electronic wave packets are excited by femtosecond optical pulses. If the carrier densities are kept low and the excitation conditions are such that excitonic effects are minimized,[26] then again carrier-carrier interaction does not play a crucial role over the time of the experiments (a few ps). Decoherence and dephasing in these systems result in the loss of DL with time. However, as with BOs, the clear signatures of DL should be observable for the first few periods of the ac field. In superlattices, for example, typical decoherence times are roughly 1–2 ps. Since the period of the ac field in BOs experiments is typically 200–300 fs,[16–18] the evidence of DL should be observable. This has been confirmed theoretically in models that include carrier-carrier interactions and dephasing,[26–29] where it has been predicted that the signature of DL should be seen in the optical absorption spectra, degenerate four-wave mixing spectra, and emitted Terahertz radiation. Therefore, in this work, we neglect the effects of Zener tunneling, carrier-carrier interactions, and decoherence so as to allow us to focus on the basic physics of DL in general dc-ac fields.

In this paper, we examine electron dynamics in a one-dimensional periodic potential in the presence of *general* periodic electric fields. We find that *two distinct types of DL* can occur in the presence of combined ac and dc fields. The first type of DL is similar to conventional EDL or ADL in a pure ac field, and occurs if the ratio ω_B/ω is a rational number and the total field meets the conditions for DL; this, therefore, requires that the *ac part of the field has a particular shape and amplitude*. The second type of DL can occur with the period $T=N\tau=Q\tau_B$ (where $N>1$) if the ratio ω_B/ω is a *noninteger rational number*. This type of DL can occur for *any shape and amplitude of the ac part of the field*. Just as for BOs, the occurrence of this type of DL only depends on the dc component of the field. We, therefore, refer to this form of DL as *quasi-Bloch oscillations* (QBOs). We note that QBOs only occur within the tight-binding approximation. However, it is more robust than ADL in that QBOs can occur beyond the nearest-neighbor approximation in most cases (if $N>2$) and in fact is valid up to the $(N-1)$th nearest-neighbor approximation.

The paper is organized as follows. In Sec. II A we present the general solution of the Schrödinger equation in a general electric field. In Sec. II B we review Floquet-Bloch theory for this system when the electric field applied to the system is periodic. In Sec. II C, we derive the conditions for EDL and ADL in a general periodic field. In Sec. III we examine EDL occurring with period $T=\tau$, while in Sec. IV A we derive the conditions for which conventional EDL occurs with period $T=N\tau$ $(N>1)$. In Sec. IV B we demonstrate the existence of QBOs occurring with period $T=N\tau$, regardless of the amplitude and shape of the ac part of the field, and examine the conditions under which QBOs occur. Finally, in Sec. V we summarize our results.

II. GENERAL THEORY OF ELECTRON DYNAMICS AND DYNAMIC LOCALIZATION

Here we examine the dynamics and dynamic localization of electrons in periodic electric fields. As is commonly done,[4–6] and as motivated in Sec. I, we treat the system in a one-band model and neglect scattering. We begin by presenting the solution of the Schrödinger equation in the presence of a general electric field. We then review the Floquet-Bloch theory and quasienergy bands that results with the application of a periodic electric field. We show that using either of these two approaches, we obtain the same two necessary and sufficient conditions for DL of the electrons.

A. General solution of the Schrödinger equation

The Hamiltonian of an electron in a one-dimensional, periodic potential of period d, in the presence of a general electric field $E(t)$, can be written in the form

$$H = H_o + eE(t)z, \tag{2.1}$$

where H_o is the Hamiltonian without the electric field. Using a one-band model, the wave function can be expanded in the basis of single-band Wannier functions, $|a_n\rangle$, as[11]

$$|\Psi(t)\rangle = \sum_n B_n(t)|a_n\rangle, \tag{2.2}$$

where label n represents the localization site of Wannier functions. Using this in the Schrödinger equation, we obtain[30,31]

$$i\hbar\dot{B}_n = \sum_m B_m(\varepsilon_{n-m} + eE(t)W_{m-n}) + nedE(t)B_n, \tag{2.3}$$

where ε_n are the Fourier coefficients of the band's energy dispersion in reciprocal space $(\varepsilon_n \equiv \sum_k \epsilon(k)\exp[ikd])$ and W_p are the matrix elements of z in the basis of the Wannier functions, such that

$$W_p \equiv \langle a_0|z|a_p\rangle. \tag{2.4}$$

The solution to Eq. (2.3) is[30]

$$B_n(t) = e^{-i[\varepsilon_0 t/\hbar + (n+W_0/d)\gamma(t)]}\sum_m A_{n-m}(t)B_m(0), \tag{2.5}$$

where

$$\gamma(t) \equiv \frac{ed}{\hbar}\int_0^t E(t)dt \tag{2.6}$$

is the dimensionless area under the curve of the electric field with respect to time t. Also

$$A_m(t) \equiv \int_{-\pi}^{\pi} \frac{dx}{2\pi} \exp\left\{ imx - i\sum_{p\neq 0} \frac{\varepsilon_{-p}}{\hbar}\tilde{\beta}_p(t)e^{ipx}\right\}, \tag{2.7}$$

where

$$\tilde{\beta}_p(t) \equiv \beta_p(t) + i\frac{\hbar W_p}{pd\varepsilon_{-p}}[e^{-ip\gamma(t)} - 1], \tag{2.8}$$

and

$$\beta_p(t) \equiv \int_0^t e^{-ip\gamma(t')}dt'. \tag{2.9}$$

This is the general solution of the Schrödinger equation in the presence of an *arbitrary* electric field. We now consider this same problem using Floquet-Bloch theory for the special case of a periodic electric field.

B. Bloch-Floquet theory and quasienergy bands

If a purely ac electric field $E(t)$, which is periodic with time period τ such that $E(t+\tau)=E(t)$ is applied to the system, then the Hamiltonian $H(t)$ is also periodic in time. This situation has been examined by a number of authors.[5,7,8,13,15] We summarize the key results of this research here to allow for comparison with the direct Schrödinger approach presented in Sec. II A.

Floquet's theorem states that the time-dependent state of the Hamiltonian for the electron in this periodic field can be written as a linear combination of the Floquet states $|\phi_k(t)\rangle$,

$$|\Psi(t)\rangle = \sum_k C_k|\phi_k(t)\rangle. \tag{2.10}$$

The $|\phi_k(t)\rangle$ can be written as

$$|\phi_k(t)\rangle = e^{-i\hat{\epsilon}_k t/\hbar}|u_{\hat{\epsilon}_k}(t)\rangle, \tag{2.11}$$

where the $\hat{\epsilon}_k$ are the *quasienergies*[13] and the $|u_{\hat{\epsilon}_k}(t)\rangle$ are time-periodic energies such that $|u_{\hat{\epsilon}_k}(t+\tau)\rangle = |u_{\hat{\epsilon}_k}(t)\rangle$. From the Schrödinger equation, we see that we must have

$$\left(H - i\hbar\frac{d}{dt}\right)|u_{\hat{\epsilon}_k}(t)\rangle = \hat{\epsilon}_k|u_{\hat{\epsilon}_k}(t)\rangle. \tag{2.12}$$

It can be shown that for a pure ac field, the Floquet states can also be chosen to be eigenstates of the translation operator for the lattice.[5,8,13] The crystal momentum k is then a good quantum number and is used to label the quasienergies and the Floquet-Bloch states. After solving (2.12), Zhao[7] and Zhao *et al.*[15] showed that the quasienergies can be written as

$$\hat{\epsilon}_k = \sum_{p\neq 0}\frac{\varepsilon_{-p}}{\tau}\beta_p(\tau)e^{ipk} + \epsilon_c, \tag{2.13}$$

where ϵ_c is a constant, independent of k.

We now consider the case where the field period is still τ, but we are looking for Floquet states with $|u_{\hat{\epsilon}_k}(t)\rangle$ that are periodic not with time τ, but rather with time $T=N\tau$. Floquet-Bloch theory[14] requires that the period T is an integer multiple Q of the Bloch period τ_B. Thus, we have

$$T = N\tau = Q\tau_B, \tag{2.14}$$

where N and Q are integers. Under these conditions, the quasienergy dispersion is written as follows:

$$\hat{\epsilon}_k = \sum_{p\neq 0}\frac{\varepsilon_{-p}}{T}\beta_p(T)e^{ipk} + \epsilon'_c, \tag{2.15}$$

where ϵ'_c is a constant that is independent of k.

One could, in principle, use the above theory to calculate the Floquet-Bloch states $|\phi_k(t)\rangle$, however, for our purposes we only need consider the quasienergy bands given in Eqs. (2.13) and (2.15). We now turn to the application of these results and the results of the Sec. II A to the problem of dynamic localization.

C. General dynamic localization

Here we consider the issue of DL when the electric field is periodic with period τ. For DL to occur, we require that the electron return to within a constant overall phase φ_c, to its initial state after some time $T=N\tau$ (where N is an integer), and such

$$|\Psi(t+T)\rangle = e^{i\varphi_c}|\Psi(t)\rangle. \tag{2.16}$$

Using the Schrödinger approach from Sec. II A, we see from Eqs. (2.2) and (2.5) that first, DL only occurs with period $T=N\tau$ if

$$\gamma(T) = 2\pi Q. \tag{2.17}$$

The second condition for DL requires that $A_n(T)=\delta_{n,0}$. From Eq. (2.7), we see that this second condition is satisfied iff

$$\tilde{\beta}_p(T) = 0 \text{ for all } p \neq 0. \tag{2.18}$$

From Eqs. (2.8) and (2.17) we can see that $\tilde{\beta}_p(T)=\beta_p(T)$, and thus, Eq. (2.18) is equivalent to

$$\beta_p(T) = 0 \text{ for all } p \neq 0. \tag{2.19}$$

Now, turning to the Floquet-Bloch theory of Sec. II B, we see from Eq. (2.14) that the theory is only valid if $T=N\tau=Q\tau_B$. If we write the electric field $E(t)$ as the sum of a dc component F_o and a purely ac component $F(t)$ from Eq. (2.6), we see that

$$\gamma(T) = 2\pi N\frac{\tau}{\tau_B}, \tag{2.20}$$

where $\tau_B=2\pi\hbar/(eF_o d)$. Thus, we obtain the requirement

$$\gamma(T) = 2\pi Q, \tag{2.21}$$

which is identical to requirement (2.17) obtained directly from the Schrödinger solution. Furthermore, we see from Eq. (2.11) that DL only occurs in Floquet-Bloch theory if $\hat{\epsilon}_k$ is a constant independent of k, where we have used the fact the the quasienergies are only defined to modulo $2\pi\hbar/T$. Using Eq. (2.15) we see that this is equivalent to the requirement

$$\beta_p(T) = 0 \text{ for all } p \neq 0. \qquad (2.22)$$

This is the same condition that we obtained in Eq. (2.19) using the direct Schrödinger solution. Thus, using either approach, the two necessary and sufficient conditions for EDL are Eqs. (2.17) and (2.19). We finally note that if $\gamma(T) = 2\pi Q$ but only $\beta_1(T)=0$, then DL occurs within the NNTB approximation (ADL)[9] since the higher Fourier coefficients ε_p ($|p|>1$) are neglected. We now employ the conditions of Eqs. (2.17) and (2.19) to investigate DL in the presence of combined general periodic ac-dc fields.

III. DL WITH PERIOD $T=\tau$ IN AC-DC FIELDS

We consider here the special case in which $N=1$ ($T=\tau$) and the field is again given by $E(t)=F(t)+F_o$. We see from Eq. (2.17) via Eq. (2.20) that for DL to occur, we must have $\tau = Q\tau_B$ or equivalently,

$$\frac{\omega_B}{\omega} = Q. \qquad (3.1)$$

Thus, the Bloch frequency must be an integral multiple of the ac frequency if DL is to occur with period τ. In other words, there must be an integer number of BOs in each ac period if DL is to occur with the same period as the ac part of the field. Furthermore, EDL only occurs for special combined ac-dc fields that satisfy Eq. (2.19), just as when a pure ac field is applied. As we showed previously,[10] this is equivalent to the condition on the electric field that

$$\sum_{m,j} \frac{1}{|\dot{\gamma}(t_{jm})|} = \frac{\hbar}{ed}\sum_{m,j} \frac{1}{|F(t_{jm}) + F_o|} = \frac{\tau}{2\pi}, \qquad (3.2)$$

where, as discussed previously,[9,10,30] the summation is over all times within a period τ at which $\gamma(t_{jm}) = x + 2\pi m$ ($-\pi \leqslant x < \pi$), and the j's count the roots of this equation for fixed m for one value of x. The physical interpretation of Eq. (3.2) was addressed more recently.[30] Requirement (3.2) also leads to the condition that the ac part of the field must be discontinuous at times when the total combined field changes sign. Thus, just as for pure ac fields,[10] to achieve EDL, one must carefully construct discontinuous electric fields that satisfy Eq. (3.2).

IV. DL WITH PERIOD $T=N\tau$ ($N>1$) IN AC-DC FIELDS

In Sec. III, we discussed DL at time, $T=\tau$. To be more general, we now consider the case where DL occurs only with period, $T=N\tau$, where N is an integer greater than 1. From Eq. (2.17), for DL to occur we require that $\gamma(T) = 2\pi Q$. This yields the condition

$$\frac{\omega_B}{\omega} = \frac{Q}{N}, \qquad (4.1)$$

where Q and N have no common factor, since otherwise DL may occur with period $T' = (N/j)\tau = T/j$, where j is the common factor. Equation (4.1) shows that the ratio of the Bloch frequency to the ac frequency must be a noninteger rational number for DL to be observed with period $T=N\tau$ ($N>1$). Considering the discussion in previous sections where DL occurs with period τ, we reach the general conclusion that *the ratio of the Bloch frequency to the ac frequency must be a rational number if DL is to occur*. We stress that for an irrational ratio of the frequencies, true DL cannot take place. However under the special initial conditions, $B_n(0) = \delta_{n,n_o}$ something that resembles DL may occur; this, however, is not *true DL* in that it relies on these particular initial conditions. We discuss this in detail in the Appendix.

Equation (4.1) is one necessary and sufficient condition for DL in combined ac-dc electric fields, while Eq. (2.19) is the other one. Now, at time $T=N\tau$, we have

$$\beta_p(T) = \int_0^{N\tau} e^{-ip\left[\omega_B t + \left(ed/\hbar\right)\int_0^t F(t')dt'\right]} dt = C_p(N)\beta_p(\tau), \qquad (4.2)$$

where the $\beta_p(\tau)$ express the effect of the field over a single period, whereas the C_p gives the effect of having N such identical periods. Here,

$$C_p(N) = \sum_{m=0}^{N-1} e^{-ip\omega_B\tau m} = e^{-ip\pi(N-1)\omega_B/\omega}\frac{\sin(Np\pi\omega_B/\omega)}{\sin(p\pi\omega_B/\omega)}. \qquad (4.3)$$

If either $\beta_p(\tau)=0$ or $C_p(N)=0$ for some or all $p\neq 0$, then $\beta_p(T)=0$ and we have DL. However, these two requirements lead to different types of DL, as is discussed in Secs. IV A and IV B.

A. Conventional DL in special ac-dc fields

Obviously, if $\beta_p(\tau)=0$, we recover something very similar to the conventional DL discussed in Sec. III in that DL requires that the ac part of the field have a specific shape and correct amplitudes. Here, we provide some numerical simulations to illustrate electron dynamics in a realistic structure, a $GaAs/Ga_xAl_{1-x}As$ superlattice. We choose a structure for which Zener tunneling is negligible over the time τ so that the single-band approximation is fulfilled. The superlattice has a period $d=10$ nm, where the well and barrier width are $d_w=9.0$ nm and $d_b=1.0$ nm, respectively; the barrier height is $V_0=250$ meV; and the electron effective mass is 0.067 that of free electron mass. For this structure, the wells are strongly coupled, with $|\varepsilon_1|=8.9$ meV, $|\varepsilon_2/\varepsilon_1|=0.1676$, $|\varepsilon_3/\varepsilon_1|=0.0472$, and $|\varepsilon_4/\varepsilon_1|=0.0164$. An electric field is applied to the superlattice. As shown in Fig. 1(a), the field is comprised of a rectangular ac component with period $\tau = 825$ fs and a dc component $F_o=1.67$ kV/cm, such that $\omega_B=\omega/3$ ($N=3, Q=1$) and $\beta_p(T)=0$ for all $p\neq 0$. We choose the rectangular-wave ac field for simplicity, but more general fields can be constructed using an equation similar to Eq. (3.2).[10]

Now, we know that EDL occurs if the two conditions, Eqs. (2.17) and (2.19), on the electric field are satisfied. However, it is instructive to also examine DL by looking at

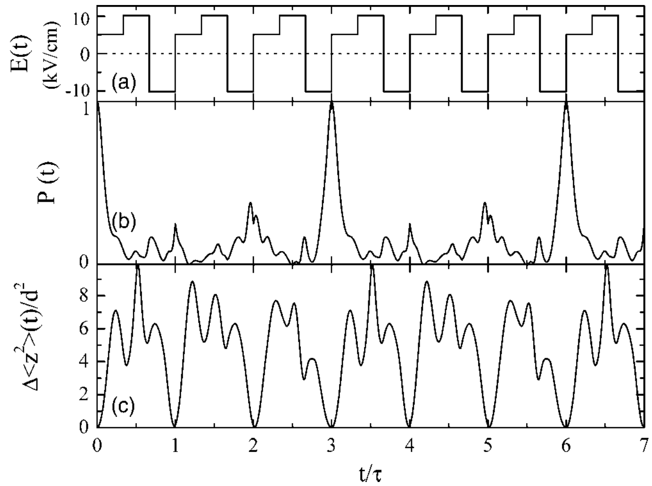

FIG. 1. (a) Combined ac-dc field for which $\beta_p(\tau)=0$ for all $p \neq 0$ and $\omega_B/\omega=1/3$; (b) time evolution of the return probability; (c) time evolution of the mean-square displacement. For this field, EDL occurs with period $T=3\tau$. At $t=\tau,2\tau,4\tau,5\tau,\ldots$, $\Delta\langle z^2\rangle(T)$ is close to, but not equal to zero.

some characteristics of the wave function. Thus, we consider the time evolution of two parameters to describe the electron dynamics: $P(t)$ and $\Delta\langle z^2\rangle(t)$. $P(t)$ is the probability of finding the electron at time t in its initial state (return probability) and is given by

$$P(t) \equiv |\langle\Psi(t)|\Psi(0)\rangle|^2. \tag{4.4}$$

The other parameter is

$$\Delta\langle z^2\rangle(t) \equiv \langle\Psi(t)|z^2|\Psi(t)\rangle - \langle\Psi(0)|z^2|\Psi(0)\rangle, \tag{4.5}$$

the mean-square displacement of the electron at time t relative to the same quantity at $t=0$. As expected, if DL occurs at time T, $P(T)=1$, and $\Delta\langle z^2\rangle(T)=0$. We note that one must be somewhat careful with these measures of DL as it does not follow that if $P(T)=1$ and $\Delta\langle z^2\rangle(T)=0$ then DL has occurred. As discussed in the Appendix, for certain initial conditions, one can obtain a periodic return to the initial state at times at which DL does not occur. However, this is not *true* DL in that it does not occur for every initial state.

In Figs. 1(b) and 1(c), we plot $P(t)$ and $\Delta\langle z^2\rangle(t)$ as a function of time t for the Full-Band (FB) calculation with initial conditions $B_n(0)=(\delta_{n,0}+\delta_{n,1})/\sqrt{2}$. These initial conditions have no special properties and are used in all numerical simulations except in the Appendix. We can see that EDL occurs at $T=3\tau,6\tau,\ldots$ for arbitrary band dispersion in the presence of this combined ac-dc field. This is similar to EDL in a purely ac field as we have discussed before[10] except that EDL occurs with a different time period.

This DL can be seen in a different way by using Floquet theory. Zhao *et al.*[15] showed that if we take τ to be the period and choose the fields such that $\beta_p(\tau)=0$ for all $p\neq 0$, then the quasienergies become

$$\hat{\epsilon}_{jk} = j\frac{2\pi\hbar}{N\tau}, \quad j=0,\pm 1,\pm 2,\ldots, \tag{4.6}$$

and are thus equally spaced, comprising a fractional Stark ladder.[15] Using Eq. (2.11), we have

$$|\Psi(N\tau)\rangle = e^{i\varphi_c}|\Psi(0)\rangle, \tag{4.7}$$

due to the rephasing of the different Floquet states at time $T=N\tau$. This shows that in the presence of such a combined ac-dc field, EDL occurs with period $N\tau$ rather than τ.

B. QBOs in general ac-dc fields

It is experimentally not easy to achieve the specific large-amplitude ac fields required to make $\beta_p(\tau)=0$, even just for $|p|=1$. Thus, we now consider a form of DL that occurs even if $\beta_p(\tau)\neq 0$ when $N>1$. From Eqs. (4.2) and (4.3), we see our second necessary and sufficient condition for DL, $\beta_p(T)=0$ is satisfied if

$$C_p(N)=0. \tag{4.8}$$

From Eq. (4.1) we know that $\omega_B/\omega=Q/N$. Using this in Eq. (4.3), we see that $C_p(N)\neq 0$ for any p if Q/N is an integer. However, if Q/N is not an integer then

$$C_p(N)=0 \text{ for all } p\neq mN, \tag{4.9}$$

and thus

$$\beta_p(T)=0 \text{ for all } p\neq mN, \tag{4.10}$$

where m is an integer. Thus, a new type of DL can be always achieved by a dc field combined with an arbitrary ac field if *the ratio of the Bloch frequency to the ac frequency is a noninteger rational number*. We refer to this as quasi-Bloch oscillations (QBOs) since this form of DL occurs regardless of the shape and the amplitude of the ac field. Here we note that QBOs only occur within the tight-binding approximation, since $\beta_p(T)=0$ only for a limited range of p, $0<|p|<N$, rather than for all $p\neq 0$ as is required for EDL. When $|p|=N$, from Eq. (4.3) we can see that $C_N=N$ and $\beta_p(T)\neq 0$ for general ac fields, i.e., for fields for which $\beta_p(\tau)\neq 0$. Therefore, we have the conclusion that QBOs *only occur within the tight-binding (TB) approximation to order of* $N-1$. For example, if $\omega_B/\omega=1/2,3/2,5/2,\ldots$ (where $N=2$), QBOs only occur within the NNTB limit. If $\omega_B/\omega=1/3,2/3,4/3,\ldots$ (where $N=3$), QBOs can occur within the next NNTB limit. To our knowledge, this has never been addressed in previous reports. One interesting case occurs when $Q=1$ such that

$$\frac{\omega_B}{\omega}=\frac{1}{N}=\frac{\tau}{\tau_B}. \tag{4.11}$$

Then QBOs occur with period τ_B that is quite similar to true BOs, but QBOs occur only within the $(N-1)$th NNTB limit.

Condition (4.1) can be understood in a semiclassical model. We previously showed that $k(t)=k_o+\gamma(t)/d$.[30] From Eq. (2.17), DL requires that $\gamma(T)$ be an integer multiple of 2π. This then results in $k(t)$ being periodic with period T. The other semiclassical equation then tells us that the average velocity is also periodic and that it has a period T. The average position of the wave packet is given by

$$\langle z\rangle(t) = z_o + id\sum_{p\neq 0}\frac{p\varepsilon_{-p}}{\hbar}\beta_p(t)e^{ipk_o d}. \tag{4.12}$$

Thus $\langle z\rangle(t)$ is periodic with time T, if the field satisfies Eq. (2.19). Alternatively, using Eq. (2.15), we obtain for the average position at time T

$$\langle z\rangle(T) = z_o + \frac{T}{\hbar}\left.\frac{\partial\hat{\epsilon}_k}{\partial k}\right|_{k=k_o}. \tag{4.13}$$

So when the quasienergy band collapses, the average particle position is periodic in T. Although these results show the agreement of the full quantum theory with the semiclassical result, they only add limited physical insight into the conditions for DL or QBOs.

We presented previously[30] a simple physical interpretation of Eq. (2.19) for EDL: the electron must spend the same amount of time at all points in the first Brillouin zone over the time τ. Such a simple physical interpretation of QBOs, where Eq. (2.19) is only satisfied for a limited set of p, does not seem possible. In the special case, $N=2$, $Q=1$, however, a simple semiclassical picture can help explain QBOs. After a time, $T/2(=\tau)$, the electron wave vector has traversed exactly half the first Brillouin Zone, i.e., $k(\tau)=k_o+\pi/d$. In the NNTB approximation, $\epsilon'(k+\pi/d)=-\epsilon'(k)$, where $\epsilon'(k)$ denotes the derivative of the band energy with respect to k. Thus using the semiclassical equation of motion, we see that $v(t+\tau)=-v(t)$. Therefore, the velocity of the electron over the first half of the period $0<t<T/2$ is reversed over the second half, $T/2<t<T$, and the electron returns to its initial position at time $t=T$. For larger N, the picture is more complicated, but the basic principle is the same: if the frequency ratio is chosen correctly the effect of the ac portion of the field averages to zero over the time T, and the electron returns to its initial state.

We now examine QBOs via numerical simulations. We consider the $GaAs/Ga_xAl_{1-x}As$ superlattice structure discussed in Sec. IV A, but now apply an *arbitrary ac field* [see Fig. 2(a)] and a dc field $F_o=1.67$ kV/cm, for which $\omega_B/\omega=1/3$ for the chosen ac period. The form of this general ac field is also used in Figs. 3–5. Fig. 2(b) shows $\gamma(t)$ of the ac part of the field (solid line) and of the total field (dashed line) in one period τ. The only feature of the field is that the ratio of the Bloch frequency to the ac frequency is 1/3 [hence $\gamma(\tau)=2\pi\omega_B/\omega=2\pi/3$ as can be seen from dashed line in Fig. 2(b)]. In Figs. 3 and 4 we plot $P(t)$ and $\Delta\langle z^2\rangle(t)$, respectively, for different approximations to the band structure $\epsilon(k)$, when the ac field has an amplitude of 4.36 kV/cm. Figures 3(a) and 4(a) present the results in the NNTB approximation ($|p|=1$). They show that $P(T)=1$ and $\Delta\langle z^2\rangle(T)=0$ at times $T=3\tau,6\tau$. The fact that, apart from a constant phase, the electron wave function at those times returns to its initial distribution shows that QBOs occur within the NNTB approximation. We also can see similar results in Figs. 3(b) and 4(b) within the next NNTB limit ($0<|p|\leq 2$). But in Figs. 3(c) and 4(c), when we take the TB approximation up to the order of $|p|\leq N=3$ (solid lines), even at $T=3\tau$, $P(3\tau)\approx 16\%$, and $\Delta\langle z^2\rangle(T)$ is substantial. Thus, the electron is delocalized strongly when we take into account the long-range coupling up to $|p|=N$. The dotted lines in Figs. 3(c) and 4(c) are calculated for the full-band dispersion. Due to the small contributions from higher orders ($|p|>3$) of ε_p, the results of the FB calculation are almost identical to the $|p|\leq 3$ result (solid lines). Therefore, like ADL, QBOs cannot be obtained for an arbitrary band structure. However, the conditions for QBOs are much less restrictive than for traditional ADL in that they occur for bands that are well described by a TB structure of order $N-1$.

We now choose a structure similar to the previous one but with $d_w=7.0$ nm and $d_b=3.0$ nm. It is a good next NNTB

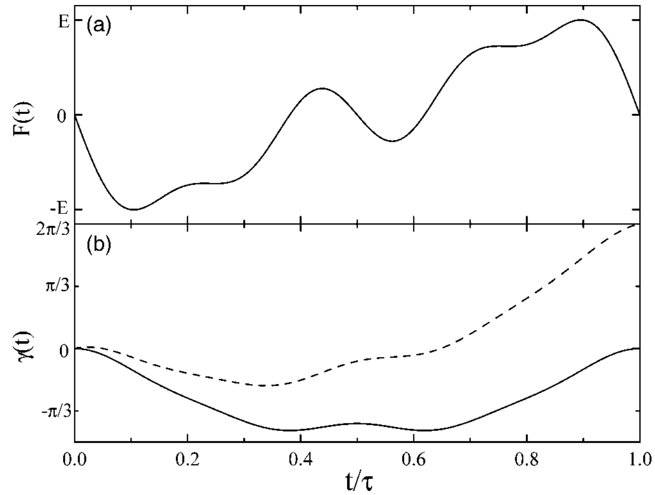

FIG. 2. (a) Ac component of the electric field in Figs. 3–5 (b) (solid line) associated dimensionless area $\gamma(t)$ of the ac component of the field in (a), and (dashed line) $\gamma(t)$ for the combined ac-dc field where $\omega_B/\omega=1/3$.

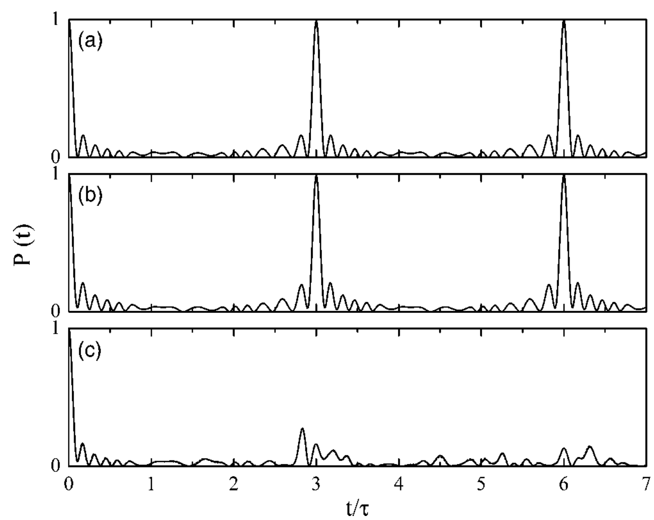

FIG. 3. Return probability $P(t)$ versus time t in the presence of the combined ac-dc field of Fig. 2, (a) within the NNTB approximation ($|p|=1$); (b) within the next NNTB approximation ($0<|p|\leq 2$); (c) within the TB approximation to third order ($0<|p|\leq 3$) (solid line) and for the FB calculation (dotted line).

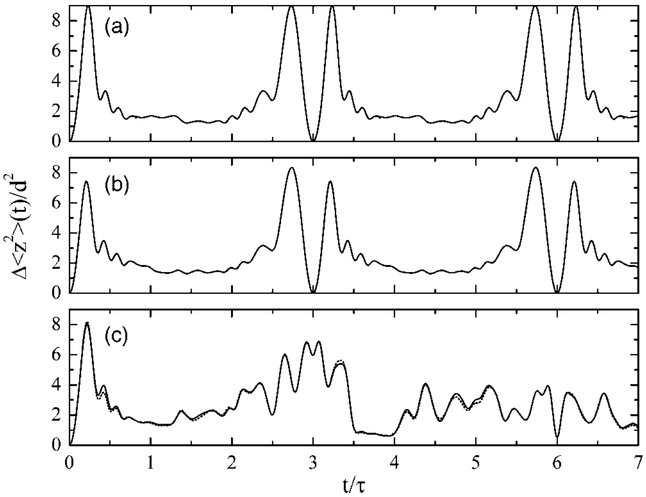

FIG. 4. Same as Fig. 3, but for the mean-square displacement $\Delta\langle z^2\rangle(t)$.

structure where only relatively weak coupling effects exist between wells beyond next-nearest neighbor ($|\varepsilon_1|=4.8$ meV, $|\varepsilon_2/\varepsilon_1|=0.0733$, $|\varepsilon_3/\varepsilon_1|=0.0087$). Figure 5 shows the time evolution of the return probability and the mean-square displacement by the FB calculation for this structure in the presence of same strong combined ac-dc field in Figs. 3 and 4. We can see that QBOs do occur and even at times $t=6\tau$ the deviation from exact DL is very small. Of course, the larger we make N, the closer the QBOs resemble exact DL. However, since QBOs occur with period $N\tau$, and in any real system there is some decay of the oscillations due to decoherence and dephasing, we have to compromise on the QBOs period T. One should therefore choose an appropriate superlattice structure so as to keep the higher-order dispersion contributions to a minimum if QBOs are to be observed.

We finish this section by comparing BOs, ADL, and QBOs so as to show the advantages of QBOs over DL in the ease with which it may be experimentally achieved. We

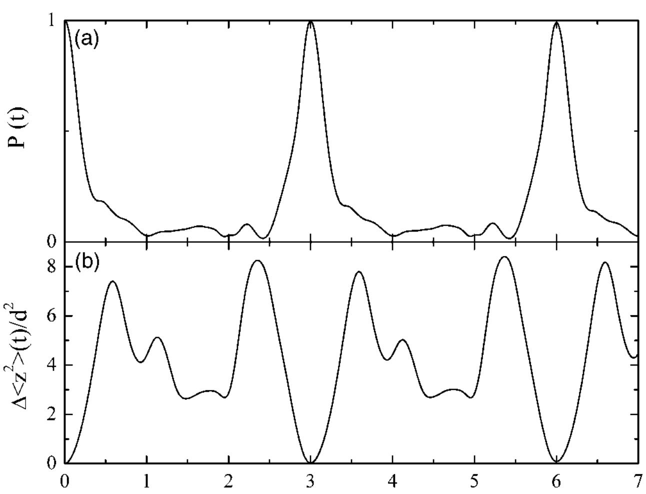

FIG. 5. (a) Evolution of the return probability $P(t)$, and (b) the mean-square displacement using a FB calculation for a relatively weak coupling structure (see text). The applied field is that shown in Fig. 2.

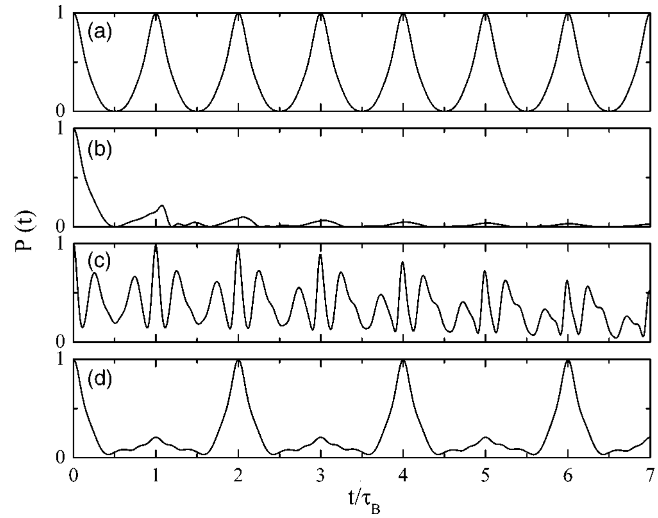

FIG. 6. Evolution of the return probability $P(t)$ for a dc field $F_o=5.02$ kV/cm and a sinusoidal field, $F(t)=E\cos(\omega t)$, with different amplitudes and frequencies: (a) $E=0$ (BOs), (b) $E=2.51$ kV/cm and $\omega=\omega_B$ (destruction of BOs), (c) $E=19.20$ kV/cm and $\omega=\omega_B$ (ADL), and (d) $E=2.51$ kV/cm and $\omega=3\omega_B/2$ (QBOs).

know that an arbitrary dc field always leads to BOs, but the addition of an ac field generally destroys BOs. In Fig. 6(a) we plot $P(t)$ when a pure dc field of $F_o=5.01$ kV/cm is applied to the same tight-binding structure as in Fig. 5. As expected, we see BOs. We now add to the dc field a sinusoidal ac field of the form $F(t)=E\cos(\omega t)$. Although such a field can never yield EDL (as it is not discontinuous), it is probably the easiest field to achieve experimentally. We first choose $E=2.51$ kV/cm and $\omega=\omega_B$. This field does not match any of the conditions for DL, ADL, or QBOs. In Fig. 6(b), we plot $P(t)$ for this field, and see that the electron relocalizations are essentially completely destroyed even by this relatively weak ac field. To obtain ADL using this sinusoidal field with the dc field of 5.01 kV/cm, we require $J_1(edE/\omega_B)=0$,[31] which requires a minimum ac amplitude of $E=19.20$ kV/cm. In Fig. 6(c) we plot $P(t)$ for this field and see that we obtain something that is close to DL with $P(m\tau_B)\simeq 1$ for $m<3$. However, even for this weak-coupling structure, in the sinusoidal field, the deviation from exact DL is very evident, and for $t\gtrsim 3\tau_B$ the return to the initial state is rather poor. We now decrease the ac amplitude back to the value in Fig. 6(b), $E=2.51$ kV/cm, but change the ac frequency from $\omega=\omega_B$ to $\omega=3\omega_B/2$ ($N=3,Q=2$). For this field, we expect QBOs with period of $T=2\tau_B$. In Fig. 6(d), we plot $P(t)$ for this field. We see that the return of the electron to its initial state is much better than it was for ADL in Fig. 6(c). We also see that the field is strong enough that it has strongly modified the electron dynamics from the pure BOs case of Fig. 6(a).

From Fig. 6 we see that QBOs are, in general, much easier to achieve experimentally than EDL or ADL. Traditional DL is difficult to achieve because one needs to apply specific fields with specific shapes and amplitudes. It is particularly difficult to achieve EDL because the electric fields must be discontinuous, or nearly discontinuous, which necessitates very high-frequency field components. Even ADL is difficult to achieve because it requires large-amplitude ac fields and only results in good DL on very weakly-coupled structures. In contrast: (i) QBOs occur for any ac field that has the required period. (ii) The ac-field amplitude required to clearly achieve QBOs is relatively weak. And (iii) good QBOs occur even for non-NNTB structures. In almost any structure in which BOs can be observed—say from current or THz radiation measurements—QBOs can also be observed. One need only tune the ac frequency or dc field amplitude so that ω_B/ω is a noninteger rational number and then increase the amplitude of the ac field. As the amplitude is increased, the period of the wave packet oscillations changes, as is clearly seen in Fig. 6. Further proof of the existence of the QBOs can be provided by then detuning the ac field, which results in the subsequent destruction of QBOs and BOs. The frequencies and amplitudes of currently available THz radiation sources are well within the range needed to observe QBOs in undoped semiconductor superlattices excited by ultrashort optical pulses.

V. SUMMARY

We have investigated electronic dynamic localization in the presence of general combined ac-dc fields. We examined the conditions under which dynamic localization occurs for times equal to an integer multiple of the period of the field. Although an electron in a periodic potential in an arbitrary dc field returns to its initial position with the period associated with the field (Bloch oscillations), the addition of an ac component to the field generally destroys the electron's relocalization. We showed how one can construct the ac component so as to recover the relocalization in agreement with traditional dynamic localization, but for general fields rather than just for sinusoidal or square-wave fields. Furthermore, we showed that if the ratio of ω_B/ω is a noninteger rational number Q/N, a form of dynamic localization—quasi-Bloch oscillations—occurs irrespective of the shape and amplitude of the ac component of the field. These quasi-Bloch oscillations are somewhat similar to Bloch oscillations, but they occur only in the tight-binding limit. However, we have demonstrated that quasi-Bloch oscillations occur beyond the common nearest-neighbor tight-binding approximation up to $(N-1)$th tight-binding approximation, and occur to a very good approximation in real structures.

The most promising systems in which to observe quasi-Bloch oscillations are in the propagation of light in coupled waveguide arrays,[22–25] the dynamics of atoms in periodic linear optical traps,[19–21] and in the dynamics of electrons in semiconductor superlattices excited by ultrashort ($\sim$100 fs) optical pulses.[16–18] Bloch oscillations have been seen in all of these systems and methods for detecting dynamic localization have also been proposed for all of these. The main obstacle to the observation of dynamic localization in electronic systems is the generation of the required ac fields, which must in general have a large amplitude and a specific temporal shape. Thus, quasi-Bloch oscillations should be considerably easier to observe experimentally than traditional dynamic localization: the structures can be more general (beyond nearest-neighbor tight-binding limit), the field shape is unimportant, and the required field amplitudes are lower.

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APPENDIX A: INITIAL CONDITIONS

We have shown that there are two necessary and sufficient conditions for DL. The first condition for DL requires that the ratio of the Bloch frequency to the ac frequency be a rational number. If the ratio is an irrational number then, strictly speaking, no DL occurs. If we make the *approximation* that the ratio can be replaced by a rational number then a form of approximate DL can occur.[15] To examine the nature of the dynamics and DL we have plotted the two quantities, $P(t)$ and $\Delta\langle z^2\rangle(t)$, versus time for some. However, as we now show, for some specific initial conditions, one may be led to the incorrect conclusion that DL occurs even when $\gamma(T)\neq 2\pi Q$.

It has been shown that the first condition of Eq. (2.17) guarantees that the phases of coefficients $B_n(t)$ at time $t=T$ are independent of n, when the second condition (2.19) is satisfied. However, for some special initial states, this condition is no longer required to obtain $|\Psi(T)\rangle = e^{i\varphi_c}|\Psi(0)\rangle$. As an example, consider the initial condition

$$B_n(0) = \delta_{n,0}, \tag{A1}$$

such that

$$|\Psi(0)\rangle = |a_0\rangle, \tag{A2}$$

then from Eq. (2.5) we have

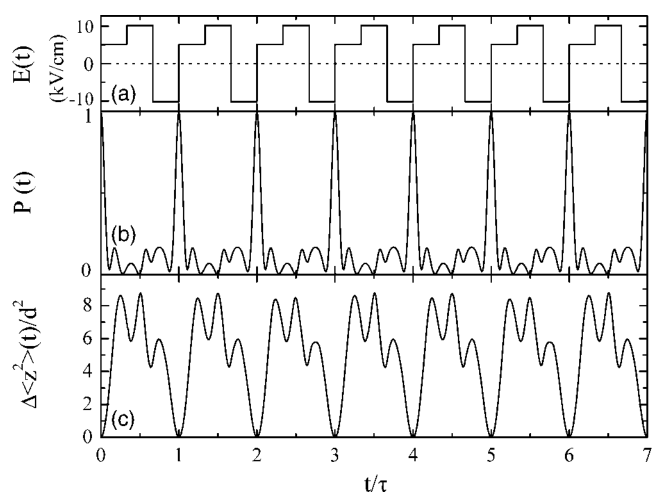

FIG. 7. Evolution of the return probability $P(t)$ and the mean-square displacement $\Delta\langle z^2\rangle(t)$ for a FB calculation for the structure and the combined ac-dc field in Fig. 1, but under the special initial condition: $B_n(0)=\delta_{n,0}$. False DL occurs at times $t=\tau, 2\tau, 4\tau, 5\tau, \ldots$.

$$B_n(t) = e^{-i[\varepsilon_0 t/\hbar + (n+W_0/d)\gamma(t)]} A_n(t). \qquad \text{(A3)}$$

If a particular electric field, for which $\tilde{\beta}_p(T)=0$, is applied, then $A_n(T)=\delta_{n,0}$, and we obtain

$$B_n(T) = e^{-i[\varepsilon_0 T/\hbar + W_0 \gamma(T)/d]} \delta_{n,0}, \qquad \text{(A4)}$$

such that

$$|\Psi(T)\rangle = e^{-i[\varepsilon_0 T/\hbar + W_0 \gamma(T)/d]} |a_0\rangle. \qquad \text{(A5)}$$

Thus, the electron returns periodically to its initial state apart from a time-dependent overall phase. This shows that DL appears to occur in a combined ac-dc field with the ac period if $\beta_p(\tau)=0$, no matter what the ratio of these two frequencies. But it is not correct! Figure 7 shows the time evolution of the return probability and the mean-square displacement in the same structure and in the presence of the same electric field as those in Fig. 1, but with the initial condition $B_n(0)=\delta_{n,0}$. Figure 7 shows that DL occurs with the period $T=\tau$ rather than the period $T=3\tau$ found in Fig. 1. We refer to the apparent DL at the times between the neighboring true DL periods, $t=\tau, 2\tau, 4\tau, 5\tau, \ldots$ as false DL because it occurs *only for specific initial conditions*. This false DL does not take place for electrons with general initial states (as in Fig. 1). Thus, *for true DL, one cannot waive the first condition for DL*, $\gamma(T)=2\pi Q$. However, we must note that no false QBOs can exist because QBOs only occur at times $T=N\tau$ where $\gamma(T)=2\pi Q$ as discussed in Sec. IV.

We finally note that as Wannier functions are not uniquely defined, then the initial condition required for false DL would not seem to be unique. However, when $\gamma(T)\neq 2\pi Q$, the condition $\tilde{\beta}_p(T)=0$, in general, depends on the ε_p and is extremely difficult, if not impossible, to achieve. However, as we discussed previously,[30] if the Wannier functions, $|a_0\rangle$, are chosen to be the maximally-localized Wannier functions, then $W_p=0$ and $\tilde{\beta}_p(t)=\beta_p(t)$. Then, the condition $\beta_p(T)=0$ is independent of the band structure and can be achieved using discontinuous fields. Thus the false DL only occurs in an arbitrary field if the electron is initially placed in the unique maximally-localized Wannier function.

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