these obvious assumptions our predictions are rather general and include the critical dynamics as well as the asymptotic temperature dependence of properties such as the central-peak half-width. In microscopic treatments, on the other hand, just the critical dynamics are usually neglected, due to insurmountable difficulties.[9,19]

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Spin and Charge Susceptibility of a Ferromagnetic Electron Gas*

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In the ferromagnetic state of an electron gas where the energy bands are spin split, the charge potential produces a spin polarization besides a charge polarization and a magnetic field produces a charge polarization besides a spin polarization. Thus, in a ferromagnetic electron gas, besides the spin susceptibility to a magnetic field χ^{mm} and the charge susceptibility to a charge potential χ^{ee}, we introduce two nondiagonal susceptibilities, the spin response to a charge potential χ^{me} and the charge response to a magnetic field χ^{em}. We present a general formulation for these four susceptibilities and discuss them within the Hartree–Fock or random-phase approximation, taking into account the long-range nature of the Coulomb interaction properly. The importance of the nondiagonal susceptibilities in analyzing observed spin and charge polarization around an impurity is pointed out.

I. INTRODUCTION

As a physical model of a metal, the electron gas has been studied extensively in recent years. Most of the studies, however, are restricted to the paramagnetic state of the electron gas. Compared with the large amount of work on the magnetic and electrical properties of the paramagnetic electron gas[1] there is very little on the magnetized state (either ferromagnetic or spin-density wave) of the electron gas. For instance, until very recently we did not have a proper expression for the wave-number-dependent magnetic susceptibility for an electron gas in the ferromagnetic state in which the energy bands of the electron gas are spin split.

In this paper we intend to discuss the magnetic and electrical responses of the electron gas in the ferromagnetic state. In the ferromagnetic state besides the ordinary magnetic susceptibility χ^{mm} and the electrical susceptibility χ^{ee} or the dielectric constant ϵ, we obtain equally important nondiagonal responses χ^{me} and χ^{em}, which are, respectively, the magnetic (spin) response to the electric disturbance and the electric (charge) response to the magnetic disturbance.[2] Note that χ^{me} and χ^{em} vanish in the paramagnetic state of the electron gas, and they become finite only in the ferromagnetic state. This comes from the fact that in the

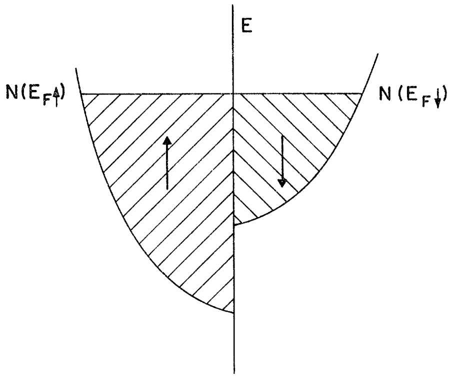

FIG. 1. Density of states of an electron gas in the ferromagnetic state. Generally the densities of states of up- and down-spin electrons at the Fermi surface, $N_+(0)$ and $N_-(0)$, are different.

spin-split state of an electron gas the spin polarization accompanies the charge polarization and vice versa.

This can be easily seen from Fig. 1. In the ferromagnetic state of the electron gas, the densities of states of up- and down-spin electrons at the Fermi surface, $N_+(0)$ and $N_-(0)$, respectively, are different. Thus the Zeeman shift of each spin band due to an external field tends to produce a change in the number of electrons besides the spin polarization. This situation may also be explained by considering the response to a δ-function-type magnetic field. As is well known, the up- and down-spin electrons are polarized in the form of Ruderman–Kittel oscillation in response to the δ-function-type magnetic field. In the paramagnetic state the amplitudes and wavelengths of these up- and down-spin electron density oscillations are the same but the signs are just opposite. Thus in the paramagnetic states the magnetic field does not produce any charge polarization. In the ferromagnetic state, however, due to the difference between $N_+(0)$ and $N_-(0)$ both the amplitudes and wavelengths of the Ruderman-Kittel-Kasuya-Yosida (RKKY) oscillations of up- and down-spin electrons are different. Thus the cancellation of charge polarization as we saw in the paramagnetic state does not occur, and an oscillating charge polarization is produced by the δ-function magnetic field. Similarly, it is easy to see that in the ferromagnetic state of an electron gas, spin polarization is produced by a charge potential.

The nondiagonal susceptibility χ^{me} was first considered by Friedel[3] for the case of zero wave number, and the general properties of the charge and spin density correlation in the ferromagnetic electron gas were recently discussed by Rajagopal, Brooks, and Ranganathan.[4] Although our approach in obtaining the susceptibilities is different, our result concerning the correlation between the charge and spin polarization is essentially the same as the result of Rajagopal, Brooks, and Ranganathan.[4]

It has been customary to use a δ-function-type screened Coulomb interaction,[5–7] instead of the real Coulomb interaction, in calculating the paramagnetic susceptibility of an electron gas whereas one must use the unscreened Coulomb interaction in discussing the dielectric constant or the motion of the charge density. We will see how this situation comes about in the course of our discussion later. Therefore, we are not allowed to use the screened-Coulomb-interaction model in calculating the magnetic, electric, or nondiagonal susceptibilities for the spin-split state of the electron gas, since even in the calculation of χ^{mm} the charge density accompanied by the spin density is involved. This is the reason why the previous calculation of χ^{mm} for the ferromagnetic state with the δ-function interaction failed to conserve the number of electrons and did not reduce to the well-known Stoner result[8] in the case of zero wave number. This point about χ^{mm} was discussed recently by a number of authors with slightly different results.[2,9–11] Our result for χ^{mm} is the same as that of Schrieffer.[9] Accordingly, in this paper we start with the bare Coulomb interaction and not the screened Coulomb interaction.

Recently, data on the elastic diffuse scattering of neutrons in ferromagnetic transition metals with impurities have been presented.[12] These data give us very detailed information on the spin polarization of the host metal electrons around an impurity. One of the purposes of this paper is to present a sound basis for analyzing these neutron data.

II. DERIVATION OF SUSCEPTIBILITIES

The Hamiltonian of an electron gas is given as

$$\mathcal{H}^0 = \sum_{l,\sigma} \epsilon_l c^\dagger_{l\sigma} c_{l\sigma} + \tfrac{1}{2} \sideset{}{'}\sum_{\substack{l,l',\kappa \\ \sigma,\sigma'}} V(\kappa) c^\dagger_{l\sigma} c^\dagger_{l'\sigma'} c_{l'-\kappa,\sigma'} c_{l+\kappa,\sigma}$$

$$= \mathcal{H}_k + \mathcal{H}_c \, . \tag{2.1}$$

$\mathcal{H}_k$, the first term of the Hamiltonian, is the kinetic energy of the conduction electrons, and $c^\dagger_{l\sigma}$ is the creation operator of a conduction electron with energy ϵ_l and spin σ (= + or −). $\mathcal{H}_c$, the second term of the Hamiltonian, is the Coulomb repulsion, where

$$V(\kappa) = 4\pi e^2/\kappa^2 \tag{2.2}$$

and the prime on the summation indicates that $\kappa = 0$ is excluded from the summation. $\mathcal{H}^0$ is the Hamiltonian of the electron gas without any perturbation. Our problem in this paper is to discuss the response of the electron gas to the magnetic and/or charge disturbances. Since it is sufficient to know the response to a general Fourier component, with wave number q, of the disturbances, we choose the perturbing potentials as follows:

$$\mathcal{H}'_m = -\mu_B H(q) \sum_l (c^\dagger_{l+} c_{l-q,+} - c^\dagger_{l-} c_{l-q,-}) , \qquad (2.3)$$

$$\mathcal{H}'_e = e U(q) \sum_l (c^\dagger_{l+} c_{l-q,+} + c^\dagger_{l-} c_{l-q,-}) . \qquad (2.4)$$

In the Hamiltonian of the magnetic disturbance $\mathcal{H}'_m$, μ_B (<0) is the Bohr magneton, and $H(q)$ is the Fourier component of the perturbing magnetic field. In the Hamiltonian for the charge disturbance $\mathcal{H}'_e$, $e\,(<0)$ is the charge of an electron, and $U(q)$ is the Fourier component of the charge potential. For the moment we assume that $q \neq 0$.

If there is no perturbation the electron density of either spin is spatially uniform and the Fourier component of the electron number density of either spin vanishes for a finite wave number $q \neq 0$. The magnetic or charge potential with a wave number q produces an electron density with the wave number q. The four susceptibilities which we will discuss in this paper are defined in terms of the perturbed electron number density $n_\pm(q)$ as follows:

$$\chi^{\mathrm{mm}}(q) = \frac{\mu_B[n^m_+(q) - n^m_-(q)]}{H(q)} , \qquad (2.5\text{a})$$

$$\chi^{\mathrm{em}}(q) = \frac{e[n^m_+(q) + n^m_-(q)]}{H(q)} \qquad (2.5\text{b})$$

which are, respectively, the spin and charge susceptibility to a magnetic field $H(q)$, and

$$\chi^{\mathrm{me}}(q) = \frac{\mu_B[n^e_+(q) - n^e_-(q)]}{-U(q)} , \qquad (2.5\text{c})$$

$$\chi^{\mathrm{ee}}(q) = \frac{e[n^e_+(q) + n^e_-(q)]}{-U(q)} \qquad (2.5\text{d})$$

which are, respectively, the spin and charge susceptibility to a charge potential $U(q)$. The superscripts m or e on the electron number density indicate the nature of the perturbation, either $\mathcal{H}'_m$ or $\mathcal{H}'_e$. Thus our problem is to calculate $n^\alpha_\pm$ corresponding to a perturbation $\mathcal{H}'_\alpha$ where α is either m or e.

In calculating $n^\alpha_\pm(q)$ in this paper, we use the double-time Green's-function method.[13] The retarded double-time Green's function of two fermion operators a and $b^\dagger$ is defined as

$$\langle a | b^\dagger \rangle^r_t = -i \langle [a(t), b^\dagger]_+ \rangle \, \Theta(t) , \qquad (2.6)$$

where $[\ ,\ ]_+$ is the anticommutator, $a(t)$ is the Heisenberg representation with the total Hamiltonian of the system $\mathcal{H}$, and $\Theta(t)$ is the step function defined by

$$\Theta(t) = \begin{cases} 1 & \text{for } t > 0 \\ 0 & \text{for } t < 0 . \end{cases} \qquad (2.7)$$

The thermal expectation value $\langle b^\dagger a \rangle$ under the Hamiltonian $\mathcal{H}$ is obtained from the Green's function by the following relation:

$$\langle b^\dagger a \rangle = -\frac{1}{2\pi i} \int_{-\infty}^{\infty} f(\omega) [\langle a | b^\dagger \rangle^r_{\omega + i0^+} - \langle a | b^\dagger \rangle^r_{\omega - i0^+}] d\omega$$
$$= -\frac{1}{\pi} \int_{-\infty}^{\infty} f(\omega) \, \mathrm{Im} \langle a | b^\dagger \rangle_{\omega + i0^+} d\omega , \qquad (2.8)$$

where $f(\omega)$ is the Fermi distribution function and $\langle a | b^\dagger \rangle^r_\omega$ is the Fourier transform of $\langle a | b^\dagger \rangle^r_t$:

$$\langle a | b^\dagger \rangle^r_\omega = \int_{-\infty}^{\infty} \langle a | b^\dagger \rangle^r_t \, e^{-i\omega t} dt . \qquad (2.9)$$

The necessary Green's function is obtained from the equation of motion

$$\omega \langle a | b^\dagger \rangle_\omega = \langle [a, b^\dagger]_+ \rangle + \langle [a, \mathcal{H}] b^\dagger \rangle_\omega , \qquad (2.10)$$

where $[\ ,\ ]$ is the commutator and we omit the superscript r indicating the retarded Green's function for simplicity. Note that throughout this paper we take the chemical potential of the system as the origin for measuring energy.

From the prescription given above [Eqs. (2.6)–(2.10)] in order to calculate the electron number density for each spin

$$n^\alpha_\pm(q) = \left\langle \sum_k c^\dagger_{k\pm} c_{k+\mathrm{q},\pm} \right\rangle, \quad \alpha = m \text{ or } e \qquad (2.11)$$

we need to obtain the Green's function of the following form:

$$\langle c_{k+q,\pm} | c^\dagger_{k,\pm} \rangle_t = -i \langle [c_{k+q,\pm}(t), \ c^\dagger_{k,\pm}]_+ \rangle \Theta(t) . \qquad (2.12)$$

In Eqs. (2.11) and (2.12) the Hamiltonian to be used in taking the thermal average or setting up the equation of motion is

$$\mathcal{H}^\alpha \equiv \mathcal{H}^0 + \mathcal{H}'_\alpha , \quad \alpha = m \text{ or } e \qquad (2.13)$$

where α should be chosen in accordance with Eq. (2.11).

In setting up the equation of motion of the form of Eq. (2.10) we should note that $\mathcal{H}'_m$ and $\mathcal{H}'_e$ [Eqs. (2.3) and (2.4)] are not much different. In $\mathcal{H}'_m$ the difference of the electron density operators of either spins appears whereas in $\mathcal{H}'_e$ the sum of the electron density operators appears. This relation enables us to handle the two cases, magnetic and charge disturbances, by a single procedure. The equation of the motion for our Green's function is given as

$$\omega \langle c_{k+q,\pm} | c^\dagger_{k\pm} \rangle_\omega = \delta_{q,0} + \langle [c_{k+q,\pm}, \mathcal{H}_k] | c^\dagger_{k\pm} \rangle_\omega$$
$$+ \langle [c_{k+q,\pm}, \mathcal{H}'_\alpha] | c^\dagger_{k\pm} \rangle_\omega + \langle [c_{k+q,\pm}, \mathcal{H}_c] | c^\dagger_{k\pm} \rangle_\omega . \qquad (2.14)$$

First we calculate the three commutators which appear on the right-hand side of Eq. (2.14). The first two commutators with $\mathcal{H}_k$ and $\mathcal{H}'_\alpha$ are simple:

$$[c_{k+q,\pm}, \mathcal{H}_k] = \epsilon_{k+q} c_{k+q,\pm} , \qquad (2.15)$$

$$[c_{k+q,\pm}, \mathcal{H}'_\alpha] = \begin{cases} \mp \mu_B H(q) c_{k\pm} & \text{for } \alpha = m \\ e U(q) c_{k\pm} & \text{for } \alpha = e . \end{cases} \qquad (2.16)$$

Note that Eq. (2.16) can be rewritten by introducing a notation

$$W_{\pm}^{\alpha} = \begin{cases} \mp\mu_B H(q) & \text{for } \alpha = m \\ eU(q) & \text{for } \alpha = e \end{cases} \tag{2.17}$$

as

$$[c_{k+q,\pm}, \mathcal{H}'_{\alpha}] = W_{\pm}^{\alpha} c_{k\pm} . \tag{2.16'}$$

The commutator with the Coulomb interaction is given as

$$[c_{k+q,\pm}, \mathcal{H}_c] = \sum_{l,\kappa,\sigma'}{}' V(\kappa) c_{l\sigma'}^{\dagger} c_{l-\kappa,\sigma'} c_{k+q+\kappa,\pm} , \tag{2.18}$$

where the summation over σ' is for $\sigma' = \pm$.

By inserting Eqs. (2.15), (2.16′), and (2.18) into Eq. (2.14) we obtain

$$(\omega - \epsilon_{k+q}) \langle c_{k+q,\pm} | c_{k,\pm}^{\dagger} \rangle_{\omega} = \delta_{q,0} + W_{\pm}^{\alpha}(q) \langle c_{k\pm} | c_{k\pm}^{\dagger} \rangle_{\omega} + \sum_{l,\kappa,\sigma'} V(\kappa) \langle c_{l,\sigma'}^{\dagger} c_{l-\kappa,\sigma'} c_{k+q+\kappa,\pm} | c_{k\pm}^{\dagger} \rangle_{\omega} . \tag{2.19}$$

The last term on the right-hand side of Eq. (2.19) introduces a new Green's function of higher order. Instead of following the equation of motion for this higher-order Green's function, we decouple the Green's function by a random-phase approximation (RPA) or Hartree–Fock approximation as follows:

$$\sum_{l,\kappa,\sigma'} V(\kappa) \langle c_{l\sigma'}^{\dagger} c_{l-\kappa,\sigma'} c_{k+q+\kappa,\pm} | c_{k,\pm}^{\dagger} \rangle_{\omega} \cong V(q) \Big[\sum_{l,\sigma'} \langle c_{l\sigma'}^{\dagger} c_{l+q,\sigma'} \rangle \Big] \langle c_{k\pm} | c_{k\pm}^{\dagger} \rangle_{\omega} (1 - \delta_{q,0}) - \Big[\sum_{\kappa}{}' V(\kappa) \langle c_{k+q+\kappa,\pm}^{\dagger} c_{k+q+\kappa,\pm} \rangle \Big] \langle c_{k+q\pm} | c_{k\pm}^{\dagger} \rangle_{\omega} - \Big[\sum_{\kappa}{}' V(\kappa) \langle c_{k+\kappa,\pm}^{\dagger} c_{k+\kappa+q,\pm} \rangle \Big] \langle c_{k\pm} | c_{k\pm}^{\dagger} \rangle_{\omega} . \tag{2.20}$$

The principal approximation used in obtaining Eq. (2.20) is to replace the electron number density operators, with wave numbers q or 0, in the Green's function by their expectation values and factorize them out. As is easily seen if the approximation of Eq. (2.20) is used, Eq. (2.19) for different k's and q's gives a closed set of equations for the problem.

We discuss the physical meaning of each term on the right-hand side of Eq. (2.20). The first term is rewritten as

$$V(q) \Big[\sum_{l,\sigma'} \langle c_{l\sigma'}^{\dagger} c_{l+q,\sigma'} \rangle \Big] \langle c_{k\pm} | c_{k\pm}^{\dagger} \rangle_{\omega} = V(q) [n_{+}^{\alpha}(q) + n_{-}^{\alpha}(q)] \langle c_{k\pm} | c_{k\pm}^{\dagger} \rangle_{\omega} . \tag{2.21}$$

Note that the above expression reduces to the form of the second term on the right-hand side of Eq. (2.19). From Eqs. (2.16), (2.17), and (2.19) it is easy to identify that Eq. (2.21) gives the contribution of the Coulomb potential due to the perturbed electron number density, $V(q)[n_{+}^{\alpha}(q) + n_{-}^{\alpha}(q)]$, which is spin independent and corresponds to $eU(q)$. It is important to note that this term diverges as q approaches zero.

In the second term on the right-hand side of Eq. (2.20), we introduce the effective exchange interaction $\tilde{V}(0)$ by the following relation:

$$\begin{aligned} \sum_{\kappa}{}' V(\kappa) \langle c_{k+q+\kappa,\pm}^{\dagger} c_{k+q+\kappa,\pm} \rangle \langle c_{k+q,\pm} | c_{k\pm}^{\dagger} \rangle_{\omega} &\equiv \tilde{V}(0) \sum_{\kappa}{}' \langle c_{k+q+\kappa,\pm}^{\dagger} c_{k+q+\kappa,\pm} \rangle \langle c_{k+q,\pm} | c_{k\pm}^{\dagger} \rangle_{\omega} \\ &= \tilde{V}(0) n_{\pm}^{\alpha}(0) \langle c_{k+q,\pm} | c_{k,\pm}^{+} \rangle_{\omega} . \end{aligned} \tag{2.22}$$

Note that the $\tilde{V}(0)$ introduced does not diverge unlike the original Coulomb potential $V(q)$ at $q = 0$. The first term of Eq. (2.22) can be interpreted as a weighted sum of occupied electron states with momentum $k + q + \kappa$ with the weight $V(\kappa)$. The weight $V(\kappa)$ diverges as $1/\kappa^2$ near $\kappa = 0$ but this divergence is cancelled by the κ^2 which comes from the density of states. Thus $\tilde{V}(0)$, the average weight, is a finite constant. Now from Eq. (2.22) it is easy to recognize that the second term on the right-hand side of Eq. (2.20) manifests the effect of the uniform exchange field which gives rise to the spin splitting of the electron energy band. This point will become clearer later.

The bracketed quantity in the last term on the right-hand side of Eq. (2.20) is rewritten by the same way as we did in Eq. (2.22), as

$$\sum_{\kappa}{}' V(\kappa) \langle c_{k+\kappa,\pm}^{\dagger} c_{k+\kappa+q,\pm} \rangle \langle c_{k\pm} | c_{k\pm}^{\dagger} \rangle_{\omega} \equiv \tilde{V}(q) n_{\pm}^{\alpha}(q) \langle c_{k\pm} | c_{k\pm}^{\dagger} \rangle_{\omega} . \tag{2.23}$$

By the same argument that we gave for Eq. (2.22) the effective exchange interaction $\tilde{V}(q)$ does not diverge for small q. Equation (2.23) resembles Eq. (2.21) but is different in that in the former the effective interaction $\tilde{V}(q) n_{\pm}^{\alpha}(q)$ depends on the spin since $n_{+}^{\alpha}(q) \neq n_{-}^{\alpha}(q)$. This term represents the effect of the exchange field associated with the wave number q component of the spin density due to the magnetic or charge disturbances.

By inserting Eqs. (2.21)–(2.23) into Eq. (2.20) and then Eq. (2.20) into Eq. (2.19) we obtain

$$\begin{aligned} [\omega - \epsilon_{k+q} - \tilde{V}(0) n_{\pm}^{\alpha}(0)] \langle c_{k+q,\pm} | c_{k,\pm}^{\dagger} \rangle_{\omega} &= \delta_{q,0} + W_{\pm(q)}^{\alpha} \langle c_{k\pm} | c_{k\pm}^{\dagger} \rangle_{\omega} \\ &\quad + V(q) [n_{+}^{\alpha}(q) + n_{-}^{\alpha}(q)] \langle c_{k\pm} | c_{k\pm}^{\dagger} \rangle_{\omega} (1 - \delta_{q,0}) \\ &\quad - \tilde{V}(q) n_{\pm}^{\alpha}(q) \langle c_{k\pm} | c_{k\pm}^{\dagger} \rangle_{\omega} (1 - \delta_{q,0}) . \end{aligned} \tag{2.24}$$

In Eq. (2.24) $n_{\pm}^{\alpha}(q)$ and $n_{\pm}^{\alpha}(0)$ are to be determined self-consistently through the procedure [Eq. (2.8)] with the Green's functions $\langle c_{k+q,\pm} | c^{\dagger}_{k\pm}\rangle_\omega$ and $\langle c_{k\pm} | c^{\dagger}_{k\pm}\rangle_\omega$. Thus Eq. (2.24) for different k's and q's constitutes a closed set of equations. Note that in obtaining Eq. (2.24) we used only one approximation contained in Eq. (2.20). In order to obtain the four susceptibilites defined in Eqs. (2.5a), (2.5b), (2.5c), and (2.5d), we only need to know the perturbed electron number density up to the first order in the perturbation. Thus as the second (and last) stage of our approximation we linearize Eq. (2.24) with respect to the perturbing potential as follows:

$$\begin{aligned}(\omega-\epsilon_{k+q,\pm})\langle c_{k+q,\pm} | c^{\dagger}_{k,\pm}\rangle_\omega &= \delta_{q,0}+W^{\alpha}_{\pm}(q)\,\langle c_{k\pm} | c^{\dagger}_{k\pm}\rangle^0_\omega \\ &\quad + V(q)[n^{\alpha}_{+}(q)+n^{\alpha}_{-}(q)]\langle c_{k\pm} | c^{\dagger}_{k\pm}\rangle^0_\omega\,(1-\delta_{q,0}) \\ &\quad - \tilde{V}(q)n^{\alpha}_{\pm}(q)\,\langle c_{k\pm} | c^{\dagger}_{k\pm}\rangle^0_\omega\,(1-\delta_{q,0})\ , \qquad (2.25)\end{aligned}$$

where the superscript 0 indicates to calculate the quantity without the effect of the perturbing potential $\mathcal{H}'_\alpha$ and the quantities without the superscript 0 are supposed to be calculated up to the first order in $\mathcal{H}'_\alpha$, and we introduced the one-particle energy of an electron with the exchange self-energy as

$$\epsilon_{k,\pm} \equiv \epsilon_k - \tilde{V}(0)n^0_{\pm}\ , \qquad (2.26)$$

where $n^0_{\pm}$ is the number of (±) spin electrons in the unperturbed state. From Eq. (2.25), for $q=0$, it is easy to see

$$\langle c_{k\pm} | c^{\dagger}_{k\pm}\rangle^0_\omega = 1/(\omega-\epsilon_{k\pm}+i0^+)\ . \qquad (2.27)$$

Now by inserting Eq. (2.26) into Eq. (2.25) we obtain the explict expression for the Green's function:

$$\begin{aligned}\langle c_{k+q,\pm} | c^{\dagger}_{k,\pm}\rangle_\omega &= \frac{\delta_{q,0}}{\omega-\epsilon_{k+q,\pm}+i0^+} + W^{\alpha}_{\pm}(q)\,\frac{1}{\omega-\epsilon_{k+q,\pm}+i0^+}\,\frac{1}{\omega-\epsilon_{k,\pm}+i0^+} \\ &\quad + V(q)\,[n^{\alpha}_{+}(q)+n^{\alpha}_{-}(q)]\,\frac{1}{\omega-\epsilon_{k+q,\pm}+i0^+}\,\frac{1}{\omega-\epsilon_{k,\pm}+i0^+}\,(1-\delta_{q,0}) \\ &\quad - \tilde{V}(q)n^{\alpha}_{\pm}(q)\,\frac{1}{\omega-\epsilon_{k+q,\pm}+i0^+}\,\frac{1}{\omega-\epsilon_{k,\pm}+i0^+}\,(1-\delta_{q,0})\ . \qquad (2.28)\end{aligned}$$

The electron number density is immediately obtained by applying the procedure of Eq. (2.8) to Eq. (2.28):

$$n^{\alpha}_{\pm}(q) = -\frac{1}{\pi}\sum_k \int_{-\infty}^{\infty} d\omega\, f(\omega)\,\mathrm{Im}\langle c_{k+q,\pm} | c^{\dagger}_{k,\pm}\rangle_{\omega+i0^+} = -\,W^{\alpha}_{\pm}(q)F_{\pm}(q) - V(q)[n^{\alpha}_{+}(q)+n^{\alpha}_{-}(q)]F_{\pm}(q) + \tilde{V}(q)n^{\alpha}_{\pm}(q)F_{\pm}(q)\ , \qquad (2.29)$$

where

$$\begin{aligned}F_{\pm}(q) &= \frac{1}{\pi}\sum_k \int_{-\infty}^{\infty} d\omega f(\omega)\,\mathrm{Im}\left(\frac{1}{\omega-\epsilon_{k+q,\pm}+i0^+}\,\frac{1}{\omega-\epsilon_{k,\pm}+i0^+}\right) \\ &= \sum_k \int_{-\infty}^{\infty} d\omega f(\omega)\left(\delta(\omega-\epsilon_{k+q,\pm})\frac{p}{\omega-\epsilon_{k,\pm}} + \frac{p}{\omega-\epsilon_{k+q,\pm}}\delta(\omega-\epsilon_{k,\pm})\right) = -\sum_k \frac{f_{k,\pm}-f_{k+q,\pm}}{\epsilon_{k\pm}-\epsilon_{k+q,\pm}} \qquad (2.30)\end{aligned}$$

is the Lindhard function, where $f_{k\pm}=f(\epsilon_{k,\pm})$ and we assumed $q \neq 0$. For $q=0$ the problem is much simpler as can be seen from Eq. (2.28). Since our result for $q\neq 0$ gives the correct $q=0$ result in the limit of $q \to 0$ as will be discussed in Sec. II, we do not consider the $q=0$ case here.

Equation (2.29) is a coupled equation for $n^{\alpha}_{+}(q)$ and $n^{\alpha}_{-}(q)$. By solving these coupled equations we obtain the electron number density for each spin:

$$n^{\alpha}_{\pm}(q) = \frac{-\tilde{F}_{\pm}(q)[1+V(q)\tilde{F}_{\mp}(q)]\,W^{\alpha}_{\pm}(q) + V(q)\tilde{F}_{+}(q)\tilde{F}_{-}(q)\,W^{\alpha}_{\mp}(q)}{1+V(q)[\tilde{F}_{+}(q)+\tilde{F}_{-}(q)]}\ , \qquad (2.31)$$

where we introduced the exchange-enhanced Lindhard function $\tilde{F}_{\pm}(q)$ as

$$\tilde{F}_{\pm}(q) = F_{\pm}(q)/[1-\tilde{V}(q)F_{\pm}(q)]\ . \qquad (2.32)$$

If we use the explicit expressions for magnetic and charge disturbances, Eq. (2.17), we obtain $n^{m}_{\pm}(q)$ and $n^{e}_{\pm}(q)$ as[14]

$$n^{m}_{\pm}(q) = \pm\,\frac{\tilde{F}_{\pm}(q)+2V(q)\tilde{F}_{+}(q)\tilde{F}_{-}(q)}{1+V(q)[\tilde{F}_{+}(q)+\tilde{F}_{-}(q)]}\,\mu_B H(q)\ , \qquad (2.31\text{a})$$

$$n^{e}_{\pm}(q) = -\,\frac{\tilde{F}_{\pm}(q)}{1+V(q)[\tilde{F}_{+}(q)+\tilde{F}_{-}(q)]}\,eU(q)\ . \qquad (2.31\text{b})$$

It is straightforward to obtain the four susceptibilities[2] defined in Eqs. (2.5a)–(2.5d) from Eqs. (2.31a) and (2.31b):

$$\chi^{mm}(q) = \mu_B^2\,\frac{\tilde{F}_{+}(q)+\tilde{F}_{-}(q)+4V(q)\tilde{F}_{+}(q)\tilde{F}_{-}(q)}{1+V(q)[\tilde{F}_{+}(q)+\tilde{F}_{-}(q)]}\ , \qquad (2.33)$$

$$\chi^{em}(q) = \chi^{me}(q) = \mu_B e\,\frac{\tilde{F}_{+}(q)-\tilde{F}_{-}(q)}{1+V(q)[\tilde{F}_{+}(q)+\tilde{F}_{-}(q)]}\ , \qquad (2.34)$$

$$\chi^{ee}(q)=e^2\,\frac{\tilde{F}_+(q)+\tilde{F}_-(q)}{1+V(q)[\tilde{F}_+(q)+\tilde{F}_-(q)]}\,. \tag{2.35}$$

We will discuss these results in detail in Sec. III.

Although in this paper we discuss only the static susceptibilities, the dynamic susceptibilities $\chi^{\alpha\alpha'}(q,\omega)$, where α, $\alpha'=m$ or e, corresponding to the above four static susceptibilities $\chi^{\alpha\alpha'}(q)$ are given simply by replacing the static Lindhard function $F_\pm(q)$ by the corresponding frequency-dependent Lindhard function $F_\pm(q,\omega)$ defined as

$$F_\pm(q,\omega)=-\sum_{\kappa}\frac{f_{k,\pm}-f_{k+q,\pm}}{\epsilon_{k\pm}-\epsilon_{k+q,\pm}+\omega+i0^+}\,. \tag{2.36}$$

In calculating $\chi^{mm}(q,\omega)$ from the Kubo formula, for instance, if we use the approximation shown in Eqs. (2.20) and (2.23) starting from the bare Coulomb interaction, Eq. (4.20) of Ref. 6 is replaced by

$$[\theta_{k\sigma}(q),\mathcal{H}_c]=V(q)\,(n_{k+q,\sigma}-n_{k\sigma})\sum_{l\sigma'}\Theta_{l\sigma'}(q)$$
$$-\tilde{V}(q)\,(n_{k+q,\sigma}-n_{k\sigma})\sum_{l}\Theta_{l\sigma}(q)\,,$$

where $\theta_{k\sigma}(q)=c^\dagger_{k+q,\sigma}c_{k\sigma}$. This replacement in Ref. 6 leads to

$$\chi^{mm}(q,\omega)=\mu_B^2\,\frac{\tilde{F}_+(q,\omega)+\tilde{F}_-(q,\omega)+4V(q)\tilde{F}_+(q,\omega)\tilde{F}_-(q,\omega)}{1+V(q)[\tilde{F}_+(q,\omega)+\tilde{F}_-(q,\omega)]}\,, \tag{2.33'}$$

where

$$\tilde{F}_\pm(q,\omega)=F_\pm(q,\omega)/[1-\tilde{V}(q)F_\pm(q,\omega)]\,. \tag{2.32'}$$

III. DISCUSSION ON SUSCEPTIBILITIES

In this section we discuss various interesting aspects of the four susceptibilities $\chi^{mm}(q)$, $\chi^{em}(q)=\chi^{me}(q)$, and $\chi^{ee}(q)$.

A. $\chi^{mm}(q)$

First we check that $\chi^{mm}(q)$ reduces to the well-known formula in the paramagnetic state of the electron gas. Since in the paramagnetic state there is no spin splitting of the band, the Lindhard function becomes independent of spins:

$$F_+(q)=F_-(q)\equiv F(q)$$
$$=-\sum_k\frac{f_k-f_{k+q}}{\epsilon_k-\epsilon_{k+q}}\,. \tag{3.1}$$

By inserting Eq. (3.1) into Eq. (2.33) we obtain

$$\chi^{mm}_{\text{para}}(q)=2\mu_B^2\,\frac{F(q)}{1-\tilde{V}(q)F(q)}\,, \tag{3.2}$$

which is the familiar wave-number-dependent exchange-enhanced susceptibility of the electron gas.[5]

Another interesting case is the $q\to 0$ limit. Since $V(q)$ diverges as $1/q^2$ as q approaches zero, whereas $F_\pm(q)$ do not, from Eq. (2.33) we obtain

$$\lim_{q\to 0}\chi^{mm}(q)=\mu_B^2\,\frac{4\tilde{F}_+(0)\tilde{F}_-(0)}{\tilde{F}_+(0)+\tilde{F}_-(0)}$$
$$=2\mu_B^2\left[\frac{1}{2}\left(\frac{1}{N_+(0)}+\frac{1}{N_-(0)}\right)-\tilde{V}(0)\right]^{-1}\,, \tag{3.3}$$

where $N_\pm(0)=F_\pm(0)$ are the density of states of (±) spin electrons at the chemical potential. Although we do not know the explicit q dependence of $\tilde{V}(q)$, from Eqs. (2.23) and (2.24) it seems natural to assume $\lim_{q\to 0}\tilde{V}(q)=\tilde{V}(0)$ in Eq. (3.3).

Equation (3.3) agrees with the Stoner susceptibility for the spin-split electron gas.[8] The wave-number-dependent longitudinal susceptibility from the spin-split bands was first discussed by Izuyama, Kim, and Kubo[6] based on the δ-function-type screened-Coulomb-interaction model. Their result, however, did not reduce to the Stoner expression [Eq. (3.3)] in the limit of $q\to 0$. This difficulty can be attributed to the use of the δ-function interaction instead of the bare Coulomb interaction. When the charge polarization is involved, as it is in the present case, the long-range nature of the Coulomb interaction must be properly taken into account to ensure charge conservation. Note that there are a number of different ways[2,9-11] to impose charge conservation as to reproduce the correct $q=0$ limit of the magnetic susceptibilities. Schrieffer's result is the same as our $\chi^{mm}(q)$, Eq. (2.33).

The spatial behavior of the electron spin polarization $\sigma(r)$ due to a δ-function-type magnetic field at the origin of coordinates, $H(r)=H\delta(r)$, can be written in terms of the longitudinal susceptibility as

$$\mu_B\sigma^m(r)=\mu_B[n^m_+(r)-n^m_-(r)]$$
$$=H\,\frac{1}{(2\pi)^3}\int\chi^{mm}(q)e^{iqr}d^3q\,. \tag{3.4}$$

When the electrons do not interact with each other and they are at the paramagnetic state the susceptibility to be used in Eq. (3.4), $\chi_0(q)$, is simply

$$\chi_0(q)=2\mu_B^2F(q)\,, \tag{3.5}$$

where the Lindhard function $F(q)$ is given by Eq. (3.1). In this case we obtain the RKKY spin polarization

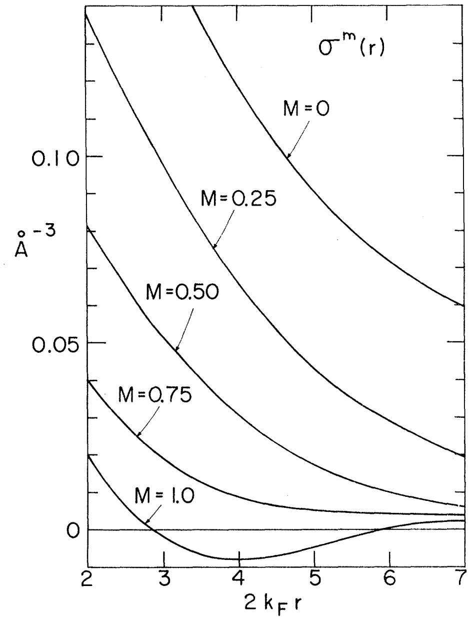

FIG. 2. Spin polarization of a ferromagnetic electron gas due to a δ-function-type magnetic field $H\delta(r)$ in the direction of the z axis for various values of the spontaneous magnetization of the electron gas $M=(n_+^0-n_-^0)/(n_+^0+n_-^0)$, where $n_\pm^0$ is the total number of the ($\pm$) spin electrons in the unperturbed state of the electron gas. Here and in Figs. 3–6 we assume a parabolic electron band, where $N(0)\tilde{V}(0)$ are determined self-consistently for various values of M: $M=0$, $N(0)\tilde{V}(0)=1.0000$; $M=0.25$, $N(0)\tilde{V}(0)=1.0047$; $M=0.50$, $N(0)\tilde{V}(0)=1.0206$; $M=0.75$, $N(0)\tilde{V}(0)=1.0553$; and $M=1.00$, $N(0)\tilde{V}(0)=1.1906$. These values for M and $N(0)\tilde{V}(0)$ are obtained from a self-consistent treatment of Eq. (2.27). k_F is fixed at 0.5×10^8/cm and the q dependence of $\tilde{V}(q)$ is neglected. We assumed $H=[N(0)\mu_B]^{-1}$ for the strength of the δ-function magnetic field.

$$\mu_B\,\sigma_{\text{RKKY}}(r)=\frac{H_0}{(2\pi)^3}\int\chi_0(q)e^{iqr}dq^3=\frac{2H_0\mu_B^2}{(2\pi)^3}\int F(q)e^{iqr}d^3q\ . \qquad (3.6)$$

When the electrons are interacting, although still in the paramagnetic state, $\chi_{\text{para}}^{\text{mm}}(q)$ [Eq. (3.2)] should be used for $\chi^{\text{mm}}(q)$ in Eq. (3.4). Then the spin polarization is no longer the simple RKKY type.[15]

Further, when the electrons are in the ferromagnetic state, $\chi^{\text{mm}}(q)$ [Eq. (2.33)] is the proper susceptibility to be used. This point was emphasized by us[16] recently, and the range of the spin polarization around a magnetic impurity observed by the elastic diffuse scattering of neutron was related to the degree of the spin splitting of the host metal conduction bands as is shown in Fig. 2. Further, the marked impurity concentration dependence of the range of spin polarization around a magnetic impurity in Pd[17] was explained by the fact that the spin split of the Pd-host-metal bands depends on the concentration of the impurities.[18] Although in these earlier analyses,[16,18] we used $\chi^{\text{mm}}(q)$ of Ref. 6, the use of the proper $\chi^{\text{mm}}(q)$ [Eq. (2.33)] changes our earlier results very little.[2]

B. $\chi^{\text{me}}(q)$

The first thing to note is that in the paramagnetic state of the electron gas

$$F_+(q)=F_-(q)=F(q),\quad \chi^{\text{me}}(q)=\chi^{\text{em}}(q)=0,$$

as it should be.

Corresponding to Eq. (3.4), the spin density produced by a charge potential $U(r)$ at the origin of the coordinates can be given as

$$\mu_B\sigma^e(r)=\mu_B[n_+^e(r)-n_-^e(r)]=-\frac{1}{(2\pi)^3}\int U(q)\chi^{\text{me}}(q)e^{iqr}d^3q\ , \qquad (3.7)$$

where $U(q)$ is the Fourier transform of $U(r)$. Note that if we assume a charge potential of $U(r)=Ze/r$, its $U(q)$ is given as

$$U(q)=4\pi Ze/q^2\ . \qquad (3.8)$$

In Fig. 3 we show a typical behavior of $\sigma^e(r)$ in the electron gas. Note the marked difference in the spatial behavior of $\sigma^m(r)$ [Fig. 2] and $\sigma^e(r)$ [Fig. 3].

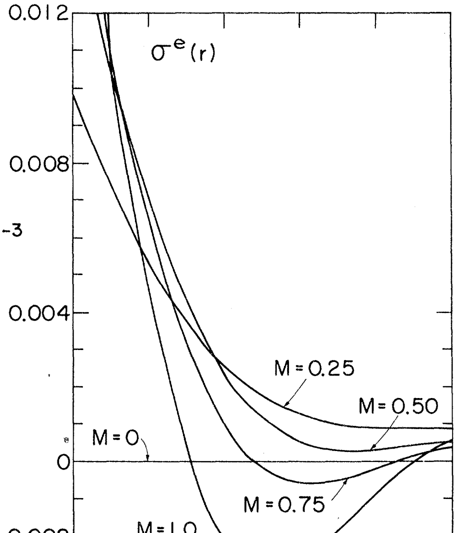

FIG. 3. Spin polarization of a ferromagnetic electron gas due to a unit point charge $U(q)=4\pi e/q^2$ as a function of M. Note for small M, the magnetic response is small but very long ranged resulting in a large net induced spin. For M close to 1, $e^2(r)$ is short ranged and oscillates in the RKKY manner, resulting in a small net induced spin.

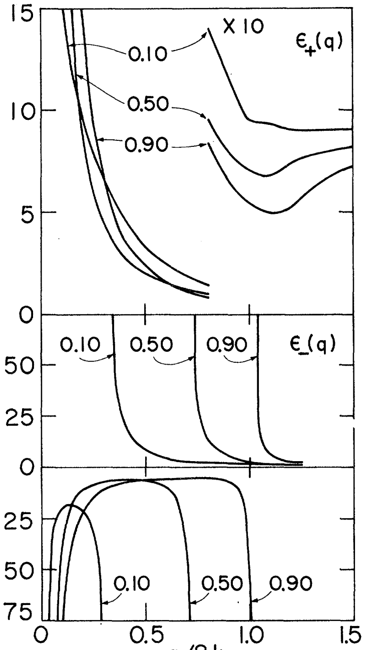

FIG. 4. Plot of the spin-dependent dielectric constants $\epsilon_+(q)$ and $\epsilon_-(q)$ for different M's. The dielectric constant of the minority spin electron $\epsilon_-(q)$ shows singular behavior and becomes negative for small q.

Recently we pointed out[19] that combination of $\sigma^m(r)$ and $\sigma^e(r)$ can account for some of the observed complicated behavior of spin polarization in the ferromagnetic iron. This point, as will be discussed more fully in a separate paper,[20] gives a very useful basis for studying the band structure of ferromagnetic transition metals.

Finally, let us examine how much conduction-electron spin polarization is produced by the screening charge around an excess charge Ze by integrating $\mu_B\sigma^e(r)$ over all the space:

$$\begin{aligned}\mu_B\int\sigma^e(r)\,d^3r &= -\frac{Ze}{(2\pi)^3}\iint\frac{4\pi}{q^2}\,\chi^{\mathrm{me}}(q)\,e^{iqr}\,d^3q\,d^3r\\ &= \mu_B\sigma^e(q=0)\\ &= -4\pi Ze\lim_{q\to 0}\frac{1}{q^2}\,\chi^{\mathrm{me}}(q)\\ &= -Z\mu_B\,\frac{\tilde{F}_+(0)-\tilde{F}_-(0)}{\tilde{F}_+(0)+\tilde{F}_-(0)}\\ &= -Z\mu_B\,\frac{N_+(0)-N_-(0)}{N_+(0)+N_-(0)-2\tilde{V}(0)N_+(0)N_-(0)}\,.\end{aligned}\tag{3.9}$$

The result of Eq. (3.9) can be understood very intuitively. As will be shown in this section in discussing $\chi^{\mathrm{ee}}(q)$, when we introduce an extra charge Ze, the source of the potential, into an electron gas, the potential produces a screening charge polarization, and the amount of the total screening charge is exactly $-Ze$. Of this screening charge of $-Ze$ some are with up spins and the others are with down spins. It is natural to expect that the numbers of (±), spin electrons are porportional, respectively, to the densities of states (±), spin electrons $N_\pm(0)$. This kind of consideration[3] leads to a total induced magnetization,

$$-Z\mu_B[N_+(0)-N_-(0)]\,[N_+(0)+N_-(0)]^{-1},$$

which is different from our correct results of Eq. (3.9) for the interacting electrons. As can be seen from Fig. 6 the conduction-electron spin polarization associated with the screening charge is greatly affected by the presence of the interaction term in the denominator of Eq. (3.9). The most remarkable fact is that the magnetization associated with the screening charge around a unit point charge can be larger than $1\mu_B$. This point is discussed more fully in Sec. III C.[21]

C. $\chi^{\mathrm{ee}}(q)$

In the ferromagnetic state the spin responses are different for each spin. This can be seen better from Eq. (2.31b) rather than Eq. (2.35). We can rewrite Eq. (2.31b) to obtain[21]

$$\begin{aligned}en_\pm^e(q)\\ &= -e^2\,\frac{F_\pm(q)}{[1-\tilde{V}(q)F_\pm(q)]\{1+V(q)[\tilde{F}_+(q)+\tilde{F}_-(q)]\}}\,U(q)\\ &\equiv \chi_{0\pm}^{\mathrm{ee}}(q)\,[-U_\pm(q)] = \chi_{0\pm}^{\mathrm{ee}}(q)\left(\frac{-U(q)}{\epsilon_\pm(q)}\right),\end{aligned}\tag{3.10}$$

where $\chi_{0\pm}^{\mathrm{ee}}(q)=e^2F_\pm(q)$ and

$$\epsilon_\pm(q)=[1-\tilde{V}(q)F_\pm(q)]\{1+V(q)[\tilde{F}_+(q)+\tilde{F}_-(q)]\}\,.\tag{3.11}$$

Equation (3.10) defines an effective dielectric constant in the ferromagnetic electron gas for each spin and can be used in a consistent manner to obtain the charge response of each spin band separately.

As is easily seen from Eq. (3.10) the spin-dependent dielectric constant $\epsilon_\pm(q)$ can be negative for small q in the ferromagnetic state of an electron gas. If we assume a parabolic electron band, for instance, the dielectric constant of the minority spin electrons, $\epsilon_-(q)$, is negative for small q as is illustrated in Fig. 4. The spatial behavior of (±) spin screening charges $n_\pm^e(r)$ around a unit positive charge is given in Fig. 5. The net screening charge $n_\pm^e(q=0)$ which can be obtained by integrating $n_\pm^e(r)$ of Fig. 5 over the space is given in Fig. 6. There we find the minority spin electrons are as a whole repulsed by a positive point charge due to the negative dielectric constant. As is seen from Fig. 6 this explains why in the linear response susceptibilities more than $1\mu_B$ per unit charge can

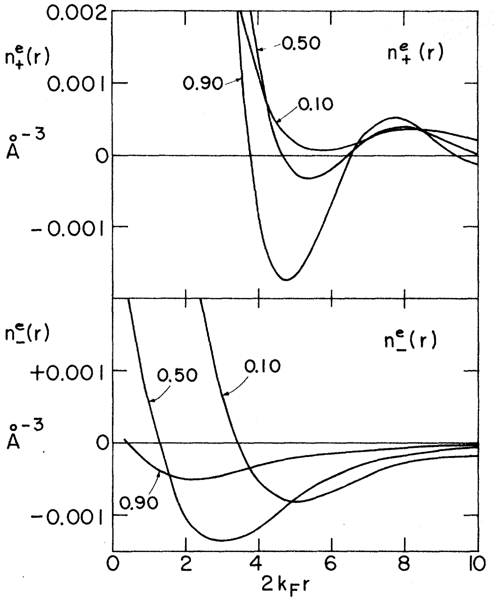

FIG. 5. Spatial behavior of the perturbed electron density $n^e_\pm(r)$ around a unit positive point charge for different magnetizations. Note there are large spatial regions near the positive point charge where the (-) spin electrons are repelled.

be induced in the screening charge around a charge potential.

We can calculate the total screening charge produced around a point charge Ze noting $U(q)=4\pi Ze/q^2$:

$$e\int[n^e_+(r)+n^e_-(r)]d^3r=-\frac{Ze}{(2\pi)^3}\iint\frac{4\pi}{q^2}\chi^{ee}(q)e^{iqr}d^3q\,d^3r$$
$$=-4\pi Ze\lim_{q\to0}\chi^{ee}(q)\frac{1}{q^2}$$
$$=-Ze\ . \tag{3.12}$$

Namely, the amount of the screening charge is just opposite in sign to the point charge Ze, and this result is independent of the magnetization of the electron gas.

We consider $\chi^{ee}(q)$ in the paramagnetic state where $F_+(q)=F_-(q)=F(q)$. In this case Eq. (2.35) reduces to

$$\chi^{ee}_{\text{para}}(q)=e^2\,\frac{2\tilde F(q)}{1+2V(q)\tilde F(q)}$$
$$=2e^2\,\frac{F(q)}{1+[2V(q)-\tilde V(q)]F(q)}\ . \tag{3.13}$$

Note that if there is no interaction at all among electrons, the charge susceptibility χ^{ee}_0 is simply the Lindhard function:

$$\chi^{ee}_0(q)=2e^2F(q)\ .$$

Using the definition for the effective dielectric constant in Eq. (3.10) we get for the paramagnetic case

$$\epsilon_+(q)=\epsilon_-(q)=\epsilon_0(q)\ ,$$
$$\epsilon_0(q)=1+[2V(q)-\tilde V(q)]F(q)\ . \tag{3.14}$$

The effect of the exchange interaction on the dielectric constant was first discussed by Hubbard[22] and its effects on the spatial dependence by Langer and Vosko.[23]

It is very convenient to define $\epsilon_\pm(q)$ by Eq. (3.10). Another possible defining equation of the dielectric constant could be

$$en_\pm(q)=-e^2\tilde F_\pm(q)\,U_{\text{eff}}(q)=-e^2\tilde F_\pm(q)\,\frac{U(q)}{\epsilon(q)}\ . \tag{3.15}$$

In this definition, exchange effects are accounted for in $\tilde F_\pm(q)$ and

$$\epsilon(q)=1+V(q)[\tilde F_+(q)+\tilde F_-(q)]\ . \tag{3.14$'$}$$

Infinities are obtained in $\epsilon_\pm(q)$ and in $\epsilon(q)$ for $q=0$ and for a finite q (see Fig. 4). The infinity at $q=0$ is nothing other than charge conservation which prevents $q=0$ disturbances. The infinity at $q\neq0$ for a given magnetization indicates that an additional wave vector besides $q=0$ cannot produce a response in a ferromagnetic metal.

D. $\chi^{em}(q)$

As we have done with the other three susceptibilities, we can study the charge polarization due to a magnetic disturbance by employing $\chi^{em}(q)$. One important thing to note is that our $\chi^{em}(q)$ does not break the electron number conservation. Let us suppose a δ-function-type magnetic field is applied at the origin of the coordinates in the electron gas of the ferromagnetic state. Then similar to Eqs. (3.9) or (3.12), the total amount of charge polarization is given as

$$e[n^m_+(q=0)+n^m_-(q=0)]=\lim_{q\to0}H(q)\chi^{em}(q)=0\ , \tag{3.16}$$

where $H(q)=H=$ const for the δ-function-type mag-

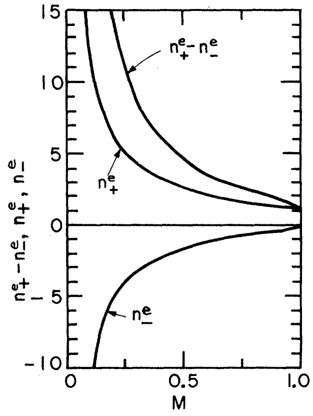

FIG. 6. Total screening charge of (±) spins around a unit positive point charge. Note that the minority spin electrons are repulsed by a positive point charge. Thus although the total screening charge is just unity $[n^e_+(q=0)+n^e_-(q=0)=1]$, the induced magnetization $\mu_B\sigma^e(q=0)=\mu_B(n^e_+(q=0)-n^e_-(q=0))$ can be larger than $1\mu_B$.

netic field. Namely, net charge polarization is zero as it should be.

Note that Eq. (3.16) is due to the divergence of $V(q)$ in the denominator of $\chi^{em}(q)$ as $q \to 0$. We can obtain $\chi^{em}(q)$ from the screened-Coulomb-interaction model[6] but in that case $V(q)$ does not diverge for small q and the charge conservation is broken.

IV. CONCLUSIONS

In this paper we discussed the spin and charge responses of a ferromagnetic electron gas to a magnetic field or a charge potential. Not only the magnetic and charge susceptibilities in the ferromagnetic state, $\chi^{mm}(q)$ and $\chi^{ee}(q)$, are different from that in the paramagnetic state, but in the ferromagnetic state there appear entirely new nondiagonal susceptibilities $\chi^{em}(q)$ and $\chi^{me}(q)$. We discussed the fundamental properties of these four susceptibilities and emphasized the importance of the nondiagonal susceptibilities. For instance, recently we noticed that the concept of the nondiagonal susceptibilities $\chi^{me}(q)$ helps relate the neutron scattering data with the band structure of ferromagnetic transition metals in a very crucial manner.[20] Also the observed negative g shift in the paramagnetic resonance of Gd^{3+} in Pd seems to be explained by the concept of χ^{me}.[24] We feel these nondiagonal susceptibilities $\chi^{me}(q)$ and χ^{em} are very useful in a variety of problems.

Due to the simplicity of our formulation, we obtained these four susceptibilities using a single unified procedure. The various roles the Coulomb interaction plays are very clearly presented in our derivation of the susceptibilities. We believe the formulation given in this paper is useful in understanding the mutual relationship between various response functions of an electron gas.

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