# Condensation kinetics of cavity polaritons interacting with a thermal phonon bath

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The kinetics of the boson condensation for micro-cavity (MC) polaritons interacting with thermal acoustic phonons is studied within rate equations, taking into account the finite MC cross-section. For a smoothly switched on cw excitation and finite polariton lifetimes we find a build-up of a large population in the lowest state of the polariton spectrum for sufficiently large deformation potential coupling at sufficiently high densities, which are still below the saturation density. The critical influence of the value of the heavy-hole deformation potential is illustrated, as well as the favorable influence of positive values of the detuning. This spontaneous polariton condensation would result in a perpendicular emission out of the micro-cavity.

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## I. INTRODUCTION

Semiconductor micro-cavities (MC) with embedded quantum wells have recently attracted considerable attention. As a result of the strong coupling between photons of the lowest cavity mode and two-dimensional (2D) quantum well excitons with the same in-plane wave vector, cavity polaritons are formed. The Rabi splitting and the dispersion of the cavity polaritons are readily observable using reflection, absorption, photoluminescence measurements.[1–3] Because the exciton component of the lower branch polaritons remains finite in the long-wavelength limit, polaritons can be scattered effectively into the lowest momentum $k_0$ state supporting a non-equilibrium condensation. All the exciton (x) scattering processes which have been studied more than twenty years ago for exciton laser gain mechanisms, namely x-phonon, x-x, and x-e scattering[4,5] have recently also been considered for the MC polariton relaxation into a condensed state.[6]

Many new phenomena have been studied in MC's: E.g., super-linear behavior in emission intensity,[7] stimulated polariton amplification,[8,9] and the suppression of the relaxation bottleneck,[10,11] which favor a condensation of cavity polaritons. Very recently Deng *et al.*[12] even attempted to measured coherence properties of the condensate in a Hanbury-Brown-Twiss experiment.

As is well-known from laser theory, a rate equation approach is the simplest way to describe the kinetics of the condensation. The Bose-Einstein condensation (BEC) kinetics for bulk excitons coupled to a thermal bath has been analyzed before.[13–15] It has been shown for $Cu_2O$[15] that under pulse excitation the BEC of ortho-exciton polaritons in the $k=0$ state of the upper branch is only a transient phenomenon, because of the scattering of the upper-branch polaritons into the states on the lower branch. On the other hand, the lower-branch polaritons cannot relax toward the lower $k$-states because of the bottleneck, caused by the rapid vanishing of the exciton component. With the suppression of the bottleneck in MCs[10,11] due to the finite x Hopfield component at $k=0$, a relaxation towards a boson condensation in the lower branch becomes possible. In order to analyze the kinetics of a spontaneous condensation, we only treat the coupling of the polaritons to thermal acoustic phonons, but do not consider the polariton-polariton scattering which has been analyzed by Ciuti *et al.*[16] Particularly under external stimulation of this process, a large population of the state $k=0$ can be achieved, which however, is driven coherently. If free-carriers are present by doping or optical excitation[6] they may support the acoustic phonon relaxation scattering.

Recently Bányai and Gartner[17] showed that the BEC occurs in a finite system at densities which are considerably lower than those in the thermodynamic limit.

Here, we investigate the rate equation for the condensation kinetics of 2D cavity polaritons by scattering with thermal acoustic phonons by taking the finite cross-section of the MC into account. Strictly speaking a condensation into a state with a strongly populated ground state in such a system is not a BEC, because in an infinite 2D system a BEC does not occur. However, due to the finite size, in particular the ground state is separated by an energy gap from the excited states. For such a spectrum also the equilibrium ideal gas theory results in a strongly populated ground state at sufficiently low temperatures and sufficiently high densities. We will present numerical solutions of the kinetic equations for a GaAs quantum well embedded in a MC with a smoothly switched on incoherent cw excitation of exciton-like polaritons in the lower branch above the bottleneck region. With finite polariton lifetimes a stationary large population in the lowest state of lower-branch polaritons can be reached at a sufficiently high concentration of polaritons, which will be shown to be still below the exciton saturation density, provided the deformation potential coupling to the phonons is strong enough. The influence of the size (i.e., its cross section) of the quantum well is studied in detail. The critical influence of the value of the deformation potential of the heavy-hole excitons on the polariton relaxation kinetics analyzed. Only if the coupling is strong enough the relaxation kinetics can result in a condensation. Furthermore, we show

 

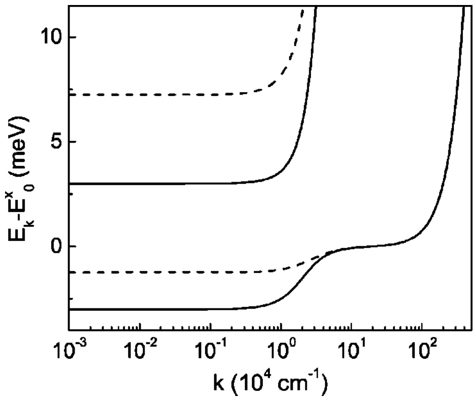


FIG. 1. Dispersion of microcavity polaritons with embeded GaAs quantum wells for $\hbar\Omega=6$ meV, $\delta=0$ meV (solid line), and $\delta=6$ meV (dashed line).

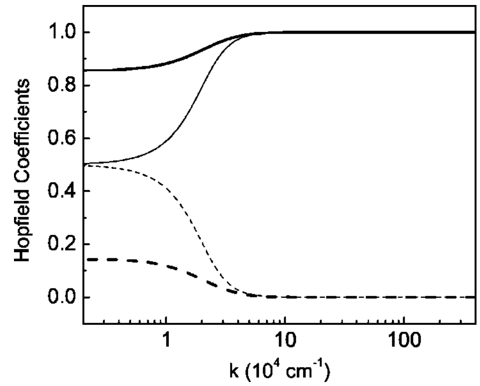


FIG. 2. Exciton Hopfield coefficients $(u_k^+)^2$ (dashed line) and $(u_k^-)^2$ (solid line) of upper and lower branch, respectively, for $\delta=0$ meV (thin line), and $\delta=6$ meV (thick line).

that a positive detuning can be used to increase the effective coupling of the lower branch polaritons to the acoustic phonons, and by these means support the occurrence of a spontaneous condensation.

## II. RELAXATION KINETICS OF MICRO-CAVITY POLARITONS

Within the two-coupled band model (considering only the 1*s* state of the heavy-hole quantum well excitons), the energies for the upper and lower cavity polariton branches are given by:[18,20]

$$E_k^{+(-)} = \tfrac{1}{2}\{E_k^c + E_k^x \pm [(E_k^c - E_k^x)^2 + (\hbar\Omega)^2]^{1/2}\}, \tag{1}$$

where

$$E_k^c = \frac{\hbar c}{n_{\mathrm{cav}}}\left[\frac{\pi}{L_c} + k^2\right]^{1/2}, \tag{2}$$

$$E_k^x = E_0^x + \frac{\hbar^2 k^2}{2(m_e + m_h)}. \tag{3}$$

Here $E_0^x$ is the fundamental exciton energy, $m_e$ ($m_h$) the electron (hole)in-plane mass, $n_{\mathrm{cav}}$ the cavity refraction index, $L_c$ the cavity length, $\Omega$ the Rabi splitting, and $k$ is the 2D momentum wave number. Figure 1 shows the cavity polariton dispersion of GaAs quantum wells embedded in cavity for Rabi splitting $\hbar\Omega=6$ meV, $n_{\mathrm{cav}}=3.43$, detuning $\delta=E_0^c-E_0^x=0$ meV (solid line) and $\delta=E_0^c-E_0^x=6$ meV (dashed line).

The corresponding Hopfield coefficients (see Fig. 2) for the transformation of polariton operators of branch $i$ ($\alpha_{\vec{k}}^i$) from excitons ($a_{\vec{k}}$) and photons ($b_{\vec{k}}$) with $\alpha_{\vec{k}}^i=u_k^i a_{\vec{k}}+v_k^i b_{\vec{k}}$, with $(u_k^i)^2+(v_k^i)^2=1$, are

$$u_k^i = \pm\frac{1}{\sqrt{1+\left(\dfrac{\Omega/2}{E_k^i - E_k^x}\right)^2}}. \tag{4}$$

Note that the exciton Hopfield coefficients are finite when $k\to 0$ so that the bottleneck effect is strongly reduced in a MC. A finite positive detuning increases the exciton component on the lower branch particularly in the region of low momenta as can be seen from the full upper line in Fig. 2.

A finite cross section will make the spectrum of the momenta discrete $k_{n_i,i}=n_1(\pi/L_1)$ with $n_i=1,2,3,\cdots$ and $S=L_xL_y$.

In the following we will use the semi-classical Boltzmann kinetics as in Refs. 13, 15, and 17–20 to investigate the 2D cavity polariton relaxation due to the scattering with thermal bulk acoustic phonons. In the rate equations for the population of the polariton states $f_{\vec{k}}^{\,i}(t)$ (the upper index is the branch index) we will treat the population of the lowest states with the lowest momentum $\vec{k}_0$ separately. Because we are interested in a large population density of this lowest state, we introduce its 2D density $n_0^i=f_{\,0}^{\,i}/S$. Here we write, e.g., $f_{\vec{k}_0}\to f_0$ for conciseness.

With this notation the polariton rate equation can be written as[15]

$$\frac{\partial}{\partial t}f_{\vec{k}}^{\,i} = G_{\vec{k}}^i - \frac{f_{\vec{k}}^{\,i}}{\tau_{\vec{k}}^i} - \sum_j \sum_{\vec{k}'\neq\vec{k}_0}\left(W_{\vec{k},\vec{k}'}^{i,j} f_{\vec{k}}^{\,i}(1+f_{\vec{k}'}^{\,j}) - (\vec{k},i\leftrightarrow\vec{k}',j)\right) - S\left(W_{\vec{k},0}^{i,j} f_{\vec{k}}^{\,i}\left(\frac{1}{S}+n_0^i\right) - W_{0,\vec{k}}^{j,i} n_0^j(1+f_{\vec{k}}^{\,i})\right) \tag{5}$$

and

$$\frac{\partial}{\partial t}n_0^i = -\frac{n_0^i}{\tau_0^i} + \sum_{j,\vec{k}\neq\vec{k}_0}\left(W_{\vec{k},\vec{k}_0}^{j,i} f_{\vec{k}}^{\,j}\left(\frac{1}{S}+n_0^i\right) - W_{\vec{k}_0,\vec{k}}^{i,j} n_0^i(1+f_{\vec{k}}^{\,j})\right) - S\left(W_{0,0}^{i,\bar{i}} n_0^i\left(\frac{1}{S}+n_0^{\bar{i}}\right) - W_{0,0}^{\bar{i},i} n_0^{\bar{i}}\left(\frac{1}{S}+n_0^i\right)\right),\quad \bar{i}\neq i, \tag{6}$$

where $S$ is the cross-section of quantum well and $1/\tau_{\vec{k}}^i$ is the radiative recombination rate. $G_{\vec{k}}^i\equiv A_{\vec{k}}^i H(t)$ is the generation rate. The transition rates $W_{\vec{k},\vec{k}'}^{i,j}$ due to the deformation potential scattering are weighted with the exciton Hopfield coefficients $u_k^i$ and $u_{k'}^j$,

$$W^{i,j}_{\vec{k},\vec{k}'}=\frac{L_z(u^i_k u^j_{k'}\Delta^{i,j}_{k,k'})^2}{\hbar\rho V u^2 q_z}B^2(q_z)D^2(|\vec{k}-\vec{k}'|)|N^p_{E^j_{k'}-E^i_k}|\theta(\Delta^{i,j}_{k,k'}-|\vec{k}-\vec{k}'|), \quad (7)$$

where

$$\Delta^{i,j}_{k,k'}=\frac{|E^j_{k'}-E^i_k|}{\hbar u}, \quad (8)$$

$$q_z=((\Delta^{i,j}_{k,k'})^2-|\vec{k}-\vec{k}'|^2)^{1/2}, \quad (9)$$

$$D(q)=D_eF\left(\frac{qm_h}{m_e+m_h}\right)-D_hF\left(\frac{qm_e}{m_e+m_h}\right), \quad (10)$$

$$B(q)=\frac{8\pi^2}{L_zq(4\pi^2-L_z^2q^2)}\sin\left(\frac{L_zq}{2}\right), \quad (11)$$

$$F(q)=(1+(qa_{2D}/2)^2)^{-3/2}. \quad (12)$$

Here $L_z$ is quantum well width, $a_{2D}$ is the exciton Bohr radius of 2D GaAs, $V$ the crystal volume, $u$ the longitudinal sound velocity, $\rho$ the mass density of the solid, $D_{e(h)}$ the deformation potentials. $|N^p_E|$ is the thermal phonon population factor for the phonon absorption or emission. For the decay rate $1/\tau^i_k$, we use the same approximation as given by Bloch and Marzin for a GaAs quantum well:[18] $1/\tau^i_k=(v^i_k)^2/\tau^c$ for $0<k<k_{cav}=6\times10^4\ \mathrm{cm}^{-1}$; $1/\tau^j_k=1/\tau^x$ for $k_{cav}<k<k_{rad}=n_{cav}E^x_0/\hbar c=2.3\times10^5\ \mathrm{cm}^{-1}$; and $1/\tau^j_k=0$ for $k>k_{rad}$, ($\tau^c$ and $\tau^x$ are the radiative lifetimes of photons and excitons, respectively.

If we set $n^i_0\equiv0$ and $1/S=0$ in Eqs. (5) and (6), we retrieve the rate equation for cavity polaritons studied in Refs. 11, 18, and 20. If we replace the expressions of $W^{i,j}_{\vec{k},\vec{k}'}$, $f^i_k$ and $n^+_0$ of the cavity polaritons in Eqs. (5) and (6) by the corresponding expressions of the bulk polaritons and set $n^-_0\equiv0$, $1/\tau^i_k=0$ we will get the rate equation for ortho-exciton polaritons in $Cu_2O$ studied in Ref. 15. If we replace the expressions of $W^{i,j}_{\vec{k},\vec{k}'}$, $f^i_k$, and $n^i_0$ of cavity polaritons in Eqs. (5) and (6) by the corresponding expressions for bulk excitons and set $1/\tau^i_k=0$, we obtain the rate equation for the bulk excitons studied in Ref. 13 or its simple version with two state model (a ground state and an excited state) studied recently in Ref. 17.

In three-dimensional (3D) it has been shown that the condensation kinetics can be started in a finite system by the spontaneous transitions contained in the first term of the population factor $(1/V)+n_0$.[13] In the thermodynamic limit $V\to\infty$ one looses these spontaneous transition rates, and a dynamical version of Bogoljubov's symmetry breaking has to be used in the form of small but finite initial values of the condensate density $n^i_0(t=0)\neq0$. Both versions have been shown to yield a similar condensation kinetics.[13] As already mentioned, in infinite 2D systems an ideal Bose gas has no BEC at finite temperatures. Therefore, we have to use a finite cross-section $S\neq0$ which starts the condensation kinetics without an initial condensate density, i.e., the initial conditions will be $n^i_0(t=0)=0$ and $f^i_k(t=0)=0$.

## III. NUMERICAL RESULTS

In this section we carry out numerical calculations using the following parameters for a MC with embedded GaAs quantum wells:[18,21,22] $E^x_0=1.515$ eV, $m_e=0.067m_0$, $m_h=0.45m_0$, $a_{2D}=10$ nm, the deformation potentials $D_e=-8.6$ eV, $D_h=5.7$ eV,[18] $u=4.81\times10^5$ cm/s, $\rho=5.3\times10^3\ \mathrm{kg/m^3}$. Particularly the value of the heavy-hole deformation potential will turn out to be crucial for the relaxation kinetics. Unfortunately, one finds in the literature a considerable spread of values for GaAs quantum wells. Bloch and Marzin[18] and Bockelmann[22] used $D_e=-8.6$ eV, $D_h=5.7+3(q_z^2/Q^2)\geq5.7$ eV. In our numerical simulation we will mainly use $D_h=5.7$ eV. Ivanov *et al.*[23,24] have approximated the expression (10) by $|D(q)|\simeq|D_e-D_h|=15.5$ eV, which is slightly greater than our value taken from Bloch and Mazin[18] and Bockelmann.[22] But also smaller values of $D_h$ are reported in the literature, e.g., $D_h=3.5$ eV by Selbmann *et al.*[30] and $D_h=2.7$ eV by Tassone *et al.*[20,31] Numerical solutions for such lower values of the deformation potential will also be presented here.

The generation rate is assumed to produce a Gauss distribution $A^i_{\vec{k}}$ of exciton on the lower branch with a width $\Delta E=0.1$ meV centered at a momentum $k_x=1.1\times10^{-1}\ \mathrm{nm}^{-1}$ above the bottleneck region.

We take into account the energy gap of the order $3\hbar^2\pi^2/2\overline{m}S^2$ of a quadratic cross section between the ground state and the first excited state into account. $\hbar^2/\overline{m}$ is given by the second derivative of the polariton dispersion (1) with respect to $k$ for small $k$ values. As an approximation we treat the spectrum of the higher states as a continuum and assume that the distribution functions are isotropic in momentum space, i.e., they are assumed to depend only on $|\vec{k}|$. Because of the stronger dispersion for small $k$ values on the lower polariton branch, the momentum of the last excited state which couples by phonon emission to the lowest state is very small for MC-QW polaritons in comparison to the situation quantum wells. Correspondingly the effect of this minimal momentum is of minor importance to relaxation kinetics of MC-QW polaritons compared with that for QW excitons.[23,24]

The switch-on of the pump rate has a ramp shape of the form $H(t)=H_0th(t/t_0)$ with $t_0=50$ ps. Unless stated otherwise, we choose $L_z=3$ nm, $T=5$ K, $\delta=E^c_0-E^x_0=1$ meV, $\tau^x=10$ ps,[26] $\tau^c=8$ ps.[6]

Note that the exciton model in Eqs. (5) and (6) is only meaningful, if the generated x density is not too large. We can evaluate an upper limit by the saturation density at which the exciton oscillator strength is completely screened by exchange interaction and phase space filling effects and, therefore, the coupling between exciton and photon is destroyed.[25] Following Schmitt-Rink *et al.*,[27] we can estimate the saturation density $n_s$ for a GaAs microcavity as

$$n_s=\frac{0.117}{\pi a_{\mathrm{eff}}^2}=1.5\times10^{11}\ \mathrm{cm}^{-2}, \quad (13)$$

where $a_{\mathrm{eff}}=a_{2D}/2$ (see Refs. 27–29). For GaAs QWs we have chosen $a_{2D}=10$ nm as given by Tassone *et al.*[20] A

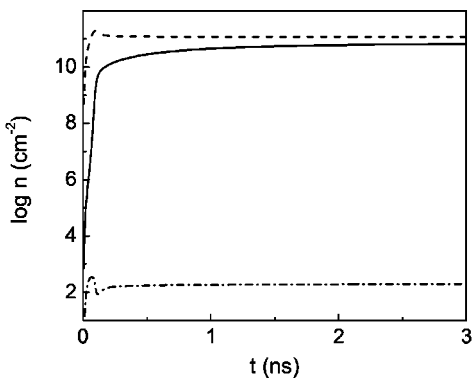


FIG. 3. Condensate densities for the lower and upper branch $n_0^-(t)$ (solid line) and $n_0^+(t)$ (dashed-dotted line), respectively, and total density (dashed line) which reaches a stationary super-critical value of $n_{tot}=1.17\times10^{11}$ cm$^{-2}$ with a quantum well cross section of $S=100$ $\mu$m$^2$.

choice of $a_{2D}=a_B/2$ would lead to a too high, unphysical saturation density, where $a_B$ is the bulk exciton radius. For CdTe, Porras *et al.*[28] and Le Si Dang *et al.*[29] have chosen $a_{2D}=a_B/2$ and this choice results in $n_s=6.7\times10^{11}$ cm$^{-2}$.

In Fig. 3 we show the lower branch condensate density which reaches a stationary value of $n_0^-=7.7\times10^{10}$ cm$^{-2}$ (solid line) and the total density $n_{tot}(t)$ (dashed line) with the supercritical stationary density of $1.17\times10^{11}$ cm$^{-2}$ for a quantum well cross section $S=100$ $\mu$m$^2$. The absolute number of condensed polaritons is thus $n_0^-S=7.7\times10^4\gg1$, which indeed is much larger than 1. We also show the density of the lowest state in the upper branch $n_0^+(t)$ (dashed-dotted line), where no appreciable population builds up as can be seen from the stationary value of $f_0^+=Sn_0=1.9\times10^{-4}$. The stationary distribution function of the higher states of the lower branch $f_k^-$ is shown in Fig. 4. It is clearly seen that the distribution function in the vicinity of $k\rightarrow k_0$ is much larger than 1.

The stationary condensate density $n_0^-$ depends on the quantum well cross-section $S$. Figure 5 shows the condensate fraction $n_0^-(t)/n_{tot}$ for $n_{tot}=1.24\times10^{10}$ cm$^{-2}$ and different cross sections $S$. The pump rate is adjusted to give for all values of $S$ the same stationary total density $n_{tot}$. One clearly sees the slow-down of the condensation kinetics and the decrease of the condensate fraction with increasing quantum well cross-section $S$. This decrease of the stationary condensate fraction is in agreement with the thermal equilibrium results for a finite 2D system as discussed, e.g., by Baumberg *et al.*[28]

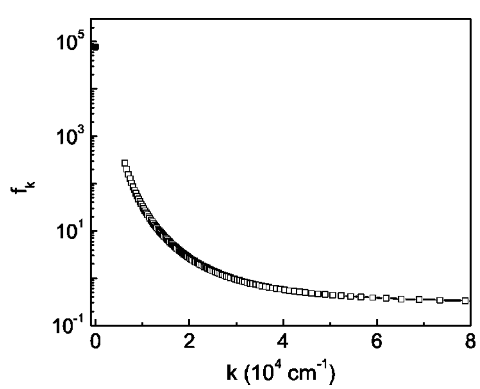


FIG. 4. Polariton distribution $f_k^-$ for the higher lying states corresponding to parameters given in Fig. 3.

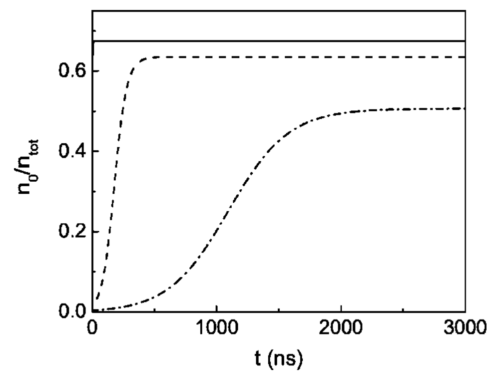


FIG. 5. Condensate fraction $n_0^-(t)/n_{tot}$ for a total density $n_{tot}=1.24\times10^{11}$ cm$^{-2}$ with different quantum well cross sections $S=100$ $\mu$m$^2$ (solid line), $S=2500$ $\mu$m$^2$ (dashed line), $S=10000$ $\mu$m$^2$ (dashed-dotted line).

Figure 6 shows the kinetics for a sub-critical density $n_{tot}=1.9\times10^8$ cm$^{-2}$. The density $n_0^-$ (solid line) is too low to achieve a condensation. The corresponding distribution function for the higher polariton states $f_k^-$ is shown in Fig. 7. In the sub-critical regime $n_0^-$ is very small and most polaritons are in the non-condensate states staying around the bottleneck as shown in Fig. 7.

Figure 8 shows the condensate density $n_0^-$ versus the total density $n_{tot}$ for various values quantum well cross-section. We can estimate the critical density $n^c$ for BEC from the abrupt increase in $n_0^-$. From Fig. 8 we find $n^c\simeq1.02\times10^{11}$ cm$^{-2}$ for $S=100$ $\mu$m$^2$, which corresponds to $n_0^-=1.5\times10^8$ cm$^{-2}$ and $f_0^-=n_0^-S=1.5\times10^2\gg1$. For a large total density the depletion of the condensate due to the increasing cross-section is relative small. Only for large cross sections (see dotted line) the reduction of the condensate density becomes large. This is in agreement with the evaluation of the total particle number and of the number of condensed particles for an ideal finite size 2D boson gas in thermal equilibrium. It is clearly seen from Fig. 8 that above the critical

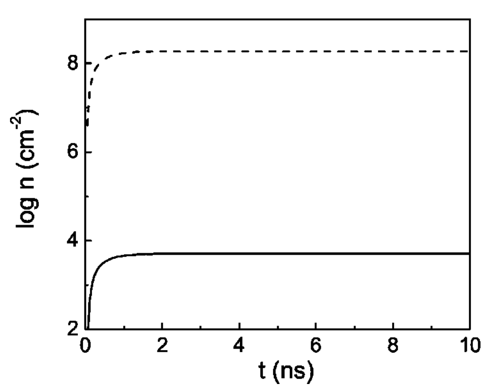


FIG. 6. Condensate density $n_0^-$ (solid line) for a sub-critical total density $n_{tot}=1.9\times10^8$ cm$^{-2}$ (dashed line) with a quantum well cross section $S=100$ $\mu$m$^2$.

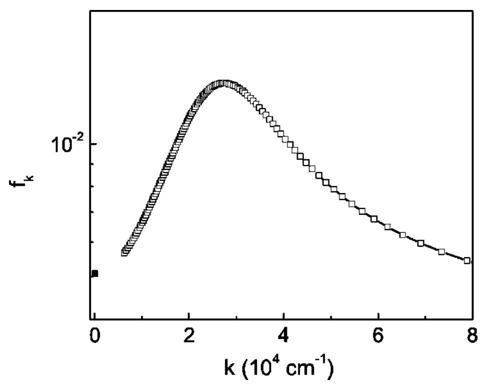


FIG. 7. Distribution $f_k^-$ for the higher states corresponding to parameters given in Fig. 7.

density we obtain a linear relation between $n_0^-$ and $n_{tot}$. The slope of the line in the linear region is almost identical to 1, i.e., all added polaritons really fall into the condensate state $k_0$.

For densities $n_{tot} > 10^{10}$ cm$^{-2}$ one should take into account the polariton-polariton scattering. We expect the inclusion of the polariton-polariton scattering would decrease the estimated value of $n^c$ in favor of the condensation. Note that Deng *et al.*[12] have experimentally increased the saturation density by using a system with 12 GaAs quantum wells placed at the anti-node positions of a GaAs/$Al_xGa_{1-x}As_x$ micro-cavity.

In Fig. 9 we display the lower branch condensate $n_0$ versus the total density $n_{tot}$ for three values of the detuning. High values of positive detuning lower the critical density considerably mainly by increasing the effective deformation potential coupling on the lower branch via the exciton Hopfield coefficient. We got a rather low value of $n^c = 2 \times 10^{10}$ cm$^{-2}$ for $\delta = 6$ meV in comparison with $n^c = 1.02 \times 10^{11}$ cm$^{-2}$ for $\delta = 1$ meV. The lowering of the critical density for the condensation can be used in situations, where a condensation would not occur say with zero detuning, e.g., because the deformation potential is too small. For the purpose of illustration we show in Fig. 10 the condensation

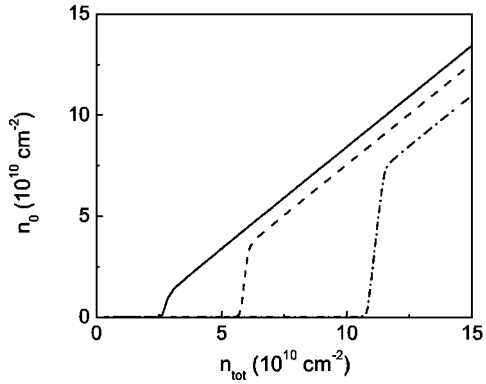


FIG. 9. Condensate density of lower branch $n_0^-$ versus total density for three values of the detuning: $\delta = 1$ meV (dashed-dotted line), $\delta = 3$ meV (dashed line) and $\delta = 6$ meV (solid line) and for $S = 100$ $\mu$m$^2$.

kinetics for various values of the heavy-hole deformation potential $D_h$ and of the detuning. Again it is seen that the decrease of the heavy-hole deformation potential can at least to some extend by compensated by an increasing detuning.

There are some differences between our work and the recent work by Porras *et al.*[28] studying CdTe micro-cavities. While we have considered low- and high-energy polaritons as full polaritons, Porras *et al.*[28] have treated high-energy polaritons as thermalized excitons. But the main difference between both studies is the scattering mechanism. In our model the phonon-polariton scattering is the only considered mechanism. In the model of Porras *et al.* the main scattering mechanism for the relaxation into the lower polariton states is an exciton-exciton scattering of the form $x + x \rightleftharpoons x + p$.

Tassone *et al.*[20] considered also only polariton-phonon scattering, but only in the range $n_{tot} \leqslant 10^{10}$ cm$^{-2}$. In this range our results confirm those of Tassone *et al.* The main difference is that we extended for comparable parameters the density range and find there a spontaneous condensation.

Selbmann *et al.*[30] who studied for 3D GaAs the relaxation kinetics with a heavy-hole deformation potential $D_h = 3.5$ eV find no spontaneous condensation. We get the same

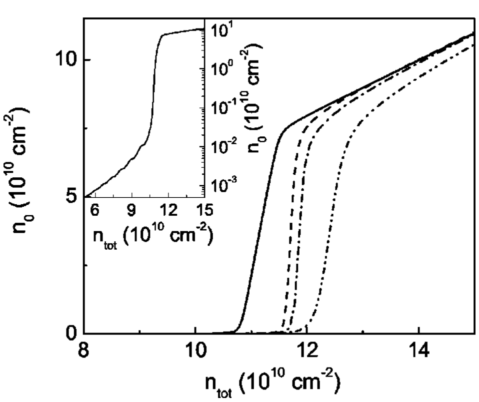


FIG. 8. Condensate density of lower branch $n_0^-$ versus total density for different cross-sections $S = 100$ $\mu$m$^2$ (solid line), $S = 900$ $\mu$m$^2$ (dashed line), $S = 2500$ $\mu$m$^2$ (dashed-dotted line) $S = 10000$ $\mu$m$^2$ (dotted line) and for $\delta = 1$ meV. The inset gives a log plot of the full line.

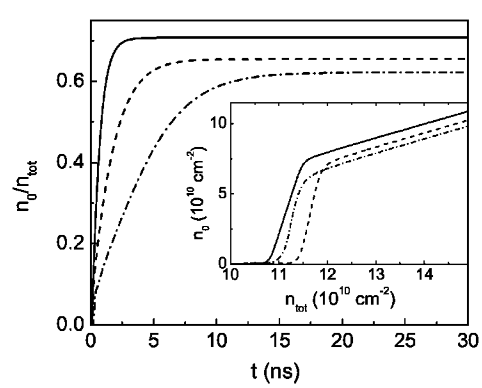


FIG. 10. Condensate density $n_0^-(t)/n_{tot}$ for a total density $n_{tot} = 1.35 \times 10^{11}$ cm$^{-2}$ for various values of the heavy-hole deformation potential and the detuning: $D_h = 5.7$ eV, $\delta = 1$ meV (solid line); $D_h = 3.5$ eV, $\delta = 3$ meV (dashed line); $D_h = 1$ eV, $\delta = 6$ meV (dashed-dotted line). Inset shows the stationary condensate density $n_0^-$ versus the stationary total density $n_{tot}$.

result for zero detuning. However, we showed that with a detuning of $\delta=3$ meV one gets for otherwise equal conditions again a spontaneous condensation. A similar conclusion holds concerning the work of Tassone *et al.*[31] in which a deformation potential $D_h=2.7$ eV does not allow a condensation for $\delta=0$ meV, but well with a detuning of, e.g., $\delta=6$ meV as can be seen from Fig. 10

In the work of Pau *et al.*[32] a too large value for the deformation potential $D_h=12$ eV has been used which naturally is very favorable for a condensation.

The results of other studies cannot be compared directly because also other scattering mechanism have been included.

In conclusion, we have presented the MC polariton kinetics due to deformation potential scattering by thermal acoustic phonons without any external stimulation for a cw excitation and for finite polariton lifetimes with finite quantum well cross sections. We have shown that with realistic parameters a large population in the lowest state of the lower polariton branch with $f_0=n_0S \ll 1$ can be obtained for total densities which are still below the saturation density. In particular, the dependence of the condensate fraction on the cross section, the heavy-hole deformation potential and the detuning has been analyzed in detail. Our analysis reveals the critical influence of the value of the heavy-hole deformation potential which explains at least to some extend the different results concerning the occurrence of a spontaneous condensation in GaAs QW-Mc's. Thus, it is a good strategy to look for QW materials with a large heavy-hole deformation potential, if one wants to achieve a spontaneous polariton condensation. At the same time we showed that the polariton-phonon coupling on the lower branch can be increased to some extend by a positive detuning, which will increase the exciton component on the lower branch, and thus will make the kinetics towards a condensation more likely. A spontaneous condensation of excitonic polaritons would result in a strong, partially coherent emission out of the micro-cavity perpendicular to its mirrors.


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