Electrical current noise of a beamsplitter as a test of spin entanglement

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We investigate the spin entanglement in the superconductor-quantum dot system proposed by Recher, Sukhorukov, and Loss, coupling it to an electronic beamsplitter. The superconductor-quantum dot entangler and the beamsplitter are treated within a unified framework and the entanglement is detected via current correlations. The state emitted by the entangler is found to be a linear superposition of nonlocal spin singlets at different energies, a spin-entangled two-particle wave packet. Colliding the two electrons in the beamsplitter, the singlet spin state gives rise to a bunching behavior, detectable via the current correlators. The amount of bunching depends on the relative positions of the single particle levels in the quantum dots and the scattering amplitudes of the beamsplitter. It is found that the bunching-dependent part of the current correlations is of the same magnitude as the part insensitive to bunching, making an experimental detection of the entanglement feasible. The spin entanglement is insensitive to orbital dephasing but suppressed by spin dephasing. A lower bound for the concurrence, conveniently expressed in terms of the Fano factors, is derived. A detailed comparison between the current correlations of the nonlocal spin-singlet state and other states, possibly emitted by the entangler, is performed. This provides conditions for an unambiguous identification of the nonlocal singlet spin entanglement.

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I. INTRODUCTION

Ever since the concept of entanglement was introduced,[1] it has been at the heart of conceptual discussions in quantum mechanics.[2–4] The discussions have mainly concerned the nonlocal properties of entanglement. Two entangled, spatially separated particles, an Einstein-Podolsky-Rosen[2] (EPR) pair, are correlated in a way that cannot be described by a local, realistic theory; i.e., the correlations give rise to a violation of a Bell Inequality.[5] In optics, the nonlocal properties of entangled pairs of photons have been intensively investigated over the last decades.[6–8] Recently, the interest has turned to possible applications making use of the properties of entangled particles. Entanglement plays an important role in many quantum computation and information schemes,[9] with quantum cryptography[10] and quantum teleportation[11,12] as prominent examples.

Compared to optics, the investigation of entanglement in solid state systems is only in its infancy. However, the controlled creation, manipulation, and detection of entanglement is a prerequisite for a large-scale implementation of quantum computation and information schemes, making it of large interest to pursue the investigation of entanglement in solid state systems. Considerable experimental[13] and theoretical[14] progress has already been made in the understanding of entangled qubits implemented with Josephson junctions.

For the entanglement of individual electrons, several important steps towards an experimental realization in mesoscopic conductors have been taken recently. A scheme for entanglement of orbital degrees of freedom was proposed in Ref. 15, allowing for control of the entanglement with experimentally accessible electronic beamsplitters.[16,17] Moreover, several proposals[15,18,19] for detecting entanglement via a violation of a Bell Inequality, expressed in terms of zero-frequency noise correlators,[20] have been put forth. Very recently, following a proposal by Beenakker *et al.*,[21] several works have discussed the possibility of electron-hole and post-selected electron-electron entanglement.[22,23] In particular, entanglement in the electrical analog of the optical Hanbury Brown–Twiss effect[24] was investigated in a mesoscopic conductor in the quantum Hall regime, transporting electrons along single edge-states and using quantum point contacts as beamsplitters.[23] Moreover, a scheme for energy-time entanglement[25] has been proposed.[26] The consequences of dephasing for orbital entanglement have been investigated[15,27,28] as well.

Earlier proposals for electronic entanglement have been based on creating and manipulating spin entanglement, in normal[29–32] as well as in normal-superconducting[33–35] systems. Spins in semiconductors have been shown[36] to have dephasing times approaching microseconds, making spins promising candidates for carriers of quantum information. However, a direct detection of spin entanglement in mesoscopic conductors is difficult. The natural quantity to measure is the electrical charge current. To investigate spin current, one thus in principle has to convert the spin current to charge current via, e.g.. spin filters. Although efficient spin filters[37] have very recently been realized experimentally,[38] there are considerable remaining experimental complications in manipulating and detecting individual spins on a mesoscopic scale. In particular, to detect the entanglement by a violation of a Bell Inequality, one needs[19] two spin filters with independent and locally controllable directions to mimic the polarizers in optical schemes.[6–8]

An alternative idea to detect spin entanglement was proposed by Burkard, Loss, and Sukhorukov,[30] and also discussed qualitatively by Maître, Oliver, and Yamamoto.[18] They proposed to use the relation between the spin and orbital part of the wave function, imposed by the anti-symmetry of the total wave function under exchange of two particles. A state with an anti-symmetric, singlet-spin wave function has a symmetric orbital wave function and vice versa for the spin triplet. When colliding the electrons in a beamsplitter, spin singlets and triplets show bunching and anti-bunching behavior, respectively. These different bunching behaviors were found to be detectable via the electrical current correlations; i.e., the properties of the orbital wave function were used to deduce information about the spin state. This approach was later extended to all moments of the current.[39] Moreover, it was recently further elaborated in Ref. 40, taking spin dephasing and nonideal beamsplitters into account.

In comparison to detecting spin entanglement via a violation of a Bell Inequality, the approach of Ref. 30, however, has a fundamental limitation. The anti-symmetric spin singlet is an entangled state, while symmetric, spin-triplet states are not neccesarily entangled. Considering, e.g., the standard singlet-triplet basis, only one of the three triplets $|\uparrow\downarrow\rangle+|\downarrow\uparrow\rangle$, $|\uparrow\uparrow\rangle$, and $|\downarrow\downarrow\rangle$ is spin entangled. However, all spin-triplet states, having the same symmetrical orbital wave function, give rise to the same anti-bunching behavior in the current correlators.[30] As a consequence, in contrast to a Bell Inequality test, the approach of Ref. 30 cannot be employed to distinguish between entangled and nonentangled triplet states. To be able to distinguish between different triplet states, one would need to consider more involved schemes, implementing in addition, e.g., single spin rotations.[31]

Despite this fundamental limitation, the approach of Ref. 30 is, due to its comparable simplicity, still of interest for entanglers emitting nonlocal spin singlets. However, the investigations in Ref. 30 were carried out assuming a discrete spectrum of the electrons and a monoenergetic entangled state incident on the beamsplitter. While giving a qualitatively correct picture of the physics, it does not quantitatively describe the situation in a conductor connected to electronic reservoirs, where the spectrum is continuous and the entangled electrons generally have a wave-packet nature; i.e., the wave function is a linear superposition of entangled electrons at different energies.[15] Moreover, the wave function in Ref. 30 was not derived considering a specific entangler; it was instead taken to be an incoming plane wave with unity amplitude. This makes the calculated current correlations inapplicable to most of the entanglers considered theoretically,[29,33,34] which operate in the tunneling regime and emit entangled states with a low amplitude.

In this paper, we revisit the approach of detection of spin-singlet entanglement presented in Ref. 30. The abovementioned shortcomings are bypassed by treating the entangler and the beamsplitter within a unified theoretical framework. As a source of nonlocal spin singlets, the superconductor-quantum dot entangler (see Fig. 1) investigated in detail by Recher, Sukhorukov, and Loss in Ref. 33, is considered. Using a formal scattering approach, the wave function of the electrons emitted from the entangler is calculated. It is found to be a linear superposition of pairs of spin-entangled electrons at different energies, a two-electron wave packet, similar to what was found for the superconducting orbital entangler in Ref. 15. The amplitude at each energy depends on the position of the single-particle levels in the dots. Both the process in which the electrons tunnel through different dots, creating the desired nonlocal EPR pair, as well as the unwanted process in which both electrons tunnel through the same dot, are investigated. In both cases the spin wave function is a singlet, preserving the spin state of the Cooper pair tunneling out of the superconductor; however, the orbital states are different.

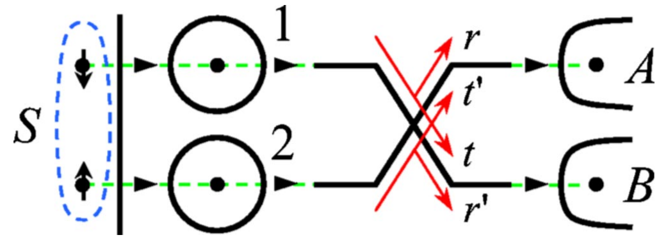

FIG. 1. Schematic picture of the system. A superconductor (S) is connected, via tunnel barriers to two quantum dots (1 and 2) in the Coulomb blockade regime. The dots are further coupled, via a second pair of tunnel barriers, to normal leads that cross in a forward scattering single-mode beamsplitter. The beamsplitter is characterized by scattering amplitudes r, t, r', and t'. On the other side of the beamsplitter, the normal leads are connected to normal electron reservoirs A and B.

The electrons emitted by the entangler are then collided in a beamsplitter and detected in two electronic reservoirs. Due to the singlet spin state, electrons tunneling through different dots show a bunching behavior when colliding in the beamsplitter. Both the auto- and cross-correlations between currents flowing into the normal reservoirs (but not the average current) depend on the degree of bunching. We find that the bunching is proportional to the wave function overlap of the two colliding electrons. This overlap depends strongly on the position of the single-particle levels in the dot, being maximal for both levels aligned with the chemical potential of the superconductors. The part of the current correlators sensitive to bunching is of the same magnitude as the part insensitive to bunching, making an experimental detection of the spin-singlet entanglement feasible.

The current correlators are independent of scattering phases and are thus insensitive to orbital dephasing. However, spin dephasing generally leads to a mixed spin state with a finite fraction of triplets. Since the spin triplets have a tendency to anti-bunch, the spin dephasing results in a reduction of the overall bunching behavior and eventually, for strong spin dephasing, to a crossover to an anti-bunching behavior. A simple expression for the concurrence, quantifying the entanglement in the presence of spin dephasing, is derived in terms of the Fano factors.

For electrons tunneling through the same dot, the wave function is a linear superposition of states for the pair tunneling through dots 1 and 2. Both the cross- and auto-correlators contain a two-particle interference term, sensitive to the position of the single-particle levels in the dots; however, in a different way than the bunching-dependent term for tunneling through different dots. In particular, the correlators depend on the scattering phases, providing a way to distinguish between the two tunneling processes by modulating, e.g., the Aharonov-Bohm phase.[33] Moreover, the phase dependence makes the correlators sensitive to orbital dephasing, while the spin part of the wave function is insensitive to dephasing.

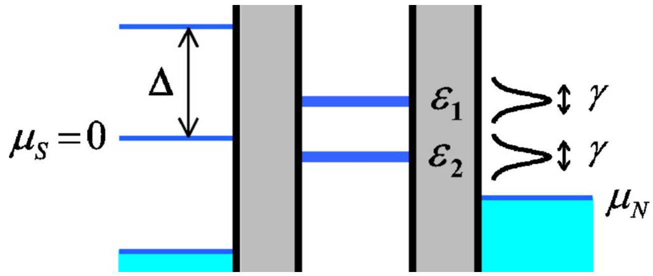

FIG. 2. Energy diagram of the entangler-beamsplitter system in Fig. 1. A bias eV is applied between the superconducting reservoir, with chemical potential $\mu_S=0$, and the normal reservoirs A and B, with the same chemical potential $\mu_N=-eV$. There is only one spin-degenerate level of each dot, with energies ε_1 and ε_2, respectively, in the energy range $-eV$ to eV. The level width γ is determined by the coupling to the normal reservoirs. The bias eV is taken to be much smaller than the superconducting gap, $eV \ll \Delta$, but so large that the broadened levels are well within the bias window, $eV-|\varepsilon_j| \gg \gamma$, $j=1,2$.

II. THE SUPERCONDUCTOR-QUANTUM DOT ENTANGLER

A schematic picture of the system is shown in Fig. 1. A superconducting (S) electrode is connected to quantum dots (1 and 2) via tunnel barriers. The dots are further contacted via normal leads to a controllable single-mode electronic beamsplitter[16] characterized by the forward scattering amplitudes r, t, r', and t'. The arms going out from the beamsplitter are connected to normal electron reservoirs A and B.

We first concentrate on a description of the entangler, the superconductor–quantum dot part of the structure in Fig. 1, investigated in great detail in Ref. 33. The entangler was also recently examined within a density matrix approach.[41] The role of the beamsplitter is discussed further below, after a discussion of the quantum state emitted by the entangler. To simplify our presentation, we carry over the notation from Ref. 33 when nothing else is stated.

An energy diagram of the superconductor–quantum dot normal lead part of the structure is shown in Fig. 2. A negative bias $-eV$ is applied to the normal reservoirs while the superconductor is grounded. The chemical potential of the superconductor is taken as a reference energy, $\mu_S=0$, giving the chemical potential of both normal reservoirs $\mu_{NA}=\mu_{NB} \equiv \mu_N=-eV$. Each dot 1 and 2 contains a single, spin-degenerate level in the energy range $-eV$ to eV, with energies ε_1 and ε_2, respectively. The level spacing in the dots is assumed to be much larger than the applied bias, so that no other levels of the dots participate in the transport. The temperature is much lower than the applied bias (but much larger than the Kondo temperature).

The tunnel barriers between the dots and the superconductor are much stronger than the tunnel barriers between the dots and the normal leads. As a consequence, the broadening γ of the levels in the dots (taken the same for both dots) results entirely from the coupling to the normal leads. The voltage is applied such that the entire broadened resonances are well within the bias window; i.e., $eV-|\varepsilon_j| \gg \gamma$ with $j=1,2$. The quantum dots are in the Coulomb blockade regime; i.e., it costs a charging energy U to put two electrons on the same dot. The ground state contains an even number of electrons in the lower lying levels.

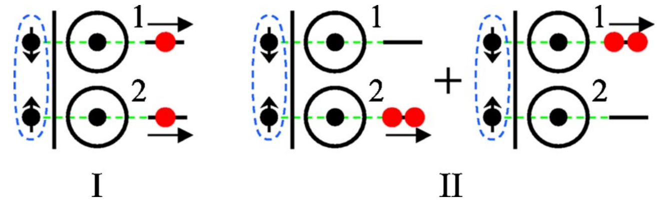

FIG. 3. Tunneling processes transporting two electrons from the superconductor to the normal leads. Process I, in which the two electrons tunnel through different dots, one through dot 1 and one through dot 2, creates the wanted EPR pair. Process II, in which the two electrons tunnel through the same dot, either both through dot 1 or both through dot 2, is unwanted.

The transport takes place as Cooper pairs tunnel from the superconductor, through the dots, and out into the normal leads. Due to the dominating tunnel barrier at the dot-superconductor interface, one pair that tunneled onto the dots leaves the dots well before the next pair tunnels. There are two distinct possibilities for the Cooper pair to tunnel from the superconductor to the normal leads, shown in Fig. 3:

• I, the pair splits and one electron tunnels through each dot, 1 and 2;

• II, both electrons tunnel through the same dot, 1 or 2.

It was shown in Ref. 33 that under the conditions stated above, all other tunneling processes could be neglected. Process I creates the wanted EPR pair, a spin-singlet state with the two electrons spatially separated. However, in an experiment, one cannot *a priori* exclude the second, unwanted process II. One thus has to investigate process II as well, to provide criteria for an unambiguous experimental identification of emission of EPR pairs.

Process I, with the two electrons tunneling through different dots, is suppressed below the single-particle tunneling probability squared, since the two electrons have to leave the superconductor from two spatially separated points; i.e., effectively breaking up the Cooper pair. The tunneling amplitude for a ballistic, three-dimensional superconductor[33,42] is $A_0 \propto \exp(-d/\xi)/(k_{FS}d)$, where d is the distance between the superconductor-dot connection points, k_{FS} the Fermi wave number in the superconductor, and ξ the superconducting coherence length. This amplitude is in general larger for lower dimensional[35] and disordered[43,44] superconductors. An investigation of the dependence of A_0 on the geometry of the contacts to the superconductor was performed in Refs. 44 and 45. We point out that ways to avoid the suppression due to pair breaking by means of additional dots have been discussed in a similar context in Ref. 46. However, since more dots complicate the calculation as well as the experimental realization, we consider the simpler geometry in Fig. 1.

Process II, with both electrons tunneling through the same dot, is suppressed by the Coulomb blockade in the dots, as $1/U$. In addition, there is a process that avoids double occupancy of the dots, but instead requires a pair breaking, leading to suppression of the order $1/\Delta$. Together, this gives an amplitude $B_0 \propto (1/U+1/\Delta\pi)$. The exact expression for the constants A_0 and B_0 in terms of tunnel amplitudes between the dots and the superconductor and the dots and the leads can be found in Ref. 33; for our purposes, these expressions are not necessary.

We point out that possible candidates for experimental realization of the proposed system are the extensively investigated[47] heterostructures with semiconductors contacted to metallic superconducting electrodes. Electron transport through double dots in semiconductor systems have been recently been reviewed,[48] with an emphasis on experimental advances.

III. THE WAVE FUNCTION OF THE SPIN-ENTANGLED ELECTRONS

To calculate the wave function of the electrons emitted from the superconductor–quantum dot entangler, we employ the formal scattering theory[49] with the Lippman-Schwinger equation expressed in terms of the transfer matrix (T-matrix). The total Hamiltonian of the system can be written as $H=H_0+H_T$, where H_0 is the Hamiltonian of the superconductor, the quantum dots, and the normal leads. The perturbation H_T describes tunneling between the superconductor, dots, and leads. The exact many-particle state $|\Psi\rangle$ satisfies the Schrödinger equation $(E-H)|\Psi\rangle=0$. In the absence of a perturbation, $H_T=0$, the system is in the ground state $|0\rangle=|0\rangle_S|0\rangle_D|0\rangle_N$, with different chemical potentials, $\mu_S=0$, and $\mu_N=-eV$. The perturbation H_T causes the electrons to tunnel from the superconductor, via the quantum dots, to the normal leads.

We use the local nature of the tunneling perturbation and take the formal scattering approach to the problem. According to this approach, the state $|\Psi\rangle$ can be obtained by solving the Lippman-Schwinger equation in Fock space, as

$$|\Psi\rangle = |0\rangle + \hat{G}(0)H_T|\Psi\rangle, \tag{1}$$

where the retarded operator $\hat{G}(E)=[E-H_0+i0]^{-1}$ gives a state describing particles going out from the scattering region. Note that the total energy of the ground state $|0\rangle$ is $E=0$. The formal solution of Eq. (1) can be written as

$$|\Psi\rangle = |0\rangle + \hat{G}(0)T(0)|0\rangle, \tag{2}$$

where

$$T(E) = H_T + H_T\sum_{n=1}^{\infty}[\hat{G}(E)H_T]^n \tag{3}$$

is the T-matrix. One then inserts a complete set of many-body states $1=\Sigma_N|N\rangle\langle N|$ with $|N\rangle$ the eigenbasis of the Hamiltonian H_0; i.e., the basis of Fock states of electrons and quasiparticles in the leads, dots, and superconductor, respectively. The quantum number N collectively denotes the energies, spins, lead and dot indices, etc., of the individual particles. The eigenenergy of the state $|N\rangle$ (i.e., the total energy of the individual particles) is E_N. This gives an expression for the state

$$|\Psi\rangle = |0\rangle - \sum_N \frac{1}{E_N - i0}|N\rangle\langle N|T(0)|0\rangle. \tag{4}$$

In the system under consideration, all relevant matrix elements[33] $\langle N|T(0)|0\rangle$ are analytic in the upper part of the complex energy plane. As a consequence, in the integration over energies of the individual particles in $|N\rangle$, the pole arising from the denominator E_N-i0 can be replaced by a $\delta(E_N)$-function, imposing a total energy $E_N=0$, equal to the chemical potential energy of the superconductor. This gives the wave function

$$|\Psi\rangle = |0\rangle - 2\pi i\sum_N \delta(E_N)|N\rangle\langle N|T(0)|0\rangle. \tag{5}$$

It was shown in Ref. 33 that, under the conditions stated above and to lowest order in coupling between the superconductor and the dots, the operator T creates from the vacuum $|0\rangle$ a two-electron spin-entangled state. As pointed out above, depending on the relation between the amplitudes A_0 and B_0, the transport of the two electrons through the same (process II) or different (process I) dots dominates. Below, we consider, for simplicity, only the limiting cases, in which either I or II is completely dominating; however, our analysis can straightforwardly be extended to a situation in which they are of comparable strength.

A. Electrons tunneling through different dots

We first consider process I, in which the amplitude for tunneling through different dots is much larger than the amplitude to tunnel through the same dot. This creates the desired EPR pair, a nonlocal spin-entangled pair of electrons. The quantities in this limit are denoted with a "I." The wave function for two spin-entangled electrons at energies E_1 and E_2 is

$$|E_1,E_2\rangle_{\mathrm{I}} = \frac{1}{\sqrt{2}}\left[b^{\dagger}_{1\uparrow}(E_1)b^{\dagger}_{2\downarrow}(E_2) - b^{\dagger}_{1\downarrow}(E_1)b^{\dagger}_{2\uparrow}(E_2)\right]|0\rangle, \tag{6}$$

where the operator $b^{\dagger}_{l\sigma}(E)$ creates an outgoing (from the dots towards the beamsplitter) electron plane wave with spin $\sigma=\uparrow,\downarrow$ and momentum $k(E)=k_F+E/\hbar v_F$ in the normal lead $l=1,2$. Here, k_F and v_F are the Fermi wave number and velocity, respectively, the same for both normal leads. The amplitude for this process was found in Ref. 33 to have a double-resonant form

$$\langle E_1,E_2|T(0)|0\rangle_{\mathrm{I}} = \frac{iA_0\gamma/\left(\pi\sqrt{2}\right)}{\left(E_1+\varepsilon_1-i\gamma/2\right)\left(E_2+\varepsilon_2-i\gamma/2\right)}. \tag{7}$$

With this, we are able to obtain the asymptotics of the outgoing spin-entangled state. To do so we substitute Eq. (7) into Eq. (5) and find

$$|\Psi_{\mathrm{I}}\rangle = |0\rangle + \int_{-eV}^{eV} dEA(E)\left[b^{\dagger}_{1\uparrow}(E)b^{\dagger}_{2\downarrow}(-E) - b^{\dagger}_{1\downarrow}(E)b^{\dagger}_{2\uparrow}(-E)\right]|0\rangle, \tag{8}$$

with

$$A(E) = \frac{A_0\gamma}{(E+\varepsilon_1 - i\gamma/2)(-E+\varepsilon_2 - i\gamma/2)}; \tag{9}$$

i.e., $A(E)=(-i\pi\sqrt{2})\langle E,-E|T(0)|0\rangle_{\mathrm{I}}$. This state is the sum of the unperturbed ground state and an entangled, two-electron state. The entangled state is a linear superposition of spin singlets at different energies, an entangled two-particle wave packet. The singlet spin state results from the singlet state of the Cooper pair, conserved in the tunneling from the superconductor. Moreover, the two electrons in each singlet have opposite energies E and $-E$ (counted from $\mu_S=0$), a consequence of the Cooper pairs having zero total energy with respect to the chemical potential of the superconductor.

Several important observations can be made regarding the state in Eq. (8). First, the properties, including the two-particle wave-packet structure, can be clearly seen by writing the wave function in first quantization. Introducing $|1,E\rangle \otimes|\uparrow\rangle$ for $b_1^{\uparrow\dagger}(E)|0\rangle$, the properly symmetrized wave function is given by (omitting the ground state $|0\rangle$)

$$|\Psi_{\mathrm{I}}\rangle = \int_{-eV}^{eV} dEA(E)\big(|1,E\rangle_\mu|2,-E\rangle_\nu + |2,-E\rangle_\mu|1,E\rangle_\nu\big) \otimes\big(|\uparrow\rangle_\mu|\downarrow_\nu\rangle - |\downarrow\rangle_\mu|\uparrow\rangle_\nu\big) \tag{10}$$

with μ,ν the particle indices. The coordinate-dependent wave function $\Psi_{\mathrm{I}}(x_\mu,x_\nu)=\langle x_\mu,x_\nu|\Psi_{\mathrm{I}}\rangle$ can then be written ($x=0$ at the lead-dot connection points) as

$$\Psi_{\mathrm{I}}(x_\mu,x_\nu) = \psi(x_\mu,x_\nu)\big(\lambda_\mu^1\lambda_\nu^2 + \lambda_\mu^2\lambda_\nu^1\big)\big(\chi_\mu^\uparrow\chi_\nu^\downarrow - \chi_\mu^\downarrow\chi_\nu^\uparrow\big) \tag{11}$$

with

$$\psi(x_\mu,x_\nu) = \frac{2\pi i\gamma A_0}{2\varepsilon - i\gamma} \times\exp\big[ik_F(x_\nu+x_\mu) - i(\varepsilon - i\gamma/2)|x_\nu - x_\mu|/\hbar v_F\big], \tag{12}$$

where, for simplicity, the case with energies $\varepsilon_1=\varepsilon_2\equiv\varepsilon$ is considered. To arrive at Eq. (11), we first introduced the wave functions $\langle x_l|E\rangle=\exp[ik(E)x_l]$, $l=\mu,\nu$, the spin spinors $\langle x_l|\uparrow\rangle=\chi_l^\uparrow$, $\langle x_l|\downarrow\rangle=\chi_l^\downarrow$, and the orbital spinors $\langle x_l|1\rangle=\lambda_l^1$, $\langle x_l|2\rangle=\lambda_l^2$, and then performed the integral over energy. The orbital spinors describe the wave function in the space formed by the lead indices 1 and 2, a pseudo spin space, as discussed in Ref. 15. We note that the beamsplitter, discussed below, only act in the orbital 12-space (i.e., spin-independent scattering). Moreover, it is the property of the state in 12-space that determines the current correlators discussed below.

As is clear from Eq. (11), the state is a direct product state between the spin and orbital parts of the wave function. The spin state is anti-symmetric under exchange of the two electrons, a singlet, while the orbital state is symmetric, a triplet. The probability to jointly detect one electron at x_μ in lead 1 and one at x_ν in lead 2 decays exponentially with the distance $|x_\mu-x_\nu|$, an effect of the two electrons being emitted at essentially the same time (separated by a small time $\hbar/\Delta$) to points $x_\mu=0$ and $x_\nu=0$, respectively. Note that the state $|\Psi_{\mathrm{I}}\rangle$, a stationary scattering state, does not describe wave packets in the traditional sense with two electrons moving out from the dots as time passes (as a solution to the time-independent many-particle Schrödinger equation, $|\Psi_I\rangle$ has a trivial time dependence). To obtain such a wave function, one must break time translation invariance by introducing a time-dependent perturbation; e.g., a variation of the tunnel barrier strength or dot-level energies in time.

In this context, it is worthwhile mentioning that such a time-dependent wave function was recently considered by Hu and Das Sarma[50] for a double-dot turnstile entangler. However, in Ref. 50, the entangled wave function was not derived from a microscopic calculation, but merely postulated. The wave function had an amplitude of order unity (no tunneling limit) and contained a double integral over energy. This is different from our wave function in Eq. (8) and, moreover, gives rise to a qualitatively different result for the currents as well as the current correlators studied below.

Second, the entangled state in Eq. (8) has just the same form as the pair-splitted state obtained in the normal-superconducting system of Ref. 15, where a scattering approach based on the Bogoliubov–de Gennes equation was used. This shows rigorously that the effect of the strong Coulomb blockade, prohibiting two electrons to tunnel through the same dot, can be incorporated in a scattering formalism by putting the amplitude for Andreev reflection back into the same dot to zero. From this observation, it follows that the rest of the calculation in the paper where the state in Eq. (8) is employed could in principle be carried out strictly within the scattering approach[51] to the Bogoliubov–de Gennes equation. However, in such a calculation the entanglement is not directly visible, which makes the interpretation of the result difficult. Instead, below, we work directly with the state in Eq. (8).

Third, it is also interesting to note the close connection between the emission of a Cooper pair and the process of spontaneous, parametric down-conversion[52] of pairs of photons investigated in optics, in which a single photon from a pump laser is split in a nonlinear crystal into two photons. From the point of view of the theoretical approach, expanding the outgoing state in a ground state and, to first order in perturbation, an emitted pair of particles, is similar to the work in, e.g., Ref. 53. The resulting state [Eq. (8)], is a spin singlet, while a state with polarization entanglement is, under appropriate conditions, produced in the down-conversion process (type II). Moreover, the emission of the two electrons is “spontaneous;” i.e., random and uncorrelated in time, in the same way as for the down-converted photons. One can also point out the perhaps less obvious relation that the two electrons emitted from the superconductor carry information about the phase of the superconducting condensate, just as the two photons carry information of the phase of the field of the pump laser. A coherent superposition of states of pairs of electrons emitted from different points of the superconductor, can give rise to observables sensitive to the difference in superconducting phase between the two emission points, as was demonstrated in Ref. 15. This has its analog in the photonic experiment with a single, coherent laser pumping two separate nonlinear crystals, presented in Ref. 54.

B. Electrons tunneling through the same dot

We now turn to process II, in which the amplitude for tunneling through the same dot is much larger than the amplitude to tunnel through different dots. The wave function for two electrons to tunnel to energies E_1 and E_2 in lead j is

$$|E_1,E_2\rangle_{\mathrm{II}} = \frac{1}{\sqrt{2}}\left[b^\dagger_{j\uparrow}(E_1)b^\dagger_{j\downarrow}(E_2) - b^\dagger_{j\downarrow}(E_1)b^\dagger_{j\uparrow}(E_2)\right]|0\rangle. \tag{13}$$

The amplitude for this process, $\langle E_1,E_2|T(0)|0\rangle_{\mathrm{II}}$, was found in Ref. 33 to have a single resonant form, different from Eq. (7), given by

$$\langle E_1,E_2|T(0)|0\rangle_{\mathrm{II}} = \frac{iB_0}{\pi 2\sqrt{2}} \times\left(\frac{1}{E_1+\varepsilon_j - i\gamma/2} + \frac{1}{E_2+\varepsilon_j - i\gamma/2}\right). \tag{14}$$

Here, for simplicity, the two dot–superconductor contacts are taken to be identical. Since the superconductor is a macroscopically coherent object, the total state is a linear combination of the states corresponding to two electrons tunneling through dot 1 and dot 2. To obtain the asymptotics of the outgoing spin-entangled state, we substitute Eq. (14) into Eq. (5) and find

$$|\Psi_{\mathrm{II}}\rangle = |0\rangle + \int_{-eV}^{eV} dE\left[B_1(E)b^\dagger_{1\uparrow}(E)b^\dagger_{1\downarrow}(-E) + B_2(E)b^\dagger_{2\uparrow}(E)b^\dagger_{2\downarrow}(-E)\right], \tag{15}$$

with

$$B_j(E) = \frac{B_0(\varepsilon_j - i\gamma/2)}{(E+\varepsilon_j - i\gamma/2)(-E+\varepsilon_j - i\gamma/2)}; \tag{16}$$

i.e., $B_j(E)=(-i\pi 2\sqrt{2})\langle E,-E|T(0)|0\rangle_{\mathrm{II}}$. Arriving at Eq. (15), we used the property $B(-E)=B(E)$ and the anti-commutation relations of the fermionic operators.

This state is a linear superposition of the states for two electrons tunneling through the same dot. Comparing to the state $|\Psi_{\mathrm{I}}\rangle$ in Eq. (11) for the two electrons tunneling through different dots, we can make the following comments. (i) Just as $|\Psi_{\mathrm{I}}\rangle$, the wave function $|\Psi_{\mathrm{II}}\rangle$ in first quantization is a product of an orbital and a spin wave function. The spin wave function is, as for $|\Psi_{\mathrm{I}}\rangle$, a singlet $\chi^\uparrow_\mu\chi^\downarrow_\nu - \chi^\downarrow_\mu\chi^\uparrow_\nu$. The orbital wave function for the simplest situation $\varepsilon_1=\varepsilon_2$ is, however, proportional to $\lambda^1_\mu\lambda^1_\nu+\lambda^2_\mu\lambda^2_\nu$, one of the Bell states, an orbitally entangled state. (ii) The state $|\Psi_{\mathrm{II}}\rangle$ is the same as would be obtained within scattering theory (as was shown in Ref. 15), taking $B_j(E)$ to be the effective Andreev reflection amplitude at dot j and assuming no crossed Andreev reflection between the dots; i.e., zero probability for an incident electron in lead 1 to be backreflected as a hole in lead 2 and vice versa.

With the state in Eq. (15) and the state for two electrons tunneling through different dots, in Eq. (8), we are in a position to analyze the transport properties.

IV. CURRENT CORRELATORS

The two electrons emitted from the dot-superconductor entangler propagate in the leads 1 and 2 towards the normal reservoirs A and B. As shown in Fig. 1, the two normal leads are crossed in a single-mode reflectionless beamsplitter. The beamsplitter is characterized by a spin- and energy-independent unitary scattering matrix connecting outgoing and ingoing operators as

$$\begin{pmatrix} b_A \\ b_B \end{pmatrix} = \begin{pmatrix} r & t' \\ t & r' \end{pmatrix}\begin{pmatrix} b_1 \\ b_2 \end{pmatrix}, \tag{17}$$

where the subscripts A and B denote towards what reservoir the electron is propagating. The electrons are then detected in the normal reservoirs A and B.

We point out that beamsplitters completely without backscattering are not easily produced experimentally. Qualitatively, electrons backscattering from the beamsplitter will be reflected from the quantum dots with unit probability (in the tunnel limit considered) and propagate towards the beamsplitter again. Due to the resonant character of the scattering at the dots, the electrons will, however, pick up energy-dependent phases. This additional energy dependence of the scattering amplitudes might modify the transport properties considerably. It should be emphasized that this effect of backscattering is generic for dot-entangler beamsplitter systems. It is therefore preferable in an experimental setup to try to minimize the backscattering at the beamsplitter.

In principle, backscattering can be incorporated quantitatively in our model; however, to keep the discussion as simple as possible, we consider only the reflectionless beamsplitter. Several aspects of backscattering from the beamsplitter were recently investigated by Burkard and Loss,[40] extending the model in Ref. 30. However, in Ref. 40, an isolated beamsplitter, not coupled to a specific entangler, was considered, and thus the problem with further scattering from the entangler was not addressed.

We also note that phases due to propagation can be included in the scattering amplitudes of the beamsplitter. In the typical system of interest, with a lateral size L in the micrometer range, the energy-dependent part of the phase $\sim L\gamma/\hbar v_F$ picked up by the electrons when propagating in the leads is negligibly small. The energy-independent part enter the results in the same way as an Aharanov-Bohm phase or a superconducting phase difference, further discussed below in connection to the treatment of tunneling through the same dot.

The properties of the electrons emitted by the entangler are investigated via the current and the zero-frequency current correlators. The electrical current operator in lead α is given by[55]

$$\hat{I}_\alpha = \frac{e}{h}\int dE dE' e^{i(E-E')t/\hbar} \times\sum_\sigma\left[b^\dagger_{\alpha\sigma}(E)b_{\alpha\sigma}(E') - a^\dagger_{\alpha\sigma}(E)a_{\alpha\sigma}(E')\right], \tag{18}$$

where $a^\dagger_{\alpha\sigma}(E)$ creates an electron plane wave incoming from the normal reservoir α with spin $\sigma=\uparrow,\downarrow$ and momentum $k(E)$. The averaged current is given by

$$I_\alpha \equiv \langle \hat{I}_\alpha \rangle, \tag{19}$$

where $\langle\cdots\rangle \equiv \langle\Psi|\cdots|\Psi\rangle$. The zero-frequency correlations between the currents in the leads α and β are

$$S_{\alpha\beta} = \int dt \langle \Delta\hat{I}_\alpha(t)\Delta\hat{I}_\beta(0) + \Delta\hat{I}_\beta(0)\Delta\hat{I}_\alpha(t)\rangle, \tag{20}$$

where $\Delta I_\alpha(t)=I_\alpha(t)-I_\alpha$ is the fluctuating part of the current in lead α. We study the two cases with electrons tunneling through different dots and the same dot separately.

V. TUNNELING THROUGH DIFFERENT DOTS

For electrons tunneling through different dots, the question is how the degree of spin-singlet entanglement is reflected in the current and current correlators. The averaged current, evaluated with the state $|\Psi_{\mathrm{I}}\rangle$ in Eq. (8), becomes

$$I_\alpha^{\mathrm{I}} = \frac{2e}{h}\int_{-eV}^{eV} dE|A(E)|^2, \tag{21}$$

which is the same for $\alpha=A$ and B. Since the two resonances ε_1 and ε_2 are well within the voltage range (i.e., $eV-|\varepsilon_1|$, $eV-|\varepsilon_2|\gg\gamma$), we get the current

$$I_\alpha^{\mathrm{I}} = \frac{2e}{h}\frac{4\pi|A_0|^2\gamma}{(\epsilon_1+\epsilon_2)^2+\gamma^2}, \tag{22}$$

just the same expression as in Ref. 33, where the leads of the entangler were contacted directly to the normal reservoirs (no beamsplitter). The current is maximal for an asymmetric setting of the resonances $\varepsilon_1=-\varepsilon_2$. This two-particle resonance reflects the fact that the two electrons in the Cooper pairs are emitted at opposite energies with respect to the superconducting chemical potential. The current contains no information about the entanglement of the emitted state. In fact, the same current would be obtained by considering a product state of one electron in lead 1 and one in lead 2, independent of their spins.

To obtain information about the entanglement, we turn to the current correlators. Inserting the expression for the state $|\Psi_{\mathrm{I}}\rangle$ into Eq. (20), following Ref. 55, we get the expressions for the auto-correlations

$$S_{AA}^{\mathrm{I}} = S_{BB}^{\mathrm{I}} = \frac{4e^2}{h}\int_{-eV}^{eV} dE\{[1+2RT]|A(E)|^2 + 2RTA(E)A^*(-E)\} \tag{23}$$

as well as the cross-correlations

$$S_{AB}^{\mathrm{I}} = S_{BA}^{\mathrm{I}} = \frac{4e^2}{h}\int_{-eV}^{eV} dE\{[T^2+R^2]|A(E)|^2 - 2RTA(E)A^*(-E)\}, \tag{24}$$

where $R=|r|^2=|r'|^2$ and $T=|t|^2=|t'|^2=1-R$. We note that the total noise S^{I} of the current flowing out of the superconductor is twice the Poissonian, i.e.,

$$S^{\mathrm{I}} = S_{AB}^{\mathrm{I}} + S_{BA}^{\mathrm{I}} + S_{AA}^{\mathrm{I}} + S_{BB}^{\mathrm{I}} = 4e(I_A^{\mathrm{I}} + I_B^{\mathrm{I}}), \tag{25}$$

describing an uncorrelated emission of pairs of electrons. This result, an effect of the tunneling limit, is different from the one in Ref. 30, where an entangled state with unity amplitude was considered and the total noise was found to be zero.

It is clear from the calculation that the second term in Eqs. (23) and (24) depends directly on the symmetry properties of the orbital wave function, and thus, due to the anti-symmetry of the total wave function, indirectly on the symmetry properties of the spin wave function. For a spin-triplet state $|\Psi_{\mathrm{I}}\rangle$, the last term in Eqs. (23) and (24) would have opposite signs. Since all the three possible triplets, with spin wave functions $\chi_\mu^\uparrow\chi_\nu^\downarrow+\chi_\mu^\downarrow\chi_\nu^\uparrow$, $\chi_\mu^\uparrow\chi_\nu^\uparrow$, and $\chi_\mu^\downarrow\chi_\nu^\downarrow$ have the same anti-symmetric orbital wave function ($\lambda_\mu^1\lambda_\nu^2-\lambda_\mu^2\lambda_\nu^1$ for $\varepsilon_1=\varepsilon_2$) they give rise to the same noise correlators. As a consequence, performing a noise correlation measurement, one can only distinguish between spin singlets and spin triplets, but not between entangled $\chi_\mu^\uparrow\chi_\nu^\downarrow+\chi_\mu^\downarrow\chi_\nu^\uparrow$ and nonentangled $\chi_\mu^\uparrow\chi_\nu^\uparrow$, $\chi_\mu^\downarrow\chi_\nu^\downarrow$ spin triplets. This was pointed out already in Ref. 30. We note that it is possible to distinguish between the different triplets in a more advanced beamsplitter scheme, using controlled single spin rotations via, e.g, a local Rashba interaction.[31] Such a scheme is straightforwardly included into our theoretical treatment. However, it demands a more involved experimental setup and is therefore not considered here; we restrict our investigation to the simplest possible system.

To investigate the properties of the current correlators in detail, the remaining integral over energy in Eqs. (23) and (24) is carried out, giving

$$\int dEA(E)A^*(-E) = \frac{4\pi|A_0|^2\gamma^3}{[(\epsilon_1-\epsilon_2)^2+\gamma^2][(\epsilon_1+\epsilon_2)^2+\gamma^2]}. \tag{26}$$

This shows that, unlike the current, the noise is sensitive to both the difference and the sum of the dot energy levels. We note that the integral of $A(E)A^*(-E)$ is manifestly positive and smaller than the integral of $|A(E)|^2$ for all ε_1, ε_2 except for $\varepsilon_1=\varepsilon_2$, when they are equal.

From these observations, we can draw several conclusions and compare our results to the results in Ref. 30.

(i) The second term in Eqs. (23) and (24), dependent on the orbital symmetry of the wave function, leads to a suppression of the cross-correlation, but to an enhancement of the auto-correlation. This is an effect of the bunching behavior of the spin singlet; i.e., the two electrons show an increased probability to end up in the same normal reservoir.[30] For a symmetric beamsplitter, $R=T=1/2$ and aligned dot levels $\varepsilon_1=\varepsilon_2$, the cross-correlations are zero (to the leading order in tunneling probability considered here). This is a signature of perfect bunching of the two electrons.

(ii) The last term in Eqs. (23) and (24) is proportional to the spectral overlap $\int dEA(E)A^*(-E)$. The spectral overlap physically corresponds to the overlap between the wave functions of the two electrons colliding in the beamsplitter. For single-particle levels at different energies $\varepsilon_1\neq\varepsilon_2$, the

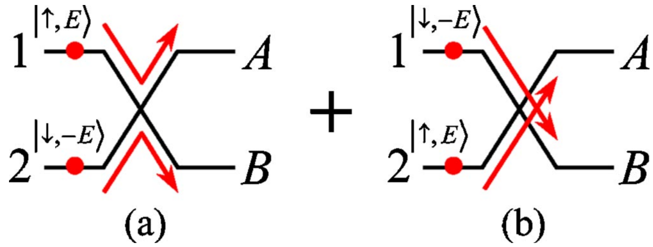

FIG. 4. Elementary scattering processes (shown at the beamsplitter) contributing to the cross-correlators S^{I}_{AB}. The two processes (a) and (b) transport a pair of electrons $|\uparrow,E\rangle$ and $|\downarrow,-E\rangle$ from the superconductor to the reservoirs A and B, respectively. The two processes, having the same initial and final state, are indistinguishable and their amplitudes must be added. The correlator S^{I}_{AB} is proportional to the integral over energy of the (energy-dependent) joint detection probability.

spectral amplitudes of the emitted electrons are centered at different energies and, consequently,[30] the Pauli principle responsible for the bunching is less efficient.

It is important to note that the last term in Eqs. (23) and (24), dependent on the bunching, generally is of the same magnitude as the first term. We emphasize that this result is qualitatively different from what was found in Ref. 30, where the bunching-dependent part of the current correlator was proportional to a Kronecker delta-function in energy, a consequence of considering a discrete spectrum. Our result clearly shows that it should be experimentally feasible to detect the bunching, and thus demonstrate that spin singlets are emitted from the entangler. We note that the same qualitative result was found in Ref. 50.

(iii) The cross-correlations are positive for any transparency of the beamsplitters (note that $R^2+T^2 \geqslant 2RT$). This is different from the result in Ref. 30, where negative cross-correlations were predicted. The negative correlations are again a result of the unity amplitude of the incoming entangled state considered in Ref. 30. In this context, we point out that positive cross-correlations have been predicted in several few-mode[56] and many-mode[57] normal-superconductor hybrid systems as well as purely normal systems in the presence of interactions.[58] In several of these cases, the positive correlations were explained with semiclassical models. Thus, the presence of positive correlations cannot itself be taken as a sign of spin entanglement.

We point out that the expression for the energy-dependent integrand of the cross-correlators in Eq. (24) can be understood in an intuitive way, by considering the elementary scattering processes contributing to the noise, shown in Fig. 4.

Let us consider the probability for the two electrons emitted from the superconductor to end up, one with spin up and energy E in reservoir A and the other with spin down and energy $-E$ in reservoir B. There are two paths the electrons can take from the superconductor to the reservoirs: (a) the electron with spin up and energy E via dot 1 and the electron with spin down and energy $-E$ via dot 2, this process having an amplitude $tt'A(E)$; (b) the electron with spin down and energy $-E$ via dot 1 and the electron with spin up and energy E via dot 2, this process having an amplitude $rr'A(-E)$. Since the two processes have the same initial and final states, they are indistinguishable and their amplitudes must be added. This gives together the energy-dependent joint detection probability $\sim|tt'A(E)+rr'A(-E)|^2=T^2|A(E)| + R^2|A(-E)|^2 + rr't^*t'^*A(E)A^*(-E) + r^*r'^*tt'A(-E)A^*(E)$. Analogously to the noise correlators for the entangler with energy-independent tunneling probabilities in Ref. 15, it is found that the noise correlator S^{I}_{AB} is simply proportional to the integral over energy of the joint detection probability. Using that the integral in Eq. (24) goes from $-eV$ to eV and that the unitarity of the scattering matrix in Eq. (17) gives $rt^*+t'r'^*=0$, we get the expression in the integrand in Eq. (24).

For the auto-correlation, a similar interpretation in terms of probabilities for two-particle scattering processes only is not possible, one also has to consider single-particle probabilities. Formally, this is the case since auto-correlations contain exchange effects between the two particles scattering to the same reservoir.

A. Fano factors

A quantitative analysis of the current correlators is most naturally performed via the Fano factors $F_{\alpha\beta} = S_{\alpha\beta}/(2e\sqrt{I_\alpha I_\beta})$. The Fano factor isolates the dependence of the noise on various parameters, not already present in the current. For the cross- and auto-correlations, respectively, we have

$$F^{\mathrm{I}}_{AB} = F^{\mathrm{I}}_{BA} = T^2 + R^2 - 2RT|H(\varepsilon_1-\varepsilon_2)|^2 \qquad (27)$$

and

$$F^{\mathrm{I}}_{AA} = F^{\mathrm{I}}_{BB} = 1 + 2RT + 2RT|H(\varepsilon_1-\varepsilon_2)|^2, \qquad (28)$$

where

$$H(\varepsilon_1-\varepsilon_2) = \frac{i\gamma}{\varepsilon_1-\varepsilon_2+i\gamma}. \qquad (29)$$

We note that only the last terms in Eqs. (27) and (28) depend on the energies ε_1 and ε_2 of the levels in the dots. In line with the discussion of the current correlations above, we point out that this energy dependence is qualitatively different from a Kronecker delta-function in energy, found in Ref. 30 as a consequence of considering a discrete spectrum.

The Fano factor as a function of energy difference $\varepsilon_1-\varepsilon_2$ is plotted for several values of transparency of the beamsplitter in Fig. 5. For the cross-correlators, the Fano factor has a minimum for the two resonant levels aligned: $\varepsilon_1=\varepsilon_2$. The value at this minimum decreases monotonically from 1 to 0 when increasing the transparency T of the beamsplitters from 0 to 0.5 (the Fano factor for transmission probability T is the same as for $1-T$). Thus, for a completely symmetric beamsplitter ($T=R=0.5$), the Fano factor is zero. This corresponds to the case of perfect bunching. For the auto-correlators, the picture is the opposite. The Fano factor has a maximum for the two resonances aligned: $\varepsilon_1=\varepsilon_2$. The value at this maximum increases monotonically from 1 to 2 when increasing the transparency T of the beamsplitters from 0 to 0.5. Thus, for a symmetric beamsplitter ($T=R=0.5$), the Fano factor is now 2.

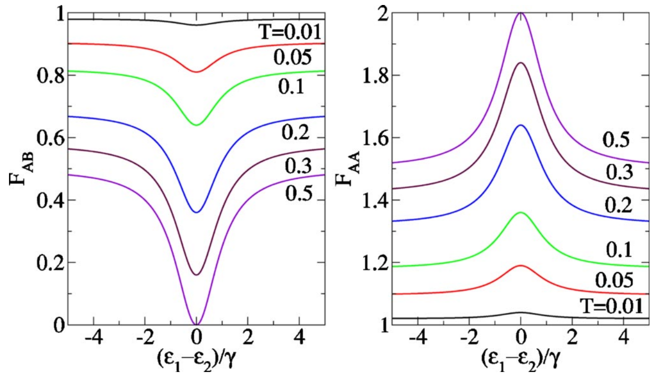

FIG. 5. The Fano factor for the cross-correlations $F_{AB}=F_{BA}$ (left panel) and auto-correlations $F_{AA}=F_{BB}$ (right panel) as a function of the normalized energy difference $(\varepsilon_1-\varepsilon_2)/\gamma$ for various beamsplitter transparencies.

B. Decoherence

Considering the robustness of the bunching behavior, an important observation is that the Fano factors in Eqs. (28) and (27) [as well as the noise correlators in Eqs. (23) and (24)] depend only on the respective transmission and reflection probabilities T and R. All information about the scattering phases, from the beamsplitter as well as from the propagation in the leads, drops out. As a consequence, the correlators are insensitive to dephasing of the orbital part of the wave function; i.e., processes that cause slow and energy-independent fluctuations of the scattering phases. This insensitivity, different from schemes based on orbital entanglement,[15,21,23,27] can be understood by considering the first quantized version [in Eq. (11)] of the wave function $|\Psi_{\mathrm{I}}\rangle$. Any orbital phase picked up by an electron in, e.g., lead 1, just gives rise to an overall phase factor of the total orbital wave function, since each term in the wave function corresponds to one electron in lead 1 and one in lead 2. Moreover, any orbital “pseudo spin flip” would imply a scattering of particles between the leads 1 and 2 and is not allowed in the nonlocal geometry.

The situation is different for spin decoherence, energy-independent spin-flip, or spin-dephasing processes tending to randomize the spin directions. Keeping the discussion completely general, in the sense that we do not consider any specific microscopic mechanism of the spin dephasing, spin decoherence modifies the Fano factors in Eqs. (27) and (28). Formally, the (mixed) state in the presence of decoherence is described by a density matrix ρ. Writing ρ in a spin singlet-triplet basis, as shown in the Appendix, only the diagonal elements ρ_{SS} (singlet) and $\rho_{T_0T_0}$, $\rho_{T_+T_+}$, $\rho_{T_-T_-}$ (triplets) contribute to the current correlators. As discussed above, all the three spin triplets give rise to the same Fano factors. The spin-triplet Fano factors are given by the spin-singlet ones in Eqs. (27) and (28) by changing the sign of the last term $2RT|H(\varepsilon_1-\varepsilon_2)|^2$; i.e., from bunching to anti-bunching. Using that the sum of the diagonal elements of the density matrix is 1 (i.e., $\rho_{SS}+\rho_{T_0T_0}+\rho_{T_+T_+}+\rho_{T_-T_-}=1$), the effect of spin decoherence is to renormalize only the part of the Fano factors dependent on the dot-level energies as

$$|H(\varepsilon_1-\varepsilon_2)|^2 \to (2\rho_{SS}-1)|H(\varepsilon_1-\varepsilon_2)|^2. \qquad (30)$$

The renormalization factor is thus the singlet weight minus the total triplet weight, $\rho_{SS}-(\rho_{T_0T_0}+\rho_{T_+T_+}+\rho_{T_-T_-})=2\rho_{SS}-1$. This clearly displays how decoherence, reducing the singlet weight and consequently increasing the triplet weight, leads to a crossover at $\rho_{SS}=1/2$ from a bunching to an anti-bunching behavior of the noise correlators. For a completely dephased spin state, with an equal mixture of singlets and triplets ($\rho_{SS}=\rho_{T_0T_0}=\rho_{T_+T_+}=\rho_{T_-T_-}=1/4$), the renormalization factor $2\rho_{SS}-1$ saturates at the value $-1/2$.

We point out that this discussion might be modified when considering other types of effects causing decoherence, such as, e.g., inelastic scattering. A more detailed investigation (see, e.g., Refs. 59), going beyond the scope of the paper, is needed to address these issues.

C. Spin-entanglement bound

In the absence of spin decoherence, the spin state of the emitted pair is a singlet, a maximally entangled state. For finite spin decoherence, this is no longer the case, and the question arises as to how to obtain quantitative information about the spin entanglement from the measurements of the current correlators.

We stress that our interest here is the spin entanglement of ρ only. However, ρ contains information about the spin part of the state as well as the energy-dependent orbital part, the wave-packet structure of the emitted pair of electrons. To quantify the spin entanglement, one thus has to consider a measurement sensitive to the spin part of ρ only (see the Appendix). One such important example is the cross-correlators between the currents in the leads 1 and 2 (i.e., without beamsplitters). It was shown in a related system in Ref. 15 that these cross-correlators are simply proportional to the probability to jointly detect one particle in lead 1 and one in 2. The wave-packet property of the emitted pair results only in an overall constant multiplying the probabilities. As a consequence, a Bell Inequality, derived in terms of the joint detection probabilities, could be formulated in terms of zero-frequency cross-correlators. In the same way, for the superconductor-dot entangler considered here, the spin entanglement of the two emitted electrons can in principle be tested via a Bell Inequality formulated in terms spin current correlators.[19] The situation is different for the beamsplitter setup, in which the Fano factors in Eqs. (27) and (28) in general depend on the wave-packet structure via the dot-level-dependent factor $|H(\varepsilon_1-\varepsilon_2)|^2$, quantifying the overlap of the two electrons when colliding. However, for $\varepsilon_1=\varepsilon_2$ [i.e., maximal overlap, $|H(0)|^2=1$] the Fano factors are independent of the wave-packet structure of the emitted electrons and are thus only sensitive to the spin part of ρ [see Eq. (30)].

The spin part of ρ can be described by the 4×4 spin density matrix ρ_σ, rigorously defined in the Appendix (note that ρ, due to the continuous energy variable, is infinitely dimensional). Formally, ρ_σ is the density matrix obtained when tracing ρ, for aligned dot levels $\varepsilon_1=\varepsilon_2$, over energies. The question is thus how to determine the entanglement of

ρ_σ. In general, knowledge of all the matrix elements is needed. This information, however cannot be obtained within our approach, since the Fano factors only provide information about the spin-singlet weight, as is clear from Eq. (30). It is nevertheless possible, as described in detail in the Appendix, to follow the ideas of Burkard and Loss[40] and obtain a lower bound for the spin entanglement.

There are several different measures of entanglement for the mixed state of two coupled spin-1/2 systems. Here, we consider the concurrence[60] C, with $C=0$ ($C=1$) for an unentangled (maximally entangled) state. To establish the lower bound, it can be shown that the concurrence $C(\rho_\sigma)$ is always larger than or equal to the concurrence $C(\rho_W)$ of the so-called Werner state,[61] described by the density matrix ρ_W. The Werner state, defined as the average of ρ_σ over identical and local random rotations, has the same singlet weight ρ_{SS} as ρ_σ. The concurrence of the Werner state has the appealing property that it is a function of the spin-singlet weight only: $C_W=\max\{2\rho_{SS}-1,0\}$.

The findings above thus lead to the simple and important result that the renormalization, Eq. (30), of the Fano factors in Eqs. (27) and (28) due to spin decoherence can be written as (for $C_W>0$)

$$|H(\varepsilon_1-\varepsilon_2)|^2 \to C_W|H(\varepsilon_1-\varepsilon_2)|^2, \tag{31}$$

where C_W thus provides a lower bound for the spin entanglement of the emitted pair of electrons [for the pure singlet $\rho_{SS}=1$, C_W and $C(\rho_\sigma)$ are equal and maximal]. Thus, as long as the Fano factors display a bunching behavior, the spin entanglement is finite ($C_W>0$). For a crossover to antibunching behavior, $C_W=0$ and one can no longer conclude anything about the entanglement of the spin state. The value of C_W can be extracted directly from the experimentally determined Fano factors, as the amplitude of the modulation of the Fano factors with respect to dot-level amplitudes $\varepsilon_1-\varepsilon_2$ divided by $2RT$. The values of R and T can be extracted independently from the Fano factors at dot levels such that $H(\varepsilon_1-\varepsilon_2)\approx 0$.

The result in Eq. (31) thus provides a simple relation between the Fano factors and the minimum spin entanglement C_W. It is clear, however, that since the Fano factors only provide information about the singlet weight, full information about the spin entanglement cannot be obtained by the beamsplitter approach employed here. It should be noted that the result in Eq. (31) is quantitatively different from what was obtained in Ref. 40, a consequence of the different states considered for the emitted electrons, as discussed above in connection with the current correlators.

VI. TUNNELING THROUGH THE SAME DOT

We now turn to the situation in which the two electrons tunnel through the same dot. To be able to distinguish this process II from process I, it is important to study the current as well as the noise in detail. The averaged current in Eq. (19), evaluated with the state in Eq. (15), becomes for reservoirs A and B,

$$I_A^{\mathrm{II}} = \frac{2e}{h}\int_{-eV}^{eV} dE\big[R|B_1(E)|^2 + T|B_2(E)|^2\big],$$
$$I_B^{\mathrm{II}} = \frac{2e}{h}\int_{-eV}^{eV} dE\big[T|B_1(E)|^2 + R|B_2(E)|^2\big]. \tag{32}$$

Since the two resonances ε_1 and ε_2 are well within the voltage range (i.e., $eV-|\varepsilon_1|$, $eV-|\varepsilon_2|\gg\gamma$), we can perform the integrals and get the current[33]

$$I_\alpha^{\mathrm{II}} = \frac{2e}{h}\pi|B_0|^2/\gamma, \tag{33}$$

which is the same for both reservoirs $\alpha=A,B$. We note that the two-particle resonance in the current, present in the pair-splitting case I, is absent due to the Coulomb blockade, as pointed out in Ref. 33. A difference from Ref. 33 is, however, that due to the absence of backscattering at the beamsplitter, there is no scattering-phase dependence of the current. Consequently, there is no dependence on a possible difference in the superconducting phase at the two emission points or an Aharonov-Bohm phase[62] due to a magnetic flux in the area between the superconductor, the dots, and the beamsplitter. It should be pointed out that this is not a generic result for normal-superconducting systems. In a situation with backscattering, which is inevitable in, e.g., the three-terminal forklike geometries, Andreev interferometers, studied extensively in both diffusive[63] and ballistic[64] conductors, the current is indeed sensitive to a superconducting phase difference as well as the Aharonov-Bohm phase.

Regarding the spin entanglement, just as for process I, no information is provided by the averaged current. The same result would have been obtained considering an incoherent superposition of two electrons in lead 1 and two in lead 2, independent of spin state. Turning to the current correlators, inserting the expression for the state $|\Psi_{\mathrm{II}}\rangle$ into Eq. (20), one gets the expressions for the auto-correlations

$$S_{AA}^{\mathrm{II}} = \frac{4e^2}{h}\int dE\{R(1+R)|B_1(E)|^2 + T(1+T)|B_2(E)|^2 + 2\mathrm{Re}\big[(r^*t')^2B_1^*(E)B_2(E)\big]\},$$
$$S_{BB}^{\mathrm{II}} = \frac{4e^2}{h}\int dE\{T(1+T)|B_1(E)|^2 + R(1+R)|B_2(E)|^2 + 2\mathrm{Re}\big[(r^*t')^2B_1^*(E)B_2(E)\big]\} \tag{34}$$

with $\mathrm{Re}[\cdots]$ denoting the real part, as well as the cross-correlations

$$S_{AB}^{\mathrm{II}} = S_{BA}^{\mathrm{II}} = \frac{4e^2}{h}\int dE\{RT\big[|B_1(E)|^2+|B_2(E)|^2\big] - 2\mathrm{Re}\big[(r^*t')^2B_1^*(E)B_2(E)\big]\}. \tag{35}$$

The integrals over $|B_j(E)|^2$ were carried out above [Eq. (33)]. Performing the integral over $B_1(E)B_2^*(-E)$ in the limit $eV-|\varepsilon_1|, eV-|\varepsilon_2|\gg\gamma$, we get

$$\int dEB_1^*(E)B_2(E) = \frac{\pi i|B_0|^2}{\epsilon_1 - \epsilon_2 + i\gamma}. \quad (36)$$

The expressions for the correlators above yield that the total noise S^{II} of the current flowing out of the superconductors is

$$S^{\mathrm{II}} = S_{AB}^{\mathrm{II}} + S_{BA}^{\mathrm{II}} + S_{AA}^{\mathrm{II}} + S_{BB}^{\mathrm{II}} = 4e\left(I_A^{\mathrm{II}} + I_B^{\mathrm{II}}\right), \quad (37)$$

twice the Poissonian, describing, just as in case I, an uncorrelated emission of pairs of electrons.

We note that, in contrast to the current and the transport properties in case I, in which the two electrons tunnel through different dots, the noise contains information about the scattering phases (via r^*t'). Quite generally, one can write

$$(r^*t')^2 = RTe^{i\phi}, \quad (38)$$

where ϕ is a scattering phase of the beamsplitter. Scattering phases picked up during propagation in the leads simply add to ϕ. As a consequence, ϕ can be modulated by, e.g., an electrostatic gate changing the length of the lead 1 or 2 or by an Aharonov-Bohm flux threading the region between the dots, the superconductor and the beamsplitter. An important consequence of this phase dependence of the current correlators is that it can be used to distinguish between tunneling via process II and between process I, since the current correlators of the latter show no phase dependence. This was pointed out in Ref. 33.

This phase dependence shows that the correlators in Eqs. (34) and (35) are sensitive to dephasing affecting the orbital part of the wave function. For complete dephasing, the last term in Eqs. (34) and (35) is suppressed. The orbital entanglement in Eq. (15), the linear superposition of states corresponding to tunneling through dots 1 and 2, is lost. This sensitivity to orbital dephasing is different from the one for process I discussed above. However, again in contrast to process I, the current correlators are insensitive to spin dephasing. This can be understood by considering the first quantized wave function $|\Psi_{\mathrm{II}}\rangle$, discussed following Eq. (16), keeping in mind that the wave function is a direct product of a spin part and an orbital part. The spin wave function is a singlet, $\chi_\mu^\uparrow\chi_\nu^\downarrow - \chi_\mu^\downarrow\chi_\nu^\uparrow$, but the orbital wave function is a combination of triplets, $\lambda_\mu^1\lambda_\nu^1 + \lambda_\mu^2\lambda_\nu^2$ for $\varepsilon_1 = \varepsilon_2$. Since no scattering between the leads is possible (i.e., no pseudo spin flip), orbital dephasing cannot change the triplet character of the orbital wave function and, as a result, the spin wave function is bound to be a singlet. Thus, the spin entanglement in $|\Psi_{\mathrm{II}}\rangle$ is protected against decoherence.

Turning to the Fano factor, the auto- and cross-correlations are

$$F_{AA}^{\mathrm{II}} = F_{BB}^{\mathrm{II}} = 1 + T^2 + R^2 + 2RT\,\mathrm{Re}\left[e^{i\phi}H(\varepsilon_1 - \varepsilon_2)\right] \quad (39)$$

and

$$F_{AB}^{\mathrm{II}} = F_{BA}^{\mathrm{II}} = 2RT - 2RT\,\mathrm{Re}\left[e^{i\phi}H(\varepsilon_1 - \varepsilon_2)\right], \quad (40)$$

respectively, where $H(\varepsilon_1 - \varepsilon_2)$ is given in Eq. (29).

The Fano factor as a function of energy difference $\varepsilon_1 - \varepsilon_2$ is plotted in Figs. 6 and 7 for several values of the transparency of the beamsplitter.

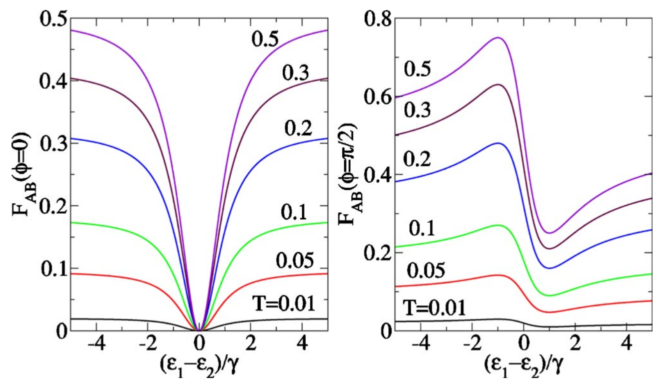

FIG. 6. The Fano factor for the cross-correlations $F_{AB} = F_{BA}$ for phase difference $\phi = 0$ (left panel) $\phi = \pi/2$ (right panel) and as a function of the normalized energy difference $(\varepsilon_1 - \varepsilon_2)/\gamma$ for various beamsplitter transparencies.

For zero phase difference $\phi = 0$, the Fano factor for the cross-correlations shows a dip for aligned resonant levels. At $\varepsilon_1 - \varepsilon_2 = 0$, the Fano factor is zero, independent of the beamsplitter transparency T. This is a signature of perfect bunching. For finite phase difference $\phi \neq 0$, the Fano factor becomes asymmetric in $\varepsilon_1 - \varepsilon_2$, showing a Fano-shaped resonance, with the minimum shifted away from $\varepsilon_1 = \varepsilon_2$.

The Fano factor for the auto-correlations, for $\phi = 0$, shows a corresponding peak for aligned resonant levels, reaching 2 for $\varepsilon_1 = \varepsilon_2$. For finite phase difference $\phi \neq 0$, the Fano factor becomes asymmetric, with the maximum Fano factor shifted away from $\varepsilon_1 = \varepsilon_2$.

We point out that, similar to case I, the integrand of the cross-correlators can be understood by considering the basic two-particle scattering processes. They are shown in Fig. 8; the general explanation is along the same line as for process I, discussed above.

VII. DISCUSSION AND CONCLUSIONS

In conclusion, we have investigated the spin entanglement in the superconductor–quantum dot system proposed by Recher, Sukhorukov, and Loss.[33] Using a formal scattering

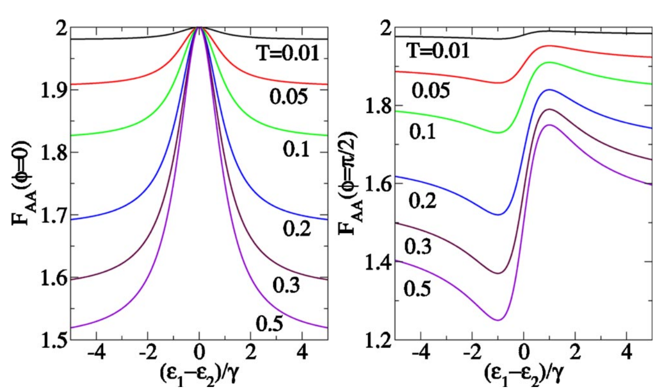

FIG. 7. The Fano factor for the auto-correlations $F_{AA} = F_{BB}$ for phase difference $\phi = 0$ (left panel) $\phi = \pi/2$ (right panel) and as a function of the normalized energy difference $(\varepsilon_1 - \varepsilon_2)/\gamma$ for various beamsplitter transparencies.

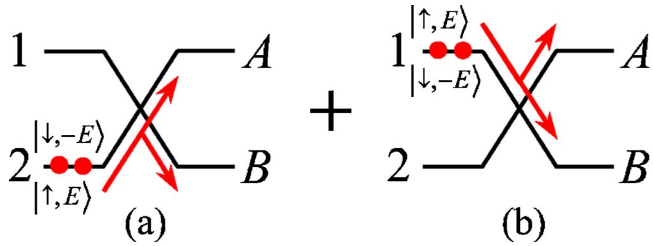

FIG. 8. Elementary scattering processes (shown at the beamsplitter) contributing to the cross-correlators S_{AB}^{II}. The two processes (a) and (b) transport a pair of electrons $|\uparrow,E\rangle$ and $|\downarrow,-E\rangle$ from the superconductor to the reservoirs A and B, respectively.

theory, we have calculated the wave function of the electrons emitted by the entangler and found that it is a superposition of spin singlets at different energies: a two-particle wave packet. Both the wave function for the two electrons tunneling through different dots, creating the desired nonlocal EPR pair, as well as the wave function for the two electrons tunneling through the same dot, were calculated.

The two electrons in the emitted pair collide in a beamsplitter before exiting into normal reservoirs. Due to the symmetrical orbital state, a consequence of the antisymmetrical singlet spin state, the electrons tunneling through different dots show a tendency to bunch. This bunching can be detected via the current correlations. It was found that the amount of bunching depends on the position of the single-particle levels in the dots as well as on the scattering properties of the beamsplitter. Importantly, the magnitude of the bunching-dependent term in the cross correlations was found to be of the same order as the bunching-independent term, implying that an experimental detection of the bunching, and thus indirectly the spin-singlet entanglement, is feasible.

The current correlators for electrons tunneling through different dots were found to be insensitive to orbital dephasing. Spin dephasing, on the contrary, tends to randomize the spin state, leading to a mixed spin state with a finite fraction of triplets. Since singlet and triplet spin states give rise to a bunching and anti-bunching behavior, respectively, when colliding in the beamsplitter, strong dephasing will suppress the bunching behavior and will eventually cause a crossover to anti-bunching. To quantify the entanglement in the presence of spin dephasing, we have derived an expression for the concurrence in terms of the Fano factors. In addition, via the current correlations, it is not possible to distinguish between entangled and nonentangled spin-triplet states, since all triplets show the same bunching behavior. This implies that the method of detecting spin entanglement via current correlations in the beamsplitter geometry has a fundamental limitation compared to the experimentally more involved Bell Inequality test.

We have also investigated the current correlations in the case in which the two electrons tunnel through the same dot. The wave function was found to be a linear superposition of states for the pair tunneling through dots 1 and 2. The cross- and auto-correlators are sensitive to the position of the single-particle levels in the dots; however, in a different way than for tunneling through different dots. Moreover, the correlators were found to be dependent on the scattering phases, providing a way to distinguish between the two tunneling processes by modulating the phase.

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APPENDIX

In the presence of spin decoherence, the state of the pair of electrons emitted through different dots can be described by a density matrix ρ, which can be written as

$$\rho = \left[\int dE|A(E)|^2\right]^{-1}\sum_{q,q'}\rho_{qq'}\int dEdE' \times A(E)A^*(E')|\Psi_q(E)\rangle\langle\Psi_{q'}(E')|, \tag{A1}$$

noting that the normalization gives $\Sigma_q\rho_{qq}=1$. The index q runs over the states in the singlet-triplet basis $\{q\}=\{S,T_0,T_+,T_-\}$; i.e.,

$$\begin{aligned}
|\Psi_S(E)\rangle &= \frac{1}{\sqrt{2}}\left[b_{1\uparrow}^{\dagger}(E)b_{2\downarrow}^{\dagger}(-E) - b_{1\downarrow}^{\dagger}(E)b_{2\uparrow}^{\dagger}(-E)\right]|0\rangle,\\
|\Psi_{T_0}(E)\rangle &= \frac{1}{\sqrt{2}}\left[b_{1\uparrow}^{\dagger}(E)b_{2\downarrow}^{\dagger}(-E) + b_{1\downarrow}^{\dagger}(E)b_{2\uparrow}^{\dagger}(-E)\right]|0\rangle,\\
|\Psi_{T_+}(E)\rangle &= b_{1\uparrow}^{\dagger}(E)b_{2\uparrow}^{\dagger}(-E)|0\rangle,\\
|\Psi_{T_-}(E)\rangle &= b_{1\downarrow}^{\dagger}(E)b_{2\downarrow}^{\dagger}(-E)|0\rangle.
\end{aligned} \tag{A2}$$

The coefficients $\rho_{qq'}$ depend in general on the nature and the strength of the spin decoherence. As pointed out in the text, only energy-independent spin decoherence is considered, and consequently the coefficients $\rho_{qq'}$ are independent of energy.

The current operators conserve the individual spins. As a consequence, the off-diagonal elements of ρ do not contribute to the noise correlators. As discussed in the text, all triplets contribute equally to the correlators. Since the singlet and triplet states contribute with opposite sign to the last term in Eqs. (27) and (28), the effect of spin decoherence on the Fano factors can be incorporated by renormalizing $|H(\varepsilon_1-\varepsilon_2)|^2\rightarrow(2\rho_{SS}-1)|H(\varepsilon_1-\varepsilon_2)|^2$, with the renormalization factor expressed in terms of ρ_{SS} only (using $\rho_{SS}+\rho_{T_0T_0}+\rho_{T_+T_+}+\rho_{T_-T_-}=1$), the weight of the singlet component in ρ.

It is a difficult (and in general not analytically tractable) problem to evaluate the entanglement of the full density matrix, since ρ contains information about both the energy-dependent orbital part of the state as well as the spin part. In particular, due to the continuous energy variable, the dimension of ρ is infinite. Here, we are, however, interested only in the spin entanglement of ρ. To determine the spin entanglement, one has to consider measurement schemes in which the observables O are sensitive only to the spin part of ρ. Such observables satisfy the property

$$\int dEdE' A(E)A^*(E')\langle\Psi_q(E)|O|\Psi_{q'}(E')\rangle = \langle\Psi_q|O_\sigma|\Psi_{q'}\rangle \int dE|A(E)|^2, \quad \text{(A3)}$$

where $|\Psi_{q'}\rangle$ are given from $|\Psi_{q'}(E)\rangle$ in Eq. (A2) by removing the energy dependence; e.g., $|\Psi_{T_+}\rangle = b^\dagger_{1\uparrow}b^\dagger_{2\uparrow}|0\rangle$. The operator O_σ is a function of the energy-independent b-operators. Using the property in Eq. (A3), we can write

$$\langle O\rangle = \mathrm{tr}[\rho O] = \left[\int dE|A(E)|^2\right]^{-1}\sum_{q,q'}\rho_{qq'} \times \int dEdE' A(E)A^*(E')\langle\Psi_q(E)|O|\Psi_{q'}(E')\rangle = \sum_{q,q'}\rho_{qq'}\langle\Psi_q|O_\sigma|\Psi_{q'}\rangle \equiv \mathrm{tr}[\rho_\sigma O_\sigma]. \quad \text{(A4)}$$

The 4×4 spin density matrix ρ_σ is thus

$$\rho_\sigma = \sum_{q,q'}\rho_{qq'}|\Psi_q\rangle\langle\Psi_{q'}|. \quad \text{(A5)}$$

It is straightforward to show that for the special ρ for aligned dot levels $\varepsilon_1=\varepsilon_2$, the current correlators in Eq. (20) are insensitive to the wave-packet structure of ρ. In this case, ρ_σ is directly obtained from ρ by tracing over energies. More generally, independent of $\varepsilon_1,\varepsilon_2$, the spin current correlators between lead 1 and 2 (i.e., in the absence of the beamsplitter) are insensitive to the wave-packet structure of ρ. These latter correlators can be used to test a Bell Inequality, along the lines of Refs. 15 and 19.

Our interest is thus to investigate the entanglement of ρ_σ, conveniently expressed in terms of the concurrence.[60] The concurrence C is defined as

$$C(\rho_\sigma) = \max\{0, \sqrt{\lambda_1}-\sqrt{\lambda_2}-\sqrt{\lambda_3}-\sqrt{\lambda_4}\}, \quad \text{(A6)}$$

where the λ_is are the real and positive eigenvalues, in decreasing order, of $\rho_\sigma\tilde{\rho}_\sigma$. The matrix $\tilde{\rho}_\sigma$ is defined as

$$\tilde{\rho}_\sigma = (\sigma_y\otimes\sigma_y)\rho^*_\sigma(\sigma_y\otimes\sigma_y), \quad \text{(A7)}$$

where σ_y are Pauli matrices, rotating locally the spins in leads 1 and 2, respectively. Importantly, in Eq. (A7), the density matrix ρ_σ is written in the spin-up/spin-down basis; i.e., $b^\dagger_{1\uparrow}b^\dagger_{2\downarrow}|0\rangle$, etc. The concurrence is $C=0$ for an unentangled state and $C=1$ for a state that is maximally entangled.

To determine $C(\rho_\sigma)$, full information about ρ_σ is needed. In the approach taken here, investigating the spin entanglement via a beamsplitter and current correlators, one cannot, however, determine all elements of the density matrix ρ_σ. As a consequence, the spin entanglement of the emitted pair cannot be determined precisely. It is nevertheless possible, following the ideas of Burkard and Loss,[40] to obtain a lower bound for the spin entanglement.

To obtain the lower bound, we first note two important properties of $C(\rho_\sigma)$. (i) $C(\rho_\sigma)$ is invariant under local rotations;[60] i.e., $C(\overline{\rho_\sigma})=C(\rho_\sigma)$ for $\overline{\rho_\sigma}=(U_1\otimes U_2)\rho_\sigma(U_2^\dagger\otimes U_1^\dagger)$, where U_1 and U_2 are unitary 2×2 matrices acting locally on the spins in leads 1 and 2, respectively. (ii) $C(\rho_\sigma)$ is a convex function,[65] $\Sigma_i p_i C(\rho_i)\geqslant C(\Sigma_i p_i\rho_i)$; i.e., for a density matrix $\rho_\sigma=\Sigma_i p_i\rho_i$, with $\Sigma_i p_i=1$, the entanglement of the total density matrix is smaller than or equal to the weighted entanglement of the parts (a consequence of information being lost when adding density matrices).

Consider the density matrix ρ_W obtained by averaging ρ_σ with respect to all possible local rotations $U\otimes U$; i.e., the same rotation in leads 1 and 2. Formally, $\rho_W=\langle(U\otimes U)\rho_\sigma(U^\dagger\otimes U^\dagger)\rangle_U$ is calculated, where $\langle\cdots\rangle_U$ denotes an average with respect to U, uniformly distributed in the group of unitary 2×2 matrices. This gives the Werner state[61]

$$\rho_W = \rho_{SS}|\Psi_S\rangle\langle\Psi_S| + \frac{1-\rho_{SS}}{3} \times\left(|\Psi_{T_0}\rangle\langle\Psi_{T_0}| + |\Psi_{T_+}\rangle\langle\Psi_{T_+}| + |\Psi_{T_-}\rangle\langle\Psi_{T_-}|\right), \quad \text{(A8)}$$

where we note that the singlet component is unaffected by the rotation $U\otimes U$. Importantly, the entanglement of the Werner state is a function of the singlet coefficient ρ_{SS} only. Using the two properties (i) and (ii) of the entanglement stated above, we can write

$$C(\rho_W) = C\left[\langle(U\otimes U)\rho_\sigma(U^\dagger\otimes U^\dagger)\rangle_U\right] \leqslant \left\langle C\left[(U\otimes U)\rho_\sigma(U^\dagger\otimes U^\dagger)\right]\right\rangle_U = \langle C(\rho_\sigma)\rangle_U = C(\rho_\sigma). \quad \text{(A9)}$$

This shows that the concurrence of the Werner state $C_W = C(\rho_W)$ provides a lower bound for the entanglement of the full spin state $C(\rho_\sigma)$. The concurrence of the Werner state is $C_W=\max\{2\rho_{SS}-1,0\}$. The renormalization of the Fano factors in Eqs. (27) and (28) due to spin decoherence can now simply be written $|H(\varepsilon_1-\varepsilon_2)|^2\rightarrow C_W|H(\varepsilon_1-\varepsilon_2)|^2$, where $C_W\geqslant 0$ is a lower bound for the concurrence of the spin state in the presence of decoherence. This is Eq. (31) in the text.

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