

Properties of Abrikosov lattices as photonic crystals

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The possibility of viewing the Abrikosov vortices arranged into two-dimensional triangular lattices in type-II superconductors in external magnetic fields, as photonic crystals for electromagnetic waves, has been investigated theoretically. Due to the Bragg diffraction on a periodic lattice, electromagnetic waves with wavelengths corresponding to lattice spacings cannot propagate in the Abrikosov lattices with different dielectric constants inside and outside the vortices. Conditions for obtaining effective properties of the Abrikosov lattices as photonic crystals by changing of Ginzburg-Landau parameters and applied magnetic fields are clarified.

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I. INTRODUCTION

For many years, the properties of superconductors in the external magnetic and electromagnetic fields at low temperature T below critical temperature T_c have attracted significant attention from both theoretical and experimental viewpoints. Since the microscopic mechanism of low-temperature superconductivity has been understood in terms of the Bardeen-Cooper-Schrieffer (BCS) theory,¹ more recently, the electromagnetic properties of novel high- T_c superconductors, such as copper-oxide high-temperature superconductors (HTSCs) and MgB_2 , have been broadly investigated.^{2,3} In superconductivity, magnetic properties are crucially important for applications because they determine the critical current and critical fields other than electrical properties, such as zero resistivity.

The classification of type-I and type-II superconductors depends on the Ginzburg-Landau (GL) parameter. The GL parameter $\kappa = \lambda / \xi$ is defined by the ratio of the London penetration depth λ relative to the coherence length ξ . The London penetration depths indicate the length scales in which static magnetic fields can penetrate into superconductors, while the coherence lengths indicate the length scales of paired electrons inside the superconductors. In type-I superconductors with $\kappa < 1/\sqrt{2}$, the complete diamagnetic Meissner effect is well known, which means that applied static magnetic fields cannot penetrate into the superconductors at a static magnetic field B below a critical magnetic field B_c . By applying stronger magnetic fields $B > B_c$, superconductivity is destroyed, and thus it disappears. In type-II superconductors with $\kappa > 1/\sqrt{2}$, on the other hand, stronger applied magnetic fields can penetrate into the superconductors in regions called vortices. States inside and outside the vortices are normal conducting ones in the vortex cores and superconducting ones with circular supercurrents around the cores, respectively. The vortices with radii of ξ appear at $B_{c1} < B < B_{c2}$, where B_{c1} and B_{c2} indicate the lower and upper critical magnetic fields, respectively, and then they take a regular arrangement. This regular arrangement of vortices is called an Abrikosov lattice.⁴ The Abrikosov lattices are com-

posed of vortices with two-dimensional triangular lattices. The behavior of superconducting vortices in the external magnetic and electromagnetic fields at microwave frequencies is a well-studied field.⁵⁻⁷ The interaction of magnetic-field components of microwave electromagnetic fields with supercurrents outside the vortices leads to the very strong absorption of microwaves at $T < T_c$ in superconductors, in small magnetic fields.⁶ This phenomenon, known as a low-field microwave absorption (also called LFMA) signal has been widely used as one of the most sensitive tests for superconductivity, in the search for new unconventional superconductors, such as fullerenes,⁶ high- T_c cuprates,⁷ and organic superconductors,⁸ and this method allows us to study in detail such features of vortex states as the vortex phase diagram⁷ or π -junction, which causes paramagnetic Meissner effects in superconductors.^{9,10} It should be noted that although electromagnetic fields penetrate only into a very thin layer of the London penetration depth in bulk superconductors, in realistic powdered or highly porous superconducting samples, the penetration of electromagnetic waves is quite sizable, so that their absorption by the vortices, which oscillate with the frequency of microwave fields and dissipate the microwave fields due to friction caused by the pinning of vortices, is quite a sizable effect. Despite this interest in microwave interaction with vortices, to our knowledge, no studies have been performed on the Abrikosov vortex lattice, i.e., on the periodic arrays of vortices. Because the dielectric constants inside and outside the vortices are slightly different, the Abrikosov lattices can also be viewed as periodic electromagnetic structures or as photonic crystals.

On the other hand, photonic crystals with dielectric periodic structures have photonic band gaps (PBGs) in which electromagnetic waves with certain frequencies cannot propagate in the photonic crystals due to the Bragg diffraction, which for the case of the largest possible PBGs should take place at the same wavelength as the Mie scattering in the same system.^{11,12}

Moreover, the periodic lattices of metallic photonic crystals composed of normal (non-superconducting) metallic spheres or wire meshes embedded into dielectric matrices

have also been widely studied.^{13–15} It has been found that a metallicity gap exists at a frequency ω below a certain cutoff frequency ω_{cut} , which is the analog of a modified effective plasmon frequency, and this gap at $0 < \omega < \omega_{cut}$ does not depend on the periodicity,^{13,14} contrary to PBGs. At $\omega > \omega_{cut}$, electromagnetic waves can propagate in metallic photonic crystals which have PBGs at proper higher frequency regions, despite the existence of losses. In this study based on the two-fluid model, we will find both a metallicity gap and a dielectric PBG in a two-dimensional superconductor with a periodic array of non-superconducting cylinders (cores of vortices) surrounded by circular superconducting currents. Again for simplicity, we will neglect the interaction of electromagnetic waves with supercurrents of vortices, as mentioned with respect to LFMA, since at the high frequencies considered here for wavelengths comparable to the typical periodicities of Abrikosov lattices, this interaction becomes much smaller than that at microwaves.^{7,16} The existence of point and linear defects in photonic crystals with PBGs causes light localization and guiding, respectively.^{17,18} For many applications, it is important to achieve the tuning of properties of photonic crystals under the influence of external factors, such as temperature and electric fields. Therefore, we have proposed various tunable photonic crystals infiltrated with functional materials, such as conducting polymers and liquid crystals, whose optical properties can be controlled by electric fields and temperature.^{19,20} The Abrikosov lattices can be another interesting case of tunable photonic crystals because of the well-known lattice spacings of vortices that are controllable by applied magnetic fields.

Therefore, we will investigate here the condition under which Abrikosov vortex lattices can be viewed as photonic crystals, that is, the condition under which the PBGs may open in the spectrum of electromagnetic waves propagating along two-dimensional type-II superconductors in an external magnetic field normal to their planes. Lattice spacings of Abrikosov lattices are of approximately 100 nm order. PBGs in dielectric photonic crystals appear for electromagnetic waves with wavelengths comparable to lattice spacings. We investigate the conditions for achieving effective PBG properties of photonic crystals in the Abrikosov lattices by the modulation of the GL parameters and applied magnetic fields. We will use for simplicity the two-fluid model in which both normal conducting and superconducting electrons exist in superconductors. Although it is known that this model does not provide a very good approximation, we believe that it may correctly capture the propagation of electromagnetic waves in the periodic lattices of vortices. At zero temperature, all of the electrons outside vortices are superconducting ones, although the electrons inside vortices are normal conducting ones. The Abrikosov lattices in superconductors with much larger thickness normal to their planes than the wavelengths of the electromagnetic waves can be seen as two-dimensional photonic crystals. In these two-dimensional photonic crystals, there exist the classifications of the transversal electric (TE) and transversal magnetic (TM) modes in which electric fields are parallel and perpendicular to the two-dimensional planes, respectively. In this paper, we treat only the TM mode for electromagnetic waves propagating in the two-dimensional planes.

II. THEORY

In order to obtain the photonic band structures of Abrikosov lattices in the TM mode, we start with the following two-dimensional differential equation for the electric field $E_z(x, y)$.

$$\frac{\partial^2 E_z(x, y)}{\partial x^2} + \frac{\partial^2 E_z(x, y)}{\partial y^2} + \frac{\omega^2}{c^2} \epsilon_{eff}(x, y; \omega) E_z(x, y) = 0, \quad (1a)$$

where

$$\epsilon_{eff}(x, y; \omega) = \epsilon \left\{ 1 - \frac{\omega_{ps}^2(x, y)}{\omega^2} - \frac{\omega_{pn}^2(x, y)}{\omega[\omega + i\gamma(x, y)]} \right\}. \quad (1b)$$

The effective dielectric constant $\epsilon_{eff}(x, y; \omega)$ is obtained from the phenomenological viewpoint of the two-fluid model.²¹ $\omega_{ps}(x, y)$ and $\omega_{pn}(x, y)$ indicate the plasma frequencies of superconducting and normal conducting electrons, respectively, and $\gamma(x, y)$ indicates the damping term in the normal conducting states. ϵ indicates the dielectric constant of superconductors. $\omega_{ps}(x, y)$ and $\omega_{pn}(x, y)$ are

$$\omega_{ps}(x, y) = \sqrt{\frac{n_s(x, y)e^2}{m\epsilon_0\epsilon}} = \frac{c}{\lambda(x, y)\sqrt{\epsilon}} \quad (2a)$$

$$\omega_{pn}(x, y) = \sqrt{\frac{n_n(x, y)e^2}{m\epsilon_0\epsilon}}, \quad (2b)$$

where $n_s(x, y)$ and $n_n(x, y)$ indicate the superconducting and normal conducting electron densities, respectively, and $\lambda(x, y)$ indicates the London penetration depth. A sum of $n_s(x, y)$ and $n_n(x, y)$ is constant, that is, $n_s(x, y) + n_n(x, y) = n$. At zero temperature, in this simple model, the electron densities inside and outside vortices are $n_s(x, y) = 0$, $n_n(x, y) = n$ and $n_s(x, y) = n$, $n_n(x, y) = 0$, respectively, and therefore, $\omega_{pn}(x, y)$ and $\omega_{ps}(x, y)$ inside and outside the vortices, respectively, are the same, while $\omega_{ps}(x, y)$ and $\omega_{pn}(x, y)$ inside and outside the vortices, respectively, are zero. At $\omega \gg \gamma(x, y)$, $\epsilon_{eff}(x, y; \omega)$ is the same inside and outside the vortices because one can neglect $\gamma(x, y)$ for such high frequencies. In other words, there are no differences, for electromagnetic waves at high frequencies are not affected by the difference between normal and superconducting states in metals, which is clearly physically correct for frequencies that are larger than the two-dimensional superconducting gaps. The effective difference in $\epsilon_{eff}(x, y; \omega)$ inside and outside the vortices, which is necessary for creating the dielectric index contrast in dielectric photonic crystals, can thus appear only at low frequencies. With respect to applications, moreover, photonic crystals without absorption are desirable, that is, the imaginary parts of $\epsilon_{eff}(x, y; \omega)$ should be small, and one should choose a superconductor with sufficiently low losses. Although γ is still not zero in realistic metals, we can assume that in a certain frequency range, i.e., $\omega \ll \gamma(x, y)$, the third term in Eq. (1b) can be neglected, since $\omega_{pn}/\gamma = \omega_{p0}/\gamma \ll 1$ is assumed inside the vortices at $\omega > \omega_{p0}$. We will show below that such parameters can be found in realistic superconducting metals, and then, the dielectric con-

trast appears between superconducting and normal conducting states.

$$\epsilon_{eff}(x, y; \omega) = \begin{cases} \epsilon & (\text{inside vortices}) \\ \epsilon \left(1 - \frac{\omega_{p0}^2}{\omega^2} \right) & (\text{outside vortices}), \end{cases} \quad (3a)$$

where

$$\omega_{p0} = \sqrt{\frac{ne^2}{m\epsilon_0\epsilon}} = \frac{c}{\lambda\sqrt{\epsilon}}. \quad (3b)$$

The effective dielectric constant outside the vortices is the simple Drude model, while that inside vortices is constant because of frequencies being sufficiently low. We carry out the estimation of $\epsilon=10$ inside vortices, and the frequencies near ω_{p0} , e.g., $\omega > \omega_{p0}$, must be considered for the effective difference of $\epsilon_{eff}(x, y; \omega)$ inside and outside the vortices. Thus, at $\omega=2\omega_{p0}$, the dielectric contrast is quite sizable, i.e., $\Delta\epsilon = \epsilon - \epsilon_{SC} = 10 - 7.5 = 2.5$, where ϵ_{SC} indicates the dielectric constant in the superconducting states.

$\epsilon_{eff}(x+R_x, y+R_y; \omega) = \epsilon_{eff}(x, y; \omega)$ is periodic with respect to the lattice vector $R_{x,y}$ generated by the primitive translation, and it may be expanded in a Fourier series on $G_{x,y}$, the reciprocal lattice vector

$$\epsilon_{eff}(x, y; \omega) = \sum_{G_x, G_y} \epsilon_{eff}(G_x, G_y; \omega) \exp[i(G_x x + G_y y)]. \quad (4)$$

Using Bloch's theorem, we may expand the electric field as

$$E_z(x, y) = \sum_{G_x, G_y} E_z(G_x, G_y) \exp[i\{(k_x + G_x)x + (k_y + G_y)y\}], \quad (5)$$

where $k_{x,y}$ is the wave vector indicating the directions of electromagnetic waves. By inserting Eqs. (4) and (5) into Eq. (1a), we obtain the matrix eigenvalue problem with respect to the frequencies.²² Therefore, the photonic band structures of Abrikosov lattices can be obtained by solving the frequencies at certain wave vectors.

In type-II superconductors, vortices with radii of ξ appear at $B_{c1} < B < B_{c2}$. Since Abrikosov lattices are triangular lattices, $B\sqrt{3}a(B)^2/2 = \Phi_0$ is satisfied, where $a(B)$ and $\Phi_0 = h/2e$ indicate the lattice spacing of triangular lattices and the fluxon, respectively. Therefore, the lattice spacing depending on the magnetic fields is

$$a(B) = \sqrt{\frac{2\Phi_0}{\sqrt{3}B}}. \quad (6)$$

The upper critical magnetic field and the coherence length satisfy $B_{c2}2\pi\xi^2 = \Phi_0$, and the upper critical magnetic field is represented as $B_{c2} = \sqrt{2}\kappa B_c$, where $B_c (> B_{c1})$ is the critical magnetic field. At strong magnetic fields, vortices constitute the Abrikosov lattices, and therefore, we investigate the properties of photonic crystals in the Abrikosov lattices at $B_c < B < B_{c2}$. Ratios of radii relative to lattice spacings and normalized plasma frequencies are important for the calculating of the photonic band structures. They are represented as

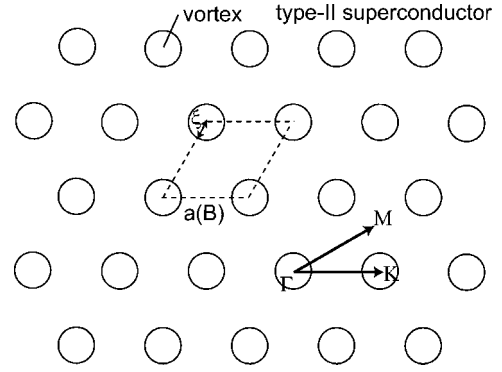


FIG. 1. Schematic diagram of Abrikosov lattices in type-II superconductors. A circle indicates the vortex with the radius of ξ , and $a(B)$ indicates the lattice spacing. The region embedded by dotted lines is the unit cell of triangular lattices. Arrows indicate directions of electromagnetic waves. The Γ , M , and K indicate high rotationally symmetric points in the first Brillouin zone in wave vectors.

$$\frac{\xi}{a(B)} = \sqrt{\frac{\sqrt{3}}{4\pi} \cdot \frac{B}{B_{c2}}} = \sqrt{\frac{\sqrt{3}}{4\sqrt{2}\pi} \cdot \frac{B}{\kappa B_c}} \quad (7)$$

and

$$\frac{\omega_{p0}a(B)}{2\pi c} = \frac{1}{\kappa} \sqrt{\frac{1}{\epsilon\sqrt{3}\pi} \cdot \frac{B_{c2}}{B}} = \sqrt{\frac{\sqrt{2}}{\epsilon\sqrt{3}\pi} \cdot \frac{B_c}{\kappa B}}. \quad (8)$$

That is, the ratios of radii relative to the lattice spacings and the normalized plasma frequencies depend on κ and B .

III. NUMERICAL CALCULATION AND DISCUSSION

Figure 1 shows a schematic diagram of Abrikosov lattices in type-II superconductors. A circle indicates the vortex with the radius of ξ , and $a(B)$ indicates the lattice spacing. The region embedded by dotted lines is the unit cell of triangular lattices. Arrows indicate the directions of the electromagnetic waves. Γ , M , and K indicate high rotationally symmetric points in the first Brillouin zone in the wave vectors.

Figure 2 shows a photonic band structure at $\kappa=1$ and $B=B_c$. Vertical and horizontal axes indicate the frequencies

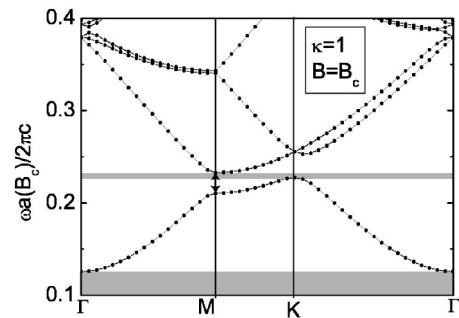


FIG. 2. Photonic band structure at $\kappa=1$ and $B=B_c$. Vertical and horizontal axes indicate frequencies and directions of electromagnetic waves, respectively. Shaded regions indicate the regions in which electromagnetic waves cannot propagate in photonic crystals in any direction. An arrow indicates a pseudo-PBG at the M point.

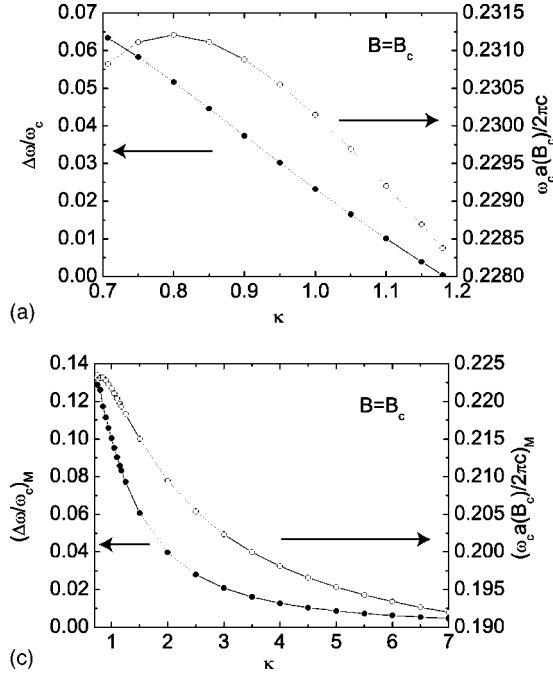


FIG. 3. GL parameter dependence of PBGs per midgaps and midgaps at $B=B_c$ for (a) omnidirectional PBGs and (b) pseudo-PBGs at the M point. Black and white points indicate PBGs per midgaps and midgaps, respectively.

and directions of electromagnetic waves, respectively. Shaded regions indicate the regions in which electromagnetic waves cannot propagate in the photonic crystals in any direction. An arrow indicates a pseudo PBG at the M point. As shown in this figure, cutoff frequencies exist due to plasma frequencies, and an omnidirectional PBG exists between the first and second photonic bands.

Omnidirectional PBGs are important properties of photonic crystals. However, pseudo-PBGs in certain directions are also valid as reflectors. Therefore, we focus our attention on the omnidirectional PBG between the first and second photonic bands and the pseudo-PBG at the M point. The pseudo PBG at the M point is valid for electromagnetic waves in the Γ – M direction in Fig. 1.

In Figs. 3(a) and 3(b), we show the GL parameter dependence of PBGs per midgaps, and the midgaps at $B=B_c$ for omnidirectional PBGs and pseudo-PBGs at the M point, respectively. Black and white points indicate the PBGs per midgaps and the midgaps, respectively. $\Delta\omega$ and ω_c indicate the PBG and the midgap, respectively. In type-II superconductors, the GL parameter is $\kappa > 1/\sqrt{2} \sim 0.707$. In Fig. 3(a), $\Delta\omega/\omega_c$ decreases linearly with increasing κ and becomes zero at $\kappa \sim 1.18$, while $\omega_c a(B_c)/2\pi c$ decreases monotonically after becoming maximum with increasing κ . In Fig. 3(b), on the other hand, $(\Delta\omega/\omega_c)_M$ and $(\omega_c a(B_c)/2\pi c)_M$ decrease monotonically with increasing κ . That is, superconductors with small GL parameters, such as Nb with $\kappa=0.78$ (0[K]), are necessary in order to obtain effective PBGs.

As evident in Eqs. (7) and (8), both the ratios of radii relative to lattice spacings and the normalized plasma frequency decrease with increasing κ . The decreases in the former and the latter mean the increase in superconducting

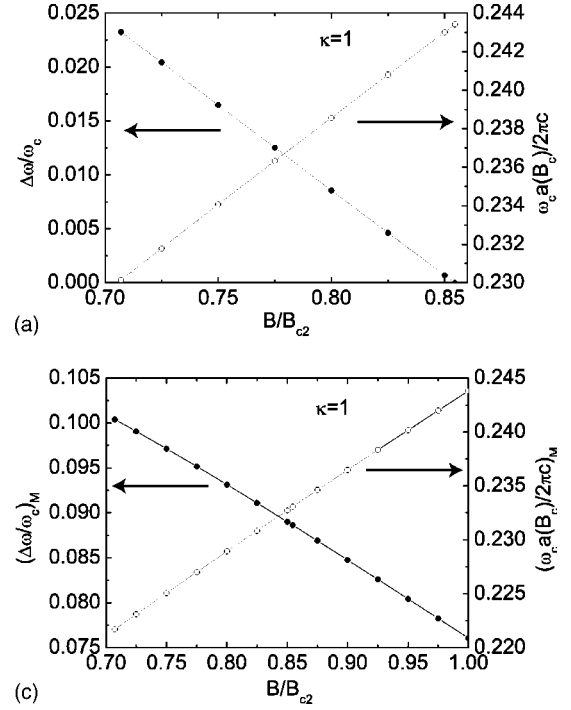


FIG. 4. Applied magnetic-field dependence of PBGs per midgaps and midgaps at $\kappa=1$ for (a) omnidirectional PBGs and (b) pseudo-PBGs at the M point. Black and white points indicate PBGs per midgaps and midgaps, respectively.

regions and the decrease in the difference in $\epsilon_{eff}(x,y;\omega)$ inside and outside the vortices for frequencies on which we focus our attention, respectively. This weakens the properties of photonic crystals and renders the PBGs smaller. Therefore, typical copper-oxide HTSCs with $\kappa \sim 100$, typical alloys with $\kappa \sim 40$ and MgB_2 with $\kappa=36.3$ (4[K]) are inadequate for obtaining effective PBGs.

In Figs. 4(a) and 4(b), we show the applied magnetic-field dependence of PBGs per midgaps and the midgaps at $\kappa=1$ for omnidirectional PBGs and pseudo-PBGs at the M point, respectively. Black and white points indicate the PBGs per midgaps and the midgaps, respectively. $\Delta\omega$ and ω_c indicate the PBG and the midgap, respectively. The critical magnetic field is $B_c=B_{c2}/\sqrt{2}\kappa \sim 0.707B_{c2}$. The applied magnetic field is assumed to be in the range of $0.707 < B/B_{c2} < 1$. In both Figs. 4(a) and 4(b), $\Delta\omega/\omega_c$ and $(\Delta\omega/\omega_c)_M$ decrease linearly with increasing B , while $\omega_c a(B_c)/2\pi c$ and $(\omega_c a(B_c)/2\pi c)_M$ increase linearly with increasing B . Particularly, $\Delta\omega/\omega_c$ becomes zero at $B/B_{c2} \sim 0.855$. That is, small applied magnetic fields are necessary to obtain effective PBGs.

As evident in Eqs. (6) and (8), lattice spacings and normalized plasma frequencies decrease with increasing B . The decreases in the former and the latter mean the increase in the frequencies of interest and the decrease in the difference in $\epsilon_{eff}(x,y;\omega)$ inside and outside the vortices for the frequencies of interest, respectively. This weakens properties of photonic crystals and renders the PBGs smaller.

In photonic crystals, reflection peak frequencies in certain directions correspond to the midgaps of pseudo-PBGs, that is, it is possible to tune the reflection peak frequencies by applying magnetic fields. By investigating the reflection peak

frequencies, moreover, one can determine the lattice spacings easily, which indicates that the applied magnetic fields can be obtained by Eq. (6).

IV. CONCLUSION

We theoretically demonstrated the properties of Abrikosov lattices as photonic crystals. When the damping terms are much larger than the plasma frequencies, the effective difference in dielectric constants inside and outside the vortices can be obtained for frequencies near the plasma frequencies. Effective properties of Abrikosov lattices as pho-

tonic crystals can be achieved when both the GL parameters and the applied magnetic fields are small. Moreover, it is possible to obtain tunable photonic crystals depending on the applied magnetic fields by using Abrikosov lattices as photonic crystals.

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