# Coulomb drag effect between two one-dimensional conductors: An integrable model with attractive and repulsive interactions

P. Schlottmann
*Department of Physics, Florida State University, Tallahassee, Florida 32306, USA*
(Received 19 May 2004; published 13 September 2004)

We calculate the critical exponent of the temperature dependence of the drag current between two parallel quantum wires. The two wires have carriers with equal mass but they may have different electron density. The electrons interact via a $\delta$-function spin-exchange potential, which is attractive or repulsive depending on whether the interacting particles are in a spin-singlet or spin-triplet state. This interaction leads to the formation of spin-singlet or spin-triplet bound states of the Cooper type (preformed hard core bosons that do not condensate). Depending on the parameters, the drag current can be parallel or opposite to the driving current. The critical exponent of the drag current is calculated using the Bethe *Ansatz* solution of the model and conformal field theory.

DOI: 10.1103/PhysRevB.70.115306 PACS number(s): 73.23.−b, 71.10.Pm, 74.20.Mn

## I. INTRODUCTION

Charge carriers moving in one conductor may induce, via the Coulomb interaction and momentum conservation, a drag current in another conductor located nearby. This drag mechanism was proposed by Pogrebinskii[1] for a semiconductor-insulator-semiconductor layer structure. Here we consider the Coulomb drag between two parallel quantum wires for ballistic electrons. In one-dimensional systems the correlations between electrons lead to charge and spin separation, the disappearance of the Fermi liquid quasiparticle pole in the excitation spectrum, and to power laws with nonuniversal critical exponents. These properties are generically referred to as Luttinger liquids.[2]

The theoretical and experimental developments of the electron-drag effect in a coupled electron system have recently been reviewed by Rojo[3] and the Coulomb drag between quantum wires is extensively reviewed in Ref. 4. We limit ourselves to investigate the drag current for ballistic carriers in linear response to the voltage applied in the driving wire, assuming that only the lowest subband in each wire is occupied. While within the Fermi liquid approach this leads to a drag current proportional to temperature,[5] the Luttinger liquid picture gives rise to nonuniversal power laws with critical exponents that depend on band filling and the interaction strength.[6–9]

The experimental results[10–13] on the Coulomb drag between parallel wires remain sparse, probably because (1) the drag voltage usually has a very small amplitude and (2) it is difficult to create parallel electrically isolated quantum wires that are sufficiently long and close enough to yield a measurable drag voltage.[4]

In a previous publication[14] we calculated the critical exponent for the $T$ dependence of the drag current between two parallel quantum wires using the Bethe *Ansatz* solutions of one-dimensional interacting electron systems. The exact solution provides the mesoscopic energy spectrum (conformal towers) and the long-time long-distance asymptotic of the drag-current correlation function can then be obtained via conformal field theory. For a repulsive interaction for carriers between wires, the momentum conservation of the interaction yields a backward momentum transfer in the drag wire, inducing this way a drag current opposite to the driving current. For an attractive interaction among carriers between the wires, on the other hand, the charges form bound states. The potential applied to the driving wire pulls the bound electrons and hence the drag and driving currents are parallel. The nonuniversal critical exponents depend on the model, the band filling and the interaction strength. We used the exact solution of the Hubbard model, the supersymmetric $t-J$ model and the gas of fermions interacting via a $\delta$-function potential.

In this paper we extend this calculation to a different model involving simultaneously attractive and repulsive interactions.[15–17] The carriers in the two conductors move in parabolic bands with equal mass and interact with each other via a $\delta$-function-like spin-exchange interaction. There are then four internal degrees of freedom, namely the two conductors and the spin. Each of the two sectors (spin and orbital) is SU(2) invariant. Depending on the sign of the interaction, the exchange is attractive (repulsive) in spin space, while it is (at the same time) repulsive (attractive) in orbital space. Here *orbital* refers to the two bands (wires). This leads to the formation of Cooper-type bound states between electrons, which, depending on the sign of the exchange, have either spin-singlet/orbital-triplet or spin-triplet/orbital-singlet symmetry. A bias potential, $\Delta$, between the two parallel conductors may split the bands, so that the electron density is not the same in the two conductors. In addition, a magnetic field $H$ may depair the Cooper pairs in the case of spin-singlet bound states and spin polarize the pairs for spin-triplet pairing. This model is then very different from those considered in Ref. 14, which all have SU($N$) or $gl(1,N)$ symmetry, i.e., the interaction is either only attractive or only repulsive.

The present model is integrable via Bethe's *Ansatz* by construction and involves four sets of rapidities to describe the ground state (four internal degrees of freedom). As a consequence, it has rich phase diagram as a function of $H$ and $\Delta$ (see Fig. 2 in Ref. 18). In this paper we discuss the critical behavior of the drag current in a few of the phases of this model for both repulsive and attractive exchange interaction.

The remainder of the paper is organized as follows. In Sec. II we briefly restate the model and its Bethe *Ansatz* solution for the ground state. We also obtain the mesoscopic corrections to the ground state energy. Using conformal field theory we obtain the drag current correlation function for repulsive spin exchange in Sec. III. In Sec. IV we consider the drag current for attractive spin exchange and conclusions are presented in Sec. V.

## II. MODEL AND BETHE *ANSATZ* SOLUTION

We consider an integrable model consisting of electrons moving in the parabolic bands of two parallel wires labeled with $m=1,2$ and interacting via a contact potential of the spin-exchange type.[15,16] The Hamiltonian is given by

$$\begin{aligned}\mathcal{H} = &\sum_{m,\sigma} \int dx\ \psi^{\dagger}_{m\sigma}(x)(-\partial^2/\partial x^2)\psi_{m\sigma}(x)\\ &+ c \sum_{m,m',\sigma,\sigma'} \int dx \int dx'\, \delta(x-x')\\ &\times \psi^{\dagger}_{m\sigma}(x)\psi^{\dagger}_{m',\sigma'}(x')\psi_{m'\sigma}(x')\psi_{m\sigma'}(x)\\ &- \Delta \sum_{m,\sigma}(-1)^m \int dx\ \psi^{\dagger}_{m\sigma}(x)\psi_{m\sigma}(x), \end{aligned} \tag{1}$$

where $\psi^{\dagger}_{m\sigma}(x)$ creates an electron with spin $\sigma$ at site $x$ in wire $m$, $c$ is the strength of the local exchange interaction, and $2\Delta$ represents the potential difference between the two wires. $\Delta$ lifts the degeneracy between the wires and introduces a difference in the electron density. We choose equal masses for the electrons in the two wires, which is also a necessary condition for the integrability of the model.

The *exchange* $\delta$-function potential is repulsive (attractive) in spin space, while attractive (repulsive) in the orbital sector for $c>0$ ($c<0$). The two-particle scattering matrix factorizes into one for the spin channels and one for the band sector[16]

$$\hat{R}(k) = \frac{k\hat{I}_{\sigma} - ic\hat{P}_{\sigma}}{k-ic}\,\frac{k\hat{I}_m + ic\hat{P}_m}{k+ic}, \tag{2}$$

where $k=k_1-k_2$ is the momentum transfer, and $\hat{I}_m$ ($\hat{I}_{\sigma}$) and $\hat{P}_m$ ($\hat{P}_{\sigma}$) denote the identity and permutation operators in the band (spin) channel, respectively. When applied to a triplet (in the spin or band sector) each of these factor yields one. Hence, the scattering matrix acts nontrivially only on band or spin singlet states. For the case of singlets in both the spin and the band sectors, the two factors cancel and there is no effective phase shift.

The scattering matrix for each of the channels (spin and band) separately satisfies the Yang–Baxter relations,[16,19] thus their product also satisfies the triangular relation. Hence, *transfer* matrices with different spectral parameters commute and can be diagonalized simultaneously, establishing the exact integrability of the model.

The eigenvalues and eigenfunctions are parametrized by three sets of *rapidities*:[16] charge rapidities $\{k_j\}_{j=1}^{N_e}$ (with $N_e$ being the total number of electrons), spin rapidities $\{\lambda_{\alpha}\}_{\alpha=1}^{M}$ ($M$ is the number of "down spins") and the band rapidities $\{\xi_{\beta}\}_{\beta=1}^{n^*}$. Here $N_e-n^*$ and $n^*$ are the number of electrons in the two wires (majority and minority), respectively (due to the potential difference $2\Delta$ the two wires have different populations). These rapidities satisfy the discrete Bethe *Ansatz* equations. For the ground state and $c>0$ the solutions are classified into four classes:[16,19] (1) $N_e-2M$ real charge rapidities of unbound itinerant electrons, (2) $M$ pairs of complex conjugated charge rapidities representing spin-singlet orbital-triplet pairs, (3) real band rapidities, and (4) interband bound states ($\xi$ strings) of length 2. For $c<0$, class (2) corresponds to spin-triplet orbital-singlet pairs, class (3) to real spin rapidities and class (4) to spin-bound states ($\lambda$ strings) of length 2.

We denote with $\epsilon_l$ the dressed energies of the rapidity bands and with $\rho_l$ the density of rapidities and their holes. Here $l=1,\ldots,4$ refers to the four classes of states indicated above. The densities satisfy the following integral equations:

$$\rho_l(\lambda) + \sum_{q=1}^{4}\int_{-B_q}^{B_q} d\lambda' K_{lq}(\lambda-\lambda')\rho_q(\lambda') = g_l(\lambda), \tag{3}$$

where $g_l$ is the driving term given by $1/(2\pi)$, $1/\pi$, 0 and 0 for $l=1,\ldots,4$, respectively. In terms of $a_n(\lambda) = (n|c|/2\pi)/[\lambda^2+(nc/2)^2]$, the integration kernels ($K_{lq}=K_{ql}$) are

$$K_{11}=0,\quad K_{12}=a_1,\quad K_{13}=-a_1,\quad K_{14}=-a_2,$$

$$K_{22}=a_2,\quad K_{23}=-a_2,\quad K_{24}=-a_1-a_3,$$

$$K_{33}=a_2,\quad K_{34}=a_1+a_3,\quad K_{44}=2a_2+a_4. \tag{4}$$

The integration limits $B_l$ are determined from the total number of carriers, the magnetization and the electron population difference between the two wires (for $c>0$)

$$\frac{N_e}{L} = \int_{-B_1}^{B_1} d\lambda\rho_1(\lambda) + 2\int_{-B_2}^{B_2} d\lambda\rho_2(\lambda),$$

$$\frac{M^z}{L} = \frac{1}{2}\int_{-B_1}^{B_1} d\lambda\rho_1(\lambda),$$

$$\begin{aligned}\frac{\mathcal{L}}{L} = &\int_{-B_1}^{B_1} d\lambda\rho_1(\lambda) + 2\int_{-B_2}^{B_2} d\lambda\rho_2(\lambda) - 2\int_{-B_3}^{B_3} d\lambda\rho_3(\lambda)\\ &- 4\int_{-B_4}^{B_4} d\lambda\rho_4(\lambda)\end{aligned} \tag{5}$$

and the energy is given by

$$\frac{E}{L} = \int_{-B_1}^{B_1} d\lambda \lambda^2 \rho_1(\lambda) + 2\int_{-B_2}^{B_2} d\lambda \left(\lambda^2 - \frac{c^2}{4}\right)\rho_2(\lambda). \qquad (6)$$

The dressed energies $\epsilon_l(\lambda)$ satisfy integral Eqs. (3) but with driving terms $h_l(\lambda)$ (for $c>0$)

$$h_1(\lambda) = \lambda^2 - \mu - H/2 - \Delta,$$

$$h_2(\lambda) = 2(\lambda^2 - c^2/4 - \mu - \Delta),$$

$$h_3(\lambda) = 2\Delta, \quad h_4(\lambda) = 4\Delta, \qquad (7)$$

where $\mu$ is the chemical potential and $H$ is the external magnetic field. Here $L$ is the length of the wires.

The generalized dressed charges form a $4\times 4$ matrix, $z_{jk}=\xi_{jk}(B_k)$, where $\xi_{jk}(\lambda)$ again satisfy the integral Eqs. (3) now with the driving term $\delta_{jk}$. The matrix of dressed charges determines the interplay between the four rapidity bands as given by the conformal towers.[19,20]

The elementary excitations of the four rapidity bands consist of particle and hole excitations of the four rapidity bands. Their energy and momentum are given by $|\epsilon_l(\lambda)|$ and $2\pi\int_{B_l}^{|\lambda|} d\lambda' \rho_l(\lambda')$, where $\lambda$ parametrizes the excitation. Close to the Fermi points the dispersion is linear in the momentum and defines a group velocity $v_l=(\partial\epsilon_l/\partial\lambda)|_{\lambda=B_l}/[2\pi\rho_l(B_l)]$. Of course, a group velocity can only be defined for rapidity bands that have a Fermi surface.

The low energy excitations of the system are given by the mesoscopic corrections to the ground state energy in terms of quantum numbers, the group velocities and the matrix of generalized dressed charges[21]

$$E = L\epsilon_\infty + \sum_l \frac{\pi v_l}{2L}\left[\sum_q (\hat{z}^{-1})_{lq}\Delta N_q\right]^2 + \sum_l \frac{2\pi v_l}{L}\left\{\left[\sum_q z_{ql}D_q\right]^2 + n_l^+ + n_l^- - \frac{1}{12}\right\}, \qquad (8)$$

where $\epsilon_\infty$ is the ground state energy density in the thermodynamic limit and the sum over $l,q$ is only over bands with Fermi points. Here $\Delta N_q$ is the departure of the number of particles in the band $q$ from the equilibrium value. Each band has two Fermi points corresponding to forward and backward moving states. $D_q$ is the backward scattering quantum number, i.e., $2D_q$ represents the difference of forward to backward moving states in each band. The $D_q$ are sensitive to the parity in each set of rapidities. Finally, $n_q^\pm$ define the low-lying particle-hole excitations about each of the Fermi points. Here $\Delta N_q$, $n_q^\pm$, and $2D_q$ always take integer values; hence $D_q$ can either be an integer or half integer depending on the initial conditions.

In terms of the quantum numbers defined above, the total momentum of the system is given by[20,21]

$$P = \frac{2\pi}{L}\sum_l [N_l D_l + n_l^+ - n_l^-]. \qquad (9)$$

From Eqs. (8) and (9) we obtain the conformal dimensions of primary fields characterized by the above quantum numbers[20]

$$\Delta_l^\pm = n_l^\pm + \left[\frac{1}{2}\sum_q (\hat{z}^{-1})_{lq}\Delta N_q \pm \sum_q z_{ql}D_q\right]^2. \qquad (10)$$

For a given set of quantum numbers, the asymptotic behavior of a correlation function for long times and large distances is proportional to

$$\prod_{l,p=\pm} e^{-2ip_{Fl}D_l}\left\{\frac{\pi T/L}{\sinh[\pi T(x - ipv_l t)/v_l]}\right\}^{2\Delta_l^p}, \qquad (11)$$

where $p_{Fl}$ is the Fermi momentum associated with the rapidity band $l$.

## III. DRAG CURRENT FOR SPIN-SINGLET PAIRING

In this section we discuss the case $c>0$ which favors the formation of spin-singlet orbital-triplet Cooper-like bound states. We first consider the simplest limit, i.e., $H=\Delta=0$, and then the situations $H=0$ with $\Delta\neq 0$ and $H\neq 0$ with $\Delta=0$.

If $H=0$ there are equal number of up-spin and down-spin electrons, i.e., $M^z=0$. For $\Delta=0$ the bands of electrons are equally populated, which corresponds to $\mathcal{L}=0$. This situation is sometimes referred to as the degenerate band limit.[16] It is easy to verify that the density of real band rapidities, $\rho_3(\lambda)$, vanishes identically, i.e., $\epsilon_3(\lambda)\equiv 0$. Similarly, the rapidity band of orbital two-strings, $\rho_4(\lambda)$, is completely filled and can be eliminated from the integral equations via Fourier transformation. The band of unpaired electrons, $\rho_1(\lambda)$, is empty. This band can only be populated by depairing Cooper pairs, i.e., an extra energy to overcome the binding energy has to be provided. Hence, excitations into this band are gapped, and the gap can only gradually be closed with an external magnetic field. For the remaining rapidity band, $\rho_2(\lambda)$ and $\epsilon_2(\lambda)$, representing the spin-singlet orbital-triplet bound states, we obtain

$$\epsilon_2(\lambda) = 2\lambda^2 - (c^2/2) - 2\mu, \quad \rho_2(\lambda) = 1/\pi. \qquad (12)$$

Hence, the pairs are *effectively free*, i.e., they are independent of other states and have a parabolic dispersion corresponding to a mass of 2. They are free (hard-core) bosons with a *symmetric* wavefunction. The chemical potential is related to the integration limit $B_2$ via $\epsilon_2(\pm B_2)=0$, i.e., $\mu=(B_2)^2-(c/2)^2$, and the number of electrons is $N_e=4B_2L/\pi$. More properties, e.g., the excitation spectrum, group velocities and critical fields, can be found in Sec. 3 of Ref. 19.

In order to obtain the critical exponent of the drag current we have to evaluate the matrix of dressed generalized charges. Since only two rapidity bands have a Fermi surface, we need to consider only $l=2$ and 4. The four functions $\xi_{lq}$ are

$$\xi_{22}(\lambda) = 1, \quad \xi_{24}(\lambda) = \int_{-B_2}^{B_2} d\lambda' G_0(\lambda - \lambda'),$$

$$\xi_{42}(\lambda) = 1/2, \quad G_0(\lambda) = [|c| 2 \cosh(\pi\lambda/|c|)]^{-1}, \tag{13}$$

so that $z_{22}=1$, $z_{24}=0$, $z_{42}=1/2$, and $z_{44}$ is given by $\psi(0)$, where $\psi(\lambda)$ satisfies

$$\psi(\lambda) + \int_0^\infty d\lambda' [2a_2(\lambda - \lambda') + a_4(\lambda - \lambda')]\psi(\lambda') = 1. \tag{14}$$

Equation (14) is of the Wiener–Hopf type and can be solved analytically, yielding $z_{44}=1/2$. The Fermi momenta of the two rapidity bands involved are $p_{F2}=(\pi/2)n$ and $p_{F4}=(\pi/4)n$, where $n=N_e/L$.

Since for $H=\Delta=0$ all electrons are bound in Cooper pairs, the current operator transfers a pair of electrons from one Fermi point to the opposite one. A drag current only exists if the paired electrons belong to different wires. Defining $O_\pm^{(s)\dagger}(x)=2^{-1/2}[\psi_{1\uparrow\pm}^\dagger(x)\psi_{2\downarrow\pm}^\dagger(x)-\psi_{1\downarrow\pm}^\dagger(x)\psi_{2\uparrow\pm}^\dagger(x)]$, where $\pm$ refers to the Fermi point, the current is given by $O_+^{(s)\dagger}(x)O_-^{(s)}(x)$. This operator neither changes the number of rapidities nor does it create particle-hole excitations at the Fermi points in either rapidity band, and hence $\Delta N_2=\Delta N_4=n_2^\pm=n_4^\pm=0$. The transfer of particles from one Fermi point to the other requires $D_2=-D_4=\pm 1$, and consequently the conformal dimensions are $\Delta_l^\pm=(z_{2l}-z_{4l})^2$, i.e.,

$$\Delta_2^\pm = [1 - 1/2]^2 = 1/4, \quad \Delta_4^\pm = 1/4. \tag{15}$$

The current correlation function, Eq. (11), has four factors and after integrating the function with respect to $x$ for equal times we obtain that the drag current is proportional to $T$. This is the same result as for a Fermi liquid, although in this case the current is carried by *free* hard core bosons (spin-paired electrons).

We now consider the more general situation of $\Delta\neq 0$ in zero magnetic field. If $H=0$ all electrons are bound in spin-singlet pairs and due to the binding energy it requires a finite energy to depair the bound states. Hence, since all spins are compensated, the rapidity bands $\epsilon_1$ and $\epsilon_3$ are still empty, and only the $\epsilon_2$ and $\epsilon_4$ bands need to be considered. The main difference with the previous case is that the $\epsilon_4$ band is not completely filled for $\Delta>0$, because the wire with less electron density does not have sufficient electrons to pair all the electrons in the other wire. Hence, some of the spin-singlet bound states are formed with both electrons belonging to the majority wire. With increasing $\Delta$ the $\epsilon_4$ band is gradually depleted and there is a critical $\Delta_c^{(s)}$ so that for $\Delta \geqslant \Delta_c^{(s)}$ the band is empty

$$\Delta_c^{(s)} = -\frac{1}{4}\int_{-B_2}^{B_2} d\lambda [a_1(\lambda) + a_3(\lambda)]\epsilon_2(\lambda). \tag{16}$$

This corresponds to the band splitting at which only one of the wires has electrons, while the other one is completely depleted. Since only one wire is populated, a drag current cannot exist for $\Delta \geqslant \Delta_c^{(s)}$.

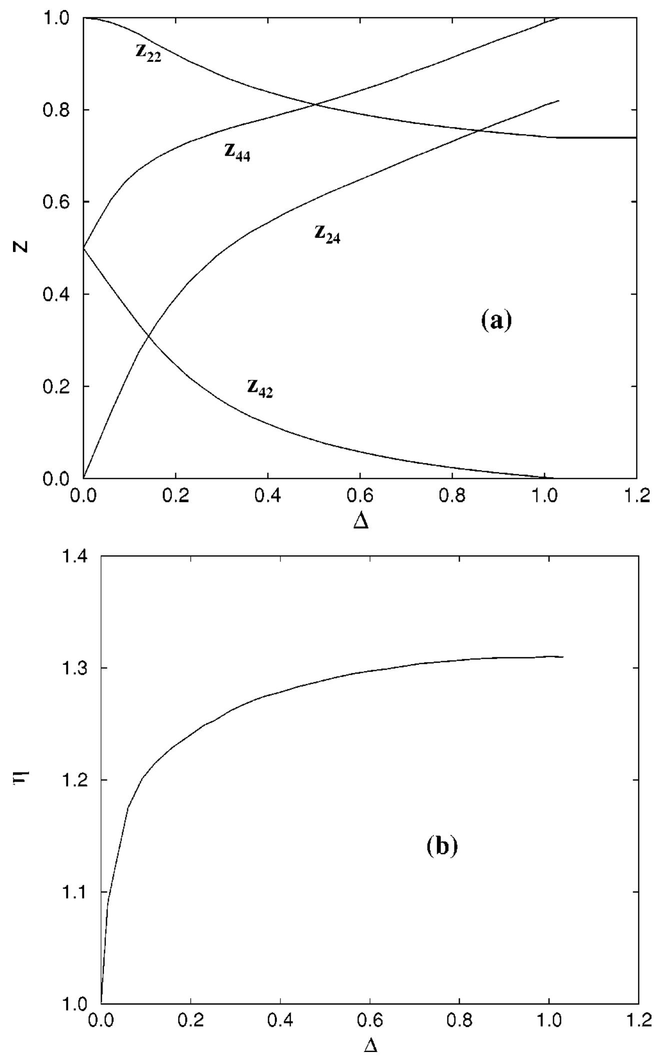


FIG. 1. (a) Components of the matrix of dressed generalized charges, $z_{lq}$, and (b) the critical exponent of the temperature dependence of the drag current as a function of the potential difference $\Delta$ between the two wires for $c=1$, $N_e/L=1.273$ and zero magnetic field.

Hence, a drag current only exists for $0\leqslant\Delta<\Delta_c^{(s)}$. Since all electrons are bound in pairs the driving and drag currents are along the same direction. The quantum numbers are still $D_2=-D_4=\pm 1$ and all others are zero. The dressed generalized charges are determined by two pairs of coupled integral equations, which have to be solved numerically. The critical exponent of the temperature dependence of the drag current is given by

$$\eta = 4(z_{22} - z_{42})^2 + 4(z_{24} - z_{44})^2 - 1, \tag{17}$$

where the $-1$ arises from the space integration of the equal time correlation function. The four components of the matrix of dressed charges are displayed in Fig. 1(a) as a function of $\Delta$ for $N_e/L=1.273$ and $c=1$. The limit $\Delta=0$ corresponds to the case discussed earlier in this section. For these parameters we have $\Delta_c^{(s)}=1.031$. Note that $z_{22}$ and $z_{42}$ (with $z_{42}$ being 0) are defined also if $\Delta>\Delta_c^{(s)}$.

The exponent $\eta$ of the current correlation function is shown in Fig. 1(b). It monotonously increases from 1.00 for $\Delta=0$ to 1.31 at $\Delta_c^{(s)}$. As $\Delta_c^{(s)}$ is approached, the exponent is essentially constant, because the minority band is almost empty and can no longer produce changes in the critical behavior. In the other limit, as $\Delta\to 0$, $\eta$ is singular and approaches the value 1 as $O[1/\ln(\Delta)]$. This singularity can be obtained analytically by reducing the Fredholm equation to a Wiener–Hopf one. The exponent is nonuniversal in the sense that it depends on the value of $c$ and the band filling, $N_e/L$.

The situation $\Delta=0$ and $H\neq 0$ is very different from the previous case, $\Delta\neq 0$ and $H=0$. It requires a finite magnetic field, larger than a critical $H_c^{(s)}$, to gradually depair the spin-singlet Cooper bound states. In other words, the Zeeman splitting first has to overcome the binding energy. The spin gap is then gradually reduced by the magnetic field and the system at $T=0$ does not respond to an external magnetic field smaller than $H_c^{(s)}$. This property is reminiscent of the Meissner effect (note that diamagnetism is not defined in one dimension), although there is no long-range order of pairs.

Hence, we have to distinguish two situations. If $H<H_c^{(s)}$ the Coulomb-drag problem is identical to the zero-field case. For $H>H_c^{(s)}$, on the other hand, the $\epsilon_1$ rapidity band (corresponding to unpaired charges) is gradually filled. Consequently, the $\epsilon_3$ rapidity band has nonzero spectral weight and is completely filled ($\Delta=0$). Hence, all four rapidity bands contribute to the critical behavior. The drag current now can have two components: One arising from the Cooper pairs, for which the drive and drag currents are parallel, and a second one due to the unpaired electrons. For the latter case the operators for the drive and drag currents are $\psi^\dagger_{1\uparrow+}(x)\psi_{1\uparrow-}(x)$ and $\psi^\dagger_{2\uparrow-}(x)\psi_{2\uparrow+}(x)$, respectively. Here one electron is pushed forward in one wire and another electron is dragged backwards in the other wire. The drag current is then opposite to the drive current. Note that this process can only occur if the total momentum is conserved, i.e., the two wires must have equal carrier density. This is only possible if $\Delta=0$. The nonzero quantum numbers for the drag current response are $D_1=-D_3=1$, and consequently the critical exponent for the $T$ dependence of the drag current is $\eta'=4\Sigma_{l=1}^4(z_{1l}-z_{3l})^2-1$. This current component exists for all $H>H_c^{(s)}$.

There is a third possibility for a drag current, namely if the pairs drag the unpaired electrons. Now the drag is opposite (backflow) to the driving current. The momentum conservation for this process requires that the momentum transfer for the unpaired electrons has the same magnitude but is opposite to that of the paired electrons. This condition on the Fermi momenta can only be satisfied for special values on the magnetic field.

If $\Delta\neq 0$ and $H>H_c^{(s)}$ the problem is considerably more complex because of the large number of different phases that are possible (see Fig. 2 of Ref. 18). On the one hand, it is qualitatively similar to the $\Delta=0$ case, only with the increased level of complication because the $\epsilon_3$ and $\epsilon_4$ rapidity bands are not completely filled, i.e., $B_3$ and $B_4$ are finite. On the other hand, all four rapidity bands contribute to the critical behavior and several processes involving more than two carriers are possible. The processes require momentum conservation (hence they are only possible for special choices of the external parameters) and will in general lead to a backflow drag current (see also Ref. 14).

## IV. DRAG CURRENT FOR SPIN-TRIPLET PAIRING

The same model for $c<0$ has quite different properties. In this case the formation of spin triplets and orbital singlets is favored. The Bethe *Ansatz* solution is the same as for $c>0$, if the spin and orbital indices are interchanged. Now the $\epsilon_2$ band corresponds to spin-triplet orbital-singlet pairs, band $\epsilon_3$ to real spin rapidities and rapidity band $\epsilon_4$ to spin-bound states ($\lambda$ strings of length 2).[17] Also the roles of $\Delta$ and the magnetic field are interchanged. Equations (3), (4), and (6), $N_e/L$, and the integral equations for the dressed generalized charges remain unchanged. The magnetization and the electron population difference between the two wires for $c<0$ are

$$\frac{M^z}{L}=\frac{1}{2}\int_{-B_1}^{B_1}d\lambda\rho_1(\lambda)+\int_{-B_2}^{B_2}d\lambda\rho_2(\lambda)-\int_{-B_3}^{B_3}d\lambda\rho_3(\lambda)-2\int_{-B_4}^{B_4}d\lambda\rho_4(\lambda),$$

$$\frac{\mathcal{L}}{L}=\int_{-B_1}^{B_1}d\lambda\rho_1(\lambda) \tag{18}$$

(the roles of $M^z$ and $\mathcal{L}$ are switched) and in Eq. (7) $\Delta$ and $H/2$ have to be interchanged.

The results of the previous section can now be taken over. For $H=\Delta=0$, the $\epsilon_1$ and $\epsilon_3$ bands are empty, while $\epsilon_4$ is completely filled, and Eq. (12) remains valid. Hence, unpaired electrons are gapped and the spin-triplet orbital-singlet pairs behave as effectively free hard-core bosons. The Fermi momenta of the $\epsilon_2$ and $\epsilon_4$ bands and the matrix of dressed generalized charges remains unchanged.

The drag current operator is $O_+^{(t)\dagger}(x)O_-^{(t)}(x)$, where $O_\pm^{(t)\dagger}(x)=2^{-1/2}[\psi^\dagger_{1\uparrow\pm}(x)\psi^\dagger_{2\downarrow\pm}(x)+\psi^\dagger_{1\downarrow\pm}(x)\psi^\dagger_{2\uparrow\pm}(x)]$. Here a spin-triplet orbital-singlet Cooper pair is transferred from one Fermi point to the other. All quantum numbers, except $D_2=-D_4=1$, are equal to zero for this process. Hence, the critical exponent for the temperature dependence of the current is again $\eta=1$.

A magnetic field gradually depopulates the band of spin two strings. This means that the number of spin-triplet bound pairs with both electrons in up-spin states increases at the expense of the other two triplet components. Note that if $\Delta=0$ all electrons are bound in spin-triplet pairs, independently of the magnetic field. This drastically differs from the situation of spin-singlet pairs ($c>0$), in which the external magnetic field breaks up the singlet pairs only if the field exceeds the critical field $H_c^{(t)}$, resembling the Meisner effect.

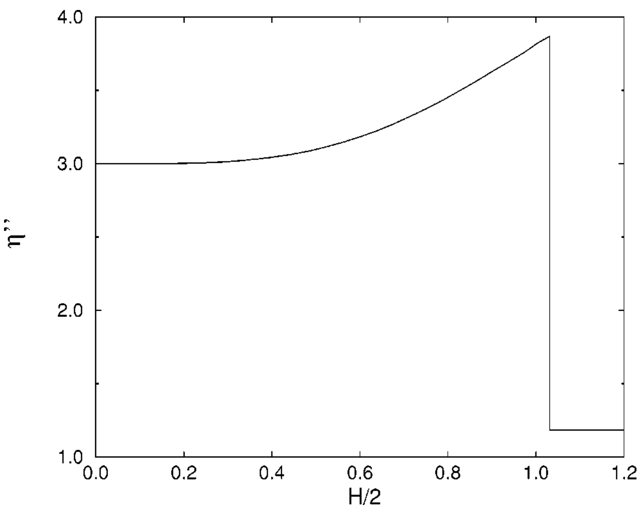


FIG. 2. Critical exponent of the temperature dependence of the drag current as a function of magnetic field for $c=-1$, $N_e/L=1.273$ and $\Delta=0$.

There are two components to the drag current, both due to the Cooper pairs and hence the driving and drag currents are parallel: (i) The one arising from the operator $O_+^{(t)\dagger}(x)O_-^{(t)}(x)$, already discussed above, with nontrivial quantum numbers $D_2=-D_4=1$, and (ii) one only involving the up-spin triplet pairs for which the operator is $\psi^\dagger_{1\uparrow+}(x)\psi^\dagger_{2\uparrow+}(x)\psi_{2\uparrow-}(x)\psi_{1\uparrow-}(x)$. For this case the only nontrivial quantum number is $D_2=1$.

The case (i) can be taken over from the previous section with only exchanging $\Delta$ and $H/2$. The $\epsilon_4$ band is gradually depleted, being full for $H=0$ and empty for $H>H_c^{(t)}$, where here $H_c^{(t)}=2\Delta_c^{(s)}$ with $\Delta_c^{(s)}$ defined in the previous section. The dressed generalized charges and the critical exponent for the $T$ dependence of the drag current are the ones shown in Figs. 1(a) and 1(b).

The mechanism corresponding to (ii) for $c>0$ only involves one quantum wire and hence a drag current cannot take place for spin-singlet Cooper pairs. The critical exponent of the drag current (ii) for $c<0$ is then $\eta''=4z_{22}^2+4z_{24}^2-1$, where $z_{22}$ and $z_{24}$ can be taken from Fig. 1(a). $\eta''$ is displayed in Fig. 2. Note that $z_{24}$ is not defined for $H>H_c^{(t)}$, so that $\eta''=4z_{22}^2-1$, and the exponent has a discontinuity at $H_c^{(t)}$.

In order to populate the $\epsilon_1$ and $\epsilon_3$ rapidity bands (these bands refer to unpaired electrons) the bias potential $\Delta$ has to exceed a minimal value given by $\Delta_c^{(t)}=H_c^{(s)}/2$, where $H_c^{(s)}$ was defined in the previous section. This is the energy required to overcome the binding energy and break up a Cooper pair. The system does not respond unless $\Delta$ is larger than $\Delta_c^{(t)}$. For $H=0$ the total magnetization remains equal to zero. All four rapidity bands contribute to this case and the matrix of dressed generalized charges involves 16 components.

For $H=0$ and $\Delta>\Delta_c^{(t)}$ the drag current has only one component arising from the Cooper pairs with the current operator $O_+^{(t)\dagger}(x)O_-^{(t)}(x)$, for which the drive and drag currents are parallel and the exponent is $\eta=4\Sigma_{l=1}^4(z_{2l}-z_{4l})^2-1$. In contrast to the $c>0$ case there is no second drag component, because the mechanism discussed in the previous section now involves only one quantum wire. However, under very special conditions (matching of Fermi momenta between the rapidity bands) the paired electrons may induce a backward drag current in the unpaired electron fluid and vice versa.

Finally, if $H>0$ and $\Delta>\Delta_c^{(t)}$ there is in addition a second component to the drag current due to Cooper pairs with only up-spin electrons. This mechanism is already discussed above and the driving and drag currents are parallel to each other. Due to the complicated phase diagram (see Fig. 2 of Ref. 17) there are also other possibilities, involving several electrons, to generate backflow drag currents through momentum conservation.

## V. CONCLUSIONS

We considered two nearby parallel quantum wires with carriers interacting via a contact potential of the spin exchange type. For $c>0$ the interaction leads to the formation of spin-singlet Cooper pairs, while for $c<0$ the pairing is of the spin-triplet type. The model has three additional parameters, namely, the electron density in each wire and the magnetization, which are controlled by the chemical potential (total number of carriers), $\Delta$ the potential difference between the wires (relative electron density) and the magnetic field. The model is integrable by construction and has been solved previously via Bethe's *Ansatz*.[15–17,19] The model is also very different from those considered in Ref. 14, since it simultaneously involves attractive and repulsive interactions in the spin and orbital sectors. The sign of the interaction is always opposite in the two sectors.

In conjunction with conformal field theory we used the Bethe *Ansatz* solution to explore possible drag currents and obtain the critical exponent of the temperature dependence of that current. The Cooper-like bound states lead to a drag current parallel to the driving current. On the other hand, unpaired electrons may produce a drag current opposite to the driving current.

The ground state of the model is described in terms of four rapidity bands. In particular, the highly symmetric situation corresponding to spin degeneracy and equal carrier density in the wires, i.e., for $\Delta=H=0$, the Cooper-like pairs act like free hard-core bosons. These the bosons are preformed and exist at any finite $T$.

For $c>0$ it requires a finite magnetic field, larger than a critical $H_c^{(s)}$, to gradually depair the spin-singlet Cooper bound states. For $H<H_c^{(s)}$ only two rapidity bands play a role, both associated with spin-singlet bound states. If $H>H_c^{(s)}$ unpaired electrons also exist and all four rapidity bands are populated. A drag current opposite to the drive current can then be induced. The process of pushing one electron forward in one wire and dragging another electron backwards in the other wire requires conservation of the total momentum. This is only possible if the two wires have equal carrier density, i.e., if $\Delta=0$. Note that linewidths of excitations in a Luttinger liquid are proportional to $T$ and smear the $\delta$ function for the momentum conservation. This allows for a small mismatch (of order $T$) between the Fermi momenta of the two wires and still have drag current.

For $c<0$, i.e., for spin-triplet pairing, the roles of the magnetic field and $\Delta$ are interchanged with respect to the $c>0$ case. Hence, it requires a minimal bias potential between the wires, $\Delta>\Delta_c^{(t)}$, to depair electrons. The unpaired electrons, however, are all placed in one wire, so that a drag current cannot occur. In a magnetic field, on the other hand, more than one component to the drag current will arise. Both correspond to the drag of paired electrons with drag and drive currents being parallel.

## ACKNOWLEDGMENTS

The support by the U.S. Department of Energy under Grant No. DE-FG02-98ER45707 and the National Science Foundation under Grant No. DMR01-05431 is acknowledged.

[1] M. B. Pogrebinskii, Fiz. Tekh. Poluprovodn. (S.-Peterburg) **11**, 637 (1977) [Sov. Phys. Semicond. **11**, 372 (1977)].
[2] F. D. M. Haldane, Phys. Rev. Lett. **45**, 1358 (1980); Phys. Lett. A **81**, 153 (1981); J. Phys. C **14**, 2585 (1981).
[3] A. G. Rojo, J. Phys.: Condens. Matter **11**, R31 (1999).
[4] P. Debray, V. N. Zverev, V. Guverich, R. Klesse, and R. S. Newrock, Semicond. Sci. Technol. **17**, R21 (2002).
[5] V. L. Gurevich, V. B. Pevzner, and E. W. Fenton, J. Phys.: Condens. Matter **10**, 2551 (1998).
[6] K. Flensberg, Phys. Rev. Lett. **81**, 184 (1998).
[7] Y. V. Nazarov and D. V. Averin, Phys. Rev. Lett. **81**, 653 (1998).
[8] O. Raichev and P. Vasilopoulos, Phys. Rev. B **61**, 7511 (2000).
[9] R. Klesse and A. Stern, Phys. Rev. B **62**, 16 912 (2000).
[10] J. S. Moon, M. A. Blount, J. Simmons, J. R. Wendt, S. K. Lyo, and J. L. Reno, Phys. Rev. B **60**, 11 530 (1999).
[11] P. Debray, P. Vasilopoulos, O. Raichev, R. Perrin, M. Rahman, and W. C. Mitchel, Physica E (Amsterdam) **6**, 694 (2000).
[12] P. Debray, V. Zverev, O. Raichev, R. Klesse, P. Vasilopoulos, and R. S. Newrock, J. Phys.: Condens. Matter **13**, 3389 (2001).
[13] M. Yamamoto, M. Stopa, Y. Tokura, Y. Hirayama, and S. Tarucha, in *Proceedings of the 25th International Conference on the Physics of Semiconductors*, Springer Procceedings in Physics, edited by N. Miura and T. Ando (Springer, Heidelberg, 2001), Vol. 87.
[14] P. Schlottmann, Phys. Rev. B **69**, 035110 (2004).
[15] A. M. Tsvelik, Zh. Eksp. Teor. Fiz. **93**, 1329 (1987) [Sov. Phys. JETP **66**, 754 (1987)].
[16] P. Schlottmann, Phys. Rev. Lett. **68**, 1916 (1992); Phys. Rev. B **49**, 6132 (1994).
[17] A. A. Zvyagin and P. Schlottmann, Phys. Rev. B **60**, 6292 (1999).
[18] A. A. Zvyagin and P. Schlottmann, Nucl. Phys. B **586**, 686 (2000).
[19] P. Schlottmann, Int. J. Mod. Phys. B **11**, 355 (1997).
[20] V. E. Korepin, N. M. Bogoliubov, and A. G. Izergin, *Quantum Inverse Scattering Method and Correlation Functions* (Cambridge University Press, Cambridge, 1993).
[21] F. Woynarovich, J. Phys. A **22**, 4243 (1989).