the origin of this strong curvature is at present not understood. The absence of a contribution to the resistivity, which varies as T^2, argues against the LSF model, and the magnetic susceptibilities do not vary with temperature in the manner that would be expected if the thermal smearing explanation were appropriate.

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PHYSICAL REVIEW B VOLUME 7, NUMBER 9 1 MAY 1973

Refraction of Thermal Neutrons by Shaped Magnetic Fields*

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The refractive bending of thermal neutrons by shaped magnetic fields has been studied experimentally and compared with that calculated from the field distribution. The small deflections, a few seconds arc, were measured on a double-crystal spectrometer utilizing matched perfect silicon crystals. Deflections were measured for a beam of polarized neutrons upon its polarization reversal for field regions of both prismatic and cylindrical configuration. The focusing and polarizing character of a magnetic lens have been observed.

I. INTRODUCTION

When slow neutrons carrying a magnetic moment μ are introduced into a region of space where a magnetic field B is present, there occurs a Zeeman splitting of the potential energy of magnitude $\pm\mu B$, corresponding to the two quantized spin states of the neutron. This occurs at the expense of the kinetic energy of the neutron, so that a velocity splitting occurs. It is convenient to portray this as a refractive-index modification, and it is easily shown that the refractive index n becomes

$$n = 1 \pm \mu B / 2E_0 , \quad (1)$$

with E_0 being the unperturbed kinetic energy of the neutron. Thus a field strength of 10 000 G (1 T) and a neutron energy of 0.0142 eV (wavelength $\lambda = 2.40$ Å) will produce a refractive-index modification of 2.12×10^{-6}. This change, although small, is of the same order of magnitude as occurs in passage of neutron radiation into solid matter, and it can be sensed and measured by suitable experimental technique. In the present study we report observations[1] of the refractive bending of a polarized neutron beam in passing through a magnetic field prism (a spatial region where the cross-sectional distribution of the magnetic field has triangular boundaries) and also through a cylindrical field region. The present experiment differs from the usual Stern–Gerlach type of experiment performed with neutrons by Sherwood *et al.*[2] and by Barkan *et al.*,[3] in that the spin splitting is transverse rather than longitudinal relative to the spin axis.

In sensing the very small refractive bending angles of 1 or 2″ arc, we have exploited the very high angular resolution of a double-crystal spectrometer utilizing a matched pair of perfect silicon crystals, both in (111) parallel orientation. The very sharp rocking curve for the second crystal (typically of width 1.4″ arc for a neutron wavelength of 2.40 Å used in the experiments) can respond to deflection effects as small as 10^{-3}″ arc occurring in the region between the crystals, as discussed in a companion study.[4] The experiments have been performed with one of the two spin-polarized beam components that emerge from a spin-splitting prism of magnetically saturated iron placed between the two crystals of the spectrometer. The angular separation of these two polarized components is sufficiently large (about 13″ arc) that one can be used to sense the later deflection produced by the pure-field prism. Provision was also available for inverting the polarization state (by resonance inversion) of the selected polarized-beam component. In this way the field deflection is reversed. Higher-order wavelength components in the monochromatic beam prepared by the first crystal were suppressed by passage through a Be-crystal filter.

II. MAGNETIC FIELD PRISM OBSERVATIONS

The prism-shaped magnetic field was obtained by taper-cutting the normal cylindrical iron-core pieces of a split electromagnet to triangular (60° equilateral with edges 4.40 cm) pole-face ends. A gap of 2.54 cm was usually used and a careful field survey of the field distribution was made with a small Bi magnetoresistance probe. There is, of course, a fringing of the magnetic field beyond

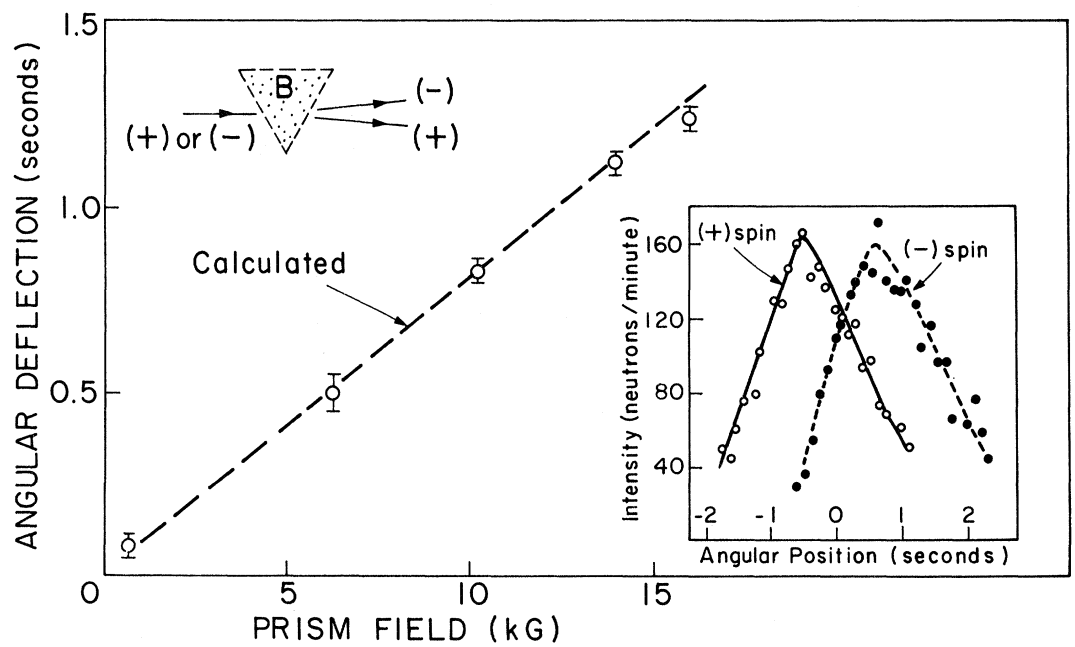

FIG. 1. Typical rocking curves of analyzing crystal (inset) showing deflection of polarized neutron beam (with polarization reversal) upon passage through a 60° prism-shaped magnetic field and deflections measured at different field strengths.

the sharp boundaries of the triangular-pole-piece area, and allowance for this along the neutron trajectory must be made. For a strictly triangular-shaped field, the angular deflection upon neutron polarization inversion for symmetrical passage becomes

$$\delta = 2\mu B E_0^{-1} \tan(\tfrac{1}{2}\phi) , \tag{2}$$

where ϕ is the full apex angle of the field prism (60° in our case). With B of value 10 000 G and a neutron wavelength of 2.40 Å, δ is calculated to be 1.02″ arc. However, the field distribution is not strictly triangular with sharp edges, and a smaller deflection is expected. It can be shown that the deflection for the case of a continuous field distribution along the neutron trajectory is expressible as

$$\delta' = \frac{2\mu}{E_0} \int \tan\alpha \left(\frac{dB}{dx}\right) dx , \tag{3}$$

where the integration is to be taken along the trajectory and α is the angle between the trajectory and the normal to the isofield lines. This integral has been evaluated numerically, using the field distribution map as measured experimentally. The effect of the present nonideal distribution is to reduce the deflection by about 10%.

Typical rocking curves measured with neutron passage through the field prism for opposite neutron polarization are shown in the inset of Fig. 1. The data were taken by a step-scanning procedure in order to minimize spectrometer drifting effects, as discussed in Ref. 4. Several determinations of the deflection upon polarization inversion were made for each of a number of central field values; the results are summarized as the experimental points in Fig. 1, along with the calculated values from Eq. (3), and are represented by the line in the figure. As expected, the effects are linear in field strength and the agreement between observed and calculated values is within experimental uncertainty.

III. CYLINDRICAL MAGNETIC FIELD BENDING

Deflection effects have also been studied by passage of the neutrons through a field region of cylindrical symmetry. Such a field distribution is obtained with the usual electromagnetic circular pole faces with, of course, some fringing beyond the geometrical boundary of the pole face. This fringing was reduced (thereby increasing the field gradient at the edges) by undercutting the inner circular area of the pole faces, leaving a sharp circular lip on the face. Again the field distribution was mapped by scanning with a small Bi magnetoresistance probe, so that the observed deflection could be compared with the calculated deflection. With this type of field symmetry, the deflection of the neutron trajectory with polarization reversal is expected to be a function of the closest distance to the center of the circular field along the trajectory.

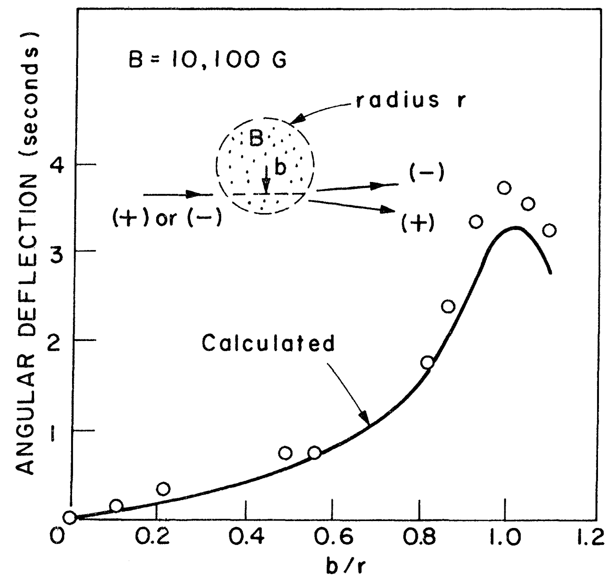

FIG. 2. Angular deflections of polarized neutron beam upon passage through a cylindrically shaped magnetic field, as a function of beam position.

Figure 2 shows the comparison between the measured deflection upon neutron polarization reversal as a function of the closest-approach distance (normalized to the pole-face radius) and that calculated using Eq. (3). The data were collected with a central field value of 10 000 G. Reasonable agreement between the measured and calculated quantities is obtained with some difficulty as the trajectory approaches tangency with the edge of the circular field area. At this position the evaluation of the integral in Eq. (3) becomes very sensitive to the field distribution, as measured with the finite-size magnetoresistance probe.

The deflection effects for the inner trajectories are approximately linear with the ray position; this would imply that the field distribution is serving as a magnetic lens. From the slope of the early part of the curve in Fig. 2, the focal length of the lens is calculated to be about 7500 m. Of course, this focal length could be greatly reduced by using superconducting magnetic field strengths and slower neutrons. It should also be noted that the magnetic lens serves as a polarizing lens, being convex for one neutron-spin state and concave for the other. Judicious shaping of the magnet pole faces could be effective in removing focusing aberrations.

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Spin-Lattice Relaxation of Al^{27} in Thulium Aluminum Garnet*

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The Al^{27} spin-lattice relaxation time T_1 has been measured between 127 and 425 °K at 8 and 14 MHz for the aluminum *a* and *d* sites in thulium aluminum garnet. Both sites exhibit T_1 dips at 275 °K which are attributed to Al^{27}-Tm^{169} cross relaxation. Below 190 °K the *a*-site T_1 magnitudes and approximately ω^{-2} frequency dependence are explained by magnetic field fluctuations caused by the Tm^{3+} ion making transitions between its two lowest crystal-field levels with a correlation time near 4×10^{-12} sec.

I. INTRODUCTION

In a previous study[1] (herein referred to as SJ) of the Tm^{169} and Al^{27} nuclear-magnetic-resonance (NMR) spectra in thulium aluminum garnet (TmAlG), the Tm^{169} NMR showed that the Tm^{3+}-ion magnetic-susceptibility tensor is very anisotropic, much more so than in thulium gallium garnet (TmGaG).[2] The paramagnetic shifts at the Al^{27} *a* and *d* sites at 1.5 °K agreed within about 10% with shifts calculated for dipolar fields from point dipoles at the Tm^{3+} sites. Above approximately 20 °K the Tm^{169} resonance was not observed because of line broadening from large fluctuating fields caused by transitions of the Tm^{3+} ions. The Al^{27} paramagnetic shifts at 27, 76, 192, 232, and 297 °K were used to infer values for the magnetic-susceptibility tensor (for a particular Tm^{3+} site) $\{\underline{\chi}\}$ at these temperatures on the assumption that the fields at the Al^{27} sites are caused by point dipoles at the thulium sites. The electric-field-gradient (efg) tensors at the Al^{27} *a* and *d* sites were found, and compared with those for corresponding sites in other rare-earth aluminum and iron garnets.

This present study[3] of Al^{27} spin-lattice relaxation time T_1 in TmAlG was conducted in the hope that T_1 would be governed by magnetic field fluctuations caused by the Tm^{3+} ions. Such relaxation has been found, but in addition we have observed faster relaxation (T_1 dips) near room temperature, where certain of the broad Tm^{169} resonances cross certain Al^{27} resonances. These results yield information on the dynamics of the thulium-ion transitions among its energy levels.

In this paper a brief experimental description is followed by a discussion in Sec. III of the relation to $\{\underline{\chi}\}$ of the Tm^{169} resonance locations inferred from the Al^{27} T_1 dips. Section IV deals with the cross relaxation responsible for these dips. From the cross-relaxation rates we infer the Tm^{169} linewidths at 8 and 14 MHz. In Sec. V, direct Al^{27} relaxation by Tm^{3+}-ion fluctuations is discussed. Inferences about the Tm^{169}-ion behavior are drawn from the Tm^{169} linewidths and from the direct Al^{27} relaxation. A general discussion appears in Sec. VI. An appendix is devoted to the calculation of quadrupolar spin-lattice relaxation for Al^{27}, which appears to be unimportant.

II. EXPERIMENTAL

The single crystal of TmAlG, which had been used in the previous NMR study,[1] was grown from a lead fluoride flux and its rotation axis was adjusted to within $\frac{1}{2}°$ by the Laue back-reflection technique.

Measurements at 8 and 14 MHz of T_1 for the Al^{27} *a* and *d* sites were made with applied field H_0 along ⟨100⟩ and ⟨111⟩ directions, respectively, and