Counting statistics of tunneling current

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The form of electron counting statistics of the tunneling current noise in a generic many-body interacting electron system is obtained and universal relations between its different moments are derived. A generalized fluctuation-dissipation theorem providing a relation between current and noise at arbitrary bias-to-temperature ratio eV/k_BT is established in the tunneling Hamiltonian approximation. The third correlator of current fluctuations S_3 (the skewness of the charge counting distribution) has a universal Schottky-type relation with the current and quasiparticle charge that holds in a wide bias voltage range, both at large and small eV/k_BT. The insensitivity of S_3 to the Nyquist-Schottky crossover represents an advantage compared to the Schottky formula for the noise power. We discuss the possibility of using the correlator S_3 for detecting quasiparticle charge at high temperatures.

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I. INTRODUCTION

Recent developments in the problem of quantum electron transport were marked by interest in the phenomenon of electric noise. The many-body theory of electron shot noise, developed by Lesovik[1] (and independently by Khlus[2]) for a point contact, was extended to multiterminal systems by Büttiker[3] and to mesoscopic systems by Beenakker and Büttiker.[4] Kane and Fisher proposed using shot noise for detecting fractional quasiparticles in a Quantum Hall Luttinger liquid.[5]

Experimental studies of the shot noise, after first measurements in a point contact by Reznikov *et al.*[6] and Kumar *et al.*,[7] focused on the quantum Hall regime. The fractional charges $e/3$ and $e/5$ were observed[8–10] at incompressible Landau level filling (see also recent work on noise at intermediate filling[11]). The shot noise in a mesoscopic conductor was observed by Steinbach *et al.*[12] and Schoelkopf *et al.*,[13] who also studied noise in an ac driven phase-coherent mesoscopic conductor.[14]

In this article we discuss a generalization of the shot noise, namely the counting statistics of fluctuating electric current. It can be defined through the probability distribution $P(q)$ of charge transmitted in a fixed time interval.[15,16] We consider ways of obtaining the distribution $P(q)$ using a fast charge integrator scheme. From the distribution $P(q)$ all moments of charge fluctuations can be calculated and, conversely, the knowledge of all moments is in principle sufficient for reconstruction of the full distribution. However, due to the central limit theorem, the high moments can be more difficult to extract from the entire distribution than the lower ones. Moreover, they are more susceptible to the distortions due to the noise generated by the electromagnetic environment.[29] Thus the high moments are difficult to access experimentally, and here we shall focus primarily on the third moment.

The counting statistics have been analyzed theoretically for a Fermi gas, in the single- and multichannel geometry,[15,17] in the mesoscopic regime,[18,19] and in the ac driven phase-coherent regime.[17,20] Charge doubling due to Andreev scattering in NS junctions was considered by Muzykantskii and Khmelnitskii,[21] and in mesoscopic NS systems by Belzig and Nazarov.[22] However, since the most interesting applications of the shot noise lie in the domain of interacting electron systems, an appropriate extension of the theory is necessary.

The problem of back influence of a charge detector on current fluctuations was considered by Lesovik and Loosen,[23] and recently by Nazarov and Kindermann.[24] Beenakker proposed an alternative way of obtaining charge statistics using photon counting.[25] Application to pumping in quantum dots was also discussed.[26]

The main result of this article is a relation between different moments of counting statistics valid *in the tunneling regime* for a generic interacting many-body system. This relation arises due to the detailed balance principle that links the rates of tunneling in opposite directions. The results are not sensitive to the properties of the tunneling charge, which can be a single electron charge or an effective charge, such as the fractional charge of a quasiparticle in a quantum Hall state $e^*=e/q$[8–10] or a Cooper pair charge $e^*=2e$.[27,28] The situation of interest is that of weak tunneling current, when subsequent tunneling events are well separated in time, so that there are no correlations between them. In this case, the tunneling current statistics takes a universal form of a bidirectional Poisson process, a mixture of two uncorrelated Poisson processes describing tunneling in the opposite directions.

Physically, the regime of interest corresponds to weak transmission for a single channel in the scattering picture, or to weak backscattering in a point contact between quantum Hall edge states. In the latter case, the backscattering current is due to tunneling between the edge states that involves quasiparticle transport through an incompressible region.

We obtain a general formula for the moments of the counting statistics of tunneling current fluctuations that holds at an arbitrary relation between temperature and bias voltage. It can be summarized in a compact form using a generating function $\chi(\lambda)=\Sigma_q P(q)e^{i\lambda q/e^*}$, with e^* the quasiparticle charge and $P(q)$ the distribution of charge transmitted during a fixed time interval. We find

$$\chi(\lambda)=\exp[(e^{i\lambda}-1)N_{1\to 2}(\tau)+(e^{-i\lambda}-1)N_{2\to 1}(\tau)], \quad (1)$$

where $N_{a\to b}(\tau)=m_{ab}\tau$ is the mean charge number transmitted from the contact a to the contact b in a time τ.

The result (1) yields a number of relations between different statistics of the probability distribution $P(q)$. The cumulants $\langle\langle \delta q^k\rangle\rangle$ (irreducible correlators) of the distribution $P(q)$ are expressed in terms of $\chi(\lambda)$ as

$$\ln \chi(\lambda)=\sum_{k=1}^{\infty}\frac{(i\lambda)^k}{k!}\frac{\langle\langle \delta q^k\rangle\rangle}{(e^*)^k}. \quad (2)$$

Using Eq. (1) one obtains

$$\langle\langle \delta q^k\rangle\rangle=(e^*)^k\begin{cases}(m_{12}-m_{21})\tau, & k \text{ odd}\\ (m_{12}+m_{21})\tau, & k \text{ even.}\end{cases} \quad (3)$$

Setting $k=1,2$ we express $m_{12}\pm m_{21}$ through the time-averaged current and the low frequency noise power[30]

$$m_{12}-m_{21}=I/e^*, \quad m_{12}+m_{21}=S_2/(e^*)^2. \quad (4)$$

It can be seen from the principle of detailed balance that the rates m_{12} and m_{21} are related by a factor $\exp(\Delta\mu/k_B T)$, where $\Delta\mu$ is the quasiparticle chemical potential difference between the reservoirs. Combined with Eq. (4) this yields a noise-current relation

$$S_2=e^*I\coth\left(\frac{\Delta\mu}{2k_B T}\right), \quad \Delta\mu=e^*V, \quad (5)$$

where V is the voltage bias. Although quite reminiscent of the fluctuation-dissipation theorem that holds in equilibrium, the relation (5) holds for tunneling in a generic nonequilibrium situation. Specifically, it can be applied to systems with an arbitrary nonlinearity of the I-V characteristic, which can arise due to the energy dependence of the quasiparticle density of states in reservoirs, due to tunneling gap, pseudogap, zero-bias anomaly, etc. Also, it holds for an arbitrary temperature/bias ratio.

A relation of the form (5) with $\Delta\mu=eV$ has been derived by Rogovin and Scalapino[32] for electron tunneling from normal metal into superconductor and, more recently, by Sukhorukov and Loss for cotunneling in a quantum dot.[33] The relation (5) with $\Delta\mu=e^*V$ has also been proposed on heuristic grounds in the work on fractional charge noise,[8,10] where it was used for fitting the results of noise measurement.

Another quantity of interest for us will be the cumulant $\langle\langle \delta q^3\rangle\rangle$ which is equal to the third correlator[31]

$$\langle\langle \delta q^3\rangle\rangle\equiv\overline{\delta q^3}=\overline{(q-\overline{q})^3} \quad (6)$$

(see Fig. 1). For this correlator Eq. (3) gives $\langle\langle \delta q^3\rangle\rangle=S_3\tau$ with the coefficient S_3 ("spectral power") related to the current I as

$$S_3\equiv\langle\langle \delta q^3\rangle\rangle/\tau=(e^*)^2 I. \quad (7)$$

We note that the relation (7) holds for the distribution (1) at any ratio of the mean number of transmitted charges $m_{12}-m_{21}$ to the variance $m_{12}+m_{21}$, i.e., at any temperature/bias ratio.

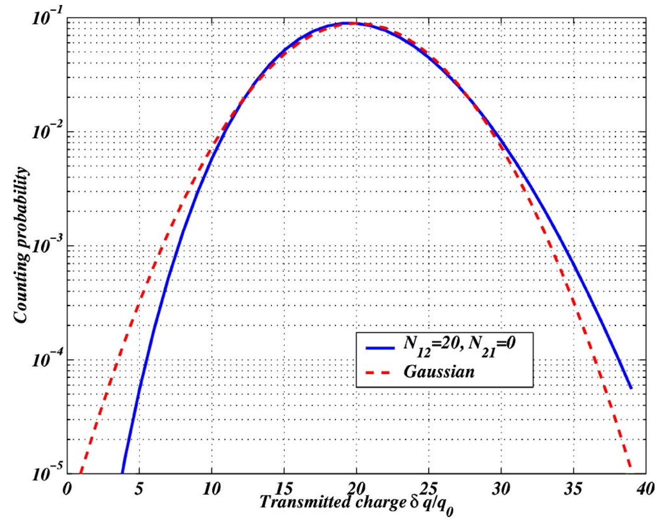

FIG. 1. The third moment (6) determines the shape of the distribution $P(q)$, namely its *skewness*. This is illustrated by a distribution of the form (1) and a Gaussian with the same mean and variance. For $S_3>0$ the peak is somewhat more stretched to the right than to the left.

The meaning of Eq. (7) is similar to that of the Schottky formula for the second correlator $S_2=\langle\langle \delta q^2\rangle\rangle=e^*I$ which is usually used to determine the effective charge e^* from the tunneling current noise. The Schottky formula is valid when charge flow is unidirectional, which means $m_{12}\gg m_{21}$ [see Eq. (3)]. The latter can be true only at sufficiently low temperatures $k_BT\ll eV$. This requirement of a cold sample at a relatively high bias voltage is the origin of a well known difficulty in the noise measurement. In contrast, the relation (7) is not constrained by any requirement on the sample temperature.

On a general basis we expect the relation (7) to hold approximately even outside the tunneling regime. Indeed, for the Nyquist noise at equilibrium all odd moments vanish. Combined with the temperature independence of $P(q)$ out of equilibrium, at $eV\gg k_BT$, this implies a dependence on the ratio eV/k_BT which is weaker than in the noise-current relation at the Nyquist-Schottky crossover. This is manifest, for instance, in the temperature independent first moment of $P(q)$ for free fermions (the Landauer formula). Below we will see that the temperature-dependent corrections to the relation (7) arise in the second order expansion in the transmission constant.

This property of the third moment, if confirmed experimentally, may prove to be quite useful for determining the quasiparticle charge. In particular, this applies to the situations when the I-V characteristic is strongly nonlinear. The nonlinearity usually makes it difficult to distinguish the variation of the second moment with current due to shot noise and due to thermal noise modified by nonlinear conductance. We stress that this is a completely general problem pertinent to any interacting system. Namely, in systems such as Luttinger liquids, the I-V nonlinearities arise at $eV\geq k_BT$. However, it is exactly this voltage that has to be applied for measuring the shot noise in the Schottky regime.

II. MICROSCOPIC ANALYSIS

Here we present and discuss a microscopic derivation of the counting distribution (1). The starting point of the analysis will be the tunneling Hamiltonian

$$\hat{\mathcal{H}} = \hat{\mathcal{H}}_1 + \hat{\mathcal{H}}_2 + \hat{V}, \tag{8}$$

where $\hat{\mathcal{H}}_{1,2}$ describe the leads and $\hat{V}=\hat{J}_{12}+\hat{J}_{21}$ is the tunneling operator. The operators $\hat{J}_{ij}$, obeying $\hat{J}_{12}=\hat{J}_{21}^{+}$, describe quasiparticle tunneling between the reservoirs 1 and 2:

$$\hat{J}_{12}(\hat{N}_1+1) = \hat{N}_1\hat{J}_{12}, \quad \hat{J}_{21}(\hat{N}_1-1) = \hat{N}_1\hat{J}_{21} \tag{9}$$

with $\hat{N}_1=\hat{Q}_1/e^*$ the quasiparticle number operator in the reservoir 1 (similar for reservoir 2). The specific form of the quasiparticle tunneling operators $\hat{J}_{12}$, $\hat{J}_{21}$ will be inessential for the most of our discussion.

The counting statistics generating function $\chi(\lambda)$ can be written[16] as a Keldysh partition function

$$\chi(\lambda) = \left\langle T_K \exp\left(-i\int_{C_{0,\tau}} \hat{\mathcal{H}}_\lambda(t)dt\right)\right\rangle, \tag{10}$$

where a counting field $\lambda(t)$ is added to the phase of the tunneling operators $\hat{J}_{12}$, $\hat{J}_{21}$ as

$$\hat{V}_\lambda = e^{(i/2)\lambda(t)}\hat{J}_{12}(t) + e^{-(i/2)\lambda(t)}\hat{J}_{21}(t). \tag{11}$$

Here $\lambda(t)=\pm\lambda$ is antisymmetric on the forward and backward parts of the Keldysh contour $C_{0,\tau}\equiv[0\to\tau\to 0]$. Equations (10) and (11) originate from the analysis of a coupling Hamiltonian for an ideal "passive charge detector" without internal dynamics.[16,24]

In what follows we compute $\chi(\lambda)$ and establish a relation with the Kubo theorem for tunneling current.[34] This theorem relates the tunneling current with an auxiliary susceptibility at finite frequency $\omega=\Delta\mu$ evaluated using the tunneling operators J_{12}, J_{21} for a system in thermodynamic equilibrium. The latter susceptibility, by Kubo theorem, can be represented as an expectation values of the commutator of the tunneling operators taken at different times.

Our first step will be to perform the usual gauge transformation turning the bias voltage into the time-dependent phase factor the tunneling operators as $\hat{J}_{12}\to\hat{J}_{12}e^{-i\Delta\mu t}$, $\hat{J}_{21}\to\hat{J}_{21}e^{i\Delta\mu t}$. Passing to the Keldysh interaction representation, we write

$$\chi(\lambda) = \left\langle T_K \exp\left(-i\int_{C_{0,\tau}} \hat{V}_{\lambda(t)}(t)dt\right)\right\rangle. \tag{12}$$

Diagrammatically, the partition function (12) is a sum of linked cluster diagrams with appropriate combinatorial factors. To the lowest order in the tunneling operators $\hat{J}_{12}$, $\hat{J}_{21}$ we only need to consider linked clusters of the second order. This gives $\chi(\lambda)=e^{W(\lambda)}$, where

$$W(\lambda) = -\frac{1}{2}\int\int_{C_{0,\tau}} \langle T_K\, \hat{V}_{\lambda(t)}(t)\hat{V}_{\lambda(t')}(t')\rangle dtdt'. \tag{13}$$

There are several different contributions to this integral, from t and t' on the forward or backward parts of the contour $C_{0,\tau}$. Evaluating them separately, we obtain

$$\begin{aligned} W(\lambda) = &\int_0^\tau\int_0^\tau \langle \hat{V}_{-\lambda}(t)\hat{V}_{\lambda}(t')\rangle dt'dt \\ &-\int_0^\tau\int_0^t \langle \hat{V}_{\lambda}(t)\hat{V}_{\lambda}(t')\rangle dt'dt \\ &-\int_0^\tau\int_t^\tau \langle \hat{V}_{-\lambda}(t)\hat{V}_{-\lambda}(t')\rangle dt'dt. \end{aligned} \tag{14}$$

We substitute the form (11) into Eq. (14) and average by pairing $\hat{J}_{12}$ with $\hat{J}_{21}$. This brings $W(\lambda)$ to the form

$$W(\lambda) = (e^{i\lambda}-1)N_{1\to 2}(\tau) + (e^{-i\lambda}-1)N_{2\to 1}(\tau) \tag{15}$$

with the rates given by

$$N_{a\to b} = \int_0^\tau\int_0^\tau \langle \hat{J}_{ba}(t)\hat{J}_{ab}(t')\rangle dtdt'. \tag{16}$$

Exponentiating (15) gives the bidirectional Poisson distribution (1).

It is instructive to link the quantities (16), and thereby the result (1), with the standard Kubo-like treatment of tunneling.[34] We consider an expression for the tunneling current operator

$$\hat{\mathcal{I}}(t) = i[\hat{Q}_1,\mathcal{H}] = -ie^*(\hat{J}_{12}(t)-\hat{J}_{21}(t)), \tag{17}$$

where the commutator is evaluated with the help of the relations (9). The Kubo theorem for the tunneling current[34] is derived by considering a linear response of the current (17) to the tunneling coupling $\hat{V}(t)=\hat{J}_{12}(t)+\hat{J}_{21}(t)$ that plays a role of an auxiliary external field. Using the canonical Kubo commutator result, one can represent the mean integrated current $\int_0^\tau \langle\hat{\mathcal{I}}(t)\rangle dt$ scaled by e^* as a time integral of a commutator,

$$\int_0^\tau\int_0^\tau \langle[\hat{J}_{21}(t),\hat{J}_{12}(t')]\rangle dtdt' = N_{1\to 2} - N_{2\to 1}. \tag{18}$$

It is convenient at this point to write the quantities (16) by expressing the expectation values in (16) through the exact eigenstates in the reservoirs 1, 2. Taking into account that the corresponding energy spectrum is continuous, we obtain rates proportional to the measurement time,

$$N_{a\to b} = m_{ab}\tau \tag{19}$$

with

$$m_{12} = 2\pi\sum_{\epsilon_1,\epsilon_2}\delta(\epsilon_2-\epsilon_1-\Delta\mu)|\langle\epsilon_2|\hat{J}_{12}|\epsilon_2\rangle|^2 e^{-\beta\epsilon_1} \tag{20}$$

with $\beta=1/k_BT$, and a similar expression for m_{21} with $1\leftrightarrow 2$ and $\Delta\mu\to-\Delta\mu$. The apparent energy nonconservation in Eq. (20) is due to the time-dependent phase factor $e^{-i\Delta\mu t}$ in the tunneling operator that was introduced above to offset the bias voltage across the contact.

With these definitions of m_{ab}, we immediately arrive at the first relation (4), $I=e^*(m_{12}-m_{21})$. To obtain the second relation (4), $S_2=(e^*)^2(m_{12}+m_{21})$, we consider the variance of the charge transmitted during time τ. In terms of the counting statistics generating function counting $\chi(\lambda)$, the variance is identified with the second moment $d^2 \ln \chi(\lambda)/d\lambda^2$. The latter can be written with the help of the microscopic definition (12) as a time integral of an averaged symmetrized product of two current operators

$$\langle\langle \delta q^2 \rangle\rangle = (e^*)^2 \int_0^\tau \int_0^\tau \langle\{\hat{\mathcal{I}}(t),\hat{\mathcal{I}}(t')\}_+\rangle dt dt' . \qquad (21)$$

The integral in (21) can be rewritten as

$$\int_0^\tau \int_0^\tau \langle\{\hat{J}_{12}(t),\hat{J}_{21}(t')\}_+\rangle dt dt' = N_{1\to 2} + N_{2\to 1} \qquad (22)$$

which immediately leads to (4).

Having the noise and current expressed in terms of the rates $N_{a\to b}$, we can establish a proportionality relation between them by noting that the microscopic formula (20) for m_{12} and an analogous formula for m_{21} imply that

$$N_{1\to 2}/N_{2\to 1} = e^{\beta\Delta\mu}. \qquad (23)$$

The universal ratio of the rates is a manifestation of the detailed balance principle. Taking the ratio of the expressions for the current and noise, we quickly obtain

$$\frac{S_2}{e^* I} = \frac{N_{1\to 2}+N_{2\to 1}}{N_{1\to 2}-N_{2\to 1}} = \coth\left(\frac{\Delta\mu}{2k_B T}\right). \qquad (24)$$

Our derivation is not constrained by linear response assumption, and thus the relation (5) holds for any tunneling I-V characteristic, no matter how nonlinear. The only assumption underpinning the detailed balance relation [Eq. (23)] is that both reservoirs are in local equilibrium. This is a weaker requirement than the condition of a true equilibrium, as it allows for the chemical potentials in both reservoirs, and even for their temperature, to be perturbed due to the presence of the tunneling current.

III. THE EFFECT OF FINITE TRANSMISSION ON S_3

The result (1), and thereby the formula (7) for the third correlator and the noise/current relation (5), are valid only at low transmission. In that the situation is similar to the Kubo formula for the tunneling current, which is valid only in the tunneling Hamiltonian approximation. To illustrate this we recall the expression for counting statistics for a single channel noninteracting Fermi system (point contact) in the presence of a dc voltage V and temperature T,[16]

$$\chi(\lambda) = \exp(-N_T \mathcal{U}_+\mathcal{U}_-), \quad N_T = \frac{\tau k_B T}{2\pi\hbar}, \qquad (25)$$

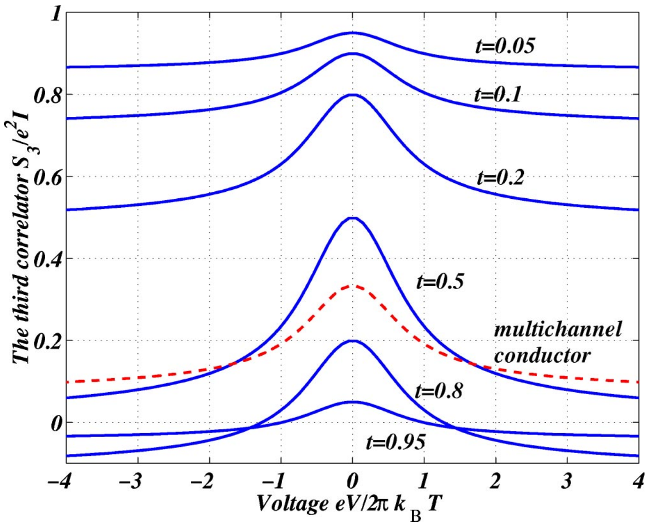

FIG. 2. The third correlator S_3 scaled by $e^2 I$, with I the time-averaged current [see Eqs. (6) and (7)], for the single channel problem (25)–(27), and for a phase-coherent mesoscopic multichannel conductor (dashed line). Note that the relation $S_3=e^2 I$ holds approximately at not too large transmission t.

$$\mathcal{U}_\pm = \mathcal{U}/2 \pm \cosh^{-1}(t\cosh(\mathcal{U}/2+i\lambda) + r\cosh(\mathcal{U}/2)), \qquad (26)$$

where τ is a measurement time, and $\mathcal{U}=eV/k_BT$. This result holds for any values of the transmission and reflection constants t and r (constrained by $t+r=1$). The formulas (25) and (26) were obtained in Ref. 16 by explicitly evaluating the Keldysh partition function in the scattering basis representation.

The third correlator $\langle\langle \delta q^3 \rangle\rangle$ can be obtained from (25) by expanding $\ln \chi(\lambda)$ in Taylor series up to $O(\lambda^3)$:

$$\langle\langle \delta q^3 \rangle\rangle = e^3 t(1-t) N_T\left(6t\frac{\sinh \mathcal{U} - \mathcal{U}}{\cosh \mathcal{U} - 1} + (1-2t)\mathcal{U}\right). \qquad (27)$$

This expression is a function of the bias-to-temperature ratio $\mathcal{U}$, and so in this case the relation (7) for the third correlator does not hold (see Fig. 2). Asymptotically

$$\langle\langle \delta q^3 \rangle\rangle = \begin{cases} e^2(1-t)I\tau, & eV \ll k_B T \\ e^2(1-2t)(1-t)I\tau, & eV \gg k_B T, \end{cases} \qquad (28)$$

where $I=(e^2/2\pi\hbar)Vt$. One can also average (27) over the universal Dorokhov's distribution of transmission in a multichannel mesoscopic metal[18] (see Fig. 2).

Equations (27) and (28) indicate nonuniversality of the relation (7) outside the tunneling regime. They also lead to an interesting qualitative prediction: At $t>0.5$ the ratio $\langle\langle \delta q^3 \rangle\rangle/I$ can become negative. Such a signature could be observed even if it proves difficult to measure $\langle\langle \delta q^3 \rangle\rangle$ quantitatively with sufficient precision. This is important in view of the difficulties in measuring the counting statistics (see below).

In the single channel problem (25), (26) the tunneling regime is realized at low transmission t. To connect with the results (1) and (7) we analyze the expression (26) at $t \ll 1$. To the lowest order in small t we have

$$\mathcal{U}_+ = \mathcal{U}, \quad \mathcal{U}_- = t\frac{e^{\mathcal{U}}(e^{i\lambda}-1)+(e^{-i\lambda}-1)}{e^{\mathcal{U}}-1}. \tag{29}$$

Substituting this in Eq. (25) we recover (1) with

$$N_{2\to 1}(\tau) = \frac{eV\tau}{2\pi\hbar}\frac{t}{e^{\mathcal{U}}-1}, \quad N_{1\to 2}(\tau) = e^{\mathcal{U}}N_{2\to 1}(\tau), \tag{30}$$

the rates of two Poisson processes. Note that the ratio of the two rates (30) is equal to $e^{\beta eV}$, as required by detailed balance.

IV. MEASUREMENT OF S_3

The measurement of the distribution $P(q)$ is a nontrivial task. Current fluctuations must be amplified with a very low noise preamplifier (e.g., the one used in Refs. 8 and 10). The amplified signal can then be digitized and analyzed with computer. This setup in principle allows to reconstruct the full statistics of transmitted charge. In practice, however, the correlators of high order become increasingly difficult to extract.

The main source of errors in the measurement of the kth cumulant $\langle\langle q^k\rangle\rangle$ of the distribution $P(q)$ is statistical. The non-Gaussian character of the amplifier noise does not present a problem, since the mean time-averaged value of the kth cumulant of the amplifier can be subtracted if known with sufficient accuracy. The measured value $\langle\langle q^k\rangle\rangle$ should be compared to its variance $\langle \delta q^k\rangle$ due to both sample and amplifier noise. The variance is expressed through the correlators of the order $2k$. Correlators of *even* order for a generic distribution can be estimated with the help of the central limit theorem, using Gaussian statistics

$$\mathrm{var}\langle \delta q^k\rangle = (\langle \delta q^{2k}\rangle)^{1/2} \simeq ((2k-1)\,!\,!)^{1/2}\langle \delta q^2\rangle^{k/2}. \tag{31}$$

The variance $\langle \delta q^2\rangle$ in Eq. (31) is the mean charge fluctuation produced by both the sample and the amplifier,

$$\langle \delta q^2\rangle = \langle\langle q^2\rangle\rangle + S_2^{(a)}\tau, \tag{32}$$

where S_2^a is the amplifier noise, expressed in A^2/Hz.

The signal-to-noise ratio for a single measurement can then be estimated as a ratio of S_k and $((2k-1)!!)^{1/2}(S_2+S_2^{(a)})^{k/2}\tau^{k/2-1}$. Repeating the measurement many times over a long time interval $\mathcal{T}=\mathcal{N}\tau$ and averaging the results will further reduce statistical fluctuations by a factor $\sqrt{\mathcal{N}}$. Thus we estimate the signal-to-noise ratio as

$$S/N \simeq \frac{S_k\sqrt{\mathcal{T}}}{((2k-1)\,!\,!)^{1/2}(S_2+S_2^{(a)})^{k/2}\tau^{(k/2-1/2)}}. \tag{33}$$

It is evident from Eq. (33) that it pays to reduce the sampling time τ. However, the measurements which are too closely spaced in time become correlated due to finite bandwidth of the input circuit. This makes the effective sampling time restricted by the parasitic capacitance of the sample, C, of both the intrinsic and stray kind, and by the effective output resistance of the sample in parallel with the input resistance of the amplifier, R, so that $\tau_{\mathrm{eff}} \simeq RC$.

To obtain a quantitative estimate of the S/N ratio for high order cumulants, let us consider the case of tunneling contact with a small transmission. We shall assume that the main source of the noise is the the resistor R thermal noise, ignoring both the shot noise produced by the sample and the amplifier noise. The signal-to-noise ratio can be estimated by replacing τ by τ_{eff} and plugging $S_2+S_2^{(a)}=2k_BT/R$ in Eq. (33).

The fluctuations frequency-dependent conversion from current into voltage is accounted for by multiplying the S_3 spectral density by the integral

$$\sum_{\omega_1+\omega_2+\omega_3=0} Z(\omega_1)Z(\omega_2)Z(\omega_3) = R^3/3\tau_{\mathrm{eff}}^2, \tag{34}$$

where $Z(\omega)=R/(1-i\omega\tau_{\mathrm{eff}})$ and $\Sigma\ldots$ denotes a triple frequency integral $(2\pi)^{-2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\ldots\delta(\omega_1+\omega_2+\omega_3)d\omega_1 d\omega_2 d\omega_3$. Simultaneously, the correlator S_2 should be multiplied by

$$\sum_{\omega_1+\omega_2=0} Z(\omega_1)Z(\omega_2) = R^2/2\tau_{\mathrm{eff}}. \tag{35}$$

Evaluating the ratio (33), finally we obtain the following estimate for the third cumulant signal-to-noise ratio:

$$S/N = \frac{e^2 I\sqrt{\mathcal{T}}}{3\sqrt{15}(k_BT/R)^{3/2}RC}. \tag{36}$$

It is clear from Eq. (36) that as the resistance R is made smaller, either by reducing the sample resistance or the amplifier input resistance, the advantage of shorter effective sampling time is overwhelmed by the increase in the thermal current fluctuations produced by the same resistor. The overall S/N ratio scales as $\sqrt{R}$. A wide band setup, such as that used in Ref. 29, is realized when R is reduced to about 50 Ohm, a typical transmission line impedance. To estimate the S/N ratio for the wide band setup, one can simply replace $1/\tau$ by the bandwidth of the measurement. Calculations similar to the above lead in this case to

$$S/N = \frac{3e^2 I f_{\max}\sqrt{\mathcal{T}}}{\sqrt{15}(4k_BT/R)^{3/2}}, \tag{37}$$

where $f_{\max}$ is the high-frequency cutoff of the amplifier.

For example, Eq. (36) predicts the ratio $S/(N\sqrt{\mathcal{T}})$ of about $1\mathrm{s}^{-1/2}$ with the current through the sample $I=1\times 10^{-8}$ A, the load resistance $R=10$ kOhm, the capacitance $C=5$ pF, and the temperature $T=4.2$ K. In comparison, Eq. (37) gives the ratio $S/(N\sqrt{\mathcal{T}})$ of $2\times 10^{-2}\ \mathrm{s}^{-1/2}$ for $R=50$ Ohm and $f_{\max}=1$ GHz for the same current and temperature.

Since the results of this work were made public,[35] the third correlator of current counting statistics have been studied in various systems by a number of authors.[36] Also, an experimental study of *voltage* fluctuations $S_3^{(V)}$ across a current-biased tunneling contact, carried out by Reulet, Senzier, and Prober,[29] indicated that the measured $S_3^{(V)}$ differs strongly from the naively expected $S_3^{(V)}=-R^3S_3$, where R is the total resistance of the sample in parallel with a load resistor. These findings triggered theoretical work[37] that analyzed the effects of environment and helped to understand the results of measurement.[29] An additional contribution to $S_3^{(V)}$ which dominates in the experiment[29] is a result of the correlation between the voltage fluctuations on the sample which stem from the environmental noise and the sample noise itself, both of which affecting the noise correlator S_2 via its voltage-dependence.

In summary, the counting statistics (1) of tunneling current is found to be universal and independent of the character of interactions. For the third correlator we obtain a generalized Schottky formula (7). This formula is valid at both large and small eV/k_BT and can be used to measure quasiparticle charge at temperatures $k_BT \geqslant eV$. The difficulties in measuring the third correlator are discussed and its feasibility is conjectured.

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