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Fluctuation-Induced Conductivity of a Superconductor above the Transition Temperature*

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Paraconductivity studies have been made on Al films in regimes of temperature and magnetic field strength, which allow a straightforward comparison with the theoretical predictions of Aslamazov and Larkin, Maki and Thompson, and Patton. The results of previous experiments in a variety of pair-breaking regimes are reviewed briefly and in some cases reinterpreted in the context of recent developments in the theory. A comprehensive interpretation of most of the existing data is possible if one considers both Patton's recent calculation and the role of thermal phonons.

I. INTRODUCTION

In spite of vigorous experimental and theoretical investigations of the effect of thermodynamic fluctuations on the electrical conductivity of a superconductor above the transition temperature, the understanding of this phenomenon was cited by Hohenberg[1] in a recent review as one of the important open problems remaining in superconductivity. The most recent experimental review of the large body of information available[2] also indicated that our understanding was incomplete. In the period since these comprehensive assessments, theoretical and experimental progress has been made which now allows a cohesive interpretation of the paraconductivity.

Recently Patton[3,4] has completed a reanalysis of the microscopic approach to the fluctuation problem including both the contribution from pair-type [Aslamazov–Larkin[5,6] (AL)] and normal-electron-type (Maki[7,8]) fluctuations. Patton's calculation analyzes the impurity vertex corrections and arrives at a prediction for the fluctuation conductivity that is finite and specified[3] without the necessity of an assumed pair-lifetime limiting effect as in the earlier treatment of the Maki term by Thompson[9,40] (MT). This calculation has been affirmed by recent theoretical[10,11] and experimental[12] studies, and is qualitatively consistent with earlier work.

Progress has also been made in our understanding of thermal phonons as a pair-lifetime limiting effect through the theoretical work of Appel[13] and through the application of Appel's approach to experimental results in several systems by Crow, Bhatnager, and Mihalisin.[14] However, while intrinsic pair breaking from thermal phonons may account for the variations in the fluctuation conductivity from one elemental species to another, it remains a quantity which is not, as yet, accurately specified by theory.

In this paper, we briefly reconsider some of the existing experimental results, present new data in a regime dominated by the effects of magnetic field, and conclude by indicating the extent to which a unified picture of the paraconductivity is now possible. We shall divide the treatment of the problem into four parts: the conductivity in the absence of applied perturbations, and the conductivity in the presence of magnetic fields, electric fields, and magnetic impurities. We shall emphasize the extent to which these aspects of the fluctuation problem allows us to distinguish between the simple AL picture, the MT approach, and the Patton calculation.

II. THEORY

A. Unperturbed Limit

Using a microscopic approach AL calculated to first order in the pair fluctuations the response function Q_w where $\vec{j}_w = -Q_w\vec{A}$, i.e., the current density arising from diagram (a) in Fig. 1. Although this calculation gives a solution for all dimensions we shall confine our attention, for the moment, to the two-dimensional (2D) case: that is, to films whose thickness d is much less than the temperature-dependent coherence length $\xi(T)=\xi(0)\tau^{-1/2}$, where $\tau=(T/T_{c0})-1$, T_{c0} being the transition temperature in the absence of pair breaking perturbations. This approach predicts that the excess conductivity above that of the normal state,

$$\sigma' = \sigma(T) - \sigma_N ,$$

should be inversely proportional to the reduced temperature

$$\sigma'_{AL} = \frac{e^2/16\hbar}{\tau d} , \qquad (1a)$$

which can be rewritten

$$\frac{\sigma'}{\sigma_N} = \frac{e^2}{16\hbar}\frac{R_\square^N}{\tau} \equiv \frac{\tau_0}{\tau} , \qquad (1b)$$

where $R_\square^N$ is the sheet resistance in the normal state and $\tau_0 = 1.52\times10^{-5}R_\square^N$. This same result follows from the time-dependent Ginsburg–Landau (GL) equations.[15–17]

Later Maki[7,8] found that a second diagram, (b) in Fig. 1, which describes the effect of ephemeral

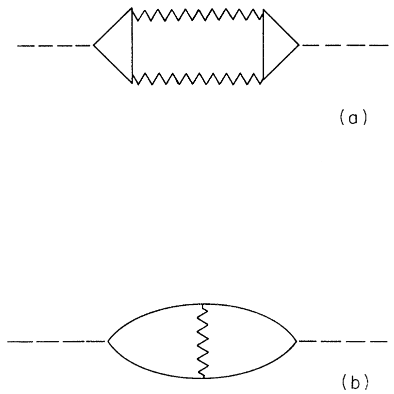

FIG. 1. One (b) and two (a) fluctuation diagrams.

Cooper pairs on the conductivity of the normal electrons, gave a contribution to the response function Q which turned out to be larger than that from the AL diagram for a bulk three-dimensional (3D) sample. For one dimension (1D) and two dimensions (2D), the corresponding calculation[9] predicted infinite conductivity for all temperatures! Thompson[9] regularized the conductivity by introducing a low momentum cutoff $q_c = \xi^{-1}\tau_c^{1/2}$, where τ_c is the reduced shift in the transition temperature

$$\tau_c = (T_{c0} - T_c)/T_{c0}$$

in the presence of pair breaking. This approach led to the following result for the conductivity from this second diagram:

$$\frac{\sigma'_{\mathrm{MT}}}{\sigma_N} = \frac{2\tau_0}{\tau} \ln\left(\frac{\tau + \tau_c}{\tau_c}\right) , \tag{2}$$

so that the total fluctuation conductivity in zero field for the 2D case becomes

$$\frac{\sigma'}{\sigma_N} = \frac{\tau_0}{\tau + \tau_c} + \frac{2\tau_0}{\tau} \ln\left(\frac{\tau + \tau_c}{\tau_c}\right) , \tag{3}$$

where the AL term has been recalculated using the Thompson cutoff q_c.

More recently Patton[3,4] has recalculated the density of states and electromagnetic response function. His careful use of impurity averaging techniques in the presence of fluctuations permits a nondivergent solution of the fluctuation-induced conductivity in reduced dimensions. For the 2D case the result is

$$\frac{\sigma'}{\sigma_N} = \frac{\tau_0}{\tau}\left[1 + \frac{2}{1-\gamma} \ln\left(\frac{2}{(\tau_0/2\tau^2 + \gamma)(1+\gamma)}\right) - \frac{2}{1+\gamma}\right] , \tag{4}$$

where $\gamma = 0.735\tau_c/\tau$ is proportional to the strength of the pair breaking perturbations. This formulation of Patton's theory applies to the temperature region not too near T_c, i.e., where $\tau > \tau_0^{1/2}$. The corresponding results from the AL, MT, and Patton theories for the 1D case are summarized by Thomas and Parks.[12]

B. Effect of Magnetic Field

Usadel[18] was the first of various workers[19,20] to calculate the effect of a strong magnetic field on the AL fluctuation conductivity. The essence of his microscopic calculation, for the case of a perpendicular magnetic field $H_\perp$, is the replacement of the simple continuous momentum values in the response function by the modified discrete set

$$q = (2n+1)\frac{2eH_\perp}{hc} .$$

For this case, the conductivity from pair-type (AL) fluctuations is

$$\sigma'/\sigma_N = (\tau_0/\tau)2a^2[\psi(\tfrac{1}{2} + a) - \psi(1 + \tfrac{1}{2}a) + 1/a] \tag{5a}$$

where ψ is the digamma function and the parameter a is related to the shift in transition temperature due to the pair breaking effect of the magnetic field

$$a(H_\perp) = \frac{T}{T_c(H_\perp)} \frac{\tau}{\ln[T_{c0}/T_c(H_\perp)]} . \tag{5b}$$

This rather complex relationship between the conductivity and the applied field can be simplified in the limit

$$T_{c0} \gg T_{c0} - T_c(H_\perp) \gg T - T_{c0} \cong 0 , \tag{6}$$

that is, for the sample temperature very near the unperturbed transition temperature and $H_\perp$ only moderately large. In this region the suppression of the transition temperature is approximately linear with applied field and the conductivity becomes

$$\frac{\sigma'_{\mathrm{AL}}(H_\perp)}{\sigma_N} = 2\tau_0\left(\frac{T}{T_{c0} - T_c(H_\perp)}\right) \tag{7a}$$

or

$$\frac{1}{R} = \frac{1}{H_\perp}\left[\frac{2\tau_0}{R_N}\left(\left|\frac{\partial H_{c\perp}}{\partial \tau}\right|\right)\right] + \frac{1}{R_N} , \tag{7b}$$

where $H_{c\perp}$ is the critical (perpendicular) magnetic field.

An analogous calculation can be performed to predict the effects of a perpendicular field on the Maki–Thompson terms. By inclusion of these terms both Patton[21] and Thompson[22] (MTP), predict that although the conductivity will again be linear in $1/H_\perp$, the slope of this line should be larger, explicitly in the limit of Eq. (6):

$$\frac{\partial \sigma'}{\partial H_\perp} = (1 + \tfrac{1}{4}\pi^2)\frac{\partial \sigma'_{\mathrm{AL}}}{\partial H_\perp} . \tag{8}$$

In the case of parallel field $H_\parallel$, the effect on the paraconductivity above T_c is much simpler to treat theoretically than that due to the perpendicular field since the density of momentum excitations available to the system does not change. One merely assumes that there is a pair-breaking interaction in addition to the intrinsic pair breaker τ_{c0} such that $\tau_c \to \tau_{c0} + \tau_{cH_\parallel} = \tau_{c0} + \frac{1}{3}[(eH_\parallel/hc)d\xi]^2$ and the results of Sec. II A hold otherwise.

C. Effect of Electric Field

Following the first explicit suggestion by Smith, Serin, and Abrahams[23] that finite electric field E could suppress the fluctuation conductivity, Schmid[24] calculated the result expected from the AL type of fluctuations. The calculation has since been repeated by several theorists[25–27] and yields for the 2D case

$$\sigma'(E) = \sigma'_{AL}(E/E_c)^{-2/3} \int_0^\infty dx \exp[-x(E/E_c)^{-2/3} - x^3], \tag{9}$$

where

$$E_c = \tau^{3/2} E_{c0} = \tau^{3/2}[16\sqrt{3}\, k_B T_c/\pi e \xi(0)]. \tag{10}$$

As pointed out independently by Hurault,[28] the behavior takes on a simple (and temperature independent) form in the limit $E \gg E_c$. In this limit, Eq. (9) gives

$$\sigma'(E) = 0.893\tau_0(E_{c0}/E)^{2/3}, \quad E \gg E_c(T), \quad \text{2D}. \tag{11}$$

The AL contribution yields a different result for one dimensional samples, which has been shown by Tucker and Halperin[29] to be, in the region not too near T_c,

$$\frac{\sigma'_{AL}}{\sigma_N} = (\pi^{1/2} I_1 R_N/L)E^{-1} \times \int_0^\infty du \sqrt{u} \exp[-u(2\sqrt{3}\, E_c/E)^{2/3} - \tfrac{1}{12}u^3], \tag{12}$$

where

$$I_1 = ek_B T_c/\pi\hbar = (6.67\times 10^{-9}\ \text{A/K})T_c.$$

For the high-E limit, i.e., $E \gg E_c$, the integral reduces to a linear combination of Airy functions and the result is

$$\frac{\sigma'_{AL}}{\sigma_N} = (1.154\pi I_1 R_N/L)E^{-1}, \quad E \gg E_c(T), \quad \text{1D}. \tag{13}$$

Maki[30] has calculated the contribution from the normal electron (Maki-type) fluctuations in finite electric field and has shown that in the large electric field limit, there is an extra contribution to the paraconductivity that is twice the AL part, independent of the dimensionality,

$$\sigma'_{MT} = 2\sigma'_{AL}, \quad E \gg E_c(T), \quad \text{1D, 2D, 3D} \tag{14a}$$

so that the total is

$$\sigma' = \sigma'_{AL} + \sigma'_{MT} = 3\sigma'_{AL}. \tag{14b}$$

(The results for all E in 2D are conveniently summarized by Kajimura and Mikoshiba.[31])

All of the results above are restricted to the region not too close to T_{c0}, because, as Ginzburg[32] and others[33] have pointed out, when the order parameter becomes too large, higher-order corrections to the linearized GL theory must be included. Following the approach of Marčelja,[34] the term in $|\psi|^4$ in the GL equation can be included self-consistently in the finite E regime, as has been done by Yamaji[26,35] for 2D and by Tucker and Halperin[29] for 1D. The interesting result of these calculations is that the restriction on the temperature region of validity of the self-consistent mean-field theory can be rewritten in terms of the restriction that σ'/σ_N not be too large. This analysis thus provides a theoretical justification for application of the theories discussed above (both for E and H) at temperatures equal to or below T_{c0}, provided that the perturbation suppresses the excess conductivity to the extent that σ' is somewhat less than σ_N. This approach also suggests that the electric-field-dominated regime can be reached at much smaller values of E for temperatures very near or below T_{c0}. Yamaji's calculations indicate that, for the 2D case, the excess conductivity σ' in the zero E limit can be used to determine the characteristic field E_c, so that Eq. (10) is replaced with

$$E_c(T) = E_{c0}(\tau_0\sigma_N/\sigma')^{3/2}, \quad \text{2D}. \tag{15}$$

Thus for a temperature below T_c, where σ'/σ_N is relatively large, the E-dominated regime may be attained at fields small enough for convenient experimental measurements. In 1D, the corresponding result, consistent with the Tucker–Halperin calculation is

$$E_c(T) = E_{c0}(\tau_1\sigma_N/\sigma')^{3/2}, \tag{16}$$

where τ_1 is the coefficient of the AL zero-field conductivity above T_c

$$\frac{\sigma'_{AL}}{\sigma_N} = \frac{\tau_1}{\tau^{3/2}} = \frac{\pi e^2}{16\hbar}\,\frac{\xi(0)R_N/L}{\tau^{3/2}}, \quad E \ll E_c(T), \quad \text{1D} \tag{17}$$

and σ' is the actual excess conductivity of the 1D sample for $E \ll E_c$.

D. Effect of Magnetic Impurities

As a consequence of Anderson's theorem[36] (that static perturbations which do not exhibit time reversal symmetry act as pair-breaking interactions), it is to be expected that magnetic impurities with a localized moment would effect the fluctuation-induced conductivity in a manner similar to an applied parallel magnetic field.[9] The impurities, then, change the pair breaking parameter $\tau_c \to \tau_{c0} + \tau_{ci}$, where τ_{ci} is proportional to the concentration of impurities according to the theory of Abrikosov and Gor'kov,[37] provided that $(1 - T_c/T_{c0})$ is much less than 1. This would tend to suppress the effect of the Maki diagram and sharpen the transition in temperature.

III. EXPERIMENT: NEW MEASUREMENTS IN THE MAGNETIC-FIELD DOMINATED REGIME

In studying the effect of a perpendicular applied field, we have looked at a series of Al thin films of varying sheet resistance and of thickness d much less than the temperature dependent coherence length. These films were prepared by flash evaporation onto Pyrex substrates after which they were scribed into zig-zag patterns to minimize edge effects and to facilitate four-probe dc voltage measurements. While immersed in the helium bath the sample temperature was monitored with a carbon resistance thermometer chosen to have high sensitivity in the vicinity of the film's transition temperature and calibrated against the vapor pressure of the liquid helium during each run. After the transition temperature in zero field was determined, a field was applied at a constant T slightly below T_{c0} to produce $R/R_N \sim \frac{1}{2}$ and was then aligned parallel to the sample plane by monitoring the sample resistance as a function of the field angle. Then, by rotating the magnet 90°, a perpendicular field alignment was achieved that was accurate to about 0.1°. As the temperature was held constant by manostat control of the pressure above the bath, the resistance of the sample was measured as a function of the applied field. In order to assure that T was in the range satisfying the criterion of [Eq. (6)], the data were taken again at several nearby temperatures as a check that the data approached a unique straight line; a good example of these measurements is shown in Fig. 2. Here we have plotted the measured conductance as a function of the inverse of the magnetic field. For $T \simeq T_{c0}$ the line becomes straight even for relatively small fields, in accordance with the theoretical predictions [Eq. (5)]. The resistance in the limit $H \to \infty$ yields the normal state resistance R_N.[38] A repeat of this measurement far from T_{c0}, i.e., at $(T - T_{c0})/T_{c0} \cong 1$, indicated that R_N was independent of T in this temperature range and that there was no evidence of magnetoresistance at the fields used.

In the second part of the experiment a direct measure was made of the depression of T_c with perpendicular magnetic field as illustrated in Fig. 3. To obtain this data, the entire resistive transition was recorded as a function of $H_\perp$ at a series of constant temperatures below T_c and a check was made that the slope $|\partial H_{c\perp}/\partial\tau|$ was independent of the ratio R/R_N (in the range $0.1 < R/R_N < 0.9$) used to define $H_c(T)$. The criterion, $H = H_c(T)$ when $R/R_N = \frac{1}{2}$, was used in the data shown here. By measuring the variation of T_c with $H_\perp$ directly, we were able to avoid ambiguities[12] involved in determining the coherence length indirectly (e.g., through the resistivity) and in theoretical questions involved in relating the coherence length to the rate of T_c depression in perpendicular field.

IV. DISCUSSION OF RESULTS

A. Magnetic Field Study

Having obtained the rate of suppression of excess conductivity as a function of the applied perpendicular magnetic field at T_{c0} in Fig. 2 and the slope of the critical field curve from Fig. 3, we can compare our results with the theoretical predictions [Eqs. (7) and (8)]. Similar data from the remainder of our samples is given in Table I.

A convenient way to compare the results with the AL or MTP theories is to plot the experimentally determined slope (of the conductivity as a function of the inverse of the field) against the slope expected from the AL theory as in Fig. 4, where we have directly determined the rate of suppression of the transition temperature. On such a plot, if the conductivity were due to the AL diagram alone, all samples should lie on a line of slope 1. All of our results, as well as the single point calculated from the data of Kajimura and Mikoshiba,[39] lie above this line, indicating an additional source of fluctuation conductivity. The amount of excess conductivity agrees reasonably well with the predictions of Patton and Thompson, e.g., the line of slope $1 + \frac{1}{4}\pi^2$. It is important to note that in this experiment the result does not vary systematically with the resistance per square of the film; the samples range over three decades of $R_\square^N$ as indicated in Table I. We would also expect to see the same ratio of excess conductivity in films with higher electron phonon coupling, and, hence, larger value of τ_{c0} as discussed in Sec. IVD, e.g., In, Sn, Pb, or amorphous Bi.

The first systematic study of the suppression of σ' with perpendicular magnetic field was performed by Serin, Smith, and Mizusaki[39] on Pb films. Their data, as well as that presented above, sup-

TABLE I. Sample parameters used in the comparison between AL and MTP theories in Fig. 4. Sample F is calculated from information in Ref. 39.

Sample	$1/R_N$ (Ω^{-1}) ($\times 10^{-4}$)	$d(1/R)/d(1/H)$ (Oe/Ω) ($\times 10^{-4}$)	$\lvert dH_c/d\tau \rvert$ (Oe)	$R_\square^N$ (Ω)
A	3.89	0.96	496	4.1
B	4.81	1.17	521	4.9
C	6.88	1.75	333	3.97
D	3.11	3.94	734	11.1
E	0.829	5.02	1870	28.0
F	0.077	3.53	2797	129.12
G	4.117	1.57	865	5.25

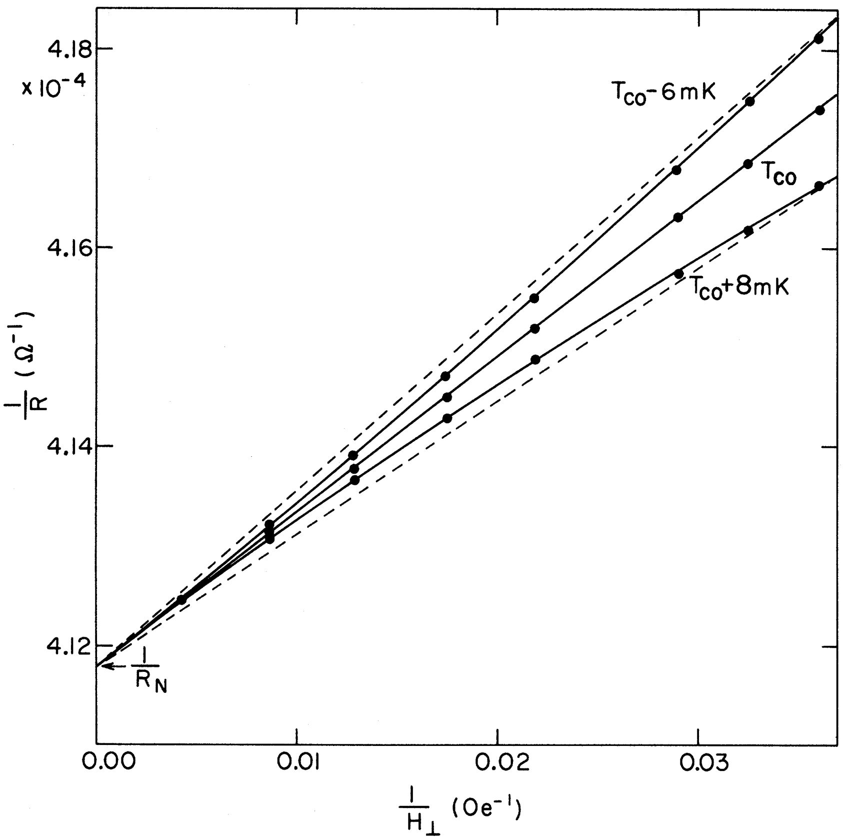

FIG. 2. Electrical conductance of sample A as a function of the inverse perpendicular magnetic field at a constant temperature near T_{c0}. The linear dependence shown for $T = T_{c0}$ is a universal prediction of fluctuation theory in this limit, but the slope is a sensitive test of the fluctuation mechanisms contributing to the excess conductivity. The $(1/R)$-axis intercept determines the normal-state resistance.

ports the functional form of σ' vs $H_\perp$, including only AL-type (pair) fluctuations calculated by Abrahams, Prange, and Stephen.[20] The magnitude of the slope in high field emphasized above was not a major goal in the Serin *et al.* study, rather, they parameterized the magnitude of the slope in terms of an effective coherence length. They did not report critical field studies of the same samples, so that further analysis or experimentation would be required for a proper comparison between the Al and Pb results.

In the parallel magnetic field case a series of experiments performed by Crow and co-workers[40] showed that for both Al and Pb films the excess conductivity due to the MT term was quenched by the application of an external pair breaker, and that this quenching agreed quantitatively with that predicted by Thompson.[9] This result has also been investigated in detail and confirmed in the case of one-dimensional microstrips by Thomas and Parks.[41,42] The experiments dealing with suppression of σ' at constant temperature (discussed above) were performed with perpendicular rather than parallel magnetic field because of the possibility that irregularities in the smoothness of the substrate could seriously affect the field-dependent resistance.

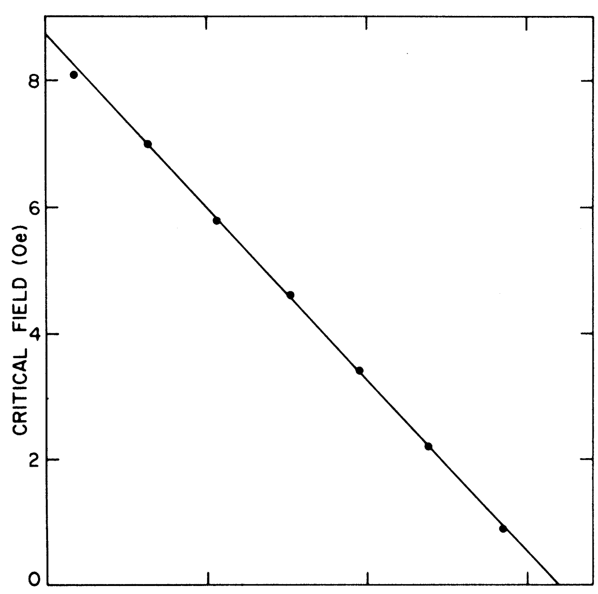

FIG. 3. Dependence of the transition temperature on perpendicular magnetic field of sample A. The slope of this curve, which is insensitive to the experimental definition of T_c (see text), is the only additional physical information needed to quantitatively compare the predicted and observed fluctuation suppression with $H_\perp$.

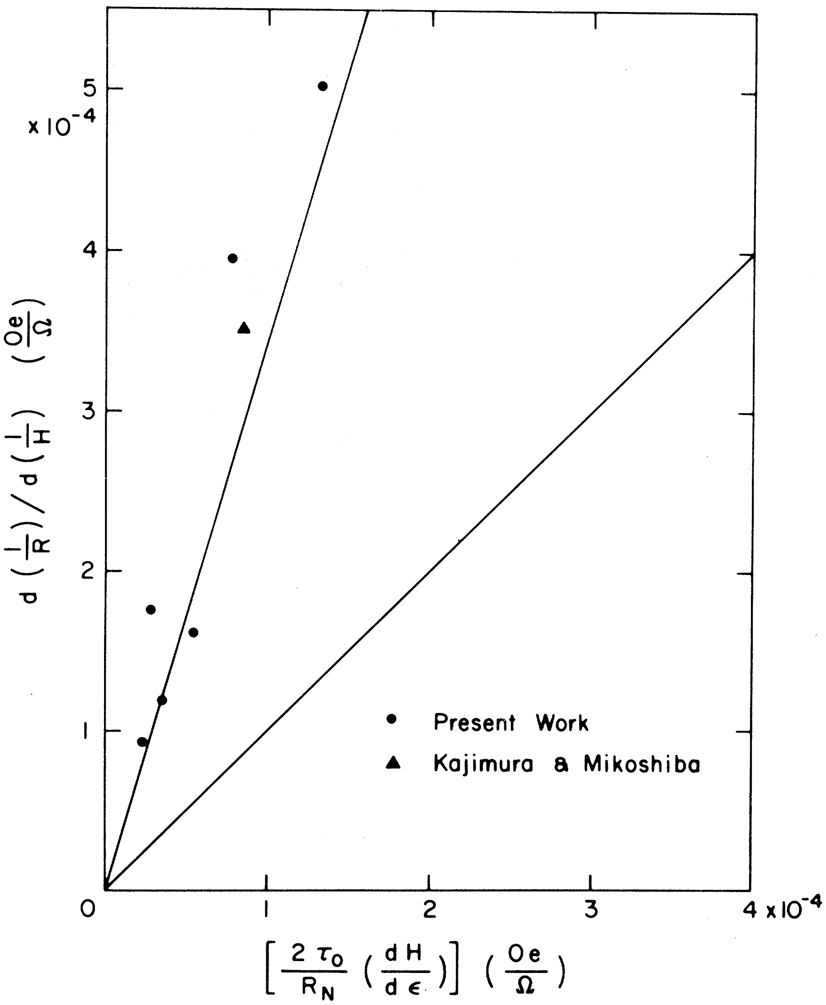

FIG. 4. Comparison of the experimentally determined slope of the conductivity as a function of the inverse of the field with that expected from the AL theory. Agreement with AL corresponds to a line slope 1, while the data strongly support the presence of MTP fluctuation contributions. The triangle is calculated from the data of Ref. 39.

B. Electric Field Dominated Case

In analogy to the regime dominated by magnetic field which we have just discussed, there is also a regime which is dominated by applied electric field. Studies of the fluctuation conductivity in the presence of large E first reported by Thomas and Parks[42] and by Klenin and Jensen,[43] indicated the correct exponent ($E^{-2/3}$) for the behavior of $\sigma'(E)$ in the limit of $E \gg E_c(T)$ for 2D films above T_{c0} [Eq. (11)]. Studies of this type are severely complicated by sample heating which results from the effect of the applied fields on the normal electrons in the samples. The heating problem was overcome in the work of Thomas and Parks[42] by the use of vacuum-insulated thin film thermometers which allowed a direct, precise monitoring of the sample temperature. This study indicated that the rate at which fluctuations were suppressed with large E was off by a factor of about 3 from the prediction based on the AL theory. With the calculation by Maki[30] of the effect of the Maki-type fluctuations in the large-E limit [as discussed above, Eq. (14)], we now understand that this factor of 3 arises from the additional pair contributions in exact analogy to the increased slope of the σ'-vs-$1/H$ curve discussed above.

The first clarification of the necessity of including the MT type fluctuation to the conductivity in finite E was made in the recent work by Kajimura and Mikoshiba.[31] They studied the first deviations from ohmic behavior as a function of temperature above T_c in 2D samples at E values small compared to $E_c(T)$ and small enough to avoid heating. From fitting the theoretical AL-MT curve to the data they obtained experimental values of $E_c(T)$ in agreement with Eq. (10).

The regime $E \gg E_c(T)$ has also been explored without difficulties due to heating by taking advantage of the greatly reduced values of $E_c(T)$ at T below or near T_{c0} as discussed above [Eqs. (15) and (16)]. Using this approach, the $E^{-2/3}$ effect, Eq. (11), has been reobserved in $2D$ by Kajimura, Mikoshiba, and Yamaji,[35] and the E^{-1} behavior expected in $1D$, Eq. (13), has been observed by Thomas.[44]

Although the inclusion of finite E in the Patton calculation has not been made at this time, and would presumably be rather complicated, it is reasonable to assume that the results in the large E limit would be exactly equivalent to the MT result, as has explicitly been shown to be the case for large H. Given this assumption, we can conclude that the experiments in finite E provide an additional source of strong support for the MTP treatment of the fluctuation conductivity.

C. Paramagnetic Impurities

As mentioned above we would expect that the lifetime of Cooper pairs will be limited by scattering sites which carry a localized moment. We would expect, then, a contribution to τ_c proportional to the dopant concentration and a resultant suppression of the MTP fluctuation conductivity relative to the AL term.

Exactly such behavior has been observed by Craven, Thomas, and Parks[45] in a series of experiments on 2D aluminum films. As the concentration of Erbium in the films was increased from 0.006 to 0.274 at.%, a dramatic sharpening of the resistive transition was observed. This behavior could be described quantitatively by the complete fluctuation theory in terms of an increasing τ_c which scaled linearly with Erbium concentration as expected for a small dopant concentrations. These results with magnetic impurities thus provide independent additional support for the MTP picture of the paraconductivity by demonstrating the apparent universality of the source of pair breaking contributions to τ_c.

D. Conductivity in Absence of Applied Perturbations

The results of Patton's calculation, e.g., Eq. (4) differ qualitatively from the results of Thompson's regularization procedure, Eq. (2), in that an additional dependence on mean free path appears. In the 2D case the result is that the quantity σ'/σ'_{AL} depends on τ_0 (or $R_\square^N$), whereas it is independent of $R_\square^N$ in the Thompson approach. Since in the limit of large pair-breaking perturbations the MT and Patton theories are equivalent (as discussed above),[46] the most definitive test is provided by studies of the unperturbed regime.

In order to make a cohesive comparison between the large amount of experimental data available and the theoretical predictions of AL, MT, and Patton, we have plotted in Fig. 5 the ratio of the excess conductivity measured at a fixed reduced temperature (e.g., $\tau = 0.03$)[47] to that expected from the AL theory as a function of the normal sheet resistance $R_\square^N$ of the samples. As noted in the caption to Fig. 5, the data represent work from various laboratories. For the Al system we have chosen the value 2×10^{-4} for the intrinsic pair breaking parameter τ_{c0}, which appears explicitly both in Thompson's and Patton's theories. This value comes from four concurring sources: (a) from fitting Patton's theory to our cleanest 2D samples, (b) from studies of clean 1D samples,[12] (c) from an extrapolation of the τ_c-vs-$R_\square^N$ data of Kajimura and Mikoshiba,[39,48] and (d) from the results of Crow *et al.*[14] As seen from Fig. 5, the Al results allow a definitive comparison of the Patton and Thompson approaches.

When we move to the other elemental systems studied, e.g., Sn and Pb, the value of τ_{c0} required to fit either Eq. (2) or Eq. (4) increases. The value, $\tau_{c0} = 0.02$, for Sn was chosen to obtain the best agreement between the data and the Patton theory. This value differs slightly from that determined from the extrapolation procedure used in Ref. 14; however, it should be noted that the latter procedure consisted of utilizing Thompson's theory and an assumed $R_\square^N$ dependence rather than Patton's theory which specifies the $R_\square^N$ dependence.

Since Patton's theory, viz. Eq. (4), is valid only in the limit of weak pair breaking, $\tau_{c0} \lesssim 0.3$, it is not appropriate in its present form to describe the results for Pb. Although early work by Naugle and Glover[49] on amorphous Bi and Ga and work by Strongin *et al.*[50] on dirty Al was reported to be in agreement with the Al theory, recent studies[51] of Bi indicate an excess conductivity that is consistently larger than predicted by AL. The smallness of σ'/σ'_{AL} for Pb as well as amorphous Bi and Ga reflects the presence of strong pair-breaking effects in these materials, which, in turn, reflects the large electron-phonon coupling strengths λ and small Debye temperatures Θ_D; note that, according to Appel,[13,14] $\tau_{c0} \propto \lambda(T/\Theta_D)^2$. The electron tunneling work of Knorr and Barth[52] demonstrates that the quantity $\lambda(T/\Theta_D)^2$ has values that are comparable in disordered Pb and Bi films and that increase as the films become disordered, which in turn would imply a decrease in σ'/σ'_{AL} through Eq. (2) or Eq. (4). Unfortunately, there is not a one-to-one correspondence between the shift in T_c of a film and the thermal pair breaker τ_{c0},[13] since there are two competing mechanisms present, namely, (a) the imaginary part of the electron-phonon interaction which leads to pairbreaking and hence a decrease in T_c and (b) the real part which does not lead to pair breaking but rather to a renormalization of the electron-phonon self-energy and an attendant increase in T_c. Thus, when the pair breaking is dominated by thermal phonons, an attempt to correlate measured shifts in T_c with τ_{c0} as the structure of films is varied is futile unless quantitative information about the phonon spectrum, e.g., as provided by electron tunneling measurements,

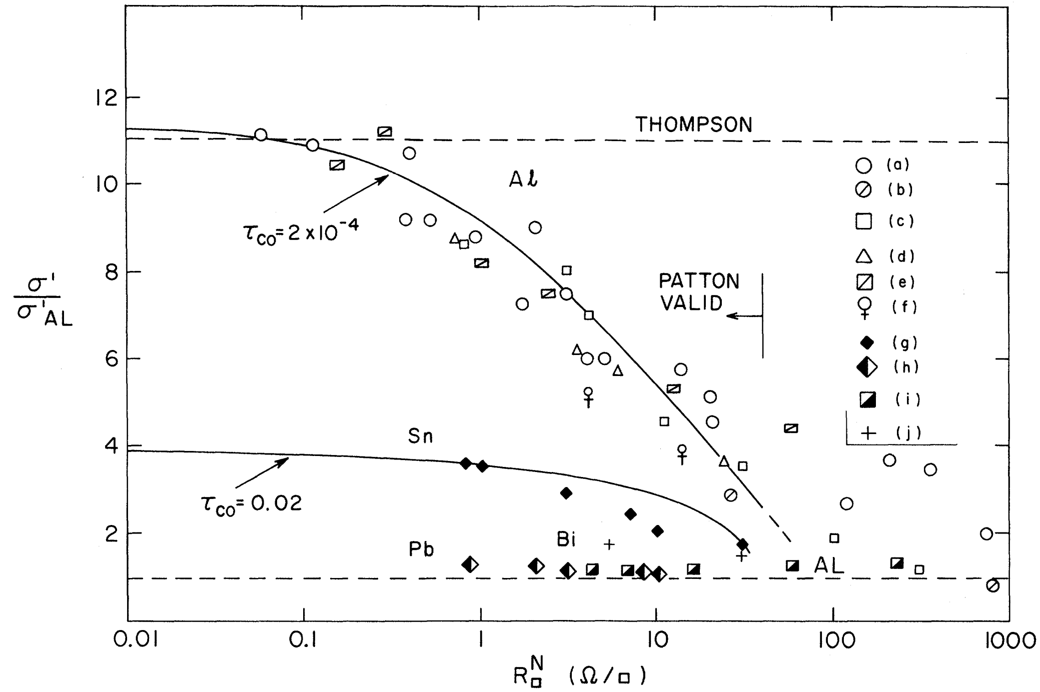

FIG. 5. Ratio of the excess conductivity measured at $\tau = 0.03$ (Ref. 47) to that expected from the AL theory as a function of the normal sheet resistance $R_{\square}^{N}$ of the samples. The data presented are a collection of the work from seven laboratories and twenty experimenters as indicated below. The solid lines are theoretical predictions by Patton [Eq. (4)] for $\tau_{c0} = 2\times 10^{-4}$ and $\tau_{c0} = 0.02$, respectively. The upper dashed line is the prediction of Thompson's theory for $\tau_c = \tau_{c0} = 2\times 10^{-4}$ [Eq. (3)]. The lower dashed line is the AL theory. The decrease in the ratio from the Al system to the Pb system is expected from thermal pair breaking as explained in Sec. IV D.

Symbol	System	Author	Ref.
(a)	Al	Rochester Expt. Group	12, 41, 44, 45, 54, 55, 56
(b)	Al	Strongin *et al.*	50
(c)	Al	Kajimura and Mikoshiba	39
(d)	Al	Crow *et al.*	14, 40
(e)	Al	Bhatnagar *et al.*	53
(f)	Al	Klenin	57
(g)	Sn	Crow *et al.*	14
(h)	Pb	Crow *et al.*	14
(i)	Pb	Mizusaki *et al.*	38, 58
(j)	Bi	Silverman and Glover	51

is available for the same films.

It appears, then, that there is reasonable agreement between the results obtained on relatively clean polycrystalline films and results obtained on amorphous or highly disordered films if we consider both Patton's theory and intrinsic pair breaking due to thermal phonons.

V. CONCLUSIONS

The new results reported herein together with the results of previous studies summarized above offer strong support for recent theoretical innovations. In the presence of moderately large pair breaking perturbations, provided by electric fields, magnetic fields, magnetic impurities or thermal phonons, the paraconductivity approaches the limit predicted by Aslamazov and Larkin. The nature of this approach is described equally well by the Maki–Thompson or Patton approach; whereas, in the limit of small pair breaking, the Patton theory provides a significant improvement over that of Maki–Thompson. The most definitive tests of these theories have come from paraconductivity studies made on aluminum, which is a nearly ideal BCS superconductor in the sense that thermal phonons have only a small effect on the superconducting properties.

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