Anomalous charge transport in triplet superconductor junctions

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Charge transport properties of a diffusive normal metal/triplet superconductor (DN/TS) junction are studied based on the Keldysh-Nambu quasiclassical Green's function formalism. Contrary to the unconventional singlet superconductor junction case, the mid-gap Andreev resonant state at the interface of the TS is shown to enhance the proximity effect in the DN. The total resistance of the DN/TS junction is drastically reduced and is completely independent of the resistance of the DN in the extreme case. Such anomalous transport accompanies a giant zero-bias peak in the conductance spectra and a zero-energy peak of the local density of states in the DN region. These striking features manifest the presence of novel proximity effect peculiar to triplet superconductor junctions.

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Physics of superconducting junctions has been one of the exciting fields of solid state physics in this decade.[1] In diffusive normal metal/conventional singlet *s*-wave superconductor (DN/CSS) junctions, it is known that the phase coherence between an incoming electron and an Andreev reflected hole plays an essential role in causing the proximity effect in the DN.[2,3] It is also known that the total resistance R of DN/CSS junctions does not follow the simple Ohm's rule, that is, $R=R_D+R_{R_D=0}$ where R_D is the resistance in the DN and $R_{R_D=0}$ is the resistance of the DN/CSS interface. The reduction of R becomes prominent for low transparent junctions, where $R<R_D+R_{R_D=0}$ is satisfied.[2] The lower limit of R is given by $R_D+R_0/2$ using the Sharvin resistance R_0, where $R_{R_D=0}=R_0/2$ is satisfied only for perfect transparent junctions.[2] Previous investigations of the proximity effect, however, are limited to DN/CSS junctions. Stimulated by the successive discovery of unconventional superconductors, we are tempted to expect novel proximity effect in junctions with unconventional pairing, e.g., *p*-wave, and *d*-wave, where pair potentials have sign change on the Fermi surface.

In unconventional superconductor junctions, reflecting the internal phase of the pair potential, charge transport becomes essentially phase sensitive. The most dramatic effect is the appearance of zero bias conductance peak (ZBCP)[4,5] in tunneling spectroscopy due to the formation of the mid-gap Andreev resonant state (MARS).[6] The origin of the MARS is due to the anomalous interference effect of quasiparticles at the interface, where injected and reflected quasiparticles feel different sign of the pair potentials.[5] It is an interesting issue to clarify the role of the MARS on the transport properties of superconducting junctions. Recently, we have developed a theory of proximity effect, which is available for a diffusive normal metal/unconventional singlet superconductor (DN/USS) junction, where USS indicates anisotropic singlet pairing like *d*-wave.[7,8] Unfortunately, however, it is revealed that the proximity effect and the MARS compete with each other in DN/USS junctions. Although the interface resistance $R_{R_D=0}$ is reduced by the MARS irrespective of the magnitude of the transparency at the interface, the resulting R is always larger than $R_0/2+R_D$. This is because the angular average of many channels at the DN/USS interface destruct the phase coherence of the MARS and the proximity effect (see Fig. 1). This destructive angular average is due to the sign change of the pair potentials felt by quasiparticles with injection angle ϕ and those with $-\phi$, where the angle ϕ is measured from the direction normal to the junction interface (see Fig. 1). However, in diffusive normal metal/triplet superconductor (DN/TS) junctions, we can escape from the above destructive average (see Fig. 1). We can expect enhanced proximity effect by the MARS. In order to study this charge transport,

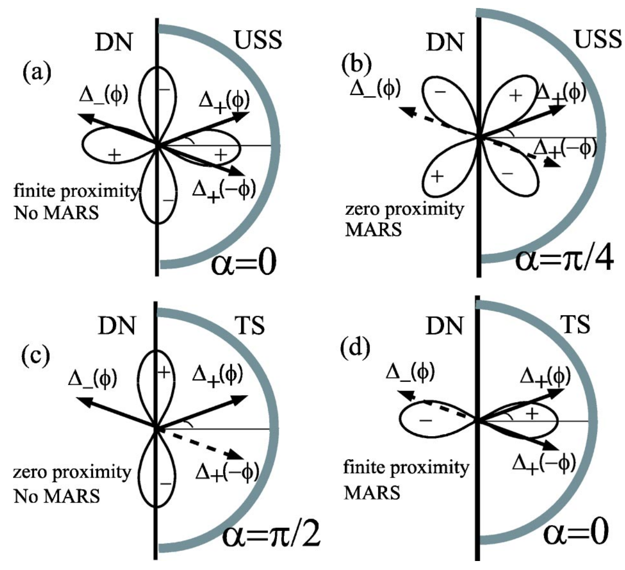

FIG. 1. Trajectories in the scattering process for outgoing (incoming) quasiparticles for the DN/USS and the DN/TS junctions and the corresponding $\Delta_{\pm}(\phi)$ are schematically illustrated. As a prototype, we choose $\Delta_{\pm}=\Delta_0\cos[2(\theta\mp\alpha)]$ for the DN/USS junctions [(a) and (b)] and $\Delta_{\pm}=\pm\Delta_0\cos[(\theta\mp\alpha)]$ for the DN/TS junctions [(c) and (d)], respectively, where Δ_0 is the maximum value of the pair potential. The measure of the proximity effect θ_0 is determined by the integration for all injection angles, i.e., the angular average over the hatched area.

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we must construct a theory for DN/TS junctions beyond preexisting ones.[7–10] This is in fact very timely since triplet superconductors have been discovered successively very recently.[11]

In the present paper, we derive a conductance formula for DN/TS junctions based on the Keldysh-Nambu (KN) Green's function formalism.[7,8] The total zero voltage resistance R in the DN/TS junctions is significantly reduced by the enhanced proximity effect in the presence of the MARS. At the same time, local density of states (LDOS) in the DN region has zero energy peak (ZEP) due to the penetration of the MARS into the DN region from the triplet superconductor (TS) side of the DN/TS interface. It is remarkable that when R_D is sufficiently larger than the Sharvin resistance R_0, R is given by $R=R_0/C_-$, which can become much smaller than the preexisting lower limit value of R, i.e., $R_0/2+R_D$. In the above, C_- is a constant completely independent of both R_D and R_B, where R_B denotes the interface resistance in the normal state. When all quasiparticles injected at the interface feel the MARS, R is reduced to be $R=R_0/2$ irrespective of the magnitude of R_D and R_B. The line shape of the bias voltage V dependent conductance $\sigma(\mathrm{eV})$ has a giant ZBCP. These features have never been expected either in DN/CSS or DN/USS junctions.

We consider a DN/TS junction with TS terminal and normal reservoir (N) connected by a quasi-one-dimensional diffusive conductor (DN) having a resistance R_D. The flat interface between the DN and the TS has a resistance R_B while the DN/N interface has zero resistance. The positions of the DN/N interface and the DN/TS interface are denoted as $x=-L$ and $x=0$, respectively, as shown in Ref. 8. We restrict our attention to triplet superconductors with $S_z=0$ that preserves time reversal symmetry. S_z denotes the z component of the total spin of a Cooper pair. It is by no means easy to formulate a charge transport of DN/TS junctions since the quasiparticle Green's function has no angular dependence by the impurity scattering in the DN. However, as shown in our previous paper,[7,8] if we concentrate on the matrix currents[9,10] via the TS to or from the DN, we can make a boundary condition of the KN Green's function. We assume that the constriction area between the DN and the TS is subdivided into an anisotropic zone in the DN, two ballistic zones in the DN and the TS, and a scattering zone, where both ballistic and diffusive regimes can be covered.[8] The sizes of the ballistic zone in the DN and the scattering zone in the current flow direction are much shorter than the coherence length.[7–9] The scattering zone is modeled by an insulating delta function barrier with the transparency $T(\phi)=4\cos^2\phi/(4\cos^2\phi+Z^2)$, where Z is a dimensionless constant and ϕ is measured from the interface normal to the junction.[8] The boundary condition for the KN Green's function in the DN $[\check{G}_N(x)]$ at the DN/TS interface is given by

$$\frac{L}{R_D}\left[\check{G}_N(x)\frac{\partial\check{G}_N(x)}{\partial x}\right]\Bigg|_{x=0_-}=\frac{-h}{2e^2R_B}\langle\check{I}\rangle, \qquad (1)$$

using matrix current $\check{I}$.[7,8] Average over the angle of injected particles at the interface is defined by

$$\langle\check{I}(\phi)\rangle\equiv\int_{-\pi/2}^{\pi/2}d\phi\cos\phi\,\check{I}(\phi)\Big/\int_{-\pi/2}^{\pi/2}d\phi\,T(\phi)\cos\phi \qquad (2)$$

with $\check{I}(\phi)=\check{I}$. The resistance of the interface R_B is given by $R_B=\langle R_0\rangle$. As shown in the Eq. (2) in our previous paper,[7] matrix current $\check{I}$ is a function of $T(\phi)$, $\check{G}_1=\check{G}_N(x=0_-)$, $\check{G}_{2+}$, and $\check{G}_{2-}$, where $\check{G}_{2+}$ ($\check{G}_{2-}$) denotes the outgoing (incoming) Green's function in the TS. The Green's functions are fixed in the "TS" terminal and in the "N" terminal, and the voltage V is applied to the "N" terminal located at $x=-L$. $\check{G}_N(x)$ is determined from the Usadel equation with Eq. (1). If we denote the retarded part of $\check{G}_N(x)$ and $\check{G}_{2\pm}$ as $\hat{R}_N(x)$ and $\hat{R}_{2\pm}$,[8] the following equations are satisfied, $\hat{R}_N(-L)=\hat{\tau}_z$, $\hat{R}_{2\pm}=(f_\pm\hat{\tau}_y+g_\pm\hat{\tau}_z)$ with $f_\pm=\Delta_\pm(\phi)/\sqrt{\Delta_\pm^2(\phi)-\epsilon^2}$ and $g_\pm=\epsilon/\sqrt{\epsilon^2-\Delta_\pm^2(\phi)}$, using the Pauli matrices. ϵ denotes the energy of the quasiparticles measured from the Fermi energy. $\Delta_+(\phi)$ $[\Delta_-(\phi)]$ is the pair potential felt by the outgoing (incoming) quasiparticles (see Fig. 1). After some algebra, we can show that $\hat{R}_N(x)$ is given by $\sin\theta(x)\hat{\tau}_x+\cos\theta(x)\hat{\tau}_z$. The spatial dependence of $\theta(x)$ in the DN is determined by

$$D\frac{\partial^2}{\partial x^2}\theta(x)+2i\epsilon\sin[\theta(x)]=0, \qquad (3)$$

with diffusion constant D in the DN. Taking the retarded part of Eq. (1), we obtain

$$\frac{L}{R_D}\frac{\partial\theta(x)}{\partial x}\Bigg|_{x=0_-}=\frac{\langle 2B_R\rangle}{R_B}, \qquad (4)$$

$$B_R=\frac{(\Gamma_1\cos\theta_0-\Gamma_2\sin\theta_0)T(\phi)}{(2-T(\phi))\Gamma_3+T(\phi)[\cos\theta_0\Gamma_2+\sin\theta_0\Gamma_1]},$$

with $\theta_0=\theta(x=0_-)$, $\Gamma_2=g_++g_-$, $\Gamma_3=1+f_+f_-+g_+g_-$, and, $\Gamma_1=i(f_+g_--g_+f_-)$. Here, we focus on $\epsilon=0$, where the left-hand side of Eq. (4) is reduced to be θ_0/R_D. We define $F_\pm(\phi)$ as $F_\pm(\phi)=\lim_{\epsilon\to 0}f_\pm=\mathrm{sign}(\Delta_\pm(\phi))$ with $\mathrm{sign}(\Delta_+(\phi))=1(-1)$ for $\Delta_+(\phi)>0(<0)$. In the following, we define the terminology as follows: when $\Delta_+(\phi)\Delta_-(\phi)>0$ is satisfied, quasiparticles are in the *conventional channels* (CC), while when $\Delta_+(\phi)\Delta_-(\phi)<0$ is satisfied, quasiparticles are in the *unconventional channels* (UC). In UC, quasiparticles feel the MARS while in CC quasiparticles do not feel the MARS. Following the above definition, $F_+(\pm\phi)=F_-(\pm\phi)$ is satisfied for CC, while for UC, $F_+(\pm\phi)=-F_-(\pm\phi)$ is satisfied. From Eq. (4), we can show that B_R is zero for CC and $B_R=iF_+(\phi)$ for UC, respectively. Then, θ_0 becomes

$$\theta_0=iR_DC_-/R_0, C_-=\int_{\mathrm{UC}}F_+(\phi)\cos\phi d\phi, \qquad (5)$$

where $\int_{\mathrm{UC}}$ means the ϕ integral only from the UC within $-\pi/2<\phi<\pi/2$. A remarkable feature is that θ_0 becomes a purely imaginary number as shown below. From Eq. (3), $\theta(x)$ at $\epsilon=0$ becomes $\theta(x)=(x+L)\theta_0/L$. Since the LDOS of the quasiparticles in the DN region renormalized by its value in normal state is given by $\rho(\epsilon)=\mathrm{Real}[\cos\theta(x)]$, $\rho(0)$ is always larger than unity except for $x=-L$. This means that $\rho(0)$ is enhanced due to the enhanced proximity effect by the MARS and has a ZEP as shown later. In the preexisting theories of DN/CSS and DN/USS junctions, θ_0 at $\epsilon=0$ is always a real number and $\rho(x)$ never exceeds unity.

In order to understand the essential difference between the DN/TS junctions and the DN/USS junctions intuitively, we consider four simplified cases, (a) the DN/CSS junctions with the CC, (b) the DN/CSS junctions with the UC, (c) the DN/TS junctions with the CC, and (d) the DN/TS junctions with the UC for all ϕ. This situation is actually realized by choosing d-wave pair potential with $\Delta_\pm(\phi) = \Delta_0 \cos[2(\phi \mp \alpha)]$ (Figs. 1(a) and 1(b)) and p-wave pair potential with $\Delta_\pm(\phi) = \pm\Delta_0 \cos(\phi \mp \alpha)$ [Figs. 1(c) and 1(d)] as a prototype of USS and TS, respectively. The relations $F_+(\pm\phi)=F_-(\mp\phi)$ and $F_+(\pm\phi)=-F_-(\mp\phi)$, are satisfied for the DN/USS and the DN/TS junctions, respectively, both for the UC and for the CC. For the DN/USS junctions with the CC, since $F_\pm(\phi)=F_\pm(-\phi)$ is satisfied, the contribution to θ_0 is not cancelled by angular averaging [Fig. 1(a)]. For the DN/USS junctions with the UC and the DN/TS junctions with the CC, since $F_\pm(\phi)=-F_\pm(-\phi)$ is satisfied, the contribution to θ_0 is cancelled by the angular average [Figs. 1(b) and 1(c)]. However, for the DN/TS junctions with UC, since $F_\pm(\phi)=F_\pm(-\phi)$ is satisfied, the contribution to θ_0 is nonzero [Fig. 1(d)]. It is remarkable that for the DN/TS junctions the CC do not contribute to the proximity effect while UC do.

The total zero voltage resistance of the DN/TS junction can be given as

$$R = \frac{R_D}{L}\int_{-L}^{0} \frac{dx}{\cos^2 \theta(x)} + \frac{R_B}{\langle I_{b0}\rangle}, \tag{6}$$

where I_{b0} is obtained from the Keldysh part of $\check{I}$ and is a complex function of $g_\pm$, $f_\pm$, θ_0, and $T(\phi)$. At $\epsilon=0$, I_{b0} becomes $1+\exp(2|\theta_0|)$ for UC, while for CC, I_{b0} becomes

$$\frac{2T^2(\phi)}{[2-T(\phi)]^2}\cos^2 \theta_0.$$

After simple algebra, R is given by

$$R = R_0\left\{\frac{\tanh \theta_{0i}}{C_-} + \frac{2}{[1+\exp(2|\theta_{0i}|)]C_+ + \cosh^2 \theta_{0i} D}\right\} \tag{7}$$

with $\theta_{0i}=-i\theta_0=R_D C_-/R_0$, $C_+=\int_{\mathrm{UC}}\cos\phi d\phi$ and

$$D = \int_{\mathrm{CC}} \frac{2T^2(\phi)}{[2-T(\phi)]^2} d\phi,$$

where $\int_{\mathrm{CC}}$ means the ϕ integral only from the CC. The resulting R is given by R_0/C_- for sufficiently large R_D/R_0 and is independent both of R_D and R_B except for the very special case with $C_-=0$. The magnitude of R can become much smaller than the preexisting lower limit value of R, i.e., $R_0/2+R_D$. This giant reduction of R is due to the enhanced proximity effect by the MARS. When all quasiparticles feel the MARS independent of ϕ, C_- is given by 2 and R becomes $R=R_0/2$ independent of R_D and R_B. This interesting situation is actually realized for a p_x-wave case, where pair potentials are given by $\Delta_\pm(\phi)=\pm\Delta_0\cos\phi$ [see Fig. 1(d)]. The above enhanced proximity effect by the MARS has never been expected in any preexisting theories.[2,7–10]

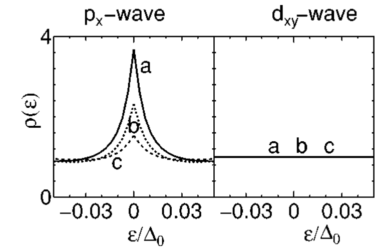

FIG. 2. $\rho(\epsilon)$ is plotted for the DN/TS junction with p_x-wave superconductor (left panel) and the DN/USS junction with d_{xy}-wave superconductor (right panel) for $Z=1.5$, $R_D/R_B=0.5$, and $E_{Th}=0.02\Delta_0$ with various x in the DN. a: $x=0_-$, b: $x=-L/4$ and c: $x=-L/2$.

In order to understand this proximity effect in detail, we focus on the LDOS $\rho(\epsilon)$ in the DN region normalized by its value in the normal state. We choose p_x-wave pairing as a prototype of the TS. As a reference, we compare the results with those for the DN/USS junction with d_{xy}-wave pair potential, where $\Delta_\pm(\phi)$ is given by $\Delta_\pm(\phi)=\pm\Delta_0\sin(2\phi)$. Although $\rho(\epsilon)$ at the TS side of the DN/TS interface and that at the USS side of the DN/USS interface both have ZEP by the formation of the MARS, $\rho(\epsilon)$ in the DN has a drastic difference between the two cases. For the p_x-wave case, the LDOS in the DN region has a ZEP. The height of the ZEP $\rho(0)$ is given by $\cosh[2R_D(x+L)/(LR_0)]$ and the order of its width is E_{Th}. On the other hand, for the d_{xy}-wave case, $\rho(\epsilon)$ is always unity independent of R_D due to the absence of the proximity effect. Contrary to the DN/TS junction, the MARS formed at the USS side of the DN/USS interface cannot penetrate into the DN. As seen from $\rho(\epsilon)$ in DN/TS junctions, the significant reduction of R originates from the penetration of the MARS into the DN. Although we have actually shown the existence of ZEP of $\rho(\epsilon)$ in the DN for p_x-wave case as a prototype, ZEP is universally expected for DN/TS junctions with the MARS at the interface independent of the detailed shape of the pair potentials of the TS. On the other hand, the LDOS in the DN region of DN/USS junctions do not have ZEP. Using this clear and qualitative difference of LDOS in the DN region between the DN/TS and DN/USS junctions, we can identify the triplet symmetry of the pair potentials Fig. 2.

Finally, we focus on the line shape of the tunneling conductance of DN/TS junctions for non zero voltage, which is defined by $\sigma(\mathrm{eV})=R_B/R(\mathrm{eV})$. We choose p_x-wave pairing as a prototype. As a reference, we compare the results with those for the DN/USS junction with d_{xy}-wave pair potential. Although both p_x-wave and d_{xy}-wave cases have similar line shapes of the voltage-dependent conductance with the ZBCP as a function of eV for ballistic junctions,[12] i.e., $R_D=0$ case, we can classify these two for $R_D\neq 0$ as shown in Fig. 3. We choose the Thouless energy E_{Th} as $E_{Th}=0.02\Delta_0$. For the p_x-wave case, since all injected quasiparticles feel the MARS, $\sigma(0)=2R_B/R_0$ is satisfied for any R_D. For $R_D\neq 0$,

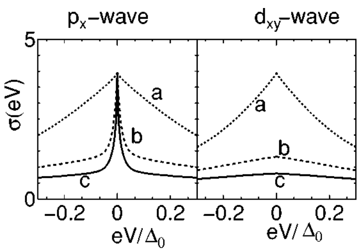

FIG. 3. σ(eV) is plotted as a function of eV for the DN/TS junctions with p_x-wave superconductor (left panel) and the DN/USS junctions with d_{xy}-wave superconductor (right panel) for $Z=1.5$ and $E_{Th}=0.02\Delta_0$. a: $R_D/R_B=0$, b: $R_D/R_B=0.5$ and c: $R_D/R_B=1$.

σ(eV) can be expressed by the summation of the broad ZBCP and the narrow one, where $\sigma_N=R_0/R_B$ denotes the angular averaged transparency of the junction. The width of the former one is proportional to $\sigma_N\Delta_0$ and the latter one is the Thouless energy. However, with the increase of R_D/R_B, σ(eV) for $|\mathrm{eV}|>E_{Th}$ is suppressed, and the ratio of $\sigma(0)$ to its background value is largely enhanced. We can call this largely enhanced $\sigma(0)$ as giant ZBCP (curve *b* or *c* in the left panel of Fig. 3). By contrast, for d_{xy}-wave case, since $\sigma(0)=2R_B/(R_0+2R_D)$ is satisfied, $\sigma(0)$ is reduced with the increase of R_D. The width of ZBCP is proportional to $\Delta_0\sigma_N$ and is not changed with the increase of R_D/R_B due to the absence of the proximity effect.

In conclusion, we have presented a theory of charge transport in the DN/TS junctions. The total resistance R can become much smaller than the preexisting lower limit value of R for DN/CSS and DN/USS junctions, i.e., $R_0/2+R_D$. As the extreme case where all injected electrons feel the MARS, R is reduced to be $R_0/2$. The significant reduction of R is due to the enhanced proximity effect by the MARS and the resulting LDOS in the DN region has a ZEP. At the same time, we can expect giant ZBCP. The above enhanced proximity effect has never been expected in DN/CSS and DN/USS junctions. We have shown in the present paper that these effects are actually realizable for DN/TS junctions with *p*-wave pair potential. We believe that these features are easily verifiable in experiments since a mesoscopic interference effect due to the proximity effect has recently been observed in high T_C cuprate junctions.[13] Similar experiments are technologically possible for junctions composed of Sr_2RuO_4, where triplet pairing is believed to be realized.[11]

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