High-Frequency Waves and Landau Fermi-Liquid Effects in Noble Metals*

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A theorem concerning the symmetry properties of the expansion coefficients of the Landau f function for anisotropic systems is proved. We then calculate the dispersion curves of the ordinary high-frequency waves (HFW) in copper in the region of small wave number $\vec{q}$, for the static field $\vec{H}_0$ and current $\vec{j}$ in the [100] direction ($\vec{q} \perp \vec{H}_0$), paying attention to the parts of the Fermi surface which touch the boundary of the Brillouin zone. We do this by reducing the transport equation to a system of linear equations, and by looking for a condition under which there exist normal-mode solutions to the system of equations. We carry out the calculation, first excluding and then including Landau Fermi-liquid effects. Qualitative differences are found between dispersion curves in alkali and noble metals.

I. INTRODUCTION

Several years ago Walsh and Platzman[1,2] observed what they called high-frequency waves (HFW) in potassium. HFW are waves which propagate in a metal placed in a static magnetic field $\vec{H}_0$ with frequency ω, which is of the order of the cyclotron frequency ω_c. It has been shown by several people[3-5] that for an isotropic metal one can extract the spin-independent part of the Landau Fermi-liquid coefficients A_n from the HFW dispersion curves. For potassium it was determined by Platzman, Walsh, and Foo that $A_2 \simeq -0.03$ and $|A_3| < 0.01$. Recently, some efforts have been made to include the effect of crystal anisotropies on HFW. Foo[6] and Alodzhants have considered ellipsoidal Fermi surfaces which do not touch the Brillouin-zone boundaries. Cheng[8] treated, in the geometry $\vec{q} \parallel \vec{H}_0$, a model Fermi surface which may have some characteristics of noble-metal Fermi surfaces. Experimentally, HFW have been observed in copper,[9] a metal with an anisotropic Fermi surface that intersects the Brillouin-zone boundary.

In this paper we calculate dispersion curves of HFW for copper propagating perpendicular to $\vec{H}_0$ in the limit of small q, where q is the wave number. Although a large part of the Fermi surface of noble metals is nearly spherical, it is far from spherical in the [111] direction where the Fermi surface intersects the Brillouin-zone boundary. This leads to qualitative differences between HFW in noble and alkali metals. The main reason for the qualitative differences is that the spherical harmonics Y_n^m are not orthogonal to each other with respect to the Fermi surface of noble metals. Thus a variety of methods which have been used for isotropic metals fail for noble metals. The method we use here is the following. We obtain from the transport equation a system of coupled linear equations of infinite order for α_{nm}, the expansion coefficients for the distribution function defined in Eq. (16). We truncate this system of equations at finite $n = n_0$ and find a condition under which there exist nontrivial solutions to the system of equations. This truncation is an approximation, and we shall first check the validity by calculating the dispersion curve for isotropic metals by this method and comparing it with the one in Ref. 3.

We use the Fermi surface and the velocity distribution for copper derived by Halse.[10] But some modification must be made so that the solution to the transport equation satisfies appropriate boundary conditions.

In Sec. II we discuss the expansion of the Landau f function for anisotropic systems. In an anisotropic system $f(\vec{k}, \vec{\sigma}; \vec{k}', \vec{\sigma}')$ depends on both $\vec{k}$ and $\vec{k}'$, not just the relative angle Ω between them. Hence we cannot expand it in Legendre polynomials; we need to expand it in a double series. In Sec. III we reduce the transport equation to a system of linear equations. In Sec. IV we discuss our numerical results, and in Sec. V we summarize our findings.

II. LANDAU INTERACTION FUNCTIONS FOR ANISOTROPIC SYSTEMS

In this section we discuss the expansion of the Landau interaction function $f(\vec{k}, \vec{\sigma}; \vec{k}', \vec{\sigma}')$ for anisotropic systems. It is customary to separate it into two parts, the spin-independent part $f(\vec{k}, \vec{k}')$ and the spin-dependent part $\zeta(\vec{k}, \vec{k}')$:

$$f(\vec{k}, \vec{\sigma}; \vec{k}', \vec{\sigma}') = f(\vec{k}, \vec{k}') + \vec{\sigma}' \cdot \vec{\sigma}\, \zeta(\vec{k}, \vec{k}') \ .$$

The following discussion is valid for both $f(\vec{k}, \vec{k}')$ and $\zeta(\vec{k}, \vec{k}')$, but we shall discuss only $f(\vec{k}, \vec{k}')$.

In anisotropic systems $f(\vec{k}, \vec{k}')$ depends on the directions of $\vec{k}$ and $\vec{k}'$ separately and not on just

the relative angle Ω between them. Hence it is not sufficient to expand f in Legendre polynomials with $\cos\Omega$ as the argument. It is most convenient to expand it in a double series of a complete set of orthogonal functions $\{\varphi_n;\ n=1,\ 2\ldots\}$ which transform according to irreducible representations of the point group G of the crystal; i.e.,

$$f(\vec{k},\vec{k}')=\sum_{n,n'}A_{nn'}\varphi_n(\vec{k})\varphi_{n'}^*(\vec{k}')\ , \tag{1}$$

where $A_{n,n'}$ are expansion coefficients.

From a physical point of view we impose the following conditions on $f(\vec{k},\vec{k}')$:

$$f^*(\vec{k},\vec{k}')=f(\vec{k},\vec{k}')\ , \tag{2}$$

$$f(\vec{k}',\vec{k})=f(\vec{k},\vec{k}')\ , \tag{3}$$

$$P_R f(\vec{k},\vec{k}')=f(\vec{k},\vec{k}')\ , \tag{4}$$

for any R in G, where P_R is the operator of the group element R. From Eq. (4) alone, we have the following theorem concerning the symmetry of $A_{n,n'}$.[11]

Theorem. Let the Landau interaction function $f(\vec{k},\vec{k}')$ be expanded in the form of Eq. (1) where $\{\varphi_n;\ n=1,\ 2\ldots\}$ is a complete set of orthogonal functions in the space of functions defined on the Fermi surface as the domain, and each φ_n transforms according to one of the rows of one of the irreducible representations of the symmetry point group G of the crystal. Let $\Gamma^{(i)}$ and $\Gamma^{(j)}$ be the ith and jth irreducible representations of G with dimensionality l_i and l_j, respectively. If $\{\varphi^r_{n_\alpha};\ \alpha=1,\ \ldots\ l_i\}$ and $\{\varphi^s_{m_\beta};\ \beta=1,\ \ldots\ l_j\}$ are sets of partner functions which transform according to the αth row of $\Gamma^{(i)}$ and βth row of $\Gamma^{(j)}$, respectively, then

$$A^{rs}_{n_\alpha m_\beta}=A^{rs}_{n_1 m_1}\delta_{ij}\delta_{\alpha\beta}\ , \tag{5}$$

provided that $f(\vec{k},\vec{k}')$ satisfies Eq. (4). The superscripts s and r distinguish different sets of functions among the complete set $\{\varphi_n\}$ which transform according to the same irreducible representation. We require that any two such sets not just belong to equivalent representations but that the representations be identical.

Proof. Since $\{\varphi_n;\ n=1,\ 2\ldots\}$ is a complete orthogonal set, the set of coefficients $\{A_{nn'}\}$ is uniquely determined for a given $f(\vec{k},\vec{k}')$. From this, and also from the fact that all φ_n transform according to irreducible representations, we have from Eq. (4)

$$\sum_{i,j}\sum_{r,s}\sum_{\alpha,\beta}\sum_{l,l'}A^{rs}_{n_\alpha m_\beta}\varphi^{(i)r}_{n_l}\Gamma^{(i)}_{l\alpha}(R)\varphi^{(j)s*}_{m_{l'}}\Gamma^{(j)*}_{l'\beta}(R)$$
$$=\sum_{i,j}\sum_{r,s}\sum_{\alpha,\beta}A^{rs}_{n_\alpha m_\beta}\varphi^{(i)r}_{n_\alpha}\varphi^{(j)s*}_{m_\beta} \tag{6}$$

for any R in G. Interchanging dummy variables on the left-hand side of Eq. (6) and comparing the coefficients on both sides, we obtain

$$\sum_{l,l'}A^{rs}_{n_l m_{l'}}\Gamma^{(i)}_{\alpha l}(R)\,\Gamma^{(j)*}_{\beta l'}(R)=A^{rs}_{n_\alpha m_\beta}\ . \tag{7}$$

There is the following orthogonality relation between Γ's:

$$\sum_R\Gamma^{(i)}_{\alpha l}(R)\Gamma^{(j)*}_{\beta l'}(R)=(h/l_i)\,\delta_{ij}\delta_{\alpha\beta}\delta_{ll'}\ , \tag{8}$$

where h is the order of the group.

Summing Eq. (7) over R and using Eq. (8) we obtain

$$(1/l_i)\sum_l A^{rs}_{n_l m_l}\delta_{ij}\delta_{\alpha\beta}=A^{rs}_{n_\alpha m_\beta}\ . \tag{9}$$

We see by Eq. (9) that the only nonvanishing coefficients are for $\varphi^r_{n_\alpha}$ and $\varphi^s_{m_\beta}$ that transform according to the *same* row of the *same* irreducible representation and that the value of the coefficient for this case is independent of the particular row of the irreducible representation, so we can choose the first row. Hence Eq. (5) follows.

This theorem is quite general and can be applied in principle for any Fermi system. However, it is difficult to find appropriate $\{\varphi_n;\ n=1,\ \ldots\}$ for complicated Fermi surfaces, because the domain of φ_n must be exactly the Fermi surface in order that the set of coefficients be determined uniquely.

III. METHOD OF CALCULATION

In this section we shall describe in detail our method of the calculation. We choose the x, y, and z axes so that they coincide with the crystal axes, and assume that the wave vector $\vec{q}$ and the static magnetic field $\vec{H}_0$ are in the x and z directions, respectively. In this geometry the system as a whole has reflection symmetry with respect to the x-y plane (note that $\vec{H}_0$ is an axial vector). It follows from this that elements of the conductivity tensor $\sigma_{yz}=\sigma_{zy}=\sigma_{xz}=\sigma_{zx}=0$; hence the ordinary and extraordinary waves are decoupled.[3] In this paper we shall restrict our discussions to the ordinary wave, i.e., the wave with $\vec{j}$ in the z direction.

We write the Fermi distribution function $n(\vec{k},\vec{r},t)$ for quasiparticles as

$$n(\vec{k},\vec{r},t)=n_0(\vec{k})-\frac{\partial n_0(\vec{k})}{\partial\epsilon_0}\,g(\vec{k},\vec{r},t)\ , \tag{10}$$

where $n_0(\vec{k})$ is the distribution function at equilibrium, and ϵ_0 is the energy of the quasiparticle without Landau interaction. Equation (10) can be regarded as the definition of $g(\vec{k},\vec{r},t)$. Then the transport equation which includes the Landau interaction is[3]

$$\frac{\partial g}{\partial t}+[\vec{v}_{\vec{k}}\cdot\vec{\nabla}-(|e|/\hbar c)(\vec{v}_{\vec{k}}\times\vec{H})\cdot\vec{\nabla}_{\vec{k}}](g+\delta\epsilon)$$
$$=-|e|\vec{v}_{\vec{k}}\cdot\vec{E}\ , \tag{11}$$

where

$$\delta\epsilon = \frac{1}{4\pi^3}\int d^3k' f(\vec{k},\vec{k}')\, g(\vec{k}')\,\delta(\epsilon_0(\vec{k}') - \epsilon_f)\ . \qquad (12)$$

$\vec{E}$ in the right-hand side of Eq. (11) is the self-consistent field associated with current $\vec{j}$, which is given by

$$\vec{j} = -(|e|/4\pi^3)\int d^3k\, \vec{v}_{\vec{k}}(g+\delta\epsilon)\,\delta(\epsilon_0(k) - \epsilon_f)\ . \qquad (13)$$

We look for a condition under which Eqs. (11)–(13) have a solution of the form

$$g(\vec{k},\vec{r},t) = g_0(\vec{k})e^{i(\vec{q}\cdot\vec{r}-\omega t)} + \text{c. c.} \qquad (14)$$

However, the solutions in general do not possess the required property

$$g_0(\vec{k}_A) = g_0(\vec{k}_B)\ , \qquad (15)$$

where $\vec{k}_A$ and $\vec{k}_B$ are points on both the Fermi surface and the Brillouin-zone boundary and are related to one another by a reciprocal-lattice vector. This condition Eq. (15) is essential for $g_0(\vec{k})$ to be physically significant. Thus we must modify the Fermi surface so that the solutions automatically satisfy Eq. (15). We consider in a periodically extended zone scheme the part of the Fermi surface which touches the zone boundary as in Figs. 1(a) and 1(b). We then translate that part so that its center coincides with the z axis [Fig. 1(c)]. This translation is merely for convenience to identify points. We identify points on the Fermi surface by means of a sphere whose center is at the origin with radius k_0, where k_0 is the k value on the Fermi surface in the $\langle 100\rangle$ directions. Corresponding to a point $\vec{k}$ on the Fermi surface we have a point X on the sphere given by the intersection with the sphere of a line drawn from the z axis parallel to the x-y plane through $\vec{k}$ [Fig. 1(c)]. The point $\vec{k}$ is specified by the polar and azimuthal angle (θ, φ) of the corresponding point X. The advantages of our coordinate system are that first, in our system electron orbits in $\vec{k}$ space with magnetic field H_0 are given by constant θ, and second, there is a one-to-one correspondence between (θ, φ) and $\vec{k}$ on the Fermi surface. This modified Fermi surface has D_{2h} symmetry which includes invariance under reflection with respect to the x-y plane but has only a twofold axis of rotation in the z direction.

We now expand g_0 in spherical harmonics. Cubic harmonics might appear more reasonable to use, but since the system we are considering does not have cubic symmetry due to the presence of $\vec{H}_0$ as well as due to the modification of the Fermi surface, spherical harmonics turn out to be more convenient. Let us write $g_0(\vec{k})$ and the Landau interaction function $f(\vec{k},\vec{k}')$ as

$$g_0(\vec{k}) = \sum_{n,m}\alpha_{n,m}Y_n^m(\hat{k}) \qquad (16)$$

and

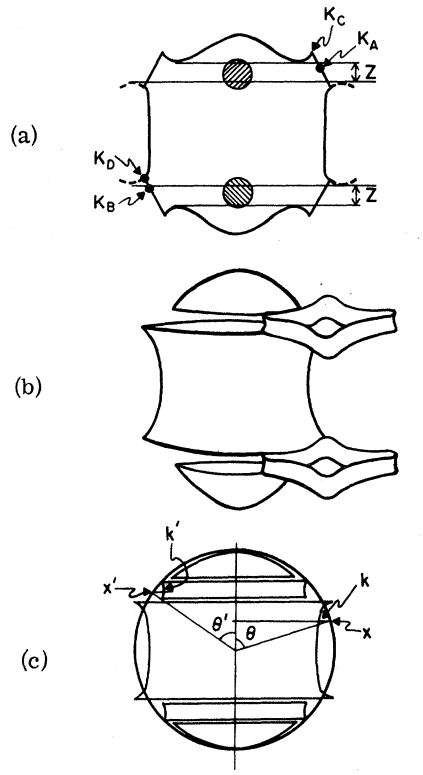

FIG. 1. Modification of the Fermi surface of noble metals: (a) Fermi surface of the original form. Points $\vec{k}_A$ and $\vec{k}_B$ are physically identical. Points in the neighborhood of k_C are translated by a reciprocal-lattice vector to the neighborhood of k_D. We shall consider the strips of width Z in the periodically extended zone scheme. (b) Electron orbits in $\vec{k}$ space in the periodically extended zone scheme for the strips of width Z. (c) The orbits considered in (b) are translated so that the center of the orbits coincides with the z axis. We identify a point $\vec{k}$ on the Fermi surface by means of a sphere whose center is at the origin with radius k_0, where k_0 is the k value on the Fermi surface in the $\langle 100\rangle$ directions. Corresponding to $\vec{k}$ we have point X on the sphere given by the intersection with the sphere of a line drawn from the z axis parallel to the x-y plane through $\vec{k}$. The point $\vec{k}$ is specified by the polar and azimuthal angles (θ, φ) of the corresponding point X.

$$f(\vec{k},\vec{k}') = \sum_{n,m}\sum_{n',m'}(n,m;n',m')\,Y_n^m(\hat{k})\,Y_{n'}^{m'*}(\hat{k}')\ . \qquad (17)$$

Here $\alpha_{n,m}$ and $(n, m; n', m')$ are expansion coefficients. In these equations the coordinate system defined above is implicit.

One can show by using Maxwell's equations that when the fields and current are of the same form as Eq. (14), $\vec{E}$ and $\vec{j}$ are related in the following way:

$$E_i = -(4\pi/\omega)i\Lambda_{ij}j_j\ , \qquad (18)$$

with

$$\Lambda=\begin{pmatrix}1 & 0 & 0\\ 0 & [1-(qc/\omega)^2]^{-1} & 0\\ 0 & 0 & [1-(qc/\omega)^2]^{-1}\end{pmatrix}. \quad (19)$$

We substitute Eqs. (12)–(14), (16), and (17) into (11) and multiply by $Y_{n'}^{m'*}(\hat{k})$. Then integrating over the Fermi surface, we obtain the following expressions for the ordinary wave ($\vec{j}$ in the z direction):

$$\sum_N \left[O_N^{N'}(-i\omega)+iq\left(P_N^{N'}+\sum_{N''}\sum_{N'''}P_{N''}^{N'}(N'';N''')O_N^{N'''}\right)+i\frac{H_0|e|}{c\hbar}\left(Q_N^{N'}+\sum_{N''}\sum_{N'''}Q_{N''}^{N'}(N'';N''')O_N^{N'''}\right)\right]\alpha_N$$
$$=ie^2\frac{4\pi}{\hbar}\frac{\omega}{\omega^2-q^2c^2}V_{N'}^{z*}\times\sum_N\left(V_N^z+\sum_{N''}\sum_{N'''}V_{N''}^z(N'';N''')O_N^{N'''}\right)\alpha_N , \quad (20)$$

with

$$O_N^{N'}=\frac{1}{4\pi^3}\int d^3k\,Y_{n'}^{m'*}(\hat{k})Y_n^m(\hat{k})\delta(\epsilon_0-\epsilon_f) , \quad (21)$$

$$P_N^{N'}=\frac{1}{4\pi^3}\int d^3k\,Y_{n'}^{m'*}(\hat{k})v_xY_n^m(\hat{k})\delta(\epsilon_0-\epsilon_f) , \quad (22)$$

$$Q_N^{N'}=\frac{1}{4\pi^3}\int d^3k\,Y_{n'}^{m'*}(\hat{k})\left(\frac{v_x^2+v_y^2}{k_x^2+k_y^2}\right)^{1/2}\times Y_n^m(\hat{k})\delta(\epsilon_0-\epsilon_f) , \quad (23)$$

and

$$V_N^z=\frac{1}{4\pi^3}\int d^3k\,v_zY_n^m(\hat{k})\delta(\epsilon-\epsilon_f) . \quad (24)$$

Here N, N', etc., stand for pairs (n, m), (n', m'), etc., and v_μ ($\mu=x, y, z$) is the μ component of $\vec{v}_{\vec{k}}$. In the numerical calculation, integrations with respect to $\vec{k}$ are changed to surface integrations; i.e.,

$$\int d^3k\,\delta(\epsilon_0-\epsilon_f)=\int_{FS}dS\,\hbar v_{\vec{k}}^{-1}, \quad (25)$$

where FS stands for Fermi surface. It should be noted in Eqs. (21) and (25) that one needs the knowledge of the magnitudes of the Fermi velocity as a function of $\vec{k}$ for the calculation of $O_N^{N'}$. The anisotropy of the Fermi velocity is one of the factors which contributes nonvanishing off diagonal elements of $O_N^{N'}$ for noble metals. However, $O_N^{N'}$, $P_N^{N'}$, and $Q_N^{N'}$ all vanish if one of Y_n^m and $Y_{n'}^{m'}$ is odd and the other is even in $\cos\theta$. Odd Y_n^m's couple to the ordinary waves, and even to extraordinary waves.

We look for conditions under which Eqs. (20)–(24) have nontrivial solutions. But they are of infinite order and difficult to handle. Therefore, we truncate Eq. (20) at finite $n=n_0$. The justification for that is the following. Since the experiment is by means of the absorption of electromagnetic waves, the mode of HFW must produce an electric field in order to be detected. In the case of a spherical Fermi surface, only the Y_1^0 mode produces $\vec{j}$, and all others do not; hence, experimentally dominant signals are from such solutions that have a large component of Y_1^0, and there is only one such solution for each subharmonic in the limit of small q. Although this is not quite true for noble metals, dominant signals are still from such solutions that have large components in small n, where n is the order of the spherical harmonic. It is reasonable to assume, even in the case of the anisotropic metal, that the coupling between modes Y_n^m with small n and $Y_{n'}^{m'}$ with large n' is small. Taking n_0 large enough, one can neglect the coupling of Y_n^m with small n to $Y_{n'}^{m'}$ with $n'>n_0$. This justifies the truncation.

The truncated Eq. (20) is a homogeneous linear system of equations for α_N. It has nontrivial solutions if and only if the determinant of the coefficient matrix vanishes. Therefore we seek the relation between H_0, ω, and q for which the determinantal equation vanishes. We do this by actually calculating the determinant for various values of H_0 for fixed ω and q.

Unfortunately, Y_n^m do not transform according to irreducible representations of the group D_{2h}; hence we cannot apply the theorem in Sec. II for this case. But their linear combinations $\mathcal{Y}_n^m$ ($-n\le m\le n$) do transform according to irreducible representations as shown in Table I, where $\mathcal{Y}_n^m$ are defined by

$$\mathcal{Y}_n^m=\begin{cases}(Y_n^m+Y_n^{-m})/\sqrt{2}, & m>0\\ Y_n^0, & m=0\\ (Y_n^{|m|}-Y_n^{-|m|})/(i\sqrt{2}), & m<0\end{cases} . \quad (26)$$

One expands $f(\vec{k}, \vec{k}')$ in $\mathcal{Y}_n^m$'s in the form of Eq. (1) and by applying the theorem finds the symmetry relations among expansion coefficients,[12] which then can be translated into the relations among $(n, m; n', m')$'s. We obtain

$$(n, m; n', m')^*=(n, m; n', m') , \quad (27)$$

$$(n, -m; n', -m')=(n, m; n', m') , \quad (28)$$

and

$$(n', m'; n, m)=(n, m; n', m') . \quad (29)$$

Thus nonzero coefficients relevant to the ordinary wave for n, $n'\le 3$ are

$$(1,0;1,0) ,$$

$$(1,0;3,0)=(3,0;1,0) ,$$

$$(1,0;3,2)=(1,0;3,-2)=(3,2;1,0)=(3,-2;1,0) ,$$

$$(2,1;2,1)=(2,-1;2,-1) ,$$

$$(2,1;2,-1)=(2,-1;2,1) ,$$

TABLE I. Irreducible representations of the group D_{2h}, according to which various $\mathcal{Y}_n^m$'s transform. The last column indicates whether the mode couples to ordinary or extraordinary waves. The table is arranged in such a way that it is apparent that D_{2h} is a direct product group of C_{2h} and C_{1h}. For the subgroup C_{2h}, the last four elements should be ignored. Hence for that, α', β' ... coincide with α, β..., respectively.

	n	m	Label	E	I	m_{xy}	c_2^z	m_{yz}	c_2^x	c_2^y	m_{xy}	Mode
$\mathcal{Y}_n^m$	even	even	α	1	1	1	1	1	1	1	1	ext
		odd	β'	1	1	−1	−1	−1	−1	1	1	ord
$n \geq m \geq 0$	odd	even	γ	1	−1	−1	1	1	−1	−1	1	ord
		odd	δ'	1	−1	1	−1	−1	1	−1	1	ext
$\mathcal{Y}_n^m$	even	even	α'	1	1	1	1	−1	−1	−1	−1	ext
		odd	β	1	1	−1	−1	1	1	−1	−1	ord
$-n \leq m < 0$	odd	even	γ'	1	−1	−1	1	−1	1	1	−1	ord
		odd	δ	1	−1	1	−1	1	−1	1	−1	ext

$$(3, 0; 3, 0) ,$$

$$(3, 2; 3, 2) = (3, -2; 3, -2) ,$$

$$(3, 0; 3, 2) = (3, 0; 3, -2) = (3, 2; 3, 0) = (3, -2; 3, 0) .$$

IV. NUMERICAL RESULTS

In this section we shall discuss our numerical results. In order to check the validity of our method we first carried out calculations for potassium with $\omega = 10^{11}$ sec^{-1} and $n_0 = 5$ and 7. The dispersion curves for the fundamental mode are shown in Fig. 2 together with the previous calculation of Ref. 3. Here $R_s = v_s/\omega$ with the Fermi velocity v_s, and the cyclotron frequency ω_c^s. The subscript s stands for "spherical Fermi surface." The agreement is very good in the region $qR_s \lesssim 6.5$ with $n_0 = 7$. This is the region where experimental data have been taken, and from which A_n can be extracted. In the region $qR_s > 6.5$ our method seems to fail. This is because the $\vec{q} \cdot \vec{\nabla}$ term in Eq. (11) is so large that it mixes the Y_2^1 mode with Y_n^m's for $n > n_0 = 7$.

We now proceed to the calculation for copper. In Fig. 3 are plotted dispersion curves for copper for small q with $n_0 = 8$ and 10, $\omega = 10^{11}$ sec^{-1}, and with vanishing Landau parameters $(n, m; n', m')$. In this case v_s and ω_c^s are the Fermi velocity and the cyclotron resonance frequency, respectively, in the *absence* of band structure. For noble metals

$$v_s = 5.685/a \text{ cm sec}^{-1}$$

and

$$\omega_c^s = 1.759 \times H_0 \times 10^7 \text{ sec}^{-1}$$

with lattice constant a in cm and the static magnetic field H_0 in gauss. For copper $a = 3.603 \times 10^{-8}$ cm. Since $O_N^{N'}$ and $Q_N^{N'}$ are not diagonal, a wave in the noble metal consists of many modes even in the limit of $q \to 0$. By actually solving the simultaneous equations by setting some $\alpha_{n,m} = 1$, one can show that, for example, branch (A) in Fig. 3 consists mainly of mode Y_2^1, but it also has components of $Y_4^{\pm 1}$, $Y_4^{\pm 3}$, Y_2^{-1}, etc., at $q = 0$, and branch (B) consists mainly of Y_4^1, but it also has components of $Y_2^{\pm 1}$, $Y_4^{\pm 3}$, Y_4^{-1}, etc. Consequently, these two branches are not degenerate at $q = 0$. This should

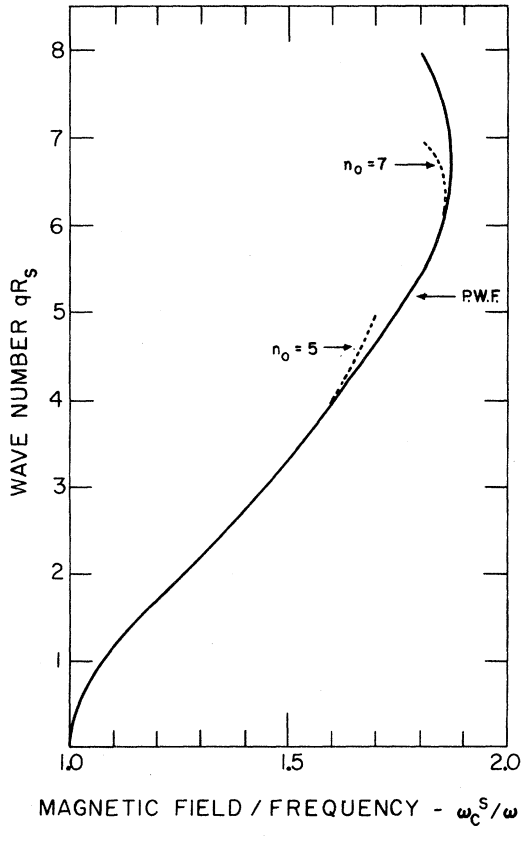

FIG. 2. Dispersion curves for potassium calculated by our method with $n_0 = 5$ and 7 for the purpose of checking the validity of this method. The previous calculation by Platzman, Walsh, and Foo (P. W. F.) (see Ref. 3) is also shown. Our method yields a correct result in the region $qR \lesssim 6.5$ with $n_0 = 7$.

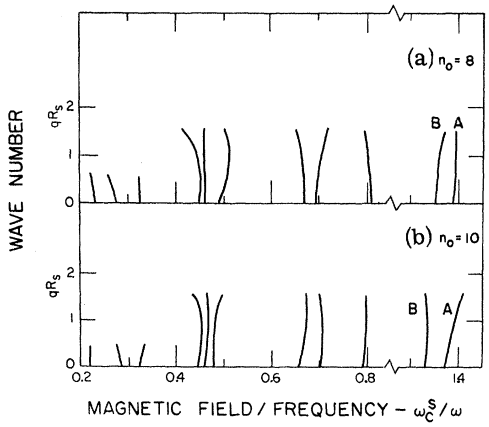

FIG. 3. Dispersion curves for copper with $\omega = 10^{11}$ sec^{-1} and all $(n, m;\ n'm') = 0$. (a) $n_0 = 8$; and (b) $n_0 = 10$. The agreement between (a) and (b) is fairly good, but may not be good enough to enable us to extract Landau coefficients from experimental data.

be compared with the fact that in an isotropic metal all branches which consist of Y_n^m with a given m are degenerate, and among these only one branch is dominant for small q. The latter statement is not applicable either for noble metals for two reasons: First, the way each mode couples to Y_1^0 is not as simple as in isotropic metals, again because $O_N^{N'}$ and $Q_N^{N'}$ are not diagonal. Second, although mode Y_1^0 produces the largest self-consistent fields for a given amplitude, other modes also produce it; thus $g_0(\vec{k})$ need not contain the Y_1^0 component in order to be detected electrically. For these reasons one has to consider many branches. However, it is qualitatively true that Y_n^m with large n fails both to couple to Y_1^0 and to produce appreciable self-consistent fields.

If one increases n_0, more branches appear. In Fig. 3 we have plotted only those branches which give similar dispersion curves for $n_0 = 8$ in Fig. 3(a) and $n_0 = 10$ in Fig. 3(b). They are the branches which consist of Y_n^m with smaller n, and should therefore give stronger signals. We see fairly good agreement between these two calculated results.

A trial has been made to include Landau parameters. Guessing from the experimental value $A_2 = -0.03$ for potassium, 6×10^{-36} would be reasonable for $(n, m; n', m')$ as an order of magnitude. n_0 has been set equal to 10 for the following calculation. First we set $(1, 0; 1, 0) = 6\times10^{-36}$ and all other $(n, m; n', m') = 0$. With these parameters we did not see appreciable change in the region of small q. This is reasonable since there are no branches in which Y_1^0 is the dominant mode in this region. Next we set $(2, 1; 2, 1) = (2, -1; 2, -1) = 6\times10^{-36}$ and all others zero. Paying attention to about a half of the branches, the results are shown in Fig. 4(b). In Fig. 4(a) some branches of Fig. 3(b) are redrawn in the same scale as in Fig. 4(b) for the purpose of comparison. The most eminent change is that the intersecting point of branch A with the x axis is shifted by about -0.03. This is consistent with the fact that Y_2^1 is the most dominant mode in A. One also notices that the shapes of branches B, C, and some others are affected. This is because they contain small components of Y_2^1. In Fig. 4(c) we show the dispersion curves with $(3, 2; 3, 2) = (3, -2; 3, -2) = 6\times10^{-36}$ and all others zero. Comparing Fig. 4(c) with Fig. 4(a), one notices that the intersecting point of branch E is shifted by about 0.015, and also the intersecting points of C and D are slightly shifted. At finite q the change of curvature of branch C is large.

It should be noted that there are two groups of branches: Branches [group (a)] whose intersecting points with the x axis are shifted by finite $(2, \pm1; 2, \pm1)$ but not by $(3, \pm2; 3, \pm2)$, and branches [group (b)] which are affected in the other way. Group (a) includes branches A, B, F, and G, while group (b) includes branches C, D, and E. The reason why there are two groups is the following. Since $O_N^{N'} = Q_N^{N'} = 0$ for $n+n' =$ odd (one can show this by Table I), the corresponding $g_0(\vec{k})$ at $q = 0$ cannot have components in Y_n^m with both even and odd n at the same time. If g_0 are made of Y_n^m with even n, then the branch belongs to group (a), and if with odd n, then the branch belongs to group (b). Hence the intersecting points with the x axis of group (a) branches are shifted by $(2, \pm1; 2, \pm1)$ but not by $(3, \pm2; 3, \pm2)$. And those of group (b) branches are shifted by $(3, \pm2; 3, \pm2)$ but not by $(2, \pm1; 2, \pm1)$. In terms of $\mathcal{Y}_n^m$, the g_0 corresponding to group (a) branches at $q = 0$ are made of $\mathcal{Y}_n^m$ with even n; therefore, under operations of the group C_{2h}, which is a subgroup of D_{2h}, g_0 transforms according to irreducible representation β (refer to Table I) although it does not transform according to any irreducible representations of D_{2h}. Similarly, g_0 at $q = 0$ of group (b) branches transforms according to irreducible representation δ of group C_{2h}.

The largest source of errors in the present calculation is probably the truncation of Eq. (11) at $n = n_0$. Comparing Figs. 3(a) and 3(b), we estimate that the errors due to the truncation are within a few percent for those branches plotted there. Another source of relatively large errors would be the uncertainty of the exact shape of the Fermi surface and the velocity distribution. They are important because off diagonal elements of $O_N^{N'}$ and $Q_N^{N'}$, which determine the mixing among different modes, are very sensitive to them. In particular, one must expect that errors in the velocity distribution would be rather large.

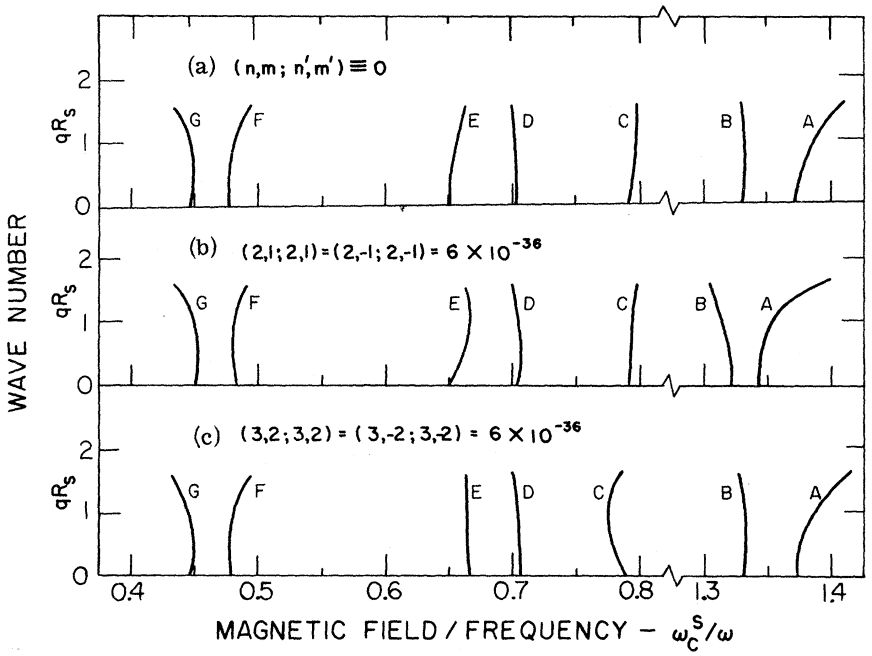

FIG. 4. Dispersion curves for copper with $n_0 = 10$ and $\omega = 10^{11}$ sec^{-1}. (a) all $(n, m; n', m') = 0$; (b) $(2, 1; 2, 1) = (2, -1; 2, -1) = 6 \times 10^{-36}$ and all others zero; and (c) $(3, 2; 3, 2) = (3, -2; 3, -2) = 6 \times 10^{-36}$ and all others zero. Shifts of intersecting points with the x axis of various branches are seen. Also curvatures at finite q values change.

V. SUMMARY

We showed that for anisotropic Fermi systems the expansion of the Landau interaction function $f(\vec{k}, \vec{k}')$ must be done in double series; by a proper choice of the basis functions one can make many of the expansion coefficients vanish due to the crystal symmetry. We then calculated dispersion curves for HFW in copper. They differ qualitatively in several ways from those for isotropic systems. (a) With $q = 0$ there are no degeneracies between modes Y_n^m with the same m but different n's even with no interaction among quasiparticles; (b) even with $q = 0$, mixing between different modes is important, and consequently each branch point is made of several different Y_n^m components; and (c) other modes than Y_1^0 can produce a self-consistent electric field; hence waves need not couple to the Y_1^0 mode in order that it be observable by experiments involving the absorption of electromagnetic waves.

At present, experimental data for copper are not available to us, so that no comparison with experiment has been made. However, a comparison for an anisotropic case might not be as simple as in an isotropic case because in the former case there are many branches which are important, and it may not be clear from which branch a given maximum or minimum in the absorption curve is coming. Therefore, in order to compare theoretical calculations quantitatively with experimental data, one probably needs to include coupling between the impinging r.f. fields and HFW, which is still an open question even for isotropic metals.

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[11]The same conclusion has been reached independently in Ref. 8.

[12]Strictly speaking, one must add further conditions on $f(\vec{k}, \vec{k}')$ so that when the original form of the Fermi surface is restored, $f(\vec{k}, \vec{k}')$ has cubic symmetry. But we do not do it here.