Current-Induced Intermediate State in Thin-Film Type-I Superconductors: Electrical Resistance and Noise*

R. P. Huebener and D. E. Gallus
Argonne National Laboratory, Argonne, Illinois 60439
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We have investigated the current-induced resistive state in superconducting strips of lead and indium carrying electrical transport currents with an average density up to 10^6 A/cm^2. Following previous high-resolution magneto-optical experiments, we report here measurements of the electrical resistance and the electrical-noise-power spectra. The experiments were performed in zero external magnetic field, using micro-strips 60-300 μm wide, 0.3-10 μm thick, and 3-6 mm long. In lead, resistive voltage steps were usually observed during variation of the current resulting in pronounced peaks of the derivative $\partial V/\partial I$. These voltage steps are caused by the abrupt creation of channels of normal phase at distinct levels of the transport current. The abrupt growth of these channels of normal phase apparently results from a magnetic instability similar to the kink instability in magnetohydrodynamics. In indium, a step structure in the resistive voltage has been observed only close to T_c, the voltage steps being usually smeared out by fluctuations in time of the number of current-generated channels of normal phase. From the magnitude of the resistive voltage steps and from the observation of dynamic magnetic coupling between two films of a sandwich structure, we conclude that, in the strips of both Pb and In, the current-induced resistive state can best be described by a dynamic model. In this model, the channels of normal phase consist of arrays of flux tubes moving rapidly from the edges to the center of the strip, where they are annihilated by flux tube arrays of opposite sign. The noise-power spectra in the current-induced resistive state usually show a frequency dependence ranging from ω^{-1} to ω^{-2} behavior. The electrical-noise data are discussed in terms of a model assuming fluctuations in time of the number of channels of normal phase.

I. INTRODUCTION

More than 30 years ago London[1] treated the current-induced intermediate state in a cylindrical type-I superconductor carrying an electrical current in axial direction. Since about the same time the destruction of superconductivity by an electrical current has been studied experimentally from the restoration of electrical resistance in superconducting wires.[2] More recently London's model has been refined[3,4] in order to account for the detailed manner[5,6] in which the resistance appears in a wire. The current-induced intermediate state has been detected in a cylinder with a Bitter method.[7]

We have investigated the current-induced intermediate state in superconducting strips of lead and indium carrying electrical transport currents with an average density up to 10^6 A/cm^2. The magnetic behavior of a superconducting strip carrying a high electrical transport current is shown schematically in Fig. 1. For small current values the current flows mainly along the edges of the strip[8,9] such that the magnetic field associated with the current is completely expelled from the specimen, except for a surface layer with the thickness of the penetration depth λ [Meissner effect, Fig. 1(a)]. Here we assume that the film thickness d is large compared to λ. As the magnetic field of the current approaches a value of the order of the critical field at the sample edge, a normal region will be created locally at the edge. The transport current will try to avoid this region by flowing around it [Fig. 1(b)]. Because of the magnetic field enhancement at the inside of this current loop, this configuration is magnetically unstable, and the normal region will grow abruptly until it reaches the center of the strip. We note that this behavior is analogous to the *kink instability* in magnetohydro-

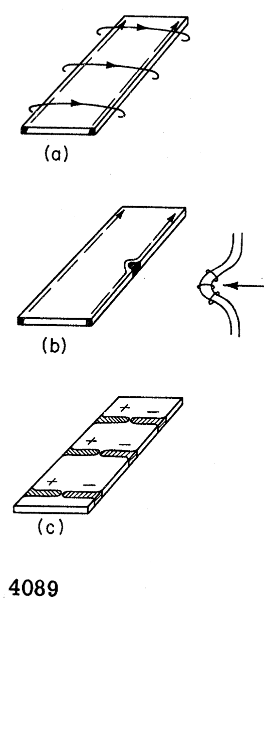

FIG. 1. Magnetic structure created in a superconducting strip by an electrical transport current: (a) Meissner effect; (b) nucleation of a domain at the edge; (c) magnetic structure at high currents.

dynamics as indicated in Fig. 1(b). At higher current levels the current-induced magnetic structure consists of many "channels" of normal phase which have grown from both sample edges to the center [Fig. 1(c)]. We note that the magnetic field in these channels has opposite sign at the two sides of the strip. In this configuration the transport current will follow a rather complicated flow pattern. It is also possible that a *flux tube* is created by the current at the sample edge which moves towards the center of the strip under the influence of the Lorentz force. In this case the channels of normal phase actually consist of arrays of flux tubes moving rapidly towards the center, where they are annihilated by flux tube arrays of opposite sign coming from the opposite edge of the strip.

The current-induced magnetic structure, outlined schematically in Fig. 1, has been verified recently from high-resolution magneto-optical experiments[10–12] performed with strips of lead and indium. However, these experiments do not differentiate clearly between the *static* (stationary normal channels) and the *dynamic* (rapidly moving flux-tube arrays) *model* of the current-induced intermediate state.

In the following we report measurements of the electrical resistance and of the noise-power spectra in the current-induced intermediate state of superconducting films of lead and indium. Our experiments are performed in zero applied magnetic field, the only field present being that associated with the transport current itself. The abrupt creation of one or more channels of normal phase at a distinct current level results often in relatively large resistive voltage steps which have been investigated with a derivative technique. Attempting to discriminate between the static and dynamic model of the current-induced resistive state we have performed a series of measurements with magnetically coupled and electrically insulated sandwich structures similar to those used originally by Giaever.[13] Both for lead and indium our experiments tend to support the dynamic model. Fluctuations in time of the number of normal channels contribute significantly to the electrical noise in high-current-carrying superconducting microstrips. In principle, the noise-power spectrum yields information on the rate and lifetime of these fluctuations. Preliminary results of this work have been reported elsewhere.[14]

II. EXPERIMENTAL

The samples were strips of lead and indium 60–300 μm wide, 0.3–10 μm thick, and 3–6 mm long. They were made by vacuum deposition (starting pressure about 10^{-6} Torr) on circular glass substrates of 22 mm in diameter using Pb and In with 99.9999% purity and a tunable mask with two razor-blade sections. The samples carried electrical current and voltage leads attached with indium solder to rather wide sections at both ends of the strips. These wide end sections were deposited after the center strip and always consisted of lead with a thickness larger than that of the center strip. The experiments were performed in zero applied external magnetic field. Electrical transport currents with an average density up to 10^6 A/cm^2 were applied to the strips. The samples were placed in a cryostat in direct contact with the liquid-helium bath, the temperature of which could be lowered below 4.2 °K by pumping. In order to protect the specimen strips against destruction by thermal runaway, a constantan-wire shunt was attached parallel to the strip. The shunt resistance was 3–5 times the normal resistance of the specimens at 4.2 °K, being at least 1–2 orders of magnitude larger than the current-induced sample resistance for the current range investigated. During the measurements of the derivative of the voltage-current curve and of the electrical noise the primary coil of a step-up transformer in the liquid-helium bath provided the protective shunt.

In order to determine whether the current-induced intermediate state consists of stationary domains of the normal phase or of rapidly moving flux-tube arrays, we have studied *magnetic coupling* in a series of sandwich configurations consisting of two superconducting strips arranged in close proximity but electrically separated from

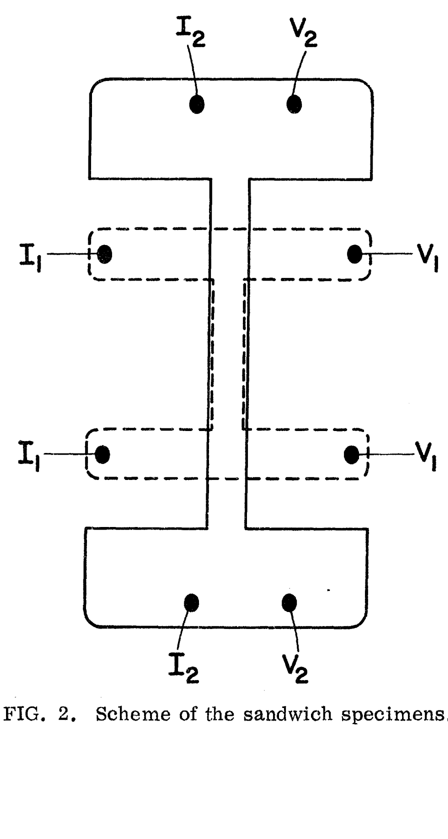

FIG. 2. Scheme of the sandwich specimens.

each other by a thin insulating film. The arrangement of these sandwiches is shown schematically in Fig. 2. The longer of the two strips connected to the leads marked V_2 and I_2 was deposited first and covered subsequently with the insulating film. The shorter strip connected to the leads marked V_1 and I_1 was then deposited on top of the insulating film. If the sandwich consisted of two superconducting films with different thickness, the thinner of the two films was deposited first in order to facilitate the electrical insulation between the two films along the edges of the bottom film. It is the geometrical step along these edges where the insulation appears most likely to break down. In most of the sandwiches the two superconducting strips had the same width. This could easily be achieved using the same tunable mask without changing its setting. Alignment of the substrate was facilitated by a ring screwed to the body of the mask into which the substrate could be inserted. For electrical insulation we used initially about 1000 Å of Al_2O_3 and finally about 1000 Å of SiO_2. Both kinds of insulators were deposited using an electron-beam-gun system. In was found helpful to deposit the insulator in two steps at an angle less than 90° between the plane of the substrate and a straight line connecting substrate and source. Rotating the substrate by 180° between the two steps resulted in a sufficient exposure of both edges of the bottom strip during the evaporation of the insulator.

In the course of the present experiments a total of twelve Pb strips, six In strips, and seven different sandwich structures have been investigated.

All electrical-resistance measurements were performed using the four-probe method as can be seen from the arrangement of current and voltage leads in Fig. 2. From the magneto-optical experiments[10–12] one expects an abrupt creation of relatively large domains of the normal phase at distinct current levels due to the instability of a small normal domain at the sample edge. This should result in relatively large resistive voltage steps in the voltage-current behavior. We have investigated these voltage steps using a conventional derivative technique. In these experiments an oscillatory component of 10–100-μA-rms magnitude was superimposed on the transport current which was of the order of several amperes. The direct current could be varied in steps as small as 10 μA using a Fluke model 382A voltage/current source. The oscillatory voltage component was detected using a step-up transformer (ratio 60:1) within the liquid-helium bath and a lock-in amplifier.

The electrical-noise measurements were performed using the scheme shown in Fig. 3. The specimen was connected to the primary coil of a step-up transformer within the liquid-helium bath (ratio 60:1). The resistance of the primary coil was at least 3–5 times the normal resistance of the specimen strip at 4.2 °K, thus providing protection of the sample against destruction by thermal runaway without appreciably shorting the sample in the current range investigated. The signal was further enhanced by a selective amplifier consisting of the type-A preamplifier and the intermediate amplifier and tuned amplifier section of a Princeton Applied Research model HR-8 operated at 1-μV sensitivity. The signal then passed through a rectifier and entered an integrator which was controlled manually. Measurements were made between 10 Hz and 20 kHz using a constant Q of 25 in the tuned-amplifier section. During the noise measurements the specimen was in direct contact with liquid helium. It was enclosed in a copper can surrounded by superconducting lead foil. The Fluke model 382A served again as current source. In order to determine the noise contribution resulting from sources in the circuit other than the specimen the two following experiments were performed. First, by replacing the electronically stabilized current supply with a series of Ni-Cd cells our results were found to remain unchanged. Second, by replacing the superconducting strip with a copper wire having a resistance similar to the current-induced specimen resistance and using the Fluke current supply the noise level was found to be equal to the background noise during the regular experiments. From this we conclude that in our experiments noise contributions from sources other than the specimens themselves were insignificant. In order to discriminate against contributions from thermal fluctuations in the specimens due to nucleate boiling of liquid helium at the sample surface, the noise measurements were usually extended below the λ-point temperature.

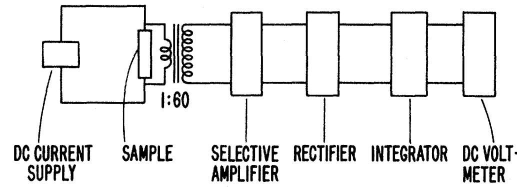

FIG. 3. Noise detection system.

For each run background noise was measured at all frequencies by reducing the sample current to a small value at which no dc resistive voltage appeared. The noise power spectrum $w(f)$ was then obtained from the following expression:

$$w(f)=\frac{\langle \delta V_f^2\rangle - \langle \delta V_0^2\rangle}{\Delta f} \quad . \tag{1}$$

Here $\langle \delta V_f^2\rangle$ and $\langle \delta V_0^2\rangle$ are the squares of the noise voltage at full current load and of the background noise voltage, respectively. Δf is the bandwidth of the detection system, which we have taken as

$$\Delta f = f/Q \; . \tag{2}$$

A calibration of the complete ac detection system and its frequency dependence has been carried out together with each individual run. For this purpose a small alternating current of known amplitude and various frequencies has been superimposed on the direct current. The calibration curve was usually quite flat in the frequency range between 50 and 2000 Hz.

III. EXPERIMENTAL RESULTS

A. Current-Induced Electrical Resistance

A typical curve for the resistive voltage in a Pb strip plotted versus the applied current is shown in Fig. 4(a). For the strip geometry the electrical resistance is seen to be gradually restored over a wide current range, in contrast to the behavior found in superconducting wires.[2,5,6] In Fig. 4(a) the slope of the curve at a current of 700 mA represents about 14% of the normal-state resistance of the sample at 4.2 °K. Curves similar to that shown in Fig. 4(a) have been found in all Pb and In strips investigated. The appearance of the first resistive voltage was always found to coincide with the creation of the first channels of normal phase as seen in the high-resolution magneto-optical experiments.[10-12] We note that above the current level, at which the first resistive voltage and the first normal channels appeared, the number of normal channels has been found to increase about linearly with increasing current. However, the width and length of the individual channel was rather independent of the current, each channel extending always to the center of the strip. Both in the Pb and the In strips, the width of the current-induced normal channels was about equal to the domain width observed in the same specimens at zero transport current with an external magnetic field applied perpendicular to the film. An example of the increase of the number of normal channels with transport current, as counted from the magneto-optical images, is shown in Fig. 5. The numbers plotted in Fig. 5 were obtained for a length of 320 μm in a lead strip with 4.0-μm thickness and 160-μm width. The normal domains were counted near the sample *edge* before some branching occurred towards the center of the strip.[10,12] From the linear behavior seen in Fig. 5 one expects the average slope of the voltage-current curve also to increase linearly with current. Such behavior has generally been found. As an example we show in Fig. 6 the increase of the derivative $\partial V/\partial I$ with current observed in a Pb strip. We note that a smooth curve such as shown in Fig. 6 is obtained only by avoiding the current values at which large resistive voltage steps occur. The structure in the derivative $\partial V/\partial I$ resulting from these voltage steps will be discussed further below.

In the Pb specimens the critical-current value,

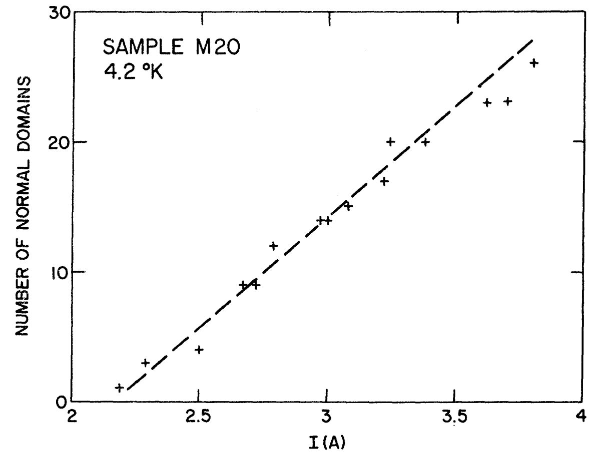

FIG. 5. Number of current-induced normal domains vs current in a Pb strip (thickness = 4.0 μm; width = 160 μm; length of observed section = 320 μm; 4.2 °K).

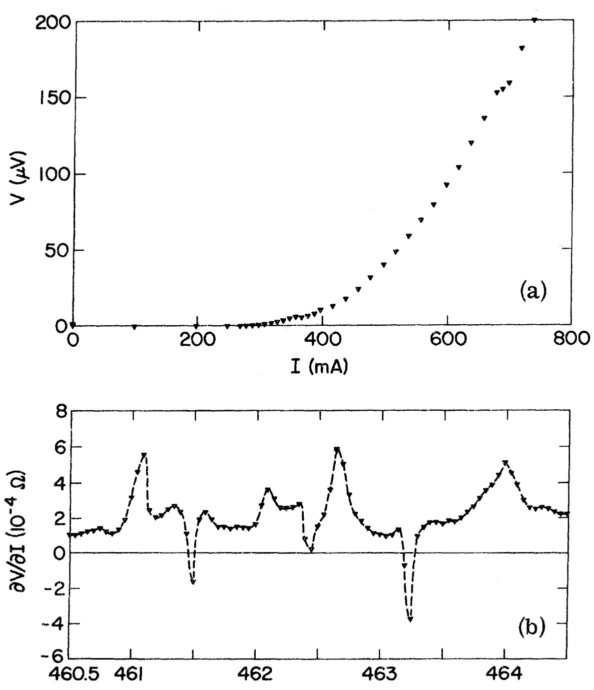

FIG. 4. Restoration of electrical resistance by a transport current in a Pb strip (sample Pb 83, thickness = 3.4 μm; width = 144 μm; length = 6.62 mm; 4.2 °K). (a) voltage vs current; (b) derivative $\partial V/\partial I$ vs current using an ac component of 24 μA rms and 320 Hz.

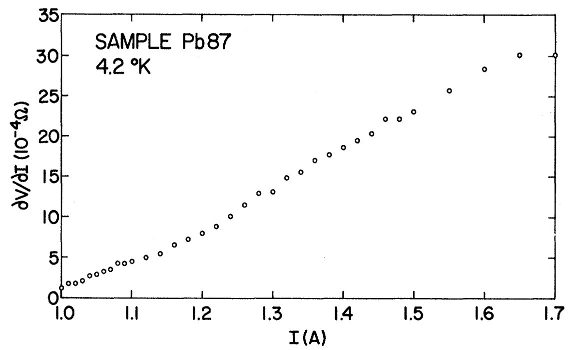

FIG. 6. Derivative $\partial V/\partial I$ vs current in a Pb strip (thickness = 0.69 μm; width = 145 μm; length = 6.5 mm; 4.2 °K; resistance at 4.2 °K in the normal state = 62×10^{-3} Ω).

at which the resistive voltage and the channels of normal phase start to appear, was generally found to increase somewhat following thermal cycling of the specimens between room temperature and 4.2 °K with the sample being exposed to air. No such change in the critical current has been observed in the In strips. The magnetic field at the edge of a current-carrying superconducting strip in the state of complete magnetic-flux expulsion [Fig. 1(a)] is given by[9]

$$H=\left(0.8\ \frac{\text{G cm}}{\text{A}}\right)\ \frac{I}{2d}\quad , \tag{3}$$

where I is the electrical current and d the thickness of the strip. In deriving Eq. (3) the penetration depth λ is assumed to be negligibly small compared to the thickness d. At the critical current value, at which electrical resistance starts to appear, the magnetic field at the sample edge is expected to be approximately equal to the critical field of the particular film. We have found that the magnetic field at the sample edge, as calculated from Eq. (3) for the critical current value, varied appreciably between the different specimens and generally ranged between 1 and about 100 times the critical field of the particular lead or indium film. In addition, the critical current was found to increase with the width of the superconducting strip, a result quite unexpected from a simple-minded interpretation of Eq. (3). These large values of the critical current and their dependence on the specimen width can be understood from the Gibbs-free-energy barrier against magnetic-flux entry at the edge, which can lead to a large enhancement of the critical entry field above the value of H_c.[15] A detailed discussion of this enhancement will be given elsewhere.[16]

The structure in the resistive voltage curve of Fig. 4(a), associated with the abrupt creation of channels of normal phase at distinct current levels, is shown in Fig. 4(b) for a small current interval. The peaks in the derivative $\partial V/\partial I$ are due to steps in the resistive voltage which coincide with an abrupt change in the current-induced magnetic structure in the strip. It is interesting that negative peaks in the derivative $\partial V/\partial I$ can also be observed occasionally. This indicates that with increasing current the magnetic structure and the current pattern can rearrange themselves such that a more favorable state of higher electrical conductivity is attained. The detailed structure in $\partial V/\partial I$ was quite reproducible in each Pb specimen during the same low-temperature run and was the same for increasing and decreasing transport current. However, the results differed between different runs when in the meantime the sample had been allowed to warm up considerably above 4.2 °K. The position of the peaks in the curves of the derivative $\partial V/\partial I$ remained unchanged while varying the frequency or amplitude of the small oscillatory component of the current.

Results similar to those of Fig. 4(b) were found in most of the Pb samples investigated. The voltage steps ΔV associated with the peaks in $\partial V/\partial I$ such as shown in Fig. 4(b) can be estimated from the integral

$$\Delta V=\int \frac{\partial V}{\partial I}\, dI\,, \tag{4}$$

the integration being performed over each peak. The value obtained from Eq. (4) can be compared with the voltage step produced in the strip if a continuous channel of normal phase extending all the way across the strip is created abruptly. In this comparison we have taken the width of the normal channel from previous magneto-optical experiments,[10–12,17] and we have assumed the domain width to increase proportional to the square root of the sample thickness. The resistive voltage due to a single normal channel can be obtained directly from the normal resistance of the strip at 4.2 °K together with the fraction of the volume occupied by the normal domain. Very often we have found the voltage steps from Eq. (4) to range only between 1 and 10% of the value estimated for a continuous normal channel. Such small values of ΔV seem to indicate that *in the Pb strips the current-induced channels of normal phase actually consist of arrays of widely spaced flux tubes moving rapidly from the edges to the center of the strip*. We shall see that this conclusion is also supported by our magnetic-coupling experiments. On the other hand in Pb we also observe voltage steps between 1 and 10 times the value calculated for a continuous normal channel its width being estimated from our previous magneto-optical experiments. These large values of ΔV may be explained by the simultaneous creation of a *group*

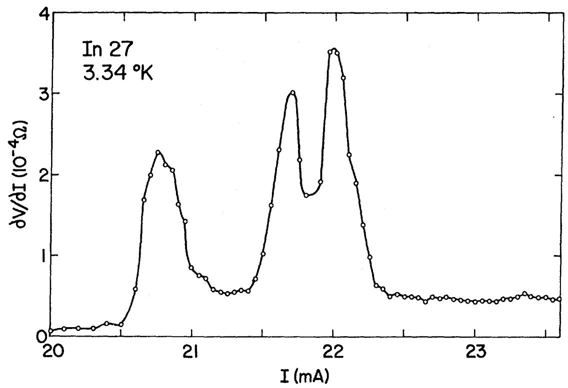

FIG. 7. Derivative $\partial V/\partial I$ vs current in an In strip (thickness = 4.5 μm; width = 154 μm; length = 6 mm; 3.34 °K; resistance at 4.2 °K = 9.2×10^{-3} Ω).

of several flux-tube arrays. Such simultaneous creation of a group of normal domains near the same location in the specimen has often been magneto-optically observed in lead.

A quantitative treatment of the detailed structure in the variation of the derivative $\partial V/\partial I$ with current for a particular specimen appears to be difficult because of its high sensitivity to small changes in the sample properties (strain, oxidation, etc.). This high sensitivity to slight variations in the sample characteristics is, of course, due to the fact, that we are dealing with a nucleation phenomenon which is amplified by the magnetic instability involved. A qualitative discussion of instabilities in a current-carrying conductor has been given recently by Azbel.[18]

Whereas the appearance of many distinct steps in the voltage-current characteristics, such as shown in Fig. 4(b), has been a general feature in the Pb microstrips, in the indium strips structure in the derivative $\partial V/\partial I$ has been observed very rarely and only close to T_c. Our measurements of the voltage-current characteristics in the In strips have been performed in the temperature range between 1.5 and 3.4 °K. An example of the derivative $\partial V/\partial I$ for indium is shown in Fig. 7. The peaks in the derivative $\partial V/\partial I$ shown in Fig. 7 were observed at 3.34 °K, the transition temperature of this specimen being 3.385 °K. With decreasing temperature the magnitude of these peaks decreased quickly, and at 3.30 °K the peaks disappeared. The voltage step ΔV associated with the double peak on the right of Fig. 7 corresponds to an increase in resistance of about 0.1% of the sample resistance at 4.2 °K. In the current-carrying In strips the derivative $\partial V/\partial I$ generally showed rather large fluctuations in time, quite in contrast to the behavior in the Pb samples. In addition, the electrical-noise power in the current-carrying In strips has been found to be 2 orders of magnitude larger than in our Pb specimens. These results seem to indicate that in the In samples the number of normal channels created by the current fluctuates rapidly and strongly in time. The measured value of the derivative $\partial V/\partial I$ represents, of course, the time average of the local slope of the $V(I)$ curve. If the number of normal channels and correspondingly the derivative $\partial V/\partial I$ fluctuate strongly with time, any step structure will be smeared out by the time averaging, and the time-averaged voltages are expected to increase continuously with current. These conclusions are further supported by our magneto-optical studies of Pb and In strips[10–12] which indicate that in In over a wide current range the current-induced magnetic structures are much less stable than in Pb.

The voltage steps which we have observed in the microstrips of Pb and In appear to be similar to the voltage instabilities reported for current-carrying whiskers of type-I superconductors[19,20] and thin-film microbridges.[21] From an inspection of the data for whiskers[19,20] it appears that the voltage steps occur at a current level for which the current-induced magnetic field around the whisker is of the order of the critical field of the specimens. However, an understanding of these results in terms of current-generated nonsuperconducting domains, similar to the situation in our experiments, is questionable in view of the small whisker diameter which is comparable to the penetration depth λ.

B. Magnetic-Coupling Experiments

As pointed out in Sec. III A, the magnitude of the resistive voltage steps in the Pb strips suggests that the current-induced resistive state can best be described by the *dynamic model* consisting of rapidly moving flux tube arrays. This conclusion is further supported by the magneto-optical observation[17] that in Pb films over a considerable magnetic field range the flux structure consists of a liquidlike mixed state of flux tubes containing many flux quanta. In the indium strips the resistive voltage steps could generally not be resolved presumably because of fluctuations in time of the number of normal channels. However, it is interesting to note that previous magneto-optical experiments with In strips of about 5-μm thickness[11] did not demonstrate the existence of a liquidlike arrangement of flux spots at any value of an externally applied magnetic field, the intermediate state structure consisting always of long extended domains of the normal phase.

A discrimination between the static and dynamic model of the current-induced resistive state in the microstrips should be possible from magnetic cou-

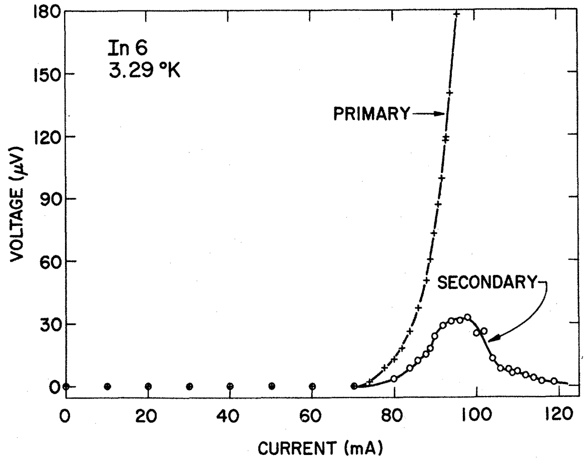

FIG. 8. Magnetic coupling in an In–Al_2O_3–In sandwich: primary and secondary voltage vs primary current (primary film: thickness = 1.0 μm, width = 175 μm, length = 6mm; secondary film: thickness = 0.39 μm, width = 175 μm, length = 10.8 mm; 1000-Å Al_2O_3 insulation; 3.29 °K).

pling experiments with a sandwich structure. If flux motion occurs in the current-carrying part (primary film) of the sandwich structure, and if the insulation between the two superconducting films of the sandwich is sufficiently thin, the flux structure in the other part (secondary film) of the sandwich should also be set in motion. This results in a voltage also in the secondary film to which no external current is applied.[13,22] The secondary voltage is also to be expected if in the primary film flux tubes of opposite sign move from the edge to the center. We have observed secondary voltages due to magnetic coupling in sandwich structures using both lead and indium as the primary strip. Again, these experiments were performed in zero external magnetic field, the only field present being that associated with the primary current. The absence of a secondary voltage above H_c or T_c confirmed in all specimens that the secondary voltages were not caused by current leakage between the two films. In Figs. 8 and 9 we show results obtained with two indium strips (samples In 6 and In 24). In these specimens magnetic coupling was investigated from the superconducting transition temperature down to 1.6 °K. The ratio of the secondary to the primary voltage (*coupling parameter*) was generally found to increase with decreasing temperature and to reach a maximum at an intermediate current above the critical current of the primary film. The reduction of the coupling parameter at higher currents may be explained by slippage between the flux structure in the primary and secondary film, which should become more likely at higher flux-tube velocities. In samples In 6 and In 24 we have observed values of the coupling parameter as large as 0.7–0.8. However, in other sandwich structures we found much smaller values of the coupling parameter or no magnetic coupling at all. It remains unclear whether the absence of a secondary voltage in some of our sandwiches was due to difficulties in achieving sufficient magnetic coupling between both films or whether it was caused by the absence of flux tube motion in the primary film. However, the appearance of secondary voltages in a series of Pb and In sandwiches tends to support the dynamic model for the current-induced resistive state in both lead and indium.

C. Electrical-Noise Power

As mentioned in Sec. III A, evidence for appreciable electrical noise generated in the current-carrying microstrips through fluctuations in the number of current-induced normal domains has been obtained from the high-resolution magneto-optical studies and the measurements of the voltage-current characteristics. In the following we

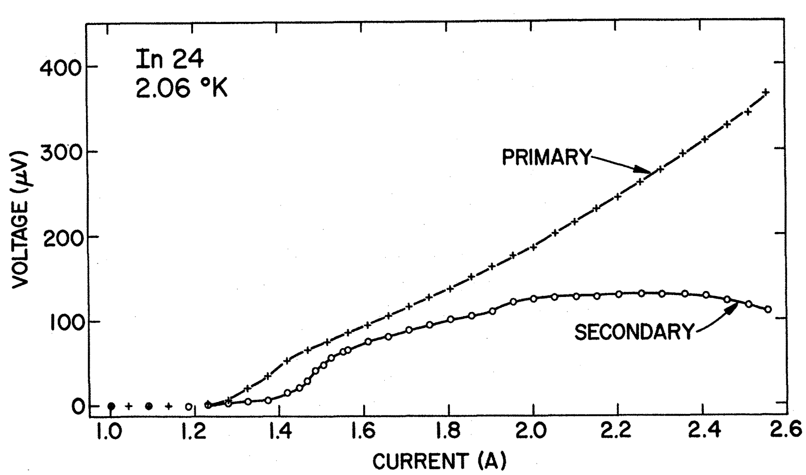

FIG. 9. Magnetic coupling in an In–SiO_2–In sandwich: primary and secondary voltage vs primary current (primary film: thickness = 5 μm, width = 167 μm, length = 6 mm; secondary film: thickness = 0.5 μm, width = 167 μm, length = 10.8 mm; 1000-Å SiO_2 insulation; 2.06 °K).

assume for simplicity that all normal domains generated in the sample by the current are identical and that it is only their *number* N which fluctuates by ΔN with time. The resistive voltage per normal channel is then, of course, also identical for all domains. The following three model spectra for the noise power are of interest.

(a) If the fluctuations ΔN in the number of normal domains are statistically independent of each other and have constant magnitude and lifetime τ_0 (pure "shot noise") the noise-power spectrum is given by[23]

$$w(\omega)=\frac{8\Phi^2\langle(\Delta N)^2\rangle_{\text{av}}}{\tau_0\omega^2}\sin^2\frac{\omega\tau_0}{2} \quad . \tag{5}$$

Here, Φ is the resistive voltage per normal channel, and ω the angular frequency. The notation $\langle\cdots\rangle_{\text{av}}$ denotes time average.

(b) Next we consider the case of statistically independent fluctuations ΔN with constant magnitude and an exponential distribution of their lifetime τ as indicated by the autocorrelation function

$$\langle\Delta N(t)\Delta N(t+\tau)\rangle_{\text{av}}=\langle(\Delta N)^2\rangle_{\text{av}}e^{-\tau/\tau_0} \; . \tag{6}$$

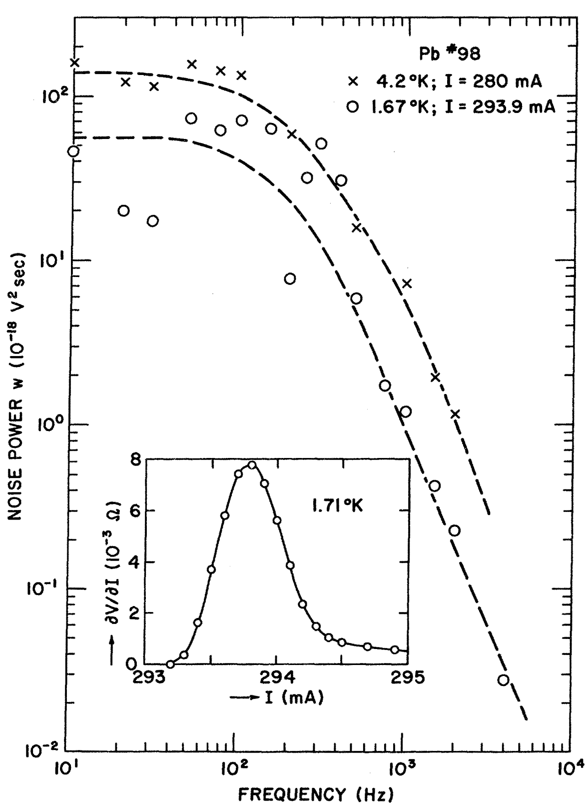

FIG. 10. Noise-power spectra at two temperatures in a Pb strip (thickness = 0.5 μm; width = 170 μm; length = 6.35 mm; resistance at 4.2 °K in the normal state = 49.5×10^{-3} Ω). The data were taken at a peak of the derivative $\partial V/\partial I$ (see inset).

Here the noise-power spectrum is[24]

$$w(\omega)=4\Phi^2\langle(\Delta N)^2\rangle_{\text{av}}\frac{\tau_0}{1+\omega^2\tau_0^2} \quad . \tag{7}$$

We note that this case is rather similar to the power spectrum of Eq. (5), except for the structure in the latter due to the factor $\sin^2(\frac{1}{2}\omega\tau_0)$.

(c) Finally, we assume statistically independent fluctuations ΔN with a distribution of their lifetime τ as indicated by Eq. (6) and with a coupling between the magnitude ΔN and the lifetime τ of the form

$$\Delta N\tau=\text{const}\equiv k \; . \tag{8}$$

A consequence of this coupling is an increasing of the noise power at high frequencies. A coupling such as given in Eq. (8) appears quite reasonable, since small fluctuations are expected to have a longer lifetime than large fluctuations. The noise power for this case shows ω^{-1} dependence at high frequencies, and the power spectrum is given by[25]

$$w(\omega)=4\nu\frac{k^2\Phi^2}{\omega^2\tau_0^2}\left[\omega\tau_0\arctan\omega\tau_0-\tfrac{1}{2}\ln(1+\omega^2\tau_0^2)\right] \; . \tag{9}$$

Here, ν is the number of fluctuations per unit time in the strip between the voltage probes.

In the Pb specimens the magnitude of the noise power and its variation with frequency and sample current has been found to vary considerably from sample to sample and to change in the same specimen due to thermal cycling between room temperature and 4.2 °K. It seems that the detailed behavior is again very sensitive to small changes in the sample characteristics. The In strips generally showed a maximum in the noise power near or somewhere above the critical current at which the first resistance appeared. Both in Pb and In the noise-power spectra very often showed a frequency dependence ranging from ω^{-1} to ω^{-2} behavior above and below the λ-point temperature. Such behavior can be attributed to flicker noise caused by a modulation of the electrical conductivity in the microstrips because of fluctuations in the number of normal domains. In various Pb and In specimens noise spectra have been observed showing very weak dependence of the noise power on frequency at low frequencies and approximately ω^{-2} behavior at high frequencies. Two typical cases are shown in Figs. 10 and 11. The data in Fig. 10 were obtained at a current value at which the derivative $\partial V/\partial I$ showed a strong maximum (see inset). From the similarity of the data in Fig. 10 above and below the temperature of the λ point it appears unlikely that the noise is related to nucleate boiling of liquid helium at the sample surface. The data in Figs. 10 and 11 show the frequency dependence of Eq. (7) and indicate a fluctuation of the number

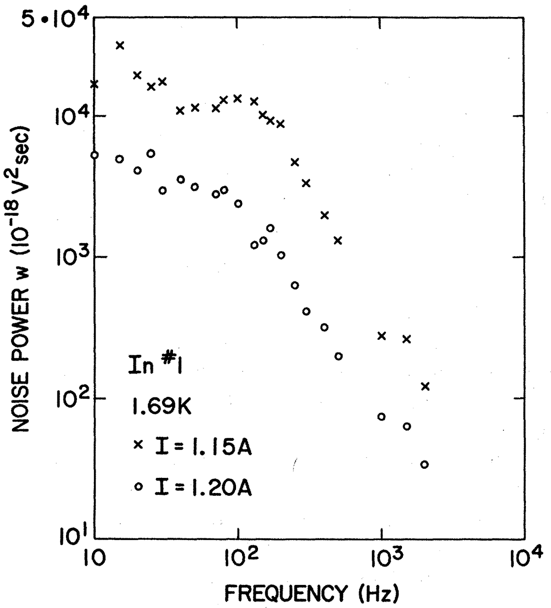

FIG. 11. Noise-power spectra at 1.69 °K in an In strip (thickness = 4.7 μm; width = 160 μm; length = 5.7 mm; resistance at 4.2 °K = 2.44 × 10^{-3} Ω).

of normal domains generated by the current, the fluctuation being characterized by a single time constant. A typical power spectrum showing ω^{-1} behavior is seen in Fig. 12.

The data shown in Fig. 10 and 11 can be analyzed in terms of Eq. (7), yielding the lifetime τ_0 of the fluctuation and the time average $\langle (\Delta N)^2 \rangle_{av}$. The results are listed in Table I. Here the quantity Φ has been calculated assuming the current-induced channels of normal phase to consist of a continuous normal domain of the width given in Table I, this width being estimated from our previous magneto-optical experiments.[11,17] In this calculation we may have considerably overestimated the voltage Φ per normal channel since we argued in Secs. III A and III B that the channels consist of moving arrays of flux tubes thereby contributing considerably less resistance than a continuous normal domain of the same width. Owing to this overestimate of Φ, the time average $\langle (\Delta N)^2 \rangle_{av}$ may be underestimated correspondingly. From τ_0 and the quantity $\langle (\Delta N)^2 \rangle_{av}$ one finds the rate ν for fluctuations by a single channel $[(\Delta N)^2 = 1]$ simply with

$$\nu = \langle (\Delta N)^2 \rangle_{av} / \tau_0 \, . \qquad (10)$$

The rate ν is also given in Table I.

The ω^{-1} behavior of the noise power as shown in Fig. 12, which has been observed in a series of specimens, may be understood in terms of the expression (9) which is based on a coupled distribution of τ and ΔN given by Eqs. (6) and (8). However, further analysis of the data using Eq. (9),

TABLE I. Lifetime τ_0, time average $\langle (\Delta N)^2{}_{av} \rangle$, and rate ν of the fluctuations of the number of current-induced normal channels obtained from the data of Figs. 10 and 11 using Eqs. (7) and (10).

Sample	Width of normal channel (μm)	τ_0 (10^{-3} sec)	$\langle (\Delta N)^2 \rangle_{av}$	ν (sec^{-1})
Pb strip No. 98 4.2 °K; I = 280 mA	0.8	1.0	1.2×10^{-2}	12
Pb strip No. 98 1.67 °K; I = 293.9 mA	0.8	1.4	3.0×10^{-3}	2.1
In strip No. 1 1.69 °K; I = 1.15 A	3	1.0	2.3	2200
In strip No. 1 1.69 °K; I = 1.20 A	3	1.0	0.54	540

including the determination of τ_0, would require an extension of our measurements to lower frequencies where $w(\omega)$ is expected to become frequency independent.

In Fig. 13 we have plotted the noise power versus the current through an indium microstrip for different temperatures. The data were obtained with the selective amplifier tuned to 320 Hz and $Q = 25$. A sharp peak in the noise power is ob-

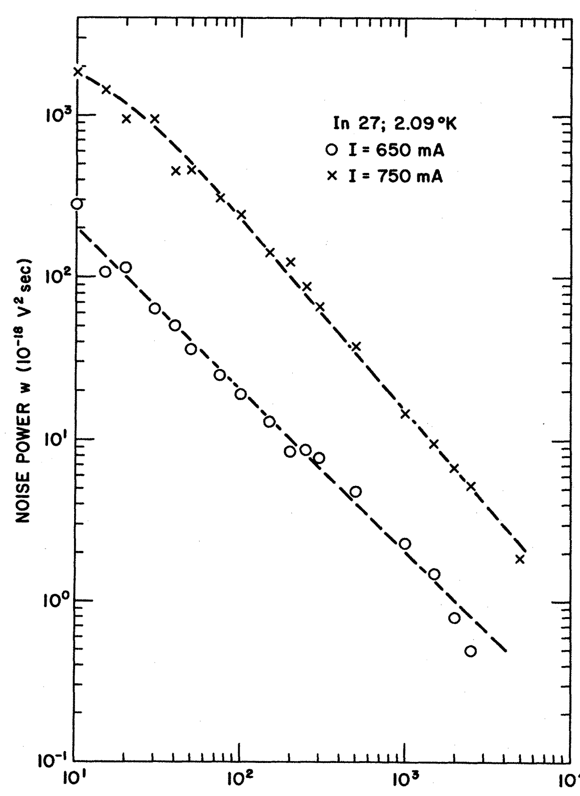

FIG. 12. Noise-power spectra at 2.09 °K and two current values in an In strip (thickness = 4.5 μm; width = 154 μm; length = 6 mm; resistance at 4.2 °K = 9.2 × 10^{-3} Ω).

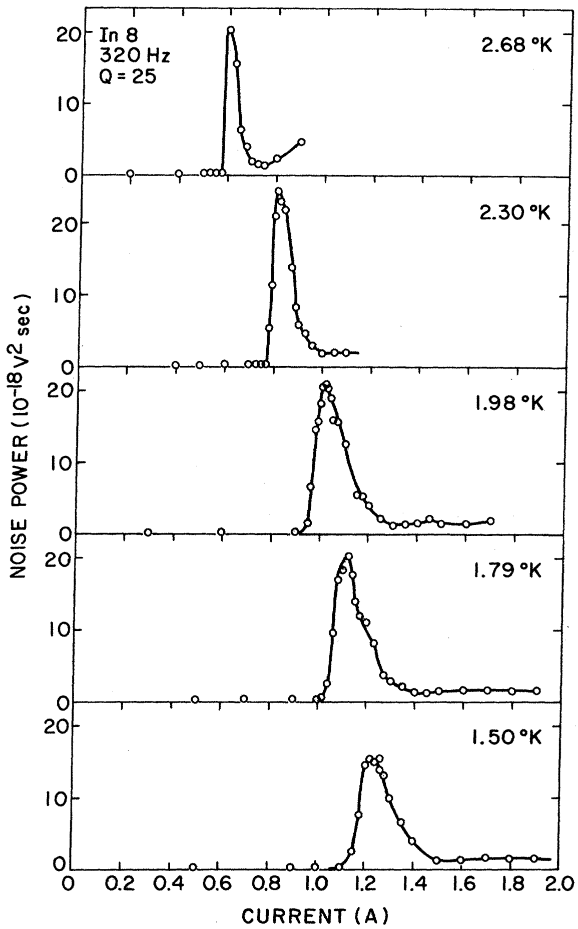

FIG. 13. Noise power for an In strip vs current for different temperatures (sample thickness = 2.0 μm; width = 190 μm; length = 6.3 mm; resistance at 4.2 °K = 4.1 $\times 10^{-3}$ Ω). Note power taken at 320 Hz with $Q = 25$.

served at all temperatures. The strong rise in the noise power has been found to occur close to the critical current at which the first resistive voltage appears. In the plots of Fig. 13 we have included the background noise power which is represented by the data at low currents. We note that the width of the peak in the noise power increases with decreasing temperature. A peak in the noise power above the critical current has been found in all indium samples. However, this peak was not always as sharp as shown in Fig. 13. The maximum in the noise power observed in the indium strips somewhere above the critical current value appears to be associated with an instability in the current-induced magnetic structure. We note that the average number of normal domains and the resistive voltage (except for some fine structure) have always been found to increase monotonically with current. However, a strong instability of the current-induced magnetic structure within a small current range above the critical current has been observed in previous magneto-optical studies of In strips.[11] A quantitative understanding of these instabilities clearly requires further investigations.

IV. CONCLUSIONS

(i) The electrical resistance in superconducting strips of lead and indium is restored by a large transport current over a wide current range in contrast to the behavior in cylindrical wires of type-I superconductors. High-resolution magneto-optical studies have shown that, during the restoration of the resistance by a transport current, channels of the normal phase are generated in the strip perpendicular to the current direction.

(ii) The creation of the channels of normal phase is a consequence of a magnetic instability similar to the kink instability in magnetohydrodynamics. It results in sharp resistive voltage steps and well-defined peaks in the derivative $\partial V/\partial I$. Whereas in lead strips the appearance of detailed structure in $\partial V/\partial I$ has been a general feature, appreciable peaks in $\partial V/\partial I$ in indium strips have been observed only close to T_c.

(iii) Above the critical current the number of channels of normal phase and the average slope of the $V(I)$ curve increase about linearly with current.

(iv) The current-induced intermediate state in strips of Pb and In can best be described by a dynamic model consisting of arrays of flux tubes moving rapidly from the edges to the center of the strip perpendicular to the current direction. This model is supported by the magnitude of the individual voltage steps and by the observation of dynamic magnetic coupling between two films of a sandwich structure.

(v) The noise-power spectra in the current-induced resistive state usually show a frequency dependence ranging from ω^{-1} to ω^{-2} behavior and may be attributed to flicker noise caused by fluctuations in the number of normal channels. From the noise power spectra the rate and lifetime of these fluctuations has been estimated. In the In strips a distinct peak in the noise power has been observed above the critical current.

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Electrical and Structural Properties of Amorphous Metal–Metal-Oxide Systems

J. J. Hauser
Bell Laboratories, Murray Hill, New Jersey 07974
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Amorphous mixtures of Al-Al_2O_3, Sn-SnO_2, Ta-Ta_2O_5, and Ni-NiO were sputtered in various concentrations at room temperature. The resistivity was measured as a function of temperature between 1 and 300 °K in three different ways; two-point and four-point contacts in a planar geometry and in a capacitance configuration. The most extensive measurements conducted on the Al-Al_2O_3 and Sn-SnO_2 systems showed that, over nine orders of magnitude in resistance the resistivity could be described by $\rho=\rho_0\, e^{(T_0/T)^{1/n}}$ with $n=4$ over a wide range of compositions in the Al-Al_2O_3 system. This resistivity behavior along with the current's voltage dependence and the ac resistivity measurements indicate that the conduction mechanism in such films may be similar to the thermally activated hopping mechanism described in other amorphous materials.

I. INTRODUCTION

The electrical and structural properties of metal–metal-oxide films have been studied in both limits: high metal concentration or granular films[1] and low metal concentration or cermet films.[2] The conduction mechanism has, however, been studied only at high temperatures and low electric fields[2–5] except in the granular system Ni–SiO_2, which was investigated at high fields and low temperatures.[6] However, little attention has been given to the conduction mechanism at low temperatures in amorphous metal–metal-oxide films. The approach used in the present study was to investigate the properties of certain metal–metal-oxide systems as a function of the oxide concentration starting from the granular-type films and ending with doped oxide films. One may expect that such a continuous approach will permit a better characterization of films which by electron or x-ray diffraction could only be described as amorphous.

From a theoretical point of view, the conduction in such amorphous materials as Ge and Si films or in 3*d* oxides was described by Mott[7] in terms of a phonon-assisted hopping mechanism. This theory was recently recast using percolation theory[8] and predicts that the electrical resistivity (ρ) should vary as $\exp(\text{const}\times T^{-1/4})$. This temperature dependence has been verified for amorphous Ge and Si [7,9–11] in the temperature range 60–300 °K. On the other hand, although this temperature dependence is observed in Fe_2O_3 [12] and NiO [13] below 60 °K, it seems to go over to a power law below 10 °K, in agreement with a recent calculation.[14]