

# First-principles calculation of the intersublattice exchange interactions and Curie temperatures of the full Heusler alloys $\text{Ni}_2\text{MnX}$ ( $X=\text{Ga, In, Sn, Sb}$ )

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The interatomic exchange interactions and Curie temperatures in Ni-based full Heusler alloys  $\text{Ni}_2\text{MnX}$  with  $X=\text{Ga, In, Sn, and Sb}$  are studied within the framework of the density-functional theory. The calculation of the exchange parameters is based on the frozen-magnon approach. Despite closeness of the experimental Curie temperatures for all four systems, their magnetism appeared to differ strongly. This difference involves both the Mn-Mn and Mn-Ni exchange interactions. The Curie temperatures,  $T_C$ , are calculated within the mean-field approximation by solving a matrix equation for a multisublattice system. Good agreement with experiment for all four systems is obtained. The role of different exchange interactions in the formation of  $T_C$  of the systems is discussed.

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## I. INTRODUCTION

Much effort is currently devoted to the search for ferromagnetic materials suitable for application in semiconductor spintronics devices. To allow efficient spin injection into the semiconductor, these materials must satisfy a number of conditions. In particular, they must have a Curie temperature noticeably higher than the room temperature, be compatible with the semiconductors used industrially, and possess a very high spin polarization of the electron states at the Fermi level.<sup>1</sup> Some of the Heusler compounds were found to have a half-metallic ground state,<sup>2,3</sup> which is characterized by a 100% carrier spin polarization.

A feature of other Heusler alloys that is useful for spintronic applications is the very small lattice mismatch with widely employed semiconductors (e.g.,  $\text{Ni}_2\text{MnIn}$  and  $\text{InAs}$ ).<sup>4-6</sup> Among further prominent physical properties of this class of materials one can mention a martensitic transformation in  $\text{Ni}_2\text{MnGa}$  that takes place below the Curie temperature.<sup>7</sup> An interesting combination of physical properties makes Heusler alloys the subject of intensive experimental and theoretical investigations, which could motivate their use as multifunctional materials in practical applications.<sup>8-14</sup>

In the present work we report the theoretical study of the exchange interactions and Curie temperature of four full Heusler compounds  $\text{Ni}_2\text{MnX}$  with  $X=\text{Ga, In, Sn, and Sb}$ . Experimentally, all of them are ferromagnetic and have similar values of the Curie temperature.<sup>20</sup>

The first important contribution to the density functional theory study of these systems was made in an early paper by Kübler *et al.*,<sup>15</sup> where the microscopic mechanisms of the magnetism of Heusler alloys was discussed on the basis of the calculation of the ferromagnetic and antiferromagnetic alignments of the Mn moments. Recently a detailed study of the magnetic interactions in  $\text{Ni}_2\text{MnGa}$  and  $\text{Ni}_2\text{MnAl}$  was reported by Enkovaara *et al.*<sup>16</sup> The authors used planar spiral configurations and have shown that the Ni sublattice plays an important role in the magnetic properties of the system.

The main purpose of the present work is a detailed study of the exchange interactions in these systems. In particular

we report a systematic study of the exchange interaction between atoms of different sublattices and show that the pattern of exchange interactions in these systems deviates strongly from the physical picture that can be expected on the basis of the experimental information available. Indeed, the common crystal structure, similar chemical composition, and close experimental values of the Curie temperature make the assumption natural in that the exchange interactions of these systems are similar. Our study shows, however, that this assumption is not correct. The exchange interactions vary widely depending on the  $X$  constituent. In particular, the intersublattice interactions change strongly from system to system. We show that different exchange interactions lead, in agreement with experiment, to similar values of the Curie temperatures. We analyze the relation between the properties of the exchange interactions and the Curie temperatures.

The paper is organized as follows. In Sec. II we present the calculational approach. Section III contains the results of the calculations and discussion. Section IV gives the conclusions.

## II. CALCULATIONAL APPROACH

The calculations are carried out with the augmented spherical waves method<sup>17</sup> within the atomic-sphere approximation.<sup>18</sup> The exchange-correlation potential is chosen in the generalized gradient approximation.<sup>19</sup> A dense Brillouin zone (BZ) sampling  $30 \times 30 \times 30$  is used. In all calculations the experimental values of the lattice parameters are used (Table I).<sup>20</sup> The radii of all atomic spheres are chosen equal.

We describe the interatomic exchange interactions in terms of the classical Heisenberg Hamiltonian

$$H_{eff} = - \sum_{\mu, \nu} \sum_{\mathbf{R}, \mathbf{R}'} J_{\mathbf{R}\mathbf{R}'}^{\mu\nu} \mathbf{s}_{\mathbf{R}}^{\mu} \cdot \mathbf{s}_{\mathbf{R}'}^{\nu}. \quad (1)$$

( $\mu\mathbf{R} \neq \nu\mathbf{R}'$ )

In Eq. (1), the indices  $\mu$  and  $\nu$  represent different sublattices,  $\mathbf{R}$  and  $\mathbf{R}'$  are the lattice vectors specifying the atoms within

TABLE I. Experimental lattice parameters and magnetic moments (in  $\mu_B$ ) of  $\text{Ni}_2\text{MnX}$  ( $X=\text{Ga, In, Sn, Sb}$ ).

	$a$ (a.u.)	Mn	Ni	$X$	Cell
$\text{Ni}_2\text{MnGa}$	11.058 <sup>a</sup>	3.570	0.294	-0.068	4.090
		(3.43 <sup>b</sup> )	(0.36 <sup>b</sup> )	(-0.04 <sup>b</sup> )	(4.11 <sup>b</sup> ) (4.17 <sup>c</sup> )
$\text{Ni}_2\text{MnIn}$	11.468 <sup>a</sup>	3.719	0.277	-0.066	4.208
$\text{Ni}_2\text{MnSn}$	11.419 <sup>a</sup>	3.724	0.206	-0.057	4.080
		(3.53 <sup>d</sup> )	(0.24 <sup>d</sup> )	(-0.03 <sup>d</sup> )	(4.08 <sup>d</sup> )
$\text{Ni}_2\text{MnSb}$	11.345 <sup>a</sup>	3.696	0.143	-0.033	3.950

<sup>a</sup>Reference 20.

<sup>b</sup>Reference 25.

<sup>c</sup>Reference 26 (expt.).

<sup>d</sup>Reference 27.

sublattices, and  $\mathbf{s}_\mathbf{R}^\mu$  is the unit vector pointing in the direction of the magnetic moment at site  $(\mu, \mathbf{R})$ . The systems considered contain three  $3d$  atoms in the unit cell with positions  $(\frac{1}{4}, \frac{1}{4}, \frac{1}{4})$  for the Mn atom and  $(0, 0, 0)$  and  $(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$  for two Ni atoms.

We employ the frozen-magnon approach<sup>21-23</sup> to calculate interatomic Heisenberg exchange parameters. The calculations involve few steps. In the first step, the exchange parameters between the atoms of a given sublattice  $\mu$  are computed. The calculation is based on the evaluation of the energy of the frozen-magnon configurations defined by the following atomic polar and azimuthal angles:

$$\theta_\mathbf{R}^\mu = \theta, \quad \phi_\mathbf{R}^\mu = \mathbf{q} \cdot \mathbf{R} + \phi^\mu. \quad (2)$$

In the calculation discussed in this paper the constant phase  $\phi^\mu$  is always chosen equal to zero. The magnetic moments of all other sublattices are kept parallel to the  $z$  axis. Within the Heisenberg model (1) the energy of such a configuration takes the form

$$E^{\mu\mu}(\theta, \mathbf{q}) = E_0^{\mu\mu}(\theta) + \sin^2 \theta J^{\mu\mu}(\mathbf{q}), \quad (3)$$

where  $E_0^{\mu\mu}$  does not depend on  $\mathbf{q}$  and the Fourier transform  $J^{\mu\nu}(\mathbf{q})$  is defined by

$$J^{\mu\nu}(\mathbf{q}) = \sum_{\mathbf{R}} J_{0\mathbf{R}}^{\mu\nu} \exp(i\mathbf{q} \cdot \mathbf{R}). \quad (4)$$

In the case of  $\nu=\mu$  the sum in Eq. (4) does not include  $\mathbf{R}=\mathbf{0}$ . Calculating  $E^{\mu\mu}(\theta, \mathbf{q})$  for a regular  $\mathbf{q}$  mesh in the Brillouin zone of the crystal and performing back Fourier transformation, one gets exchange parameters  $J_{0\mathbf{R}}^{\mu\mu}$  for sublattice  $\mu$ .

To determine the exchange interactions between the atoms of two different sublattices  $\mu$  and  $\nu$ , the frozen-magnon configurations [Eq. (2)] are formed for both sublattices. The Heisenberg energy of such configurations takes the form

$$E^{\mu\nu}(\theta, \mathbf{q}) = E_0^{\mu\nu}(\theta) + \sin^2 \theta [J^{\mu\mu}(\mathbf{q}) + J^{\nu\nu}(\mathbf{q})] + 2 \sin^2 \theta \text{Re} J^{\mu\nu}(\mathbf{q}), \quad (5)$$

where  $E_0^{\mu\nu}(\theta)$  is a  $\mathbf{q}$ -independent part. Calculating  $[E^{\mu\nu}(\theta, \mathbf{q}) - E^{\mu\nu}(\theta, 0)]$  and subtracting single-sublattice contributions known from the previous step one finds  $[\text{Re} J^{\mu\nu}(\mathbf{q}) - \text{Re} J^{\mu\nu}(\mathbf{0})]$ . The back Fourier transformation of this expression gives for  $\mathbf{R} \neq \mathbf{0}$  the following combinations of the interatomic exchange parameters:  $J_{\mathbf{R}}^{\mu\nu} \equiv \frac{1}{2}(J_{0\mathbf{R}}^{\mu\nu} + J_{0(-\mathbf{R})}^{\mu\nu})$ . In general, one needs to perform similar calculations for different phases  $\phi^\mu$  in Eq. (2) to get another linear combination of the parameters to be able to separate them. For the systems considered these calculations can be avoided by taking into account the symmetry of the lattice (see below).

$J_{\mathbf{R}}^{\mu\nu}$  does not contain information about the interaction of the atoms within the first unit cell corresponding to  $\mathbf{R}=\mathbf{0}$ . These exchange parameters can be found in the calculations for magnetic configurations periodic with the periodicity of the lattice. The atoms in the unit cell are separated into two groups. Within each group the moments are parallel. The moments from the different groups form an angle  $\theta$ . The energies of such magnetic configurations are expressed through the sums  $J_0^{\mu\nu} \equiv \sum_{\mathbf{R}} J_{0\mathbf{R}}^{\mu\nu}$ . Since the sums  $\sum_{\mathbf{R} \neq 0} J_{0\mathbf{R}}^{\mu\nu}$  are known from the preceding step the parameters with  $\mathbf{R}=\mathbf{0}$  become accessible. The symmetry relation  $J_{00}^{\mu\nu} = J_{0\mathbf{R}}^{\mu\nu}$  for  $\mathbf{R} = (0, \frac{1}{2}, \frac{1}{2})$  allows us to split the sums  $J_{\mathbf{R}}^{\mu\nu}$  to individual parameters.

The Curie temperature is estimated within the mean-field approximation (MFA) for a multisublattice material by solving the system of coupled equations<sup>24</sup>

$$\langle s^\nu \rangle = \frac{2}{3k_B T} \sum_{\nu} J_0^{\mu\nu} \langle s^\nu \rangle, \quad (6)$$

where  $\langle s^\nu \rangle$  is the average  $z$  component of  $\mathbf{s}_\mathbf{R}^\nu$ . Equation (6) can be represented in the form of the eigenvalue matrix problem

$$(\Theta - T\mathbf{I})\mathbf{S} = 0, \quad (7)$$

where  $\Theta_{\mu\nu} = (2/3k_B)J_0^{\mu\nu}$ ,  $\mathbf{I}$  is a unit matrix and  $\mathbf{S}$  is the vector of  $\langle s^\nu \rangle$ . The largest eigenvalue of matrix  $\Theta$  gives the value of the Curie temperature.<sup>24</sup>

### III. RESULTS AND DISCUSSION

In Table I we present calculated magnetic moments. For comparison, the available experimental values of the moments and the results of previous calculations are presented. The relative variation of the Mn moment is small. On the other hand, the moment of Ni and  $X$  atoms show strong relative variation and are in  $\text{Ni}_2\text{MnSb}$  about two times smaller than in  $\text{Ni}_2\text{MnGa}$  or  $\text{Ni}_2\text{MnIn}$ . The values of the magnetic moments are in good agreement with the results of previous calculations.

The calculated Heisenberg exchange parameters are presented in Fig. 1. As mentioned in the Introduction, the assumption that the closeness of the experimental Curie temperatures is the consequence of the similarity of the exchange interactions is not confirmed by the calculations.

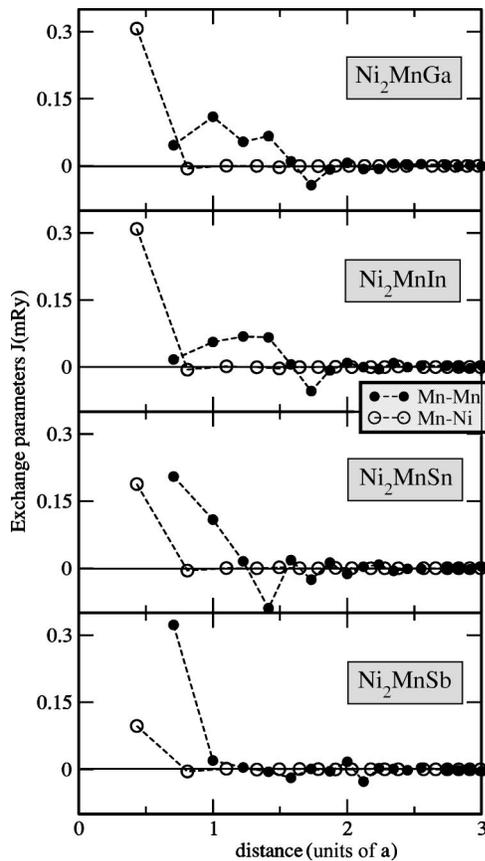


FIG. 1. The parameters of the Mn-Mn (intrasublattice) exchange interactions and Mn-Ni (intersublattice) exchange interactions in  $\text{Ni}_2\text{MnX}$  ( $X=\text{Ga}, \text{In}, \text{Sn}, \text{Sb}$ ). The distances are given in the units of the lattice constant. The significance of the oscillations of the exchange parameters is verified by varying the  $\mathbf{q}$  mesh in the frozen-magnon calculations.

We obtain strong dependence of the exchange interactions on the type of  $X$  atom. For  $X=\text{Ga}$  and  $X=\text{In}$ , which belong to the same column of the Mendeleev's table (see inset in Fig. 2), we obtain a similar pattern of Heisenberg exchange parameters. On the other hand, for  $X$  atoms belonging to different columns the changes in the exchange interactions are very strong (Fig. 1). These changes concern both the Mn-Mn intrasublattice interactions and the Ni-Mn intersublattice interaction.

Considering the Mn-Mn interactions, we notice that in  $\text{Ni}_2\text{MnGa}$  and  $\text{Ni}_2\text{MnIn}$  the interaction with the coordination spheres from the first to the fourth is positive. The interaction with the first coordination sphere is weaker than with the following ones. The interaction with the fifth sphere is very small. The interaction with the sixth sphere is negative. The interaction with further coordination spheres is very weak.

In  $\text{Ni}_2\text{MnSn}$  the interaction with the first sphere strongly increases compared with  $\text{Ni}_2\text{MnGa}$  and  $\text{Ni}_2\text{MnIn}$ . On the other hand, the interaction with the third sphere becomes small. The interaction with the fourth sphere is strongly negative. The interaction with further neighbors are weak.

The trend observed in transition from  $\text{Ni}_2\text{MnGa}$  and  $\text{Ni}_2\text{MnIn}$  to  $\text{Ni}_2\text{MnSn}$  becomes even stronger in the case of

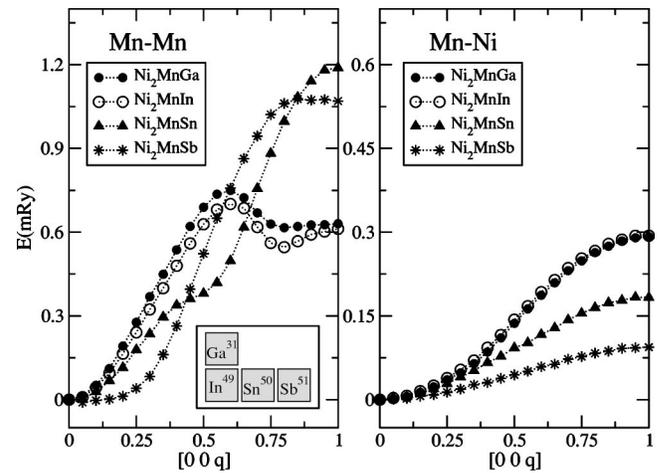


FIG. 2. Frozen-magnon energies as a function of the wave vector  $\mathbf{q}$  (in units of  $2\pi/a$ ) in  $\text{Ni}_2\text{MnX}$  for Mn-Mn (left) and Mn-Ni interactions (right).

$X=\text{Sb}$ . Here the interaction with the first neighbor increases further and is the only strong exchange interaction between the Mn atoms.

The intersublattice Mn-Ni interaction behaves very differently. A sizable interaction takes place only between nearest neighbors. This interaction is very strong in  $\text{Ni}_2\text{MnGa}$  and  $\text{Ni}_2\text{MnIn}$  and quickly decreases for  $X=\text{Sn}$  and, especially,  $X=\text{Sb}$ .

To reveal the physical origin of the strong difference in the exchange parameters of these systems we plot in Fig. 2 the frozen-magnon energies as a function of wave vector  $\mathbf{q}$  for one direction in the Brillouin zone.<sup>28</sup> We remind the reader that the interatomic exchange parameters are Fourier transforms of the  $E(\mathbf{q})$  functions and therefore reflect their form. Indeed,  $E(\mathbf{q})$  curves for  $\text{Ni}_2\text{MnGa}$  and  $\text{Ni}_2\text{MnIn}$  are close to each other, leading to a similar set of interatomic exchange parameters (Fig. 1). These curves deviate strongly from a simple cosinusoid having a maximum at  $\mathbf{q}$  about 0.6 and a rather weak variation at  $\mathbf{q} > 0.6$ .<sup>29</sup> The complexity of  $E(\mathbf{q})$  means that several Fourier components are needed to describe the features of the function. This is reflected in the Heisenberg parameters of  $\text{Ni}_2\text{MnGa}$  and  $\text{Ni}_2\text{MnIn}$ .

On the other hand, the  $E(\mathbf{q})$  curve of  $\text{Ni}_2\text{MnSb}$  is well described by one cosinusoid (Fig. 2) that results in a single large Mn-Mn exchange parameter (Fig. 1). The  $E(\mathbf{q})$  of  $\text{Ni}_2\text{MnSn}$  assumes an intermediate position from the viewpoint of the complexity of the function. This property is also reflected in the exchange parameters (Fig. 1).

Note that the character of the  $\mathbf{q}$  dependence of the energy illustrated by Fig. 2 is a consequence of the properties of the electronic structure of the compounds. Indeed, in Fig. 3 we see that the electronic structures of  $\text{Ni}_2\text{MnGa}$  and  $\text{Ni}_2\text{MnIn}$  are similar. Transition along the row In-Sn-Sb leads to increasing difference in the electron spectrum. This increasing difference can be traced back to the change in the number of valence electrons: a Sb atom has two more valence electrons than In and one more electron than Sn. As the result an important difference in the electron structure of the system is

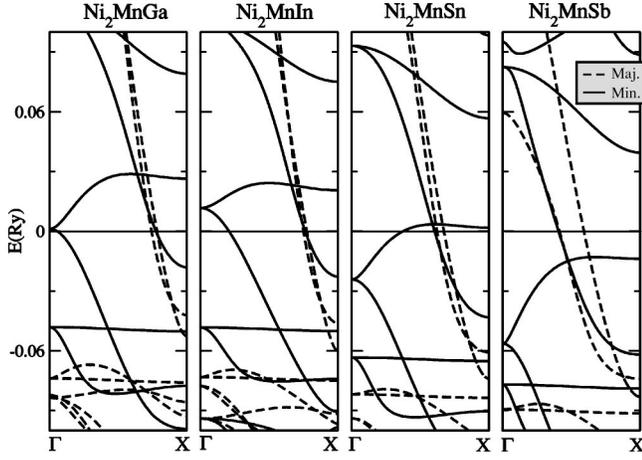


FIG. 3. Band structure of  $\text{Ni}_2\text{MnX}$  along the high symmetry line ( $\Gamma-X$ ).

a relative shift of the Fermi level to a higher energy position in the sequence In-Sn-Sb. This shift is clearly seen in the density of states (DOS) presented in Fig. 4. The positions of the same features of the DOS in different systems are well described by linear functions with negative angle coefficients. For the Mn peaks all three lines are almost parallel. This means that the change in the Mn DOS from system to system can be treated as a rigid shift with respect to the Fermi level. In the case of the Ni DOS the situation is more complicated since, besides the variation of the electron number, an additional influence on the peak positions is exerted by the variation of the Ni atomic moment (see Table I).

The  $E(\mathbf{q})$  curves determining the Mn-Ni interactions are presented in Fig. 2. The form of the curves is in all cases close to a cosinusoid. Therefore, only one exchange parameter has sizable value. The strength of the interaction is in correlation with the value of the magnetic moment of the Ni atoms.

The interatomic exchange parameters are used to evaluate the Curie temperature. If only the Mn-Mn interactions are taken into account we obtain values shown in Table II. Despite very strong difference in the Mn-Mn exchange parameters in these systems the difference in the corresponding Curie temperatures is not very large. The explanation for this result is the property that in MFA to a one-sublattice ferromagnet the value of the Curie temperature is determined by the sum of the interatomic exchange interactions  $J_0 = \sum_{\mathbf{R} \neq 0} J_{0\mathbf{R}}$ .  $J_0$  gives the average value of  $E(\mathbf{q})$  and is less sensitive to the detailed form of the  $E(\mathbf{q})$  function.

The comparison of the Curie temperatures calculated with the use of the Mn-Mn exchange parameters only with experimental Curie temperatures shows that the agreement with experiment is not in general good. In the case of  $\text{Ni}_2\text{MnGa}$  the error is about 30%.

Account for intersublattice interactions improves the agreement with experimental  $T_C$  values considerably (Table II). This shows that the Ni moment provides a magnetic degree of freedom which plays an important role in the thermodynamics of the system.

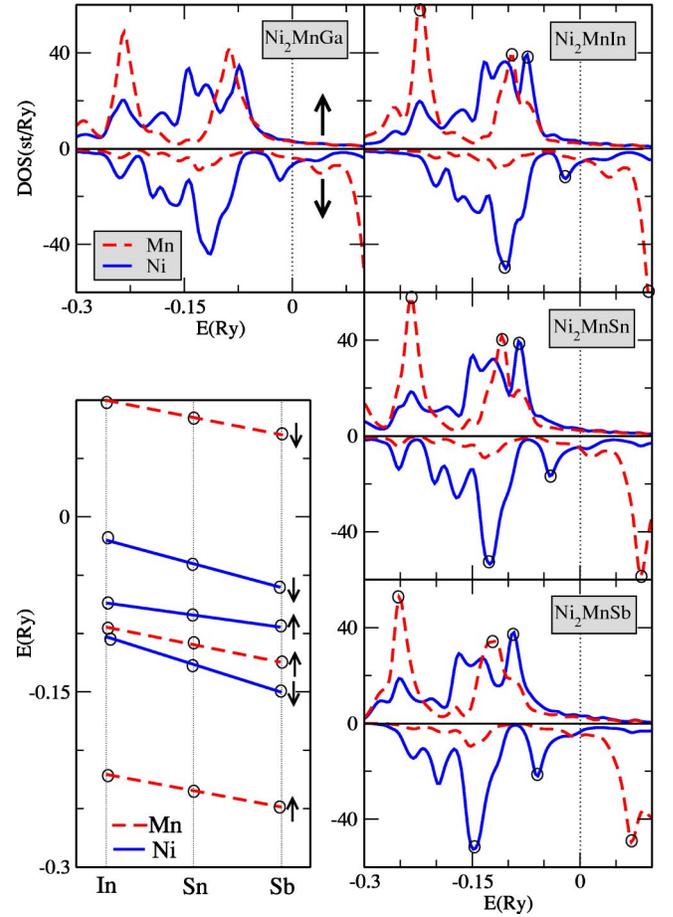


FIG. 4. Spin-projected partial density of states of  $\text{Ni}_2\text{MnX}$ . The separate graph in the lower left-hand part of the figure shows the variation of the positions of a number of the density-of-states (DOS) peaks for the In-Sn-Sb series of compounds. The lines in this graph are guides for the eye. The energies are measured with respect to the Fermi level of the corresponding system. The small open circles on the DOS curves mark the positions of the peaks. Arrows show spin projections. The analysis of the states at the Fermi level shows that the main contributions comes from the 3d states of Mn and Ni.

#### IV. CONCLUSION

In conclusion, we have systematically studied exchange interactions and Curie temperatures in Ni-based full Heusler alloys  $\text{Ni}_2\text{MnX}$  ( $X = \text{Ga}, \text{In}, \text{Sn}, \text{Sb}$ ) within the parameter-free density-functional theory. Our calculations show that despite

TABLE II. Mean-field estimation of the Curie temperatures for  $\text{Ni}_2\text{MnX}$  ( $X = \text{Ga}, \text{In}, \text{Sn}, \text{Sb}$ ). The experimental Curie temperature values are taken from Ref. 20.

	$T_C^{\text{Mn-Mn}[MFA]}$ (K)	$T_C^{\text{[MFA]}}$ (K)	$T_C^{\text{[Expt.]}}$ (K)
$\text{Ni}_2\text{MnGa}$	302	389	380
$\text{Ni}_2\text{MnIn}$	244	343	315
$\text{Ni}_2\text{MnSn}$	323	358	360
$\text{Ni}_2\text{MnSb}$	343	352	365

the similarity of the Curie temperatures of these systems there is a strong difference in the underlying magnetic interactions. This difference involves both the Mn-Mn and Mn-Ni exchange interactions, which depend strongly on the X constituent. The Curie temperatures are calculated within the mean-field approximation to the classical Heisenberg Hamiltonian by solving a matrix equation for a multisublattice system. Good agreement with experiment for all four systems is obtained.

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- <sup>1</sup>H. Ohno, *Science* **281**, 951 (1998).  
<sup>2</sup>R. A. de Groot, F. M. Mueller, P. G. van Engen, and K. H. J. Buschow, *Phys. Rev. Lett.* **50**, 2024 (1983).  
<sup>3</sup>I. Galanakis, P. H. Dederichs, and N. Papanikolaou, *Phys. Rev. B* **66**, 174429 (2002).  
<sup>4</sup>K. A. Kilian and R. H. Victora, *J. Appl. Phys.* **87**, 7064. (2000).  
<sup>5</sup>J. Q. Xie, J. W. Dong, J. Lu, C. J. Palmström, and S. McKernan, *Appl. Phys. Lett.* **79**, 1003 (2001).  
<sup>6</sup>M. Kurfiss and R. Anton, *J. Alloys Compd.* **361**, 36 (2003).  
<sup>7</sup>P. J. Webster, K. R. A. Ziebeck, S. L. Town, and M. S. Peak, *Philos. Mag. B* **49**, 295 (1984).  
<sup>8</sup>L. Ritchie, G. Xiao, Y. Ji, T. Y. Chen, C. L. Chien, M. Zhang, J. Chen, Z. Liu, G. Wu, and X. X. Zhang, *Phys. Rev. B* **68**, 104430 (2003).  
<sup>9</sup>M. P. Raphael, B. Ravel, M. A. Willard, S. F. Cheng, B. N. Das, R. M. Stroud, K. M. Bussmann, J. H. Claassen, and V. G. Harris, *Appl. Phys. Lett.* **79**, 4396 (2001).  
<sup>10</sup>S. Picozzi, A. Continenza, and A. J. Freeman, *Phys. Rev. B* **66**, 094421 (2002).  
<sup>11</sup>S. Plogmann, T. Schlathöler, J. Braun, M. Newmann, Y. M. Yarmoshenko, M. Yablonskikh, E. I. Shreder, E. Z. Kurmaev, A. Wrona, and A. Slebarski, *Phys. Rev. B* **60**, 6428 (1999).  
<sup>12</sup>J. Kübler, *Phys. Rev. B* **67**, 220403 (2003).  
<sup>13</sup>G. A. de Wijs and R. A. de Groot, *Phys. Rev. B* **64**, 020402 (2001).  
<sup>14</sup>M. P. Raphael, B. Ravel, Q. Huang, M. A. Willard, S. F. Cheng, B. N. Das, R. M. Stroud, K. M. Bussmann, J. H. Claassen, and V. G. Harris, *Phys. Rev. B* **66**, 104429 (2002).  
<sup>15</sup>J. Kübler, A. R. Williams, and C. B. Sommers, *Phys. Rev. B* **28**, 1745 (1983).  
<sup>16</sup>J. Enkovaara, A. Ayuela, J. Jalkanen, L. Nordström, and R. M. Nieminen, *Phys. Rev. B* **67**, 054417 (2003).  
<sup>17</sup>A. R. Williams, J. Kübler, and C. D. Gelatt, *Phys. Rev. B* **19**, 6094 (1979).  
<sup>18</sup>O. K. Andersen, *Phys. Rev. B* **12**, 3060 (1975).  
<sup>19</sup>J. P. Perdew and Y. Wang, *Phys. Rev. B* **45**, 13 244 (1992).  
<sup>20</sup>P. J. Webster and K. R. A. Ziebeck, in *Alloys and Compounds of d-Elements with Main Group Elements*, P. 2, edited by H. R. J. Wijn, Landolt-Börnstein, New Series, Group III, Vol. 19/c (Springer, Berlin, 1988), pp. 75–184.  
<sup>21</sup>N. M. Rosengaard and B. Johansson, *Phys. Rev. B* **55**, 14 975 (1997).  
<sup>22</sup>S. V. Halilov, H. Eschrig, A. Ya. Perlov, and P. M. Oppeneer, *Phys. Rev. B* **58**, 293 (1998).  
<sup>23</sup>L. M. Sandratskii and P. Bruno, *Phys. Rev. B* **67**, 214402 (2003).  
<sup>24</sup>P. W. Anderson, *Theory of Magnetic Exchange Interactions: Exchange in Insulators and Semiconductors*, in *Solid State Physics*, edited by F. Seitz and Turnbull (Academic Press, New York), Vol. 14, pp. 99–214.  
<sup>25</sup>A. Ayuela, J. Enkovaara, and R. M. Nieminen, *J. Phys.: Condens. Matter* **14**, 5325 (2002).  
<sup>26</sup>J. Worgull, E. Petti, and J. Trivisonno, *Phys. Rev. B* **54**, 15 695 (1996).  
<sup>27</sup>A. Ayuela, J. Enkovaara, K. Ullakko, and R. M. Nieminen, *J. Phys.: Condens. Matter* **11**, 2017 (1999).  
<sup>28</sup>The energy of the frozen magnon with a given  $\mathbf{q}$  can be seen as a result of a complex interaction of the ferromagnetic states separated by vector  $\mathbf{q}$  in the reciprocal space [L. M. Sandratskii, *Adv. Phys.* **47**, 1 (1998)]. This interaction is stronger if the states have close energies and weaker for the states separated by a large energy interval. Note that the DOS is an integrated characteristic and does not contain information about the distribution of the states in the reciprocal space. Therefore the study of the DOS is not sufficient for understanding of the character of the  $\mathbf{q}$  dependence of the frozen-magnon dispersion.  
<sup>29</sup>Recently Bungaro *et al.* [C. Bungaro, K. M. Rabe, and A. D. Corso, *Phys. Rev. B* **68**, 134104 (2003)] reported calculations of the phonon dispersion in Ni<sub>2</sub>MnGa and found a nonmonotonic behavior qualitatively similar to the behavior obtained by us for the frozen-magnon dispersion of Ni<sub>2</sub>MnGa and Ni<sub>2</sub>MnIn (Fig. 2). They also studied the properties of the Fermi surface and discussed the absence of the coupling of the nesting parts of the Fermi surface to the phonons. The features of the Fermi surface can be important for forming the details of the frozen-magnon dispersions such as the maximum at  $\mathbf{q} \sim 0.6$  for Ni<sub>2</sub>MnGa and Ni<sub>2</sub>MnIn. Our calculations show, however, that the ground state is ferromagnetic. No instability connected with formation of a spin-density wave ground state is detected. [See also A. T. Zayak, P. Entel, J. Enkovaara, A. Ayuela, and R. M. Nieminen, *Phys. Rev. B* **68**, 132402 (2003).]