Theory of microwave fluctuations in a degenerate nonequilibrium electron gas

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The aim of the paper is to analyze the possibilities of macroscopic manifestation, through microwave noise, of the degeneracy of three- or two-dimensional electron gas in bulk semiconductors and in semiconductor structures. Analytic investigation is performed of current fluctuations and noise temperatures in a current-carrying state of a degenerate two- or three-dimensional electron gas. In the electron-temperature approximation, applicable at sufficiently high electron densities, the spectral intensities of current fluctuations and the noise temperatures of the nonequilibrium electron gas in a macroscopic spatially uniform sample are proved to be expressible in terms of the electron temperature and the current-voltage characteristics of the channel in the same manner as for Boltzmann statistics, but only at not-too-high applied electric fields ("warm electrons"). The relations promise to be useful while interpreting now widely studied noise properties of degenerate two-dimensional electron gas, e.g., in GaN-based heterostructure channels.

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I. INTRODUCTION

Investigation of fluctuation and noise properties of three- and two-dimensional electron gas (3DEG and 2DEG) has proved to be a powerful tool for diagnostics of *nonequilibrium electron systems* (see Refs. 1 and 2). Relations between *noise* and *transport characteristics*, if available, are just what experimentalists need to interpret their results on noise in the current-carrying channels of the modern semiconductor structures. Of course, no *general* relationship exists between fluctuation and transport characteristics of a nonequilibrium state. However, so-called *electron-temperature approximation*, valid at sufficiently high electron densities, drastically simplifies description of a nonequilibrium electron gas, opening a way for analytic investigation of noise properties of such a system[3–14] (see also Ref. 15, as well as Ref. 1, pp. 147–154, and references therein). The analytic expressions for spectral intensities of *spatially uniform current fluctuations*, i.e., fluctuations averaged over the sample's volume, for *macroscopic nondegenerate spatially uniform hot-electron* system were obtained as early as in 1967 by Kogan and Shulman.[3] The result, obtained in the electron-temperature approximation, was quite attractive since it expressed noise in terms of electron temperature and current-voltage characteristics, thus constituting a specific nonequilibrium *noise-response relation*. True, the relation was obtained at the cost of very specific conjectures about the properties of Langevin sources in the energy-balance and quasi-momentum-balance equations. However, three years later, avoiding special conjectures, Shulman[5] rederived the relation on the basis of the kinetic theory of fluctuations developed just before.

Namely, the self-consistent *semiclassical kinetic theory of fluctuations*, which took into account *interelectron [electron-electron (e-e)] collisions*, was presented in 1969 simultaneously in two equivalent forms. Equations, in $\mathbf{p}$ space and in $(\mathbf{r},\mathbf{p})$ space, for correlation functions of occupancies of one-electron states ("*kinetic equations for the moments*") were derived *ab initio* (by using the diagrammatic approach) by Gantsevich, Gurevich, and Katilius.[16–18] Kogan and Shulman,[19] basing themselves on the presumption of *independence of collision events*—the presumption indispensable for validity of the Boltzmann equation—obtained so-called *Boltzmann-Langevin equations*. From these equations, treated in the electron-temperature approximation, not only the above-mentioned fluctuation-response relation for *spatially uniform fluctuations* followed, but also the "*quasimacroscopic*" ("quasi-hydrodynamic") Langevin-type equations[4] were obtained for *spatially nonuniform fluctuations* of slowly changing variables—for *large-scale low-frequency* fluctuations of *electron density* and *electron temperature*. On the basis of these equations, a thorough investigation of spectra of quasimacroscopic electron-density and electron-temperature fluctuations and their cross-correlation was performed.[6] The results enabled to predict peculiarities of the spectra of electromagnetic waves scattered from the collisional solid-state plasma.[6] Most of work in the electron-temperature approximation for macroscopic systems was done for the Boltzmann statistics,[4–10] not taking into account possible degeneracy of the electron system.

In contrast to macroscopic systems, in small *mesoscopic* samples the *shot noise* reveals itself. In the framework of the semiclassical kinetic theory of fluctuations,[17–19] the analytic description of the shot noise is obtainable quite naturally[11–14,20–26] [see also review article[27] (Sec. VI)]. As a result of these efforts, now more or less universal expressions for shot-noise power in terms of conductance are known. While calculating noise in mesoscopic systems, the electron-temperature approximation was proved to be quite useful.[11–14]

Shot noise in a macroscopic sample, of the dimensions *larger* than the inelastic relaxation lengths, is suppressed *by inelastic electron-phonon scattering*.[11,12,20,24,28–32] What remains is the "nonequilibrium thermal" noise (we use the term to accentuate its distinction from the shot noise)—the noise, characteristic for macroscopic spatially uniform hot-electron systems we spoke about in the first paragraphs of this section. Spatially uniform current fluctuations in these systems are conditioned by fluctuations of occupancies of the one-electron states (fluctuations in **p** space). In other words, as the conductor's length is increased from small values, much smaller than the electron energy relaxation length, up to values remarkably exceeding the electron energy relaxation length, the current noise in a conductor varies from the level set by the theory of shot noise in a mesoscopic sample to the level predicted by the theory of nonequilibrium fluctuations in macroscopic conductors. In the case of a macroscopic conductor with sufficiently *dense but nondegenerate* electron gas, the noise level and noise spectrum are given just by Shulman's relation.

The natural question arises: to what extent, if any, Shulman's result, obtained in the framework of the Boltzmann statistics, is applicable also to a macroscopic degenerate electron systems. The answer is not obvious, especially for the following reason. Interparticle collisions create a correlation between occupancies of one-particle states[16–18,33–35] (for a review, see Refs. 15 and 36, and Ref. 1 Chap. 4; for a summary, see Ref. 37). The nontrivial general result of the kinetic theory of fluctuations is the conclusion that in a nonequilibrium state this *additional, or kinetic, correlation* can contribute to the spectral intensities of current fluctuations. In a nondegenerate case, this contribution was proved[5] to be expressible in terms of the macroscopic transport coefficients.

The aim of this paper is to obtain, when possible, expressions for noise in macroscopic spatially uniform sample, containing the nonequilibrium *degenerate* electron gas, in terms of the measurable transport characteristics of the sample—to derive these expressions basing oneself on the semiclassical kinetic (microscopic) theory of fluctuations. Our investigation demonstrates that, in general, theoretical description of noise properties of a degenerate electron gas is much more complicated than in the nondegenerate case. Even in the electron-temperature approximation, the above-mentioned noise-transport relations are violated by the degeneracy due to a rather complicated, in comparison with the nondegenerate situation, character of the additional correlation. The expressions for the sources of the additional correlation in a degenerate state were derived from the first principles by Kagan[38] (see also Muradov[39]). Our main result is that, in the electron-temperature approximation, the additional correlation, being as complicated as it is, manifests itself in the spectral intensity of electric current fluctuations, *independent of the degree of degeneracy*, only through the terms proportional to the *fourth and higher powers* of the applied electric field strength, E^4. As a result, at not too high electric fields E, the macroscopic relations between the noise characteristics (noise temperatures) and transport (current-voltage) characteristics *survive* in the electron-temperature approximation even in the degenerate system.

The result is applicable to three- and two-dimensional electron gas, and thus can prove to be equally useful while investigating bulk semiconductors and modern semiconductor structures containing two-dimensional electron gas. In many cases of interest, thanks to a comparatively *weak nonlinearity* of the current-voltage characteristics of a structure in question, the warm-electron region of electric fields is rather well pronounced. Of course, in order to avoid generation-recombination noise and $1/f$ fluctuations, the frequencies at which the noise measurements are performed should not be too low: as a matter of fact, the theory we are developing is applicable at *microwave* frequencies. By now, a great amount of experimental results on microwave noise, especially in up-to-date semiconductor structures and new semiconductor compounds, important for microelectronics, is available at these frequencies (see Refs. 1 and 40). The theoretical results predicting features of microwave noise in macroscopic systems can prove to be advantageous.

The paper is organized as follows. Resume of the kinetic theory of spatially uniform fluctuations in macroscopic degenerate hot-electron system is presented in Sec. II. Electron-temperature approximation and limits of its applicability to degenerate electron gas are described in Sec. III. Necessary results on fluctuations in a macroscopic nondegenerate (semi)conductor obtained earlier in the electron-temperature approximation are collected in Sec. IV. In Secs. V and VI, the original results on transverse and longitudinal current fluctuations in the degenerate nonequilibrium electron gas are presented and analyzed. In Sec. VII, the macroscopic noise-response relations for degenerate warm-electron gas are obtained. Sections VIII and IX contain discussion of results and conclusions.

II. FLUCTUATIONS IN A MACROSCOPIC SPATIALLY UNIFORM DEGENERATE HOT-ELECTRON SYSTEM

As noticed in the Introduction, we restrict ourselves to investigation of *macroscopic spatially uniform* hot-electron systems. Spatially uniform *current* fluctuations in these systems are conditioned by fluctuations—in **p** space—of occupancies of the one-electron states. The kinetic equations for correlation functions of fluctuations of occupancies of one-electron states in a *nondegenerate* 3D electron gas were derived from first principles in Refs. 16 and 17. These equations led to now well-known expression for the *spectral intensities* of semiclassical fluctuations of the occupancies in the nonequilibrium system:[18]

$$(\delta n_{\mathbf{p}}\delta n_{\mathbf{p}'})_{\omega} = (-i\omega + B_{\mathbf{p}})^{-1}(i\omega + B_{\mathbf{p}'})^{-1}(B_{\mathbf{p}} + B_{\mathbf{p}'})[\overline{n_{\mathbf{p}}}\delta_{\mathbf{pp}'} - I^{\mathrm{ee}}_{\mathbf{pp}'}\{\overline{n},\overline{n}\}]. \quad (1)$$

Here B is the evolution operator of the electron system (the operator of the linearized Boltzmann equation), $\overline{n_{\mathbf{p}}}$ is the stationary distribution function, **p** is the electron quasimomentum, and $I^{\mathrm{ee}}_{\mathbf{pp}'}$ is the *source term describing creation, due to electron-electron collisions, of correlation among occupancies of the one-electron states in a nondegenerate system*:

$$-I^{\mathrm{ee}}_{\mathbf{pp}'}(n,n) = \sum_{\mathbf{kk}'} W^{\mathbf{kk}'}_{\mathbf{pp}'}(n_{\mathbf{k}}n_{\mathbf{k}'} - n_{\mathbf{p}}n_{\mathbf{p}'}), \quad (2)$$

where $W^{\mathbf{kk}'}_{\mathbf{pp}'}$ is the transition probability for the electron-electron collision carrying two electrons from the states **k** and **k**′ into the states **p**, **p**′. We suppose that $W^{\mathbf{kk}'}_{\mathbf{pp}'} = W^{\mathbf{pp}'}_{\mathbf{kk}'}$.

The source $I^{\mathrm{ee}}_{\mathbf{pp}'}$, creating the correlation between occupancies of the one-electron states in nonequilibrium nondegenerate case, was introduced in Ref. 16 (see also Ref. 33). Generally speaking, each interelectron ("two-particle," "binary") collision create an equal-time correlation between the occupancies of two final (as well as two initial) electron states.[33] However, at *thermal equilibrium* the average correlating *fluxes*, in the quasimomentum space, *into the pair* of one-electron states and *out of it* cancel each other, so that the additional correlation does not reveal itself through the fluctuation properties of the electron gas in equilibrium.[16] On the contrary, in a nonequilibrium (e.g., current-carrying) state the fluxes in question do not balance each other, and the observable characteristics of fluctuations in the electron gas, such as spectral intensities of current fluctuations and noise temperatures, generally speaking, become sensitive to the above-mentioned correlation.

This reasoning, in fact, is valid independent of a degeneracy of the electron gas. Generalization for the degenerate system was achieved by Kagan[38] and Muradov.[39] By using the diagrammatic approach, they obtained the following expression for the spectral intensities of semiclassical fluctuations of the occupancies in the degenerate nonequilibrium system:

$$(\delta n_{\mathbf{p}}\delta n_{\mathbf{p}'})_\omega = (-i\omega + B_{\mathbf{p}})^{-1}(i\omega + B_{\mathbf{p}'})^{-1}(B_{\mathbf{p}} + B_{\mathbf{p}'})[\overline{n_{\mathbf{p}}}(1 - \overline{n_{\mathbf{p}'}})\delta_{\mathbf{pp}'} + L_{\mathbf{pp}'}], \tag{3}$$

where $L_{\mathbf{pp}'}$ are the source terms describing creation, due to electron-electron collisions, of correlation among occupancies of the one-electron states in a degenerate system. According to Refs. 38 and 39, each of the source terms $L_{\mathbf{pp}'}$ in the case of degenerate statistics consist of a few summands:

$$L_{\mathbf{pp}'} = L^{(1)}_{\mathbf{pp}'} + L^{(2)}_{\mathbf{pp}'} + L^{(3)}_{\mathbf{pp}'}. \tag{4}$$

Explicitly,

$$L^{(1)}_{\mathbf{pp}'} = \sum_{\mathbf{kk}'} W^{\mathbf{kk}'}_{\mathbf{pp}'}[\overline{n_{\mathbf{k}}}\overline{n_{\mathbf{k}'}}(1-\overline{n_{\mathbf{p}}})(1-\overline{n_{\mathbf{p}'}}) - \overline{n_{\mathbf{p}}}\overline{n_{\mathbf{p}'}}(1-\overline{n_{\mathbf{k}}})(1-\overline{n_{\mathbf{k}'}})]. \tag{5}$$

By summing the source term $L^{(1)}_{\mathbf{pp}'}$, as given by Eq. (5), over $\mathbf{p}'$, we obtain, with the minus sign, the ordinary electron-electron collision term entering the Boltzmann equation for the degenerate gas.

The source term $L^{(1)}_{\mathbf{pp}'}$ is the only one surviving in the nondegenerate situation:

$$L^{(1)}_{\mathbf{pp}'} \to -I^{\text{ee}}_{\mathbf{pp}'}(n,n). \tag{6}$$

The main property of the source terms $L^{(i)}_{\mathbf{pp}'}(i=1,2,3)$ is that they *vanish at thermal equilibrium* both for the Boltzmann statistics (see Refs. 1 and 15–18) and the Fermi-Dirac statistics (see Refs. 38 and 39). In thermal equilibrium, the additional (kinetic) correlation cannot exist in principle since it would violate the *fluctuation-dissipation theorem* valid in equilibrium independently of the type of the statistics and of the interaction in the system.

It is easy to check directly that each of the sources $L^{(i)}_{\mathbf{pp}'}$ at equilibrium vanishes. For example, one can see that $L^{(1)}_{\mathbf{pp}'}$ given by Eq. (5) after insertion of the Fermi-Dirac functions reduces to zero. This is so because the probabilities W contain the δ functions ensuring the fulfillment of the energy-conservation law in the collision event.

The source term $L^{(2)}_{\mathbf{pp}'}$ is expressible through the term $L^{(1)}_{\mathbf{pp}'}$ in the following way:[38,39]

$$L^{(2)}_{\mathbf{pp}'} = -(n_{\mathbf{p}} + n_{\mathbf{p}'})L^{(1)}_{\mathbf{pp}'}. \tag{7}$$

We are not presenting here the explicit expression for the source term $L^{(3)}_{\mathbf{pp}'}$ being, in its turn, a sum of two terms (see Refs. 38 and 39). The common feature of these terms, as well as of the term $L^{(2)}_{\mathbf{pp}'}$, is that they *vanish in the nondegenerate case*. Indeed, the terms $L^{(2)}_{\mathbf{pp}'}$ and $L^{(3)}_{\mathbf{pp}'}$ are at least of the third order in occupation numbers, while the term $L^{(1)}_{\mathbf{pp}'}$, given by Eq. (5), is of the second order.

Generalization to spatially nonuniform fluctuations is straightforward,[38,39] though the calculations will become remarkably more complicated because of *screening effects*. For that reason, we restrict ourselves here to the investigation of the temporal behavior of spatially uniform fluctuations (fluctuations averaged over the sample's volume) in *macroscopic spatially uniform hot-electron* systems. The equations for space-independent fluctuations in a spatially uniform macroscopic sample cannot explicitly contain the terms describing the screening effects. These can enter the theory only through collision probabilities, the explicit form of which is not too important to us.

III. ELECTRON-TEMPERATURE APPROXIMATION FOR DEGENERATE STATISTICS

We investigate a degenerate two- or three-dimensional spatially uniform electron gas rather weakly interacting with the thermal bath (lattice). For sufficiently high electron densities, frequent *e-e* collisions control the *shape* of the electron *energy* distribution, making it *rather close* to the Fermi-Dirac distribution even at nonequilibrium conditions:

$$\overline{n}(\varepsilon_{\mathbf{p}}) = n_{\text{T}}(\varepsilon_{\mathbf{p}}) + \Delta\overline{n}(\varepsilon_{\mathbf{p}}), \tag{8}$$

where

$$n_{\text{T}}(\varepsilon_{\mathbf{p}}) = \left[\exp\frac{\varepsilon_{\mathbf{p}} - \mu}{k_{\text{B}}T} + 1\right]^{-1}, \tag{9}$$

$$|\Delta\overline{n}(\varepsilon_{\mathbf{p}})| \ll n_{\text{T}}(\varepsilon_{\mathbf{p}}). \tag{10}$$

In Eqs. (8)–(10), $\varepsilon_{\mathbf{p}}$ is the electron energy, $\mathbf{p}$ is the electron quasimomentum, and k_{B} is the Boltzmann constant. The situation is referred to as *electron-temperature approximation*, and the parameter T being referred to as *electron temperature*. The electron-temperature approximation to be applicable, the characteristic time of redistribution of energy (and of quasimomentum) inside the electron system, τ_{ee}, should be small as compared with the electron-temperature relaxation time τ_{T} (the time of relaxation of the energy of the electron system due to its interaction with the thermal bath): $\tau_{\text{ee}} \ll \tau_{\text{T}}$. To be able to proceed analytically, we suppose that *the electron quasimomentum relaxation* due to interaction with the thermal bath is fast enough, faster than the interelectron relaxation:

$$\tau_{\mathbf{p}} \ll \tau_{\mathrm{ee}} \ll \tau_{\mathrm{T}}. \tag{11}$$

The situation where the inequality (11) holds will be referred to as *case of electron temperature*. Of course, the situation can be realized only provided $\tau_{\mathbf{p}} \ll \tau_{\mathrm{T}}$, that is, when electron system's quasimomentum relaxation rate due to interaction with the lattice exceeds quite remarkably the corresponding rate of energy relaxation. Then the antisymmetric part of the electron distribution is small enough:

$$|\overline{n^{-}}(\mathbf{p})| \ll n_{\mathrm{T}}(\varepsilon_{\mathbf{p}}). \tag{12}$$

In the stationary current-carrying state, the electron temperature is determined by the balance of gain and loss of energy by the electron system,

$$E^2\tilde{\sigma}(T,T_0) = P(T,T_0), \tag{13}$$

where $\tilde{\sigma}=j/E$ is the "static" chord conductivity and P is the rate at which the electron system loses energy through interaction with the thermal bath (lattice) supposed to remain at the temperature T_0. For simplicity we consider an isotropic medium (for 2DEG, in the absence of the external electric field all directions in the interface are supposed to be equivalent; the dc electric field $\mathbf{E}$ is applied in the 2DEG plane).

The inverse time of relaxation of small deviations of the electron temperature from the stationary value determined by Eq. (13) is easily found to be given by the expression

$$\tau_{\mathrm{T}}^{-1} = \frac{1}{C_{\mathrm{e}}}\left(\frac{dP}{dT} - E^2\frac{d\tilde{\sigma}}{dT}\right), \tag{14}$$

where

$$C_{\mathrm{e}} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}}\frac{d\overline{n}(\varepsilon_{\mathbf{p}})}{dT} \tag{15}$$

is the *specific heat* of the (degenerate) electron gas.

Naturally enough, the electron-temperature concept can work only for frequencies

$$\omega \ll 1/\tau_{\mathrm{ee}}. \tag{16}$$

In what follows, this condition is supposed to be fulfilled.

For such frequencies, the ac small-signal conductivities are given by expressions formally coinciding with those known from the nondegenerate case: the *transverse*, with respect to the stationary current, ac small-signal conductivity equals the "static" chord one

$$\sigma_{\perp}(\omega) = \tilde{\sigma}, \tag{17}$$

while the real part of the *longitudinal* ac small-signal conductivity is easily shown to be given by the expression

$$\sigma'_{\parallel}(\omega) \equiv \mathrm{Re}\,\sigma_{\parallel}(\omega) = \tilde{\sigma}\left[1 + \frac{2\tau_{\mathrm{T}}E^2 d\tilde{\sigma}/dT}{C_{\mathrm{e}}(1+\omega^2\tau_{\mathrm{T}}^2)}\right]. \tag{18}$$

Let us emphasize that, in the electron-temperature case limited by the inequalities (11) and (16), expressions (17) and (18) are valid *independently* of the possible degeneracy of the electron system.

IV. FLUCTUATIONS IN ELECTRON-TEMPERATURE APPROXIMATION IN A NONDEGENERATE ELECTRON GAS

The expressions for spectral intensities of current fluctuations and noise temperatures of the nondegenerate 3D electron gas in the *electron-temperature approximation*, Eq. (11), were derived from the above-mentioned kinetic equations for correlation functions of occupancies of one-electron states by Shulman.[5] We present here the simple and quite natural expression for the spectral intensity of *transverse* fluctuations of current density:

$$(\delta j_{\perp}^2)_{\omega} = \frac{2k_{\mathrm{B}}T\tilde{\sigma}}{V_0}. \tag{19}$$

Here V_0 is the two- or three-dimensional volume.

To obtain a comparatively simple explicit expression for the spectral intensity of *longitudinal* fluctuations of current density for nondegenerate electron gas[5] (see also Refs. 15 and 10)

$$(\delta j_{\parallel}^2)_{\omega}^{\mathrm{B}} = \frac{2k_{\mathrm{B}}T\tilde{\sigma}}{V_0}\left\{1 + \frac{\dfrac{\sigma_{\parallel}}{\tilde{\sigma}} - 1}{1+\omega^2\tau_{\mathrm{T}}^2}\left[1 + \frac{\left(\dfrac{\sigma_{\parallel}}{\tilde{\sigma}} - 1\right)}{4\left(1 - \dfrac{T_0}{T}\right)}\right]\right\} \tag{20}$$

a weak inelasticity of scattering of electrons by the thermal bath was assumed side by side with the inequalities (11) and (16):

$$|\Delta\varepsilon| \ll \varepsilon \tag{21}$$

($\Delta\varepsilon$ is the characteristic change of the electron energy ε upon a collision). The longitudinal differential conductivity of the electron gas,

$$\sigma_{\parallel} \equiv \frac{dj}{dE} = \sigma'_{\parallel}(\omega)_{\omega\tau_{\mathrm{T}} \ll 1}, \tag{22}$$

made its appearance in Eq. (20). We use the terms static and differential, having in mind measurement of the current-voltage characteristics at frequencies small as compared to $1/\tau_{\mathrm{T}}$ but large as compared to the frequencies of the generation-recombination and other low-frequency processes.

V. FLUCTUATIONS IN A DEGENERATE ELECTRON GAS IN A TRANSVERSE DIRECTION

As mentioned above, the kinetic equations for correlation functions of occupancies of one-electron states in a *degenerate* electron gas were derived from the first principles in Ref. 38 (see also Ref. 39). We claim that we succeeded in deriving, from these equations, the expressions for spectral intensities of current fluctuations and noise temperatures of the degenerate 3D or 2D electron gas in the electron-temperature approximation defined by the inequalities (11), (16), and (21). (The derivation in full detail will be presented elsewhere.) The expression (19) for the spectral intensity of transverse fluctuations of current density proves to remain valid also for the degenerate electron gas. It follows from the definition of the noise temperature and from Eqs. (17) and (19) that the noise temperature measured in the transverse direction,

$$T_{\perp}^{(n)}(\omega) = \frac{(\delta j_{\perp}^2)_{\omega} V_0}{2k_B \mathrm{Re}\ \sigma_{\perp}(\omega)}, \tag{23}$$

equals the electron temperature:

$$T_{\perp}^{(n)}(\omega) = T. \tag{24}$$

We find that, in the sufficiently wide region of frequencies defined by Eq. (16), the transverse noise temperature is frequency independent (white noise). Measurement of the transverse noise temperature is a direct way to measure the electron temperature.

VI. LONGITUDINAL FLUCTUATIONS IN A DEGENERATE ELECTRON GAS

Contrary to the transverse fluctuations, no comparatively simple relation, of the type of Eq. (20), can be obtained for longitudinal fluctuations of a current in a degenerate nonequilibrium state of the electron gas, provided that the heating of the latter with regard to the thermal bath (lattice) is more or less arbitrary. The reason is a described-above additional correlation among the occupancies of the one-electron states, created by interelectron collisions, revealing itself in a nonequilibrium state[16–18] (see also Refs. 15, 1, and 9).

However, for a *nondegenerate* gas a special and very advantageous simplification takes place in the electron-temperature case, when the inequalities (11), (16), and (21) hold. Namely, the contribution of the additional correlation to the intensity of current fluctuation spectra is expressible in terms of the kinetic (transport) characteristics ("one-particle characteristics") of the electron system.[5] This is a specific feature of the electron-temperature approximation applied to a nondegenerate system, enabling, e.g., to express the spectral intensity of longitudinal current fluctuations through the electron temperature and differential conductivities [see Eq. (20)].

Contrary to this, we claim that for a degenerate electron gas in the electron-temperature approximation only the more complicated expression for the spectral intensity of longitudinal fluctuations of current density can be derived on the basis of the equations of fluctuational kinetics. The formula obtained by us sounds

$$(\delta j_{\parallel}^2)_{\omega} = \frac{2k_B T\tilde{\sigma}}{V_0}\left\{1 + \frac{\dfrac{\sigma_{\parallel}}{\tilde{\sigma}} - 1}{1+\omega^2\tau_T^2}\left[1 + \frac{\dfrac{\sigma_{\parallel}}{\tilde{\sigma}} - 1}{4\tilde{\sigma}E^2}\left(\frac{C_e T}{\tau_T} + \frac{\Lambda_{ee}}{2k_B T}\right)\right]\right\}. \tag{25}$$

Here Λ_{ee} is the "source" of *correlation of energies due to interelectron collisions*:

$$\Lambda_{ee} = \sum_{\mathbf{pp}'} \varepsilon_{\mathbf{p}}\varepsilon_{\mathbf{p}'} L_{\mathbf{pp}'}, \tag{26}$$

where $L_{\mathbf{pp}'}$ are source terms describing creation, due to interelectron collisions, of correlation among occupancies of the one-electron states in a degenerate system (Sec. II). Naturally enough, in the electron-temperature case these sources enter the observable noise characteristics only through the energy correlation source Λ_{ee} given by Eq. (26). In other words, in the framework of the electron-temperature approach, additional correlation can reveal itself only in macroscopic samples, *vanishing in mesoscopics*. In particular, the use by Kozub and Rudin,[12] while calculating the shot noise, of the expression (3) with term responsible for additional correlation omitted *de facto* proves to be justified.

Each of the source terms $L_{\mathbf{pp}'}$ in the case of degenerate statistics consist of three summands [Eq. (4)]. Accordingly, the energy correlation source Λ_{ee} also is a sum of three items:

$$\Lambda_{ee} = \Lambda_{ee}^{(1)} + \Lambda_{ee}^{(2)} + \Lambda_{ee}^{(3)}, \tag{27}$$

where only the term

$$\Lambda_{ee}^{(1)} = \sum_{\mathbf{pp}'} \varepsilon_{\mathbf{p}}\varepsilon_{\mathbf{p}'} L_{\mathbf{pp}'}^{(1)}, \tag{28}$$

survives in the limit of the Boltzmann statistics.

Generally speaking, in a degenerate case in nonequilibrium all the terms in Eq. (4), as well as in Eq. (27), should be taken into account (though they are not necessarily of equal significance). Away of equilibrium, the energy distribution differs from the Fermi-Dirac function. The deviation, though small in the electron-temperature case [see Eq. (10)], is rather effective in creation of additional correlation thanks to a high frequency of interelectron collisions [$\tau_{ee} \ll \tau_T$, see Eq. (11)]. The relative effectiveness of creation of additional correlation in the electron-temperature case for nondegenerate 3DEG was shown in Ref. 5, for nondegenerate 2DEG—in Ref. 9. In a case of a weak inelasticity of the scattering of electrons by the thermal bath [Eq. (21)], the term $\Lambda_{ee}^{(1)}$ *was expressed in terms of the one-particle kinetic characteristics*:

$$\Lambda_{ee}^{(1)} = 2k_B T^2\left[\frac{E^2\tilde{\sigma}}{T - T_0} - \frac{C_e}{\tau_T}\right]. \tag{29}$$

In a nondegenerate case, as was explained above,

$$\Lambda_{ee,B}^{(2)} = \Lambda_{ee,B}^{(3)} = 0, \tag{30}$$

and

$$\Lambda_{ee,B} = \Lambda_{ee,B}^{(1)}, \tag{31}$$

while in a degenerate case the term Λ_{ee} does not reduce to $\Lambda_{ee}^{(1)}$ since the contribution of the remaining terms on right-hand side of Eq. (27) can prove to be essential. The structure of the terms $L_{\mathbf{pp}'}^{(2)}$ and $L_{\mathbf{pp}'}^{(3)}$ is too complicated to leave a hope for expressing $(\Lambda_{ee}^{(2)} + \Lambda_{ee}^{(3)})$ in terms of the macroscopic observables—one-particle kinetic coefficients. In contrast to a nondegenerate system, the noise and transport characteristics of the degenerate system are not directly interrelated even in the otherwise rather simple electron-temperature case. Intrinsic *two-particle nature of fluctuation phenomena in the nonequilibrium system with interparticle collisions is clearly pronounced in the degenerate case*.

In principle, one can separate the contribution of the additional correlation, i.e., extract the values of the correlation source Λ_{ee}, by inserting the results of the noise and transport measurements into Eq. (25). Moreover, the difference between the values of the source Λ_{ee} thus obtained and the values of $\Lambda_{ee}^{(1)}$ as given by Eq. (29) could help to estimate the actual role of degeneracy in formation of the additional correlation. Indeed, provided that the applicability of the electron-temperature approximation is checked independently (this may be a separate task), the difference between Λ_{ee} and $\Lambda_{ee}^{(1)}$, if obtained from the experimental data, could be a direct evidence of degeneracy of the electron system and an indirect measure of the degree of the degeneracy.

VII. NOISE IN A "WARM" DEGENERATE ELECTRON GAS

The situation is simplified drastically in the important case of not-too-strong heating of electrons, when the electron temperature T and the temperature of the thermal bath T_0 differ not too remarkably:

$$\Delta T \equiv T - T_0 \ll T_0. \tag{32}$$

As was mentioned in the preceding section, the correlation source terms $L_{\mathbf{pp}'}^{(i)}$ vanish at equilibrium, so that the quantity Λ_{ee} given by Eq. (26) also vanishes. As a result, the first nonvanishing term in the expansion of Λ_{ee} in a Taylor series in the strength of the applied electric field E is of the second order in the field:

$$\Lambda_{ee} \propto E^2 \text{ for small } E. \tag{33}$$

In the same region of fields in the electron-temperature case the excess temperature ΔT defined by Eq. (32) is proportional to E^2 (the so-called "warm-electron" region). It follows that

$$\Lambda_{ee} \propto \Delta T \tag{34}$$

as far as

$$\Delta T/T_0 \ll 1. \tag{35}$$

A distinctive feature of the electron-temperature case is that the coefficient, standing in front of the additional correlation source Λ_{ee} in the expression (25) for the spectral intensity of current fluctuations, in the warm-electron region *is also proportional to* E^2. Indeed, at *equilibrium in the case of nearly elastic scattering of electrons by the lattice* the fluctuations of the electron temperature and fluctuations of the current (of the drift velocity, or of the total quasimomentum of the electron system) *are not coupled*. The coupling arises only in the nonequilibrium, at small deviations from equilibrium being proportional to the difference of the electron and lattice temperatures, ΔT. As a result, the contribution of the additional correlation into electric noise appears only as a term of the order of $\Delta T^2 \propto (E^2)^2$. This important physical consequence of the basic inequalities (11) and (21) ensuring the validity of the electron-temperature approximation has been noticed earlier[7,15] for the nondegenerate electron gas. Now we see that this feature survives also in degenerate gas, enabling to neglect the additional correlation effect on the current fluctuations provided that we confine ourselves to the first nonvanishing nonequilibrium-correction terms in the expressions obtained for the spectral intensity of the current fluctuations, Eqs. (25) and (19).

By expanding Eqs. (25) and (19) in a Taylor series in $\Delta T/T_0$ and retaining only two leading terms, we have

$$(\delta j_\parallel^2)_\omega = \frac{2k_B T_0 \sigma_0}{V_0}\left\{1 + \frac{\Delta T}{T_0}\left[1 + \eta_0\left(1 + \frac{2+\eta_0}{1+\omega^2\tau_{T0}^2}\right)\right]\right\}, \tag{36}$$

$$(\delta j_\perp^2)_\omega = \frac{2k_B T_0 \sigma_0}{V_0}\left[1 + \frac{\Delta T}{T_0}(1+\eta_0)\right]. \tag{37}$$

Here $\sigma_0 \equiv \tilde{\sigma}_{T=T_0}$ is a zero-field conductivity, and η_0 is the zero-field value of the electric sensitivity with respect to heating of the electron system:

$$\eta_0 \equiv \left(\frac{d\ln\tilde{\sigma}}{d\ln T}\right)_{T=T_0}. \tag{38}$$

The zero-field value of the electron-temperature relaxation time τ_T defined by Eq. (14) is denoted as τ_{T0}.

It follows from Eqs. (17) and (18) that the corresponding expressions for longitudinal and transverse ac small-signal conductivities are

$$\sigma_\parallel'(\omega) = \sigma_0\left\{1 + \frac{\Delta T}{T_0}\eta_0\left[1 + \frac{2}{1+\omega^2\tau_{T0}^2}\right]\right\}, \tag{39}$$

$$\sigma_\perp(\omega) = \tilde{\sigma} = \sigma_0\left[1 + \frac{\Delta T}{T_0}\eta_0\right]. \tag{40}$$

Let us remind that the validity of the inequalities (11) and (16) is indispensable for validity of Eqs. (36), (37), (39), and (40).

Now we are in a position to calculate, from the latter expressions, the *longitudinal and transverse noise temperatures of the degenerate warm electron gas* in the microwave region, directly measurable experimentally. For longitudinal temperature we obtain

$$T_\parallel^{(n)}(\omega) = \frac{(\delta j_\parallel^2)_\omega V_0}{2k_B\sigma_\parallel'(\omega)} = T_0\left\{1 + \frac{\Delta T}{T_0}\left[1 + \frac{\eta_0^2}{1+\omega^2\tau_{T0}^2}\right]\right\}, \tag{41}$$

for transverse one,

$$T_\perp^{(n)}(\omega) = \frac{(\delta j_\perp^2)_\omega V_0}{2k_B\sigma_\perp(\omega)} = \frac{(\delta j_\perp^2)_\omega V_0}{2k_B\tilde{\sigma}} = T = T_0\left[1 + \frac{\Delta T}{T_0}\right] \tag{42}$$

[see Eqs. (17) and (19)].

An important property of the electric noise in a current-carrying state is its anisotropic character. The anisotropy of the noise temperature is characterized by the ratio $T_\parallel^{(n)}(\omega)/T_\perp^{(n)}(\omega)$. It follows from Eqs. (41) and (42) that

$$\frac{T_{\parallel}^{(n)}(\omega)}{T_{\perp}^{(n)}(\omega)} = 1 + \frac{\Delta T}{T_0}\frac{\eta_0^2}{1+\omega^2\tau_{T0}^2}, \tag{43}$$

that is, the longitudinal noise temperature of the degenerate warm electron gas *exceeds* the transverse one.

We see that, in the electron-temperature approximation at low frequencies $\omega\tau_{T0} \ll 1$, the anisotropy of the noise temperatures in the warm-electron region is determined solely by the coefficient of the electric sensitivity of the system with respect to the heating of electrons, defined by Eq. (38). The circumstance that the coefficient η_0 *squared* enters the ratio of longitudinal and transverse noise temperatures of the degenerate warm electron gas is also worthy to be emphasized. Indeed, in many important cases the sensitivity coefficient η_0 is rather small (GaAs, wide-gap semiconductors, and their two-dimensional structures). It follows that in such cases in the warm-electron region in the electron-temperature approximation the anisotropy of the noise temperature is anomalously small, being proportional to η_0^2 independently of the degree of degeneracy. This is in contrast to the anisotropy of the spectral intensity of current fluctuations, as well as the anisotropy of the ac small-signal conductivity: the ratio $(\delta j_{\parallel}^2)_{\omega}/(\delta j_{\perp}^2)_{\omega}$, as well as $\sigma_{\parallel}'(\omega)/\sigma_{\perp}(\omega)$, contains the terms proportional to η_0. For the Boltzmann statistics, these peculiarities of the noise in a weakly heated electron gas have been noticed in literature earlier (see Ref. 1, p. 154, and Refs. 15 and 7).

It is worthy to pay attention to the following circumstance. One can see from the structure of the expressions (36), (37), and (39)–(41) that these expressions de facto are valid as far as the inequality

$$\eta_0 \Delta T/T \ll 1, \tag{44}$$

rather than the inequality (35), is fulfilled: the expansion of the kinetic coefficients and fluctuation characteristics in fact takes place in terms of the parameter $\Delta T(\eta_0/T)$.

The sensitivity coefficient η_0, as mentioned above, in many actual cases is small ($\eta_0 \ll 1$). In those cases, the parabolic field dependence of the conductivities [Eqs. (39) and (40)] and fluctuation characteristics [Eqs. (36), (37), and (41)] occurs in the essentially wider field region than it could seem from the first glance.

VIII. DISCUSSION

We have demonstrated that the degeneracy effects violate the macroscopic noise-response relationship valid, in the electron-temperature case, for the nondegenerate nonequilibrium electron gas. However, the violation takes place only outside the warm-electron region. One can say that, in the "macroscopic" sense, the degeneracy *reveals itself* only at moderately high applied electric fields. Outside the warm-electron region (of course, provided that the electron system still remains degenerate) the existence of macroscopic relations between the "one-electron" properties and "two-electron" (noise) phenomena is scarcely probable. Calculation of the noise characteristics becomes extremely complicated, because the direct calculation of effects on the macroscopic noise of the contribution of the additional correlation terms $\Lambda_{ee}^{(2)}$ and $\Lambda_{ee}^{(3)}$ proves to be indispensable. To calculate these effects, in contrast to the effect of the additional correlation term $\Lambda_{ee}^{(1)}$, one should know the non-Fermi–Dirac corrections arising in the energy distribution in the nonequilibrium. The correlation sources linearized with respect to these corrections in general are not small. This can be illustrated by the expression for the term $\Lambda_{ee}^{(1)}$ as given by Eq. (29), obtained avoiding the explicit knowledge of the corrections in question. The contribution of the terms $L_{\mathbf{pp}'}^{(2)}$ and $L_{\mathbf{pp}'}^{(3)}$ does not seem to be essentially smaller than that of the terms $L_{\mathbf{pp}'}^{(1)}$, but this contribution cannot be calculated on the same lines as the contribution of the terms $L_{\mathbf{pp}'}^{(1)}$. As a result, the analytic calculation of the total effect of the additional correlation at higher fields seems to be impossible. In such a situation, Monte Carlo analysis (cf. Ref. 41) would be very instructive, as well as the above-mentioned possible extraction of the contributions of $\Lambda_{ee}^{(1)}$ and $(\Lambda_{ee}^{(2)}+\Lambda_{ee}^{(3)})$ from the experiment.

In many cases, e.g., in GaN and GaN-based structures, the warm-electron region is wide enough (nonlinearity of the current-voltage characteristics is weak) for the noise-response relation valid in that region to be useful. We have in mind the situations when the warm-electron region stretches up to the fields where new noise sources intervene, such as strongly inelastic scattering, intervalley noise, and inter-subband and real-space transfer noise (in 2DEG), demanding some other approaches. In such cases the electron-temperature fluctuations are most interesting just in the warm-electron region. Another note is that the electron system, when degenerate at zero-field (equilibrium) conditions, being heated by the applied electric field tends to become nondegenerate. This also means that the region of not-too-high electric fields is practically important while thinking about the noise in nonequilibrium degenerate electron systems.

Our main positive result is that, for warm electrons, the macroscopic relations established in the electron-temperature approximation for nondegenerate statistics in fact are valid independently of the degree of degeneracy as far as electron-temperature approximation works. The degree of degeneracy cannot be found simply comparing the noise and transport properties of the weakly heated electron system. On the other hand, this means that experimentalists when using the noise-response relations should not worry too much about the possible degeneracy of the electron system in the sample under investigation, at least in the warm-electron region. If the macroscopic noise and transport characteristics are measured, the validity of the relations in question can be checked in practice. In other words, one will be able to verify, to some extent, the validity of the approximations leading to those relations. Remarkable deviations would definitely mean that either the electron-temperature approximation does not work or, if it is independently known that it should work and if deviations appear only outside the warm-electron region, that the electron system is degenerate.

IX. CONCLUSIONS

The analysis of fluctuation phenomena in moderately dense degenerate nonequilibrium gas is performed. Analytic expressions for spectral intensities of spatially uniform current fluctuations in two- and three-dimensional electron gas (2DEG and 3DEG) in a degenerate macroscopic spatially uniform current-carrying sample are obtained in so-called electron-temperature approximation. The latter approximation works when electron scattering by lattice is nearly elastic and interelectron collisions shape the electron energy distribution. In contrast to earlier investigated nondegenerate nonequilibrium case, where the macroscopic noise characteristics (spectral intensities of current density fluctuations, noise temperatures of the electron gas) were proved to be expressible in terms of transport coefficients and electron temperatures, in a degenerate electron gas such a type of macroscopic relationship is shown to exist only for so-called "warm" electrons [Eqs. (36), (37), and (39)–(41)], that is, provided that terms of fourth and higher powers in an applied electric field strength are neglected. A relative loss of generality of macroscopic relations in a degenerate case is caused by a more complicated than in a nondegenerate case nature of the additional correlation created by interelectron collisions in a nonequilibrium state.

However, the obtained macroscopic noise-response relationship valid for degenerate weakly heated electron system can prove to be just what is needed for practical purposes. The results are equally applicable to three- and two-dimensional electron gas. Having in mind the actual needs of modern microelectronics and nanoelectronics, this circumstance can prove to be essential.

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