Voltage-dependent conductance and shot noise in quantum microconstrictions with single defects

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Influence on nonlinear conductance and shot noise of metallic microconstriction due to interference between electron waves scattered on single impurities and/or a barrier is studied theoretically in this paper. It is shown that these characteristics are nonmonotonic functions of the applied bias V.

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I. INTRODUCTION

Single defects have a strong influence on physical properties of mesoscopic systems. A different kind of defects arise during manufacturing of mesoscopic conductors and investigation of its influence on the transport properties has been of practical significance. On the other hand, study of the contributions by single defects to kinetic coefficients makes it possible to obtain the most detailed information on the electron scattering processes which are very important for fundamental science. Point contacts and quantum microconstrictions (quantum wires) are one of the classes of mesoscopic systems, which are extensively investigated both theoretically and experimentally (see Refs. 1 and 2). The electrical conductance G of a constriction is proportional to the number N of propagating electron modes (the number of discrete energy levels $\varepsilon_n < \varepsilon_F$ of transverse quantization, ε_F is the Fermi energy), where single mode contribution is equal to $G_0 = 2e^2/h$. Changing the contact diameter d results in a different number of occupied levels ε_n and $G(d)$ exhibits a step-like change of its value with a step size equal to G_0. This effect is a manifestation of the quantum size effect in metals, which was predicted by Lifshits and Kosevich.[3] However scattering processes on defects could decrease probability $T_n < 1$ for the transmission of the nth mode and the conductance at zero temperature $T=0$ and an applied voltage of $V \to 0$ should be described by the Landauer-Buttiker formula.[4,5]

Shot noise is an important characteristic of the transport properties of mesoscopic conductors.[1,2,6] It originated from the time-dependent current fluctuations. Kulik and Omelyanchouk[7] noticed that the shot noise in a ballistic contact vanishes in the quasiclassical approximation if there is no electron scattering. In quantum microconstriction these fluctuations arise from the quantum-mechanical probability of electrons to be transmitted through the constriction. At $T=0$, bias at the contact $V \to 0$ and for low frequencies $\omega \to 0$ the shot noise is described by[1]

$$S(0) = 2eVG_0 \sum_{n=1}^{N} T_n(1 - T_n). \tag{1}$$

In perfect ballistic contacts where the transmission probability for every mode T_n is one, the shot noise is fully suppressed. However, even for an adiabatic ballistic constrictions near the values of its diameter, at which the highest energy levels ε_N is close to ε_F, the probability T_N is smaller than one.[8] According to Eq. (1) at small bias the shot noise is a linear function of voltage V.

Conductance of a quantum microconstriction containing different types of single defects has been investigated theoretically.[9–18] The most remarkable effects which manifest electron scattering process in mesoscopic constrictions with only a few point-like defects are: (i) quantum interference directly transmitted through the contact electron waves and electron waves scattered by the defects and a barrier in the contact; (ii) dependence of electron scattering amplitude on a defect position in the constriction. The first effect causes nonmonotonic dependence of the point-contact conductance on the applied bias, which was observed experimentally[20,21] and theoretically considered in papers.[9,21] Recently, experimental observations of conductance oscillations in quantum contact have been reported Ref. 22. The second effect is responsible for the contact size dependence of the Kondo anomaly.[16,19] This dependence is due to nonhomogeneity of the local density of electron states across the diameter of microconstriction. In numerical simulations[17] the influence of "dirty" banks on the conductance of a quantum point contact has been considered. Authors had predicted suppression of the conductance fluctuations near the edges of the steps of the function $G(d)$. This effect has been experimentally observed in Ref. 21 and explained by decreasing of the interference terms in the conductance under the conditions that the contact diameter d is closed to the jump of $G(d)$.

The most important feature of the ballistic microconstriction is splitting of the Fermi surface by applied voltage.[23] Effectively, there are two electronic waves moving in opposite directions with energies difference at each point of the constriction by exactly the bias energy eV. Because of this difference in the electron energies $\varepsilon \pm eV/2$, a value of a wave vector $k_z(\varepsilon \pm eV/2)$ along the constriction depends on eV. As mentioned above, the effect of quantum interference between directly transmitted and scattered waves is defined by relative phase shift $\Delta\varphi = 2k_z\Delta z$ of the wave functions, where (Δz is a distance between scatterers) and dependence on $k_z(\varepsilon \pm eV/2)$ results in oscillations of transmission probabilities $T_n(V)$ as functions of V. In this paper we consider

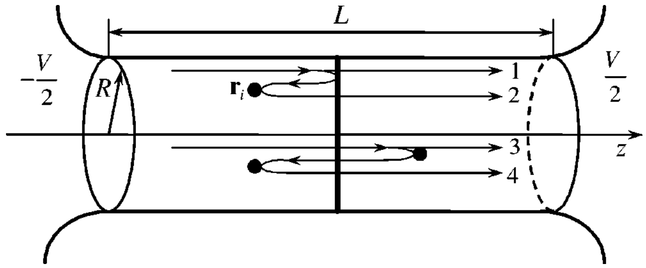

FIG. 1. A model of quantum constriction in the form of long channel adiabatically connected to bulk metallic reservoirs. Trajectories (1–4) of electrons, which are scattered by defects and a barrier are shown schematically.

the influence of the interference effect on the conductance and shot noise in long quantum microconstrictions with a few defects and the potential barrier.

The paper is organized as follows. In Sec. II the model of microconstriction and the basic equations are discussed. In Sec. III the voltage dependence of conductance and shot noise is studied. Two cases are considered: single impurity in the constriction with a barrier and two impurities in the constriction without the barrier. The expressions for Green's function for these cases are given. Also the results of numerical calculations are presented in this section. We finally summarize our results in Sec. IV.

II. MODEL OF MICROCONSTRICTION AND FORMULATION OF THE PROBLEM

We consider the quantum microconstriction in the form of a long channel with smooth boundaries and a diameter $2R$ comparable with the Fermi wavelength λ_F (Fig. 1). A length of the channel L is much larger than R. We assume that the channel is smoothly (over Fermi length scale) connected to bulk metal banks to which the voltage $eV \ll \varepsilon_F$ is applied. At the center of the constriction a potential barrier $[U(z) = U\delta(z)]$ is situated in the vicinity of which there are a few point-like defects at positions $\mathbf{r}_i$. The Hamiltonian of the system can be written as

$$\hat{H} = \frac{\hat{\mathbf{p}}^2}{2m^*} + U\delta(z) + g\sum_i \delta(\mathbf{r} - \mathbf{r}_i), \tag{2}$$

where $\hat{\mathbf{p}}$ is a momentum operator, m^* is an effective mass of an electron and g is a constant of electron–impurity interaction ($g > 0$, a repulsive impurity). In a ballistic channel without the barrier and defects ($U = g = 0$) the wave functions and energies of the eigenstates inside the channel can be separated to transversal and longitudinal parts with respect to the constriction axis z:

$$\Psi_\alpha(\mathbf{r}) = \frac{1}{\sqrt{L}} \psi_{\perp\beta}(\mathbf{R}) e^{ik_z z}, \tag{3}$$

$$\varepsilon_\alpha = \varepsilon_\beta + \frac{\hbar^2 k_z^2}{2m^*}, \tag{4}$$

where $\alpha = (\beta, k_z)$ is a full set of quantum numbers consisting of two discrete quantum numbers $\beta = (m, n)$, which define the discrete energies ε_β of conducting modes, and k_z is the wave vector along the z axis; $\mathbf{r} = (\mathbf{R}, z)$. The transversal part $\psi_{\perp\beta}(\mathbf{R})$ of the wave function satisfies zero boundary conditions at the surface of the constriction. The functions $\Psi_\alpha(\mathbf{r})$ are orthogonal and normalized.

By definition the noise power spectrum is

$$S_{ab}(\omega) = \frac{1}{2} \int dt e^{i\omega t} \langle \Delta\hat{I}_a(t)\Delta\hat{I}_b(0) + \Delta\hat{I}_b(0)\Delta\hat{I}_a(t)\rangle, \tag{5}$$

where $\Delta\hat{I}_a(t) = \hat{I}_a(t) - I_a$; $\hat{I}_a(t)$ is the current operator in the right $(a, b = R)$ or left $(a, b = L)$ lead; $I_a = \langle \hat{I}_a \rangle$ is the average current in the lead a; brackets $\langle \ldots \rangle$ denotes the quantum statistical average of a system in thermal equilibrium. In this paper we will only consider zero frequency noise $S_{ab}(0)$. Note that due to current conservation $I \equiv I_L = I_R$ we have $S \equiv S_{LL} = S_{RR} = -S_{LR} = -S_{RL}$.

A general formula for the current I through a quantum contact at arbitrary voltage was obtained by Bagwell and Orlando[24] (see also Ref. 25):

$$I = \frac{2e}{h} \int d\varepsilon T(\varepsilon, V) \times (f_L - f_R), \tag{6}$$

where is the transmission coefficient of electrons through the constriction

$$T(\varepsilon, V) = Tr[\hat{t}^\dagger(\varepsilon, V)\hat{t}(\varepsilon, V)], \tag{7}$$

and $f_{L,R}(\varepsilon) = f_F(\varepsilon \pm eV/2)$ is the distribution function of electrons moving in the contact from left (f_L) or right (f_R) banks; $f_F(\varepsilon)$ is the Fermi function, $\hat{t}(\varepsilon, V)$ is a scattering matrix. In general case the function $T(\varepsilon, V)$ depends on the applied voltage V because electron scattering leads to the appearance of nonuniform electrical field inside the constriction.[26] This field has to be calculated self-consistently from the electroneutrality equation. In an almost ballistic microconstriction containing a few scatterers and δ-function potential barrier of the small amplitude U the electrical field is small and we neglect its effect, assuming that the electrical potential drops off at the ends of the constriction.

In the same approximation the noise spectrum $S(0)$ is given by[1,2]

$$\begin{aligned} S(0) = \frac{2e^2}{h} \int d\varepsilon (&Tr[\hat{t}^\dagger(\varepsilon)\hat{t}(\varepsilon)\hat{t}^\dagger(\varepsilon)\hat{t}(\varepsilon)] \\ &\times [f_L(1 - f_L) + f_R(1 - f_R)] \\ &+ Tr\{\hat{t}^\dagger(\varepsilon)\hat{t}(\varepsilon)[\hat{I} - \hat{t}^\dagger(\varepsilon)\hat{t}(\varepsilon)]\} \\ &\times [f_L(1 - f_R)] + f_R(1 - f_L)), \end{aligned} \tag{8}$$

where $\hat{I}$ is the unit matrix. The first term in Eq. (8) corresponds to thermal fluctuations (the equilibrium, or Nyquist-Johnson noise) and vanishes if the temperature $T \to 0$. If the bias is applied to the constriction the second part of this equation remains finite at $T = 0$, and describes the shot noise.

Calculation of the transport properties of the quantum constriction can now be done by determination of the scattering matrix $\hat{t}(\varepsilon)$. Elements of scattering matrix $t_{\beta\beta'}$ can be expressed by means of the advanced Green's function $G^{+}(\mathbf{r},\mathbf{r}';\varepsilon)$ of the system:[27]

$$t_{\beta\beta'}(\varepsilon) = -\frac{i\hbar^2 k_{\beta'}}{m^*} G^{+}_{\beta\beta'}(z,z';\varepsilon), \quad z \to -\infty, z' \to +\infty, \tag{9}$$

where

$$k_\beta(\varepsilon) = \frac{1}{\hbar}\sqrt{2m^*(\varepsilon - \varepsilon_\beta)} \tag{10}$$

is an absolute value of electron wave vector corresponding to the quantum energy level ε_β; $G_{\beta\beta'}(z,z';\varepsilon)$ are components of the expansion of Green's function on the full set of wave functions corresponding to the transverse motion of electrons

$$G^{+}(\mathbf{r},\mathbf{r}',\varepsilon) = \sum_{\beta\beta'} \psi_{\perp\beta}(\mathbf{R})\psi^{*}_{\perp\beta'}(\mathbf{R}')G^{+}_{\beta\beta'}(z,z',\varepsilon). \tag{11}$$

The matrix elements $t_{\beta\beta'}(\varepsilon)$ describe the transmission probabilities for carriers incident in channel β in the left lead L and transmitted into channel β' in the right lead R. The Green's function satisfies the Dyson's equation:

$$G(\mathbf{r},\mathbf{r}',\varepsilon) = G_b(\mathbf{r},\mathbf{r}',\varepsilon) + g\sum_i G_b(\mathbf{r},\mathbf{r}_i,\varepsilon)G(\mathbf{r}_i,\mathbf{r}',\varepsilon), \tag{12}$$

where $G_b(\mathbf{r},\mathbf{r}';\varepsilon)$ is the Green' function of ballistic microconstriction with the barrier in the absence of defects. It can be found from the equation

$$G_b(\mathbf{r},\mathbf{r}';\varepsilon) = G_0(\mathbf{r},\mathbf{r}';\varepsilon) + U\int d\mathbf{R}'' G_0(\mathbf{r};\mathbf{R}'',z''=0;\varepsilon) \times G_b(\mathbf{R}'',z''=0;\mathbf{r}';\varepsilon), \tag{13}$$

where

$$G^{+}_0(\mathbf{r},\mathbf{r}';\varepsilon) = \sum_\beta \frac{m^*}{i\hbar^2 k_\beta}\psi_{\perp\beta}(\mathbf{R})\psi^{*}_{\perp\beta}(\mathbf{R}')e^{ik_\beta|z'-z|} \tag{14}$$

is the Green's function in the absence of impurities and the barrier. Substituting the expansions (11) and (14) into Eq. (13) and taking into account the orthogonality of functions $\psi_{\perp\beta}(\mathbf{R})$ for the coefficients $G^{+}_{b\beta}(z,z';\varepsilon)\delta_{\beta\beta'}$ of $G^{+}_b(\mathbf{r},\mathbf{r}';\varepsilon)$ in the expansion Eq. (11) we obtain the algebraic equation

$$G^{+}_{b\beta}(z,z';\varepsilon) = \frac{m^*}{i\hbar^2 k_\beta}[e^{ik_\beta|z'-z|} + Ue^{ik_\beta|z|}G^{+}_{b\beta}(0,z';\varepsilon)]. \tag{15}$$

Taking this equation at $z=0$ we find $G^{+}_{b\beta}(0,z';\varepsilon)$ and finally $G^{+}_{b\beta}(z,z';\varepsilon)$ is given by

$$G^{+}_{b\beta\beta'}(z,z';\varepsilon) = \frac{m^*}{i\hbar^2 k_\beta}(e^{ik_\beta|z'-z|} + r_\beta e^{ik_\beta(|z'|+|z|)}), \tag{16}$$

where

$$r_\beta = -\frac{im^*U}{\hbar^2 k_\beta + im^*U} = \cos\varphi_\beta e^{i\varphi_\beta}, \tag{17}$$

is the amplitude of reflected wave,

$$\varphi_\beta(\varepsilon) = \arcsin\left[\frac{1}{\sqrt{1+(m^*U/\hbar^2 k_\beta)^2}}\right]. \tag{18}$$

The amplitude t_β of the transmitted wave can be evaluated through r_β from the continuity of electron wave function at $z=0$:

$$t_\beta = r_\beta + 1 = \frac{\hbar^2 k_\beta}{\hbar^2 k_\beta + im^*U} = i\sin\varphi_\beta e^{i\varphi_\beta}. \tag{19}$$

The same functions r_β and t_β can be found from the solution of the one-dimensional Schrödinger equation of a system with δ-function barrier $U\delta(z)$.[28]

Equation (12) can be solved exactly for any finite number of defects. For that Eq. (12) should be written at all points $\mathbf{r}_i$ of the defect positions and the functions $G(\mathbf{r}_i,\mathbf{r}';\varepsilon)$ are found from the system of i algebraic equations.

By using the matrix elements Eq. (9) the conductance $G = dI/dV$ of the microconstriction as well as the shot noise $S(0)$ can be calculated.

III. VOLTAGE DEPENDENCE OF CONDUCTANCE AND SHOT NOISE

In order to illustrate the effect of quantum interference of scattered electron waves on the conductance and the shot noise we present the results for two cases: (i) single impurity in the constriction with a barrier; (ii) two impurities in the constriction without the barrier. For the first case the Green's function takes the form:

$$G(\mathbf{r},\mathbf{r}';\varepsilon) = G_b(\mathbf{r},\mathbf{r}';\varepsilon) + \frac{gG_b(\mathbf{r},\mathbf{r}_1;\varepsilon)G_b(\mathbf{r}_1,\mathbf{r}';\varepsilon)}{1 - gG_b(\mathbf{r}_1,\mathbf{r}_1;\varepsilon)}, \tag{20}$$

where $\mathbf{r}_1$ is the position of the impurity, a Green's function $G_b(\mathbf{r},\mathbf{r}';\varepsilon)$ is defined by Eqs. (11) and (16). In the case of only two impurities present inside the ballistic microconstriction, solution of Eq. (12) is

$$G(\mathbf{r},\mathbf{r}';\varepsilon) = G_0(\mathbf{r},\mathbf{r}';\varepsilon) + \frac{1}{1 - G_1(\mathbf{r}_1;\varepsilon)G_1(\mathbf{r}_2;\varepsilon)G_0^2(\mathbf{r}_1,\mathbf{r}_2;\varepsilon)} \times \sum_{i,k=1,2;i\neq k}\{G_1(\mathbf{r}_i;\varepsilon)G_0(\mathbf{r},\mathbf{r}_i;\varepsilon)[G_0(\mathbf{r}_i,\mathbf{r}';\varepsilon) + G_1(\mathbf{r}_k;\varepsilon)G_0(\mathbf{r}_i,\mathbf{r}_k;\varepsilon)G_0(\mathbf{r}_k,\mathbf{r}';\varepsilon)]\}, \tag{21}$$

where

$$G_1(\mathbf{r}_i;\varepsilon) = \frac{g}{1 - gG_0(\mathbf{r}_i,\mathbf{r}_i;\varepsilon)}, \tag{22}$$

and $G_0(\mathbf{r},\mathbf{r}',\varepsilon)$ is the Green's function of the ballistic microconstriction Eq. (14). Using Eqs. (20) and (21) it is easy to find the transmission probabilities $t_{\beta\beta'}$ (9).

At zero temperature the nonlinear conductance $G(V)$ and the noise power $S(0,eV)$ are given by following expressions:

$$G(V) = \sum_{\beta\beta'} \left[\left| t_{\beta\beta'}\left(\varepsilon_F + \frac{eV}{2}\right) \right|^2 + \left| t_{\beta\beta'}\left(\varepsilon_F - \frac{eV}{2}\right) \right|^2 \right], \tag{23}$$

$$S(0,eV) = \sum_{\beta\beta'\beta''\beta'''} \int_{\varepsilon_F - eV/2}^{\varepsilon_F + eV/2} d\varepsilon \{ t^*_{\beta\beta'}(\varepsilon) t_{\beta'\beta''}(\varepsilon) \times [\delta_{\beta''\beta'''}\delta_{\beta'''\beta} - t^*_{\beta''\beta'''}(\varepsilon) t_{\beta'''\beta}(\varepsilon)]\}. \tag{24}$$

In order to explain analytical results we present the expansion of the transmission coefficient (7) on the constant of electron–impurity interaction g up to linear in g term for the constriction with one impurity at point $\mathbf{r}_1 = (\mathbf{R}_1, z_1)$ and the barrier

$$T(\varepsilon) = \sum_\beta |t_\beta|^2 \left\{ 1 - \frac{2m^* g}{\hbar^2 k_\beta} |r_\beta| |\psi_{\perp\beta}(\mathbf{R}_1)|^2 \times \cos(2k_\beta z_1 + \varphi_\beta) \right\}, \quad \varepsilon > \varepsilon_\beta, \tag{25}$$

where t_β, r_β, and phase φ_β are defined by Eq. (17), (18), and (19). This formula is valid for $2m^* g/\hbar^2 k_\beta \ll 1$, i.e., far from the end of the step of conductance, where $k_\beta \to 0$. The oscillatory term in Eq. (25) originates from the interference between directly transmitted wave (trajectory 1 in Fig. 1) and the wave, which is once reflected by the barrier and after one reflection from the impurity passes through the contact (trajectory 2 in Fig. 1). The amplitude of the oscillations depends on the local density of electron states $\nu_\beta(\mathbf{R}_1, \varepsilon) = m^* |\psi_{\perp\beta}(\mathbf{R}_1)|^2 / [\hbar^2 k_\beta(\varepsilon)]$ in the point, in which the impurity is located. At certain points $\nu_\beta(\mathbf{R}, \varepsilon)$ can be equal to zero and a defect located near such a point contributes very little to the oscillatory addition of βth mode to the $T(\varepsilon)$. In particular, impurities at the surface $\mathbf{R} = \mathbf{R}_s$ do not influence oscillations of $T(\varepsilon)$, because $\psi_{\perp\beta}(\mathbf{R}_s) = 0$. As a result of the reflection from the barrier the oscillations have the additional phase φ_β. Its dependence on the energy ε leads to nonperiodicity of oscillations of function $T(\varepsilon)$. Equation (25) could be used to calculate the dependence of oscillation amplitudes on the contact diameter. If the diameter is increased and approaches the end of the conductance step, the energy of the transverse quantum mode ε_β is decreased [see, for example, Eq. (28) for cylindrical geometry]. The wave number k_β (10) is increased and according to Eq. (17) the modulus of the reflection probability $|r_\beta|$ is decreased. In the opposite situation (the radius is decreased) the decreasing of k_β leads to decrease of the transmission probability $|t_\beta|$ Eq. (19). In both cases amplitude of the oscillations of $T(\varepsilon)$ is decreased.

Similar expansion of $T(\varepsilon)$ for the constriction with two defects at points $\mathbf{r}_1 = (\mathbf{R}_1, z_1)$ and $\mathbf{r}_2 = (\mathbf{R}_2, z_2)$ without barrier is

$$T(\varepsilon) = \sum_{\beta\beta'} \left\{ \delta_{\beta\beta'} - 2\left(\frac{m^* g}{\hbar^2}\right)^2 \frac{1}{k_\beta k_{\beta'}} \sum_{i=1,2} \left[|A^{(ii)}_{\beta\beta'}|^2 + \mathrm{Re} \sum_{i\neq j=1,2} A^{(ii)}_{\beta\beta'} A^{(jj)}_{\beta\beta'} \exp[(k_\beta + k_{\beta'})(z_j - z_i) + \varphi_\beta + \varphi_{\beta'}] \right] \right\}, \quad \varepsilon > \varepsilon_\beta, \varepsilon_{\beta'}; \tag{26}$$

where

$$A^{(ii)}_{\beta\beta'} = \psi_{\perp\beta}(\mathbf{R}_i) \psi^*_{\perp\beta'}(\mathbf{R}_i). \tag{27}$$

A last term in square brackets describes the interference effect between trajectory 3 in Fig. 1 and trajectory 4, which corresponds to two scattering by different impurities. It depends nonmonotonically on the energy ε. Energy dependence of the transmission coefficient $T(\varepsilon)$ manifests itself in nonmonotonic dependence of conductance and shot noise on the applied bias eV.

The general expression for components $t_{\beta\beta'}(\varepsilon)$ (9) calculated using Green's functions [Eqs. (20) and (21)] takes into account a multiple electron scattering by impurities and barrier. It is valid for any values of parameters. We will illustrate such a situation presenting plots for the voltage dependencies of conductance and shot noise for some values of the parameters, which could be related to experiments.

For numerical calculations we used a model of cylindrical channel where in formulas (3) and (4):

$$\psi_{\perp\beta}(\rho, \varphi) = \frac{1}{\sqrt{\pi} R J_{m+1}(\gamma_{mn})} J_m\left(\gamma_{mn} \frac{\rho}{R}\right) e^{im\varphi}, \tag{28}$$

$$\varepsilon_{mn} = \frac{\hbar^2 \gamma_{mn}^2}{2m^* R^2}, \tag{29}$$

where we used the cylindrical coordinates $\mathbf{r} = (\rho, \varphi, z)$; γ_{mn} is n th zero of Bessel function J_m. Also, dimensionless parameters are introduced

$$\tilde{g} = \frac{m^* g}{\pi R^2 \hbar^2 k_F}, \quad \tilde{U} = \frac{m^* U}{\hbar^2 k_F}, \tag{30}$$

where k_F is the Fermi wave vector. We have performed the calculations for $\tilde{g} = 1$ and $\tilde{U} = 0.5$ For such values of these parameters the amplitude of conductance oscillations is close to a value which was observed in Ref. 22. For the radius $2\pi R = 2.9\lambda_F$ (one mode channel) the first energy level $\varepsilon_{0,1} < \varepsilon_F$ is comparatively far from the Fermi energy and for $2\pi R = 3.45\lambda_F$ this level is closed to ε_F. For a larger value of radius ($2\pi R = 5\lambda_F$) there are two open quantum modes with energies $\varepsilon_{0,1}$, $\varepsilon_{\pm1,1} < \varepsilon_F$. In order to illustrate different reasons for the appearance of conductance oscillations, in Figs. 2 and 3 we show the dependencies of the conductance on the applied voltage for the channel without the barrier ($U=0$) containing two impurities and for the channel with the barrier and a single impurity. By comparison of the different curves in Figs. 2 and 3 we observe that the amplitude of conductance oscillations is decreased for radius value ($2\pi R = 3.45\lambda_F$) corresponding to the end of a first step in conduc-

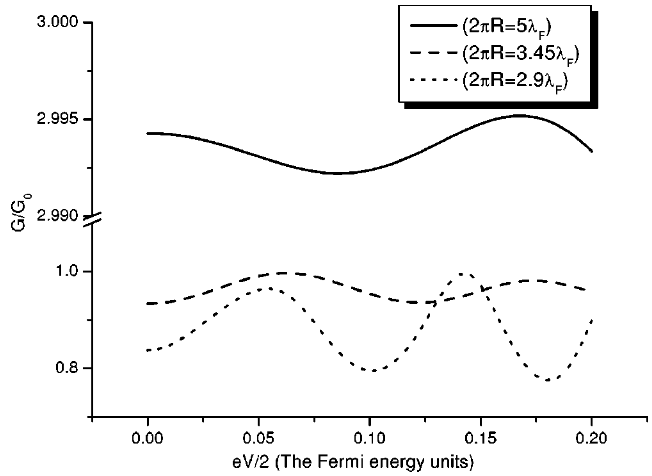

FIG. 2. Dependencies of the conductance on the applied voltage for a channel containing two impurities for different values of radius; impurity positions are $2\pi\rho_1 = 0.3\lambda_F$ and $2\pi\rho_2 = 0.4\lambda_F$, $2\pi(z_1 - z_2) = 35\lambda_F$, $\varphi_1 = \varphi_2$.

tance. In Figs. 4 and 5 the voltage dependencies of noise power are plotted. We notice that, as seen in Fig. 4, for one mode channel the shot noise is a strongly nonmonotonic function of V. Similar to behavior observed for the conductance, the amplitude of the oscillations of the shot noise is decreased near the end of the first step $(2\pi R = 3.45\lambda_F)$. For the two mode channel the $S(V)$ is almost a linear function that can be explained by the effect of a superposition of oscillations with different periods. In the contact with the barrier the main part of the shot noise $S_0(V)$ originates from electron reflection from the barrier potential $[S(V) = S_0(V), \text{if } g=0)]$, if and is the monotonic function of V. A small nonlinearity of this function arises from the energy dependence of the transmission probability. The interference of electron waves in the presence of a defect leads to nonmonotonic additions, which we show in Fig. 5.

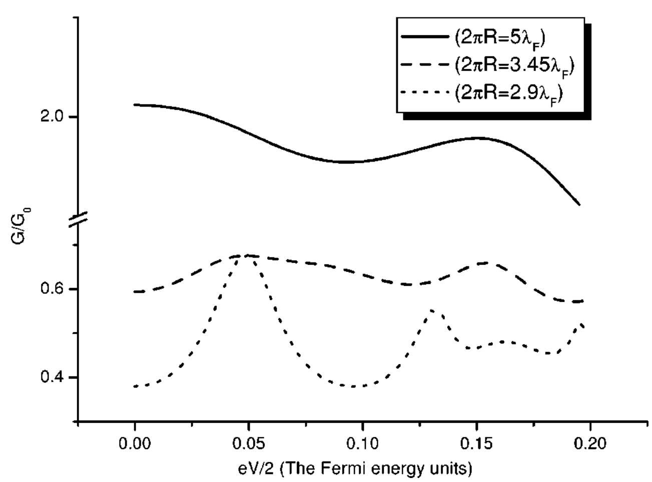

FIG. 3. Dependencies of the conductance on the applied voltage for a channel containing a single impurity and a barrier for different values of radius; the impurity position is $2\pi\rho_1 = 0.3\lambda_F$, $2\pi z_1 = 35\lambda_F$.

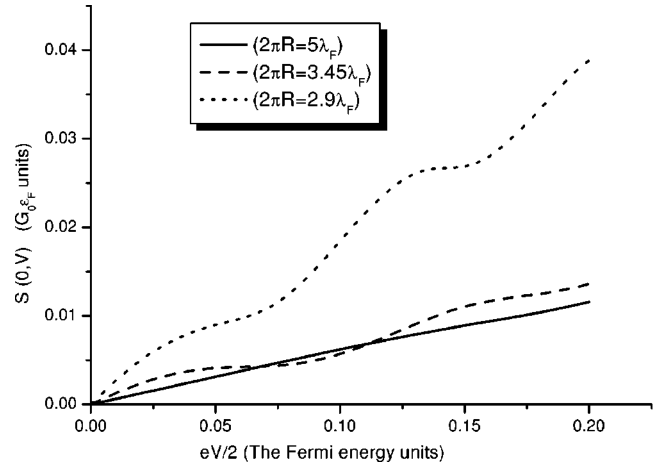

FIG. 4. Voltage dependencies of noise power on the applied voltage for a channel containing two impurities for different values of radius; impurity positions are $2\pi\rho_1 = 0.3\lambda_F$ and $2\pi\rho_2 = 0.4\lambda_F$, $2\pi(z_1 - z_2) = 35\lambda_F$, $\varphi_1 = \varphi_2$.

IV. CONCLUSION

We have studied theoretically the voltage dependence of the conductance G and the shot noise power S in a quantum microconstriction in the form of a long channel (quantum wire). The effect of quantum interference of electron waves scattered by single defects and the potential barrier inside the constriction, is taken into account. In the framework of our model we have obtained an analytical solution for the problem and found dependencies of G and S on such important parameters as a constriction diameter, a constant of electron–impurity interaction, an amplitude of the barrier potential and positions of impurities. In general, these dependencies are complex and are defined by the expression of transmission probability $t_{\beta\beta'}$ (9) by means of Green's functions [Eqs. (20) and (21)]. For a small constant g of electron–impurity interaction and far from the step in conductance the part of the

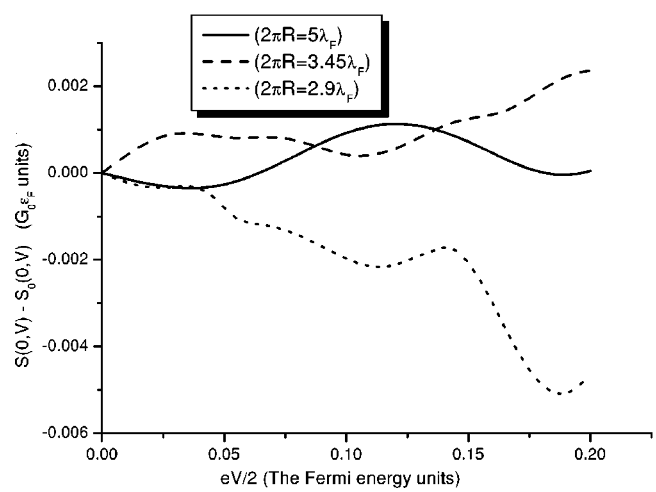

FIG. 5. Voltage dependencies of the nonmontonic part of noise power on the applied voltage for the channel containing a single impurity and a barrier for different values of radius; the impurity position is $2\pi\rho_1 = 0.3\lambda_F$, $2\pi z_1 = 35\lambda_F$.

total transmission coefficient $T(\varepsilon)$ (25), which is due to the interference effect, is proportional to g and to the amplitude of the reflected from the barrier wave r_β [see, Eq. (17)]. As a result, at small g and U the interference part of conductance and shot noise is proportional to gU or g^2 (for $U=0$) for any number of defects.

We have shown that conductance and noise are oscillatory functions on the applied bias V and have come to the conclusion that the experimentally observed suppression of conductance oscillations[21] could be explained by energy dependence of the transmission probability of electrons through the constriction. In the framework of our model this suppression of conductance oscillations can be explained in the following way: The oscillatory part of conductance is decreased with the decreasing of amplitude r_β of reflected from the barrier wave. The reflection probability r_β from the barrier has the minimal value, if the energy of quantum mode ε_β is close to Fermi level $\varepsilon_\beta \lesssim \varepsilon_F$. It is demonstrated that in one mode constriction containing only impurities the shot noise power is a strongly nonlinear function of V. In a contact with the barrier the almost linear dependence $S(V)$ has a small oscillatory addition.

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