Nernst effect, quasiparticles, and *d*-density waves in cuprates

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(Received 22 December 2003; revised manuscript received 20 April 2004; published 4 August 2004)

We examine the possibility that the large Nernst signal observed in the pseudogap regime of hole-doped cuprates originates from quasiparticle transport in a state with *d*-density wave (DDW) order, proposed by S. Chakravarty *et al.* [Phys. Rev. B **63**, 094503 (2001)]. We find that the Nernst coefficient can be moderately enhanced in magnitude by DDW order, and is generally of negative sign. Thus, the quasiparticles of the DDW state cannot account for the large and positive Nernst signal observed in the pseudogap phase of the cuprates. However, the general considerations outlined in this paper may be of broader relevance, in particular to the recent measurements of Bel *et al.* in $NbSe_2$ and $CeCoIn_5$ [Phys. Rev. Lett. **91**, 066602 (2003); *ibid.* **92**, 217002 (2004)].

DOI: 10.1103/PhysRevB.70.054503 PACS number(s): 74.72.−h, 72.15.Jf

I. INTRODUCTION

Much of the original interest in the pseudogap phenomena in high-temperature superconductors (HTSC) stemmed from the belief that it represented, in some way, a vestige of the superconducting state,[1] and thus could offer insights as to the latter's origins. In recent years, however, fluctuations of other orders, such as spin and charge density waves, have been detected in some of these materials,[1,2] suggesting that a proper understanding of the pseudogap phase requires incorporating these ordering instabilities. A proposal by Chakravarty *et al.*[3] goes one step further: according to them the pseudogap is a consequence of long range *d*-density wave (DDW) order, a pattern of circulating currents. At the mean-field level, this unconventional density wave order possesses quasiparticles, and much of its potential for explaining pseudogap phenomena stems from the changes in the single particle spectrum due to the breaking of translational symmetry.[4–10]

The discovery[11] of a large Nernst effect in hole-doped cuprates posed a new major challenge for the theoretical description of the pseudogap regime. The Nernst effect (see Sec. II) is anomalously large in the nonsuperconducting state of underdoped samples, extending to rather high temperatures above T_c.[11–14] This surprising result contrasts with conventional materials where the effect is generally small in the normal state.

In superconductors (both conventional and high-temperature), a large Nernst signal is observed below T_c as field induced vortices become depinned and float down the thermal gradient, their motion producing a transverse voltage by phase slips.[15] By continuity, therefore, the observed signal in the pseudogap may be associated with collective fluctuations of the superconducting order parameter. In this spirit, Ong and collaborators have interpreted their results as evidence for vortices above T_c.[11] Recently, a detailed analysis by Sondhi, Huse, and one of us[16] has shown that superconducting fluctuations can produce a sizable effect in the cuprates (see also Ref. 17). Other works[18–20] have also suggested, in one way or another, that the Nernst effect is a result of collective phenomena of superconducting origin.

In contrast, here we consider whether single particle transport can be a source of an enhanced Nernst signal. This is done first on rather general terms, which should be applicable to different systems (including spin and charge density waves). We then consider in detail whether the onset of DDW order can by itself account for the experimental observations in HTSC. We also discuss two experiments by Bel and coworkers, in which a large Nernst signal is observed in the normal state of $NbSe_2$,[21] and, very recently, in a heavy fermion compound $CeCoIn_5$.[22]

We begin by re-examining the conventional theory of transport in metals in Sec. II, focusing in particular on the Nernst phenomena it predicts. In Sec. III we compute the effects of the DDW order. Experimental results are discussed in the following section. We close with a short Summary and an Appendix.

II. GENERAL CONSIDERATIONS

The Nernst effect is the off-diagonal component of the thermopower tensor Q, measured in the absence of electric flow and with magnetic field B in the $\hat{\mathbf{z}}$ direction (in the cuprates, perpendicular to the copper-oxide planes). The thermopower tensor is given by

$$\mathbf{E} = Q \cdot \boldsymbol{\nabla} T = \sigma^{-1} \cdot \alpha \cdot \boldsymbol{\nabla} T, \tag{1}$$

where σ and α are the conductivity and Peltier (thermoelectric) conductivity tensors, respectively. The relation $Q_{xy} = -Q_{yx}$ generally holds only when the system has isotropic transport tensors (e.g., $\sigma_{xx} = \sigma_{yy}$). In this case, the Nernst signal is given by

$$Q_{xy} = \frac{E_y}{(-\nabla T)_x} = \frac{\alpha_{xy}\sigma_{xx} - \alpha_{xx}\sigma_{xy}}{\sigma_{xx}^2 + \sigma_{xy}^2}. \tag{2}$$

Here, we follow the sign convention such that the Nernst signal arising from vortex flow in a superconductor is positive. We will be mainly concerned with situations where $Q_{xy} \propto B$ over a wide range of fields, and therefore concentrate on the calculation of the Nernst coefficient $\nu = Q_{xy}/B$.

Quasiparticle contribution to the Nernst effect is usually argued to be small, as it is strictly zero in the simple Drude model due to the "Sondheimer cancellation"[12] between the two terms in Eq. (2). Generally, in any realistic system, such a cancellation will be incomplete. In this section we delineate the factors determining the magnitude of Q_{xy} under general conditions of validity of the Boltzmann theory of transport.[23,24]

Solving the Boltzmann equation at low temperature T, the thermoelectric tensor α is related to the conductivity tensor σ through

$$\alpha = -\frac{\pi^2}{3}\frac{k_B^2 T}{e}\frac{\partial \sigma}{\partial \mu}, \tag{3}$$

where μ is the chemical potential, and $-e<0$ is the electron charge.[25] Using Eq. (2), the Nernst coefficient of a degenerate Fermi liquid may then be reduced to

$$\nu = -\frac{\pi^2}{3}\frac{k_B^2 T}{eB}\frac{\partial \Theta_H}{\partial \mu}. \tag{4}$$

Here, $\Theta_H = \sigma_{xy}/\sigma_{xx}$ is the Hall angle to linear order in magnetic field. The Nernst coefficient is thus encoded in the dependence of Θ_H on μ.

We proceed by assuming a constant scattering time τ, in order to focus on the role of Fermi surface geometry in the Nernst effect, returning to the details of τ below. In two dimensions and to linear order in magnetic field the longitudinal and Hall conductivities are expressed in terms of integrals over the Fermi surface,

$$\sigma_{xx} = e^2\tau \int \frac{d^2k}{(2\pi)^2} v_x^2 \delta(\epsilon_{\mathbf{k}} - \mu), \tag{5}$$

$$\sigma_{xy} = -\frac{e^3 B\tau^2}{\hbar c}\int \frac{d^2k}{(2\pi)^2}\left(v_x^2\frac{\partial v_y}{\partial k_y} - v_x v_y \frac{\partial v_y}{\partial k_x}\right)\delta(\epsilon_k - \mu), \tag{6}$$

where $\mathbf{v} = (\partial\epsilon_{\mathbf{k}}/\partial\mathbf{k})/\hbar$ is the velocity of the quasiparticle (and spin and band indices are suppressed).[26]

It is convenient to rearrange the expression for the Nernst coefficient, Eq. (4), as

$$B\nu = -\frac{\pi^2}{3}\frac{k_B}{e}\frac{a^2}{\ell_B^2}\frac{k_B T\tau}{\hbar}\frac{\partial \Upsilon}{\partial \mu}. \tag{7}$$

Here, $k_B/e \approx 86\ \mu$V/K is the only dimensionful factor, setting the natural scale for a thermopower measurement. The reduction factor involving the lattice spacing $[a]$ and the magnetic length $[\ell_B = (\hbar c/eB)^{1/2}]$ encodes the field's weakness on a scale natural to the system. This is multiplied (apart from the numerical factor) by the ratio of the thermal energy and relaxation rate, and by the dimensionless derivative $\partial\Upsilon/\partial\mu$. The energy scale Υ is constructed by formally stripping the weak-field Hall angle of its dependence on magnetic field and relaxation,

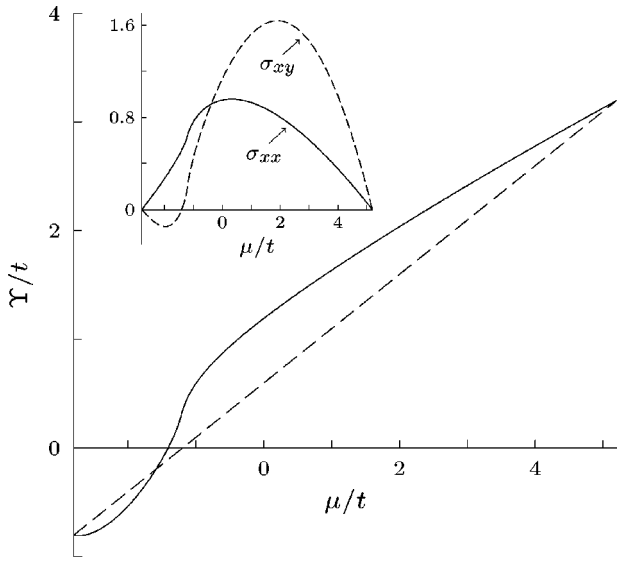

FIG. 1. The energy scale Υ (Hall angle in appropriate units) is compared against the naive estimate of a uniform slope over the entire band range, from 0 to 2 electrons per site (using the same hopping parameters as in Sec. III, $t'=0.3t$; half-filling is at $\mu=-0.66t$). In the inset we present σ_{xx} and σ_{xy} in units $(e^2/\hbar)(t\tau/\hbar)$ and $(e^2/\hbar)(t\tau/\hbar)^2(a^2/\ell_B^2)$, respectively.

$$\Upsilon = \frac{\ell_B^2}{a^2}\frac{\hbar}{\tau}\Theta_H. \tag{8}$$

For an isotropic Fermi surface of holes $\Theta_H = \omega_c\tau$ (with the cyclotron mass given by $m = \hbar k_F/v_F$), and $\Upsilon = \hbar^2/ma^2$. Finally, it is important to note that when $\partial\Upsilon/\partial\mu > 0$ the Nernst coefficient is negative.

We now consider a tight-binding model on the square lattice with nearest and next-nearest neighbor hopping parameters (t-t' model) as a simple, concrete example with which to explore Eq. (7), and also for future use in Sec. III. The free electron expression for Υ suggests a simple way of approximating the slope of $\Upsilon(\mu)$ by its average over the entire band,

$$\overline{\frac{\partial \Upsilon}{\partial \mu}} = \frac{1}{W}\frac{\hbar^2}{a^2}\left(\frac{1}{m_e} + \frac{1}{m_h}\right) = \frac{1}{2}. \tag{9}$$

Here, m_e (m_h) is the electron (hole) mass near the bottom (top) of the band, W is the overall bandwidth, and the result is independent of the hopping parameters. Remarkably, this crude estimate is exceedingly accurate (see Fig. 1) and for practical purposes $\partial\Upsilon/\partial\mu$ is a number of order 1.[27] With $\partial\Upsilon/\partial\mu \approx 1$, the estimate for the Nernst coefficient then boils down to

$$\nu \approx -100\frac{k_B T\tau}{\hbar}\frac{\mathrm{nV}}{\mathrm{KT}}, \tag{10}$$

where the last expression is evaluated using $a = 5$ Å. Remarkably, this estimate is applicable even in the limit where the effective band dispersion is essentially that of free particles (e.g., for $\mu > 4t$ in Fig. 1), despite the Sondheimer cancellation. The results for the conductivities (inset of Fig. 1) can

also be used to extract the components of the Peltier tensor and to compute the Nernst coefficient via Eq. (2).

Equation (10) suggests that quasiparticles will generally make a finite, typically negative contribution to the Nernst coefficient. The magnitude of the effect is controlled by the product $k_B T\tau/\hbar$. This can be small, e.g., when the scattering is strong or at low temperatures in the impurity dominated regime, or large, e.g., in clean systems with only moderate inelastic scattering. In this latter regime the Boltzmann theory predicts that the range of magnetic fields over which Q_{xy} is linear diminishes, with the crossover to $Q_{xy}\sim 1/B$ taking place at $\omega_c\tau\approx 1$. Although in practice this crossover between weak-field and large-field regimes need not be sharp (or simple), in cases where such behavior in $Q_{xy}(B)$ can be observed as the temperature is lowered, it can serve as an independent evidence of coherent quasiparticle transport.[28]

Up to now our discussion explicitly assumed that all relaxation processes can be summarized by a single scattering time, independent of the energy or momentum of the quasiparticles. This simplified analysis can be readily generalized to three dimensions [the factor of $1/2$ in Eq. (9) becomes $1/3$]. The analysis in the next section proceeds along these lines and focuses on the contribution to the effect coming from the DDW induced changes in Fermi surface, which is very much in the spirit of the proposal of Refs. 3–6.

We believe, however, that a more detailed modelling of relaxation may be necessary, both in HTSC and other cases. For example, we have completely neglected issues such as the dependence of the scattering time on energy or location on the Fermi surface, as well as the difference in the relaxation of electrical and energy currents[23] (which may lead to different scattering times, as may be the case for scattering by phonons). Clearly, these issues cannot be addressed without a specific material in mind. We defer our discussion of particular experiments until Sec. IV. Here, we focus on the energy dependence of the scattering time, for which a general statement is possible.

Consider, for simplicity, a nearly isotropic Fermi surface (this corresponds to the regime $\mu > t$ in Fig. 1). One may then estimate

$$\Theta_H = \frac{a^2}{\ell_B^2}\frac{\tau(\mu)}{\hbar}\frac{1}{2\pi N(\mu)a^2}, \tag{11}$$

where $N(\mu)$ is the density of states at the Fermi surface. If the scattering is primarily due to weak quenched disorder (or phonons above the Debye temperature) the corresponding rate is approximated by $1/\tau(\mu)\propto N(\mu)$. Then, properly accounting for the μ dependence of τ amounts to an additional factor of 2 in the estimate of the Nernst coefficient.[29] More generally, even for inelastic processes, one expects the scattering rate to be an increasing function of the electronic density of states, and therefore act to enhance the estimate in Eq. (10).

III. THE DDW STATE

The DDW state is specified by the following Hamiltonian[3–6]

$$H = \sum_s \int_{\mathrm{BZ}} \frac{d^2k}{(2\pi)^2}(\epsilon_{\mathbf{k}} c^\dagger_{\mathbf{k},s} c_{\mathbf{k},s} + i\Delta_{\mathbf{k}} c^\dagger_{\mathbf{k},s} c_{\mathbf{k}+\mathbf{Q},s} + \mathrm{h.c.}), \tag{12}$$

where $c^\dagger_{\mathbf{k},s}$ is the creation operator for a quasiparticle with momentum $\mathbf{k}$ and spin s, and $\epsilon_{\mathbf{k}}$ is the effective quasiparticle dispersion. When present, the DDW order parameter gives rise to a potential, $\Delta_{\mathbf{k}}$, connecting states separated by the ordering wave vector $\mathbf{Q}=(\pi,\pi)$, creating two bands in a reduced Brillouin zone. The eigenvalues of this Hamiltonian are

$$\epsilon^{\pm}_{\mathbf{k}} = \tfrac{1}{2}(\epsilon_{\mathbf{k}} + \epsilon_{\mathbf{k}+\mathbf{Q}}) \pm \tfrac{1}{2}\sqrt{(\epsilon_{\mathbf{k}} - \epsilon_{\mathbf{k}+\mathbf{Q}})^2 + 4\Delta^2_{\mathbf{k}}}. \tag{13}$$

The essential ingredient in transport calculations is the particle current operator. In the basis in which the Hamiltonian is diagonal, the current operator is given by

$$\mathbf{j} = -e\sum_s \int_{\mathrm{RBZ}} \frac{d^2k}{(2\pi)^2}\chi^\dagger_{\mathbf{k},s}\begin{pmatrix} \nabla_{\mathbf{k}}\epsilon^+_{\mathbf{k}}/\hbar & \mathbf{v}^{\mathrm{inter}}_{\mathbf{k}} \\ \mathbf{v}^{\mathrm{inter}}_{\mathbf{k}} & \nabla_{\mathbf{k}}\epsilon^-_{\mathbf{k}}/\hbar \end{pmatrix}\chi_{\mathbf{k},s}. \tag{14}$$

The off-diagonal elements

$$\mathbf{v}^{\mathrm{inter}}_{\mathbf{k}} = \frac{1}{\hbar}\frac{(\epsilon_{\mathbf{k}} - \epsilon_{\mathbf{k}+\mathbf{Q}})\nabla_{\mathbf{k}}\Delta_{\mathbf{k}} - (\nabla_{\mathbf{k}}\epsilon_{\mathbf{k}} - \nabla_{\mathbf{k}}\epsilon_{\mathbf{k}+\mathbf{Q}})\Delta_{\mathbf{k}}}{\sqrt{(\epsilon_{\mathbf{k}} - \epsilon_{\mathbf{k}+\mathbf{Q}})^2 + 4\Delta^2_{\mathbf{k}}}} \tag{15}$$

result in interband contributions to transport. However, if the energy gap to the second band is larger than $k_B T$ and $\hbar/\tau$, these interband contributions may be neglected and the Boltzmann equation recovered for dc transport (see Appendix for an example where interband and intraband contributions must be treated on equal footing).

Following common practice we approximate $\epsilon_{\mathbf{k}} = -2t(\cos k_x + \cos k_y) + 4t' \cos k_x \cos k_y$ and $\Delta_k = \Delta(\cos k_x - \cos k_y)$ (in this section we set $a=1$). For the effective band-structure parameters we shall use $t=0.3$ eV and $t'=0.3t$, with the chemical potential in the range $-t \lesssim \mu \lesssim -0.75t$. In the absence of DDW this choice of parameters is consistent with the character of the Fermi surface observed in ARPES (Ref. 10) in the doping range of 5%–20%. The choice of Δ requires some care as the effect of DDW order is sensitive to the filling. For clarity we loosely classify the different regimes as "weak," "moderate," "ambipolar," and "Dirac." The names are meant to reflect qualitatively the character of the Fermi surface in each of the regimes (see Fig. 2).

The Dirac DDW occurs when Δ is large in the vicinity of half filling. Its spectrum consists of Dirac points at $(\pm\pi/2, \pi/2)$. The ambipolar regime can occur at moderately strong Δ, in the vicinity of half-filling. Here the Fermi surface consists of three well formed sheets, two holelike centered about $(\pm\pi/2, \pi/2)$ and one electronlike centered about $(\pi, 0)$, and the system is thus composed of two types of carriers. We discuss these two regime in the Appendix, as we do not believe either of them is realized in the doping range where enhanced Nernst effect is observed. The Fermi surfaces of both of these states are sufficiently remarkable to be easily ruled out based on the available ARPES data.[10] Experimentally, neither well defined electron pockets in the an-

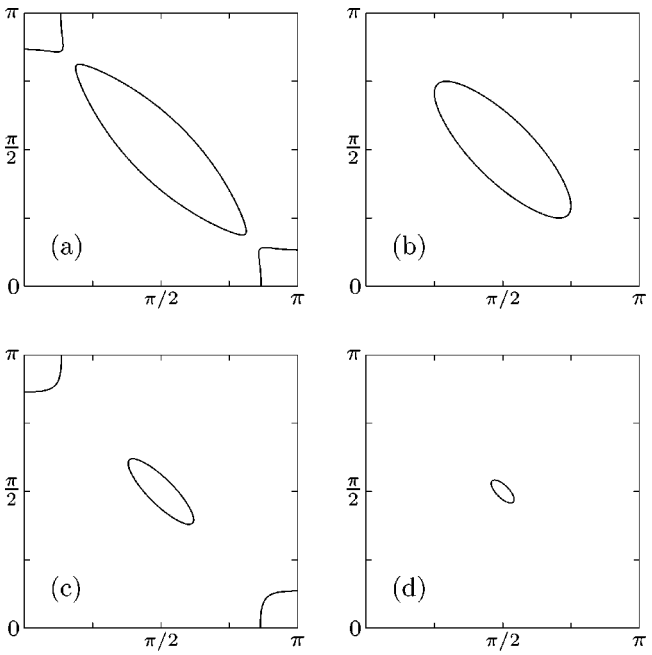

FIG. 2. Fermi surface in the upper right quadrant of the Brillouin zone, for parameter values representing the four regimes described in the text: (a) "weak" regime ($\Delta=0.05t$, $\mu=-0.9t$), (b) "moderate" regime ($\Delta=0.25t$, $\mu=-0.9t$), (c) "ambipolar" regime ($\Delta=0.35t$, $\mu=-0.4t$), (d) "Dirac" regime ($\Delta=0.75t$, $\mu=-0.2t$).

tinodal direction nor Dirac nodes (with the Fermi energy essentially at the node) are of relevance to the normal state of hole-doped cuprates.

In their stead, in this section, we present results for DDW order in the weak (high temperature) and moderate (low temperature) regimes (for which we use, as representative values, $\Delta=0.05t$ and $0.25t$, respectively). Qualitatively, the former regime is where DDW order just begins to set in by disconnecting the Fermi surface into two hole pockets [closed about $(\pm\pi/2,\pi/2)$] and one electron pocket [closed about $(\pi,0)$]. Given the rather short scattering times at these elevated temperatures it is not clear whether such minute changes in the Fermi surface geometry can be discerned (and motivated) from ARPES. However, this regime is virtually unavoidable as one begins one's descent into the pseudogap. It is worth remarking that since the electronlike pocket is only such in its topology (and name), e.g., its contribution to σ_{xy} is holelike, the effect of the DDW order is expected to be rather insignificant. At larger values of Δ and away from half-filling, the regime with moderate DDW order is intended as a caricature of the pseudogap at low temperatures, near the superconducting T_c. Here the electron pocket is absent and only two hole pockets ("arcs"[5]) remain. Perhaps this is the regime of most interest, as the behavior in this regime provides much of the motivation for introducing DDW order to explain the pseudogap in the first place.

Our results in these two regimes (as well as for $\Delta=0$) are presented in Fig. 3. Using Eqs. (5) and (6) we numerically calculated σ_{xx} and σ_{xy}, extracted the Hall angle (and Υ), and evaluated its derivative which appears in the result for the Nernst effect, Eq. (7). Even for the moderate regime, $\Delta=0.25$, the result for $\partial\Upsilon/\partial\mu$ [panel (d)] shows that the overall enhancement in magnitude of the Nernst coefficient is at most by a factor of 3. Moreover, the sign of the Nernst signal (which is opposite to the sign of $\partial\Upsilon/\partial\mu$) remains negative. These are the central results of this section, and are important for the comparison with experiment in Sec. IV.

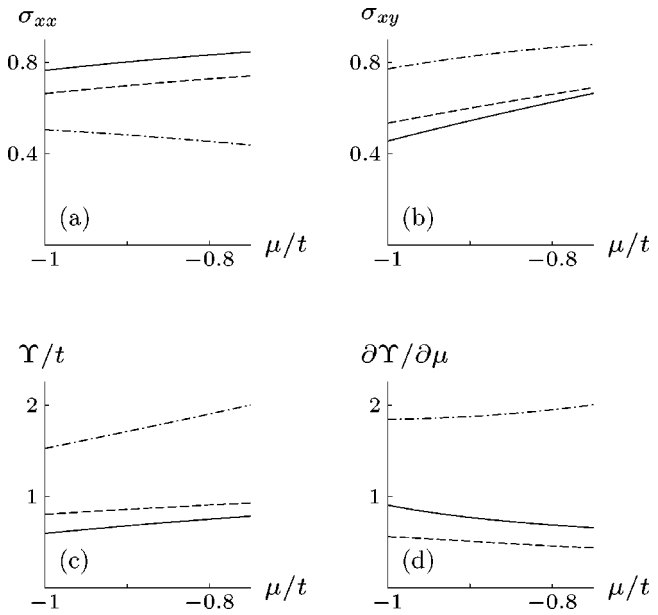

FIG. 3. (a) Conductivity [in units of $(e^2/\hbar)(t\tau/\hbar)$], (b) Hall conductivity [in units of $(e^2/\hbar)(t\tau/\hbar)^2(a^2/\ell_B^2)$] (c) energy scale (or Hall angle) Υ, and (d) $\partial\Upsilon/\partial\mu$ that enters the Nernst coefficient. All quantities are given as a function of chemical potential (in the physically relevant range) for $\Delta=0$ (solid line), $\Delta=0.05$ (dashed line), and $\Delta=0.25$ (dashed-dotted line). Note that a positive $\partial\Upsilon/\partial\mu$ implies a negative Nernst coefficient [see Eq. (7)].

Two other points are perhaps worth noting in these results. First, as anticipated, the effect of a weak DDW is very small (solid vs. dashed lines in Fig. 3). Second, we note that for moderate DDW σ_{xx} is decreasing as μ is increasing, and that this is the main source of the enhanced $\partial\Upsilon/\partial\mu$. This is the beginning of the trend which becomes especially pronounced in the Dirac regime of the model (see Appendix).

IV. EXPERIMENTS

A. HTSC and DDW order

We are now in a position to compare transport properties of the DDW scenario with experimental data, obtained from different underdoped HTSC.[11–14] Consider first the high temperature regime, where the observed effect is small, with a magnitude of about 25 nV/KT, usually negative, roughly independent of temperature, and linear in the applied magnetic fields over a very wide range. These experimental findings are consistent with our discussion of Sec. II (i.e., without DDW), where we argued for a generically negative sign of the effect. To fit the actual magnitude of the effect with our calculations requires taking $k_BT\tau/\hbar\approx0.2$, which is probably somewhat small for the cuprates. Additional factors which may affect the Nernst signal and were not taken into account may include the short scattering time of quasiparticles in the

antinodal region and a dependence of the scattering time on energy (see below). A separate measurement of the contribution of the two terms in Eq. (2) (Refs. 11 and 12) reveals a significant Sondheimer cancellation in some cases (in itself suggestive of the quasiparticle origin of the signal), but not in others, indicating that these additional factors may be more material dependent.

As the temperature is lowered, below an onset temperature which is well separated from the superconducting transition temperature T_c, the Nernst coefficient begins to increase. The signal becomes large and positive, reaching a value of about 1 μV/KT near T_c (depending on material, the increase in magnitude is typically by a factor of 50-100). There is no Sondheimer cancellation in this case, as all of the increase arises from the first term in Eq. (2), while the second goes down to zero. Deviations from linearity in the magnetic field dependence are observed at moderate fields in some samples. Below T_c the line shapes are no longer linear at small fields as one enters the flux flow regime.[30]

The question is then whether the enhancement of the Nernst signal above T_c can be ascribed to DDW order. The results of Sec. III make such prospect unlikely. First, the Nernst coefficient we find when the DDW order is introduced remains *negative*, opposite in sign to the effect in experiment. Second, the enhancement in its magnitude due to modifications of the Fermi surface by the DDW potential is rather modest, a factor of 3 at most. The Nernst effect in the DDW phase is therefore not fundamentally different than what is expected from general notions of quasiparticle transport. Given that $T\tau$ does not change appreciably in the relevant temperature regime (see below) the overall magnitude of the predicted signal is significantly smaller than 1 μV/KT.

At this point it is important to consider whether this conclusion may be modified by changing any of the underlying assumptions. We have investigated to some extent the dependence on different choices of effective band parameters (beyond those reported in this paper), with no change to our conclusions. In addition to band structure, it is important to consider the role of scattering. Indeed, the magnitude of the Nernst signal depends on $T\tau$. This quantity may increase by a factor of 3 in the pseudogap regime as inferred from Hall data[31] (it is independent of temperature according to the conductivity data[32]). We note that even if the scattering time increases dramatically when the temperature is lowered, the magnitude of the Nernst signal will increase, but the sign will not change.

Another assumption we have made is to ignore the contribution arising from the scattering time's dependence on energy. This contribution may be of either sign, and, in particular, if $\partial\tau/\partial\mu<0$ there is an additional contribution to the Nernst signal which is of positive sign. Can this contribution be so strong so as to overwhelm the band structure contribution and lead to a large positive signal? While unlikely, we note that this assumption leads to other discrepancies with the data. In particular, this would require a large contribution to the second term in Eq. (2) (equal to one-half of the contribution to the first term). However, clearly this is not seen in the experiment.[11,12]

B. Other Nernst experiments

Our simple estimate in Sec. II, Eq. (10), suggests that the Nernst effect due to quasiparticles may be quite large if $k_BT\tau/\hbar \gg 1$. Why is it, then, that large Nernst coefficients are not typically observed in conventional metals, where $k_BT\tau/\hbar$ is expected to grow as the system becomes more coherent, its magnitude limited only by the sample's purity? Much of the data in metals[33] is collected in the temperature regime dominated by classical phonons. Here, the resistivity varies linearly with temperature, while the Nernst signal does not show a strong dependence on temperature (with a few notable exceptions, e.g., Ni). There is a considerable variation in the magnitude of the Nernst signal (e.g., $\nu=-0.5$ nV/KT in Pb, $\nu=120$ nV/KT in Nb). In this regime oftentimes $k_BT\tau/\hbar<1$,[24] making our simple estimate in Sec. II broadly consistent with the data (although not a substitute for a detailed material-specific modeling). The situation at lower temperatures is less clear, even if we restrict our attention to electron-phonon scattering (so that the resistivity varies as T^5). In this regime electric and heat currents have different relaxation rates (as deduced from electrical and thermal conductivities), leading to a violation of the Wiedemann-Franz law.[23] This difference might lead to a large conductivity in the denominator of Eq. (2), and hence a suppression of the Nernst signal.

Following the discovery in the hole-doped cuprates, relatively large Nernst signals have been documented in other strongly correlated materials.[21,22,34,35] Perhaps the most obvious difference between these and conventional materials, as far as the Nernst effect is concerned, is in the strong temperature dependence of the observed signal. Since in at least some of these materials the transport is apparently due to quasiparticles, it is of interest to consider the corresponding Nernst measurements in the light of our results. While far from a complete theory the discussion of Sec. II does allow for a few general observations.

In $NbSe_2$ (for 7 K$\lesssim$T$\lesssim$60 K),[21] the Nernst signal is relatively large and negative (except very close to T_c), and hence is naturally attributed to quasiparticles. The Nernst coefficient reaches a maximum, $\nu\approx-0.12$ μV/KT at $T\approx20$ K, roughly the same temperature at which the Hall number passes through zero. The behavior of the Hall number is apparently sensitive to the charge density wave induced reconstruction of the Fermi surface, which occurs below $T_{\mathrm{CDW}}=32.5$ K. As recognized by the authors of Ref. 21, $R_H=0$ suggests an ambipolar origin of the signal due to bands of oppositely charged carriers, reminiscent of that encountered in semiconductors.[36] We note, however, that a simple ambipolar picture leads to a positive Nernst signal (see Appendix), opposite to the experimentally observed signal. It is also interesting to note that while the Hall number remains essentially constant for $T>T_{\mathrm{CDW}}$ the Nernst coefficient grows appreciably (up to 75% of its max value). This suggests to us that the enhanced Nernst coefficient here owes its existence as much (if not more) to enhanced coherence in the system as to an ambipolar compensation. Alternately, perhaps, the Nernst measurement is sensitive to incipient CDW order, more so than R_H.[39]

Yet a larger Nernst signal was reported very recently in the normal state of $CeCoIn_5$, a heavy fermion compound.[22]

Here, the Nernst signal is also negative and can be as high as 2 μV/KT at low temperatures, suggestive of a highly coherent state (large τ). As discussed in Sec. II, an increased coherence is expected to be accompanied by a reduction of the linear response regime (in magnetic field). Indeed, the measurements in $CeCoIn_5$ are highly suggestive of such behavior.[37]

V. CONCLUSIONS

The pseudogap is typically envisaged as a strongly fluctuating crossover (possibly quantum critical) regime where superconductivity and possibly other orders are present in some form, yet not fully condensed. Theories implementing this intuitive picture are necessarily subtle and complex (and rarely applicable directly to experiments). The DDW proposal is a breath of fresh air in that regard, here the pseudogap phase is a true phase of matter where the standard crisp notions of order parameter and quasiparticles apply and can be used to make predictive statements. In particular, most of the effects of DDW order discussed to date can be linked directly to the changes of the Fermi surface geometry due to the breaking of the translational symmetry. Implications of such Fermi surface reconstruction for thermodynamic and dynamic properties of the system (including transport) were addressed in the past (see, e.g., Ref. 4) and found consistent with the observed phenomenology.

In this work we have extended this investigation, focusing in particular on the Nernst effect. Our main result is that quasiparticle transport inside the DDW phase cannot explain the Nernst phenomena observed in hole-doped cuprates.

This conclusion is based on a Boltzmann theory based calculation.[38] The Nernst coefficient has a contribution originating from the changes of the Fermi surface shape, which we compute explicitly, and one from the changes in the scattering rate (treated as a phenomenological input). The first contribution alone predicts a negative Nernst coefficient, which is in contradiction with the experiment. It is possible that upon inclusion of the second term the overall sign can be reversed, however, we presented arguments for why the overall magnitude cannot approach the experimentally observed signal. We believe, therefore, that the physics of large Nernst effect in the cuprates lies elsewhere.

Our general analysis can be applied, albeit qualitatively, to the recent experiments of Bel *et al.* on $NbSe_2$ and $CeCoIn_5$. It suggests that the data in these materials is not inconsistent with a quasiparticle based interpretation, although a more detailed analysis is required. Likewise, it would be interesting to examine other cases (including, e.g., conventional density waves and other strongly correlated systems) along similar lines.

ACKNOWLEDGMENTS

We thank Kamran Behnia, Paul Chaikin, Sudip Chakravarty, Eduardo Fradkin, Leonid Glazman, David Huse, Steve Kivelson, Andy Millis, Phuan Ong, Weida Wu, and especially Shivaji Sondhi for numerous enlightening discussions and for their valuable input. This research was supported by the NSF through Grants No. EIA 02-10736 (IU), DMR 99-78074, DMR 02-13706, and by the David and Lucile Packard foundation.

APPENDIX: DDW ORDER AND PARTICLE-HOLE SYMMETRY

The discussion of DDW order in Sec. III was limited by its potential relevance to the cuprates. In this section, we explore other regimes of the DDW Hamiltonian, Eq. (12), that may be of broader interest (they may also arise in conventional density waves). Particle-hole symmetry (either exact, $\sigma_{xy}=\alpha_{xx}=0$, or approximate) plays an important role in both of the regimes considered below.

1. Ambipolar DDW

At moderately strong DDW order and in the vicinity of half-filling (we take here $\Delta=0.35t$ and $\mu=-0.4t$ as representative values), the DDW model produces a well defined electronic pocket at $(\pi,0)$, in addition to the hole pockets at $(\pm\pi/2,\pi/2)$ [see Fig. 2(c)]. The Nernst coefficient in this regime is positive and modestly enhanced. For example, for the parameters above, $\partial\Upsilon/\partial\mu=-1.99$. The reason for this behavior may be traced to the existence of two types of carriers in the system.

As a representative model of this behavior consider two oppositely charged but otherwise identical species of free carriers. We then have

$$\sigma_{xx}=(n_h+n_e)\frac{e^2\tau}{m},\quad \sigma_{xy}=(n_h-n_e)\frac{e^2\tau}{m}\omega_c\tau, \tag{A1}$$

leading to a Nernst coefficient

$$\nu=\frac{2\pi^2}{3}\frac{k_B^2T}{eB}\frac{n_en_h}{(n_e+n_h)^2}\left(\frac{1}{\epsilon_F^e}+\frac{1}{\epsilon_F^h}\right)\omega_c\tau, \tag{A2}$$

where $\partial n_e/\partial\mu=n_e/\epsilon_F^e$, appropriate for two dimensions, was used (and similarly for the hole band). This ambipolar Nernst signal is maximal when the bands are exactly compensated, and is positive. Substituting $n_e=n_h=k_F^2/4\pi$ and rewriting

$$B\nu=\frac{2\pi^2}{3}\frac{k_B}{e}\frac{k_BT\tau}{\hbar}\frac{1}{(k_F\ell_B)^2}, \tag{A3}$$

we arrive at an expression which can be compared against the lattice result in Eq. (7). Provided we loosely identify $k_F\sim 1/a$, the comparison suggests that the ambipolar Nernst effect need not be particularly larger then a signal from a single band of carriers: long scattering times are essential for either Eq. (7) or Eq. (A3) to produce a substantial Nernst coefficient. In situations when k_F and a are unrelated Eq. (A3) may lead to significant enhancements of Nernst signals (as may be the case of Fermi pockets associated with spin-density waves[39]).

2. Dirac DDW

Near half-filling and with strong DDW order ($\Delta>0.6t$), the dispersion around the Fermi surface approaches that of a

Dirac particle, with dispersion $\epsilon_k^{\pm} = \pm\hbar\sqrt{(v_F k_x)^2 + (v_\Delta k_y)^2}$, where the momenta here are measured relative to $(\pm\pi/2, \pi/2)$ along the diagonals of the Brillouin zone. The density of states of the Dirac Hamiltonian in two dimensions vanishes at the node, $N(\epsilon) = |\epsilon|/(2\pi\hbar^2 v_x v_y)$, giving rise to a host of unusual properties. To simplify the discussion we shall consider an isotropic case, $v_x = v_y = v$.

Provided the chemical potential (or scattering time) is sufficiently large ($\mu\tau > \hbar$) we can still neglect interband scattering, and use Boltzmann theory in the weak field regime. We then have

$$\sigma_{xx} = \frac{e^2\tau}{4\pi\hbar^2}|\mu|, \quad \sigma_{xy} = \frac{e^3 B\tau^2 v^2}{4\pi\hbar^2 c}\mathrm{sign}(-\mu). \tag{A4}$$

Here the dissipative conductivity is proportional to the density of states at the Fermi energy (hence the factor of $|\mu|$), while the Hall conductivity can be obtained by multiplying the conductivity by the Hall angle

$$\Theta_H = \omega_c\tau\,\mathrm{sign}(-\mu) = \frac{eB\tau}{m^* c}\mathrm{sign}(-\mu) = -\frac{eBv^2\tau}{\mu c}, \tag{A5}$$

where $m^* = \hbar k_F/v$. As a result σ_{xy} is constant on either side of the node. Finally, the Nernst coefficient

$$\nu = -\frac{\pi^2}{3}\frac{k_B^2 T v^2}{c}\frac{1}{\mu^2} \tag{A6}$$

mirrors the divergence of the Hall angle (as $\mu \to 0$) [cf. Eq. (4)]. As discussed in Sec. II, this enhancement is necessarily accompanied by a reduced regime of linear response in magnetic field (with Q_{xy} decreasing with magnetic field for $\omega_c\tau \gtrsim 1$). It is also interesting to note that for the linearized Dirac spectrum used here $\alpha_{xy} = 0$ and the large Nernst coefficient comes entirely from the enhancements in the second term in Eq. (2) due to reduced conductivity.

Eventually, for $\mu\tau < \hbar$, the Boltzmann equation is no longer applicable, and interband contributions to transport become important [see Eq. (14)]. Instead, a careful evaluation of Kubo formulae treating the interband and intraband terms on equal footing is necessary.[40,41] Qualitatively, the singularities of the conductivity tensor at $\mu = 0$ implied by the Boltzmann analysis above become smoothed. In particular, the Hall angle [Eq. (A5)] rapidly changes its value as a function of μ, passing through 0 at $\mu = 0$, consistent with the particle-hole symmetry at the node. The Nernst coefficient at the node may then become large and positive [cf. Eq. (4)], arising entirely from an unusually large[41] α_{xy} and "universally"[40,42] small $\sigma \sim e^2/\hbar$,

$$B\nu = c_1\frac{k_B}{e}\frac{k_B T\tau}{\hbar}\frac{Bev^2\tau^2}{c\hbar} = c_1\frac{k_B}{e}\frac{k_B T\tau}{\hbar}\left(\frac{\epsilon_0\tau}{\hbar}\right)^2, \tag{A7}$$

where c_1 is a numerical constant. The energy ϵ_0 can be identified as the energy gap between two lowest Landau levels of the Dirac Hamiltonian (in the absence of disorder).

Equation (A7) seemingly opens the door to a positive Nernst signal which can become very large in a clean system (as $\nu \propto \tau^3$).[43] Here, we note that the possibility of observing a large Nernst signal is very restricted (even if τ is large). First, the chemical potential must happen to be very close to the node, $|\mu| \hbar/\tau$. Second, assuming $\mu = 0$, a Nernst signal which is linear in magnetic field will only be observed provided $\epsilon_0 \lesssim \hbar/\tau$. In other words, in a very clean system, the Nernst signal as a function of magnetic field will have a large initial slope, and will very rapidly reach its maximal value of $Q_{xy} \approx k_B^2 T\tau/e\hbar$ (as in the case of Boltzmann results above).

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[28] The presence of open orbits can lead to different large B behavior. We thank W. Wu for pointing this out to us.

[29] A somewhat different situation can arise if the mean free path of quasiparticles is fixed and independent of their momentum, e.g., due to scattering off strong impurities (Ref. 39). In this case $1/\tau(\mathbf{k})=v_{\mathbf{k}}/\ell$, where ℓ is the average distance between impurities. This can lead to an enhanced Nernst signal in two dimensions.

[30] Below T_c, for weak magnetic field, the vortices are pinned and $\nu=0$. When the magnetic field is increased above a threshold, the Nernst signal Q_{xy} develops and quickly becomes large and positive as the vortices become more dense and easier to depin (Ref. 11). A similar nonlinearity is expected for low fields in the dependence of the measured electric field on temperature gradient; above a certain threshold, the temperature gradient will be sufficiently strong to depin the vortices (Ref. 15).

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[38] We have not explicitly considered the effects of collective behavior of the DDW order, which is not expected to modify our conclusions. The DDW order is not charged, and provided its collective modes remain gapped they will presumably only renormalize the effective parameters of the model and appropriate scattering times. The latter were treated by us as phenomenological inputs.

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