Spin relaxation in a two-dimensional electron gas in a perpendicular magnetic field

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We consider the problem of spin relaxation in a two-dimensional electron gas (2DEG) in a perpendicular magnetic field. We assume that the spin relaxation is induced by the Rashba spin-orbit (SO) interaction, which appears due to the inversion asymmetry of the confining potential. Our solution is based on a microscopic evaluation of the spin-density response function of the 2DEG with impurities and SO interaction. We derive explicit expressions for the transverse and longitudinal spin-relaxation rates. Our analysis shows, in particular, that the spin-relaxation rates exhibit *magnetoquantum oscillations*, which are analogous to the Shubnikov–de Haas oscillations of the electrical resistivity. These oscillations can be observed, for example, in time-resolved optical experiments.

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I. INTRODUCTION

The study of spin relaxation in semiconductor two-dimensional electron gases (2DEG's) is an important area of the emerging field of *spintronics*.[1] The dominant spin-relaxation mechanism in these systems can typically be associated[2] with the Rashba spin-onbit (SO) interaction,[3,4] which exists due to the inversion asymmetry of the confining potential in semiconductor heterostructure based 2DEG systems. A simple semiclassical picture of the spin relaxation, due to Dyakonov and Perel (DP),[2,5] is that since the Rashba interaction has the form of a momentum-dependent magnetic field $\lambda\hat{z}\cdot[\boldsymbol{\tau}\times\mathbf{p}]$, it induces precession of the spin of a moving electron. This precession leads to the randomization of the spin at long times. The DP spin relaxation rate is given, to leading order in λ, by the simple expression $1/\tau_{sf}\sim(\lambda p_F/\hbar)^2\tau$, where p_F is the Fermi momentum and τ is the elastic scattering time. Quasiclassically, $\lambda p_F/\hbar$ is simply the spin-precession frequency, associated with the Rashba field. The distinguishing feature of the DP spin relaxation is that the relaxation rate is smaller in more disordered systems, since impurity scattering disrupts the Rashba spin precession by randomizing the electron's momentum.

It is well known that transport processes in 2DEG systems are strongly influenced by the application of a perpendicular magnetic field. The perpendicular field quantizes electron's energy spectrum, which manifests spectacularly in Shubnikov-de Haas (SdH) oscillations of the resistivity and eventually leads to the quantum Hall effect. In this paper we address the question of how a perpendicular magnetic field influences the Rashba SO interaction-induced spin relaxation in semiconductor 2DEG systems. This problem has been addressed before by several authors, both in the semiclassical[6–9] and in the quantum limits,[10–15] but a complete and rigorous analysis is still lacking.

In this paper we will provide such an analysis. We will mainly be interested in the regime of moderate magnetic fields, such that $\omega_c\tau\lesssim 1$ (here ω_c is the cyclotron frequency), and $\epsilon_F\gg\hbar\omega_c$. Our analysis can, however, be generalized to the quantum Hall limit, when $\epsilon_F\sim\hbar\omega_c$ and $\omega_c\tau$ is large. Our theory is based on a microscopic evaluation of the spin density response function of a disordered 2DEG system with the Rashba SO interaction in a perpendicular magnetic field. We obtain explicit analytical expressions for the longitudinal and transverse spin relaxation rates:

$$\frac{1}{\tau_z}=\frac{8\lambda^2 m\epsilon_F\tau/\hbar^2}{1+(\omega_c\tau)^2},\quad \frac{1}{\tau_\perp}=\frac{1}{2\tau_z},$$

which reduce to the well-known Dyakonov-Kachorovskii[2] expressions in the zero-field limit. We show, in particular, that the application of a perpendicular magnetic field leads to *quantum oscillations* of the spin relaxation rate, which are analogous to the well-known quantum oscillations of transport coefficients. These oscillations arise from the magnetic field dependence of the scattering time τ, which will be calculated below.

The paper is organized as follows. In Sec. II we calculate the disorder-averaged Green's function of a 2DEG system with the Rashba SO interaction in a perpendicular magnetic field using the self-consistent Born approximation (SCBA). In Sec. III the spin-density response function of our system is calculated by summing SCBA self-energy and ladder vertex corrections to the bare polarization diagram. The transverse and longitudinal spin relaxation rates are then extracted from the poles of this response function.

II. DENSITY OF STATES AND DISORDER SELF-ENERGY

The simplest single-particle Hamiltonian describing the dynamics of electrons in a semiconductor 2DEG in a perpendicular magnetic field can be written as

$$H_0=\frac{\boldsymbol{\pi}^2}{2m}+\lambda\hat{z}\cdot[\boldsymbol{\tau}\times\boldsymbol{\pi}]-\frac{\Delta_z}{2}\tau^z, \tag{1}$$

where $\boldsymbol{\pi}=\mathbf{p}+(e/c)\mathbf{A}$ is the kinetic momentum, $\boldsymbol{\tau}$ is the spin operator, and $\Delta_z=g\mu_B B$ is the Zeeman energy. The second term in Eq. (1) describes the Rashba SO interaction. Hamiltonian H_0 can be easily diagonalized[3,16–19] if one notices that the Rashba term mixes only neighboring Landau levels with opposite spin directions. The eigenstates are thus given by

$$|n,a\rangle = u_{na}|n,\downarrow\rangle + v_{na}|n-1,\uparrow\rangle, \quad (2)$$

where $a=1,2$ and $|n,\sigma\rangle$ is the nth Landau level eigenstate with spin $\sigma=\uparrow,\downarrow$. The corresponding eigenenergies are

$$\epsilon_{na} = \hbar\omega_c n + (-1)^a \sqrt{\left(\frac{\hbar\omega_c+\Delta_z}{2}\right)^2 + 2\lambda^2 m\hbar\omega_c n}, \quad (3)$$

where $\omega_c = eB/mc$. Note that $n=1,2,\ldots$ for $a=1$, but $n=0,1,\ldots$ for $a=2$. The amplitudes u_{na} and v_{na} are given by

$$u_{na} = i\sqrt{\frac{1}{2} + \frac{(-1)^a(\hbar\omega_c+\Delta_z)/2}{\sqrt{(\hbar\omega_c+\Delta_z)^2/4 + 2\lambda^2 m\hbar\omega_c n}}},$$

$$v_{na} = (-1)^{a+1}\sqrt{\frac{1}{2} + \frac{(-1)^{a+1}(\hbar\omega_c+\Delta_z)/2}{\sqrt{(\hbar\omega_c+\Delta_z)^2/4 + 2\lambda^2 m\hbar\omega_c n}}}. \quad (4)$$

Using the above basis of single-particle eigenstates, the Hamiltonian of the 2DEG system in the presence of impurity scattering potential can be written as

$$H = \sum_{nak} \epsilon_{na} c^\dagger_{nak} c_{nak} + \sum_{nak,n'a'k'} V_{na,n'a'}(k,k') c^\dagger_{nak} c_{n'a'k'}, \quad (5)$$

where index k denotes orbit-center quantum numbers in the Landau gauge and

$$V_{na,n'a'}(k,k') = \sum_\sigma \int d\mathbf{r}\Psi^*_{nak}(\mathbf{r}\sigma)V(\mathbf{r})\Psi_{n'a'k'}(\mathbf{r}\sigma), \quad (6)$$

are the matrix elements of the impurity potential $V(\mathbf{r})$ in the basis (2). The wave functions $\Psi_{nak}(\mathbf{r}\sigma)$ are given by

$$\Psi_{nak}(\mathbf{r}\uparrow) = v_{na}\phi_{n-1,k}(\mathbf{r}),$$

$$\Psi_{nak}(\mathbf{r}\downarrow) = u_{na}\phi_{n,k}(\mathbf{r}), \quad (7)$$

where $\phi_{n,k}(\mathbf{r})$ are the Landau-level eigenfunctions in the Landau gauge.

Our analysis of the spin relaxation in a system, described by Eq. (5), is based on a calculation of the spin density response function, which is similar to Ando's calculation of the conductivity of a 2DEG in a perpendicular field[20] (see Ref. 19 for a related recent work on magnetotransport properties of Rashba 2DEG systems). Our calculation is technically more complicated than Ando's due to the presence of the SO interactions and also due to the fact that vertex corrections to the polarization bubble, which vanish in the case of the conductivity calculation, are crucial in our case.

Following Ando, we use the SCBA to find the disorder-averaged Green's function. This is a very good approximation for the high Landau-level filling factors that we are assuming. The SCBA equation for the retarded disorder self-energy in our case reads

$$\Sigma^R_{na}(\epsilon) = \sum_{n'a'} \Gamma_{na,n'a'} G^R_{n'a'}(\epsilon), \quad (8)$$

where

$$\Gamma_{na,n'a'} = \sum_{k'} \langle V_{na,n'a'}(k,k') V_{n'a',na}(k',k)\rangle, \quad (9)$$

and

$$G^R_{na}(\epsilon) = \frac{1}{\epsilon - \epsilon_{na} - \Sigma^R_{na}(\epsilon)}, \quad (10)$$

is the retarded SCBA Green's function. The angular brackets in Eq. (9) denote disorder average.

Matrix elements of Γ can be easily evaluated for δ-function impurity potential and are given by

$$\Gamma_{na,n'a'} = \frac{\hbar^2\omega_c}{2\pi\tau_0}(|u_{na}|^2|u_{n'a'}|^2 + |v_{na}|^2|v_{n'a'}|^2), \quad (11)$$

where τ_0 is the elastic impurity scattering time in the absence of the magnetic field.

Equation (8) is too complicated to be solved analytically without approximations. Fortunately, only very minor simplification is needed to make this equation solvable. The main complication is the dependence of the matrix Γ on the Landau-level index n. This dependence is, however, inessential, since it appears entirely due to the term $2\lambda^2 m\hbar\omega_c n$ in Eq. (4). The dependence on n in this term can be safely ignored, since the Rashba SO interaction can be assumed to be a weak perturbation, in the sense that $\lambda p_F \ll \epsilon_F$. In this case we can make a replacement $n \to \epsilon_F/\hbar\omega_c$ in the Rashba term in Eqs. (3) and (4), which makes the amplitudes u_{na} and v_{na} and, consequently, Γ, independent of n.[19] This simple approximation allows us to solve Eq. (8) analytically.

It is convenient to introduce the following new notation. Let

$$\Delta = 2\sqrt{\left(\frac{\hbar\omega_c+\Delta_z}{2}\right)^2 + 2\lambda^2 m\epsilon_F}. \quad (12)$$

Then $\epsilon_{na} = \hbar\omega_c n + (-1)^a\Delta/2$. Also, define

$$u_1 = i\cos(\vartheta), \quad v_1 = \sin(\vartheta),$$

$$u_2 = i\sin(\vartheta), \quad v_2 = -\cos(\vartheta), \quad (13)$$

where

$$\cos(\vartheta) = \sqrt{\frac{1}{2} - \frac{\hbar\omega_c+\Delta_z}{2\Delta}}. \quad (14)$$

Using this notation, the matrix elements of Γ are given by

$$\Gamma_{11} = \Gamma_{22} = \frac{\hbar^2\omega_c}{2\pi\tau_0}\left[1 - \tfrac{1}{2}\sin^2(2\vartheta)\right],$$

$$\Gamma_{12} = \Gamma_{21} = \frac{\hbar^2\omega_c}{\pi\tau_0}\sin^2(2\vartheta). \quad (15)$$

Let us also make the usual assumption that the real part of the disorder self-energy can be absorbed into the chemical potential. Then SCBA equation can be written as

$$\mathrm{Im}\,\Sigma_a^R(\epsilon) = \sum_{n,a'} \Gamma_{aa'} \mathrm{Im}\, G_{na'}^R(\epsilon) = -2\pi^2\ell^2 \sum_{a'} \Gamma_{aa'} \varrho_{a'}(\epsilon), \tag{16}$$

where

$$\varrho_a(\epsilon) = -\frac{1}{2\pi^2\ell^2} \sum_n \mathrm{Im}\, G_{na}^R(\epsilon), \tag{17}$$

is the density of states and $\ell = \sqrt{\hbar c/eB}$ is the magnetic length. Writing out Eq. (17) explicitly, we have

$$\varrho_a(\epsilon) = -\frac{1}{2\pi^2\ell^2} \times \sum_{n=0}^{\infty} \frac{\mathrm{Im}\Sigma_a^R(\epsilon)}{[\epsilon - \hbar\omega_c n - (-1)^a\Delta - \hbar\omega_c\delta_{a,1}]^2 + [\mathrm{Im}\Sigma_a^R(\epsilon)]^2}. \tag{18}$$

The sum over Landau-level indices in the above equation can be done using the Poisson summation formula:[21]

$$\sum_{n=0}^{\infty} f(n) = \frac{f(0)}{2} + \sum_{p=-\infty}^{\infty} \int_0^{\infty} dn\, f(n) e^{2\pi i p n}. \tag{19}$$

Up to this point, our calculations were applicable to magnetic fields of general strength. To proceed further, we will restrict ourselves to the regime when $\epsilon_F\tau/\hbar \gg 1$ and $\epsilon_F \gg \hbar\omega_c$. (The calculation we present can be done in other regimes as well, but the actual procedure will be a little different.) This allows us to neglect the term $f(0)/2$ in Eq. (19). Then, changing the integration variable to $x = \hbar\omega_c n - \epsilon + (-1)^a\Delta + \hbar\omega_c\delta_{a,1}$ and extending the lower limit of integration to $-\infty$, which is justified by the above assumptions, we obtain

$$\varrho_a(\epsilon) = -\frac{1}{2\pi^2\ell^2} \sum_{p=-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{dx}{\hbar\omega_c} \times \exp\left(2\pi i p \frac{x + \epsilon - (-1)^a\Delta - \hbar\omega_c\delta_{a,1}}{\hbar\omega_c}\right) \frac{\mathrm{Im}\,\Sigma_a^R(\epsilon)}{x^2 + [\Sigma_a^R(\epsilon)]^2}. \tag{20}$$

Calculating the above integral, we obtain

$$\varrho_a(\epsilon) = \frac{m}{2\pi\hbar^2}\left[1 + 2\sum_{p=1}^{\infty} e^{-\pi p/\omega_c\tau_0} \times \cos\left(2\pi p \frac{\epsilon - (-1)^a\Delta - \hbar\omega_c\delta_{a,1}}{\hbar\omega_c}\right)\right], \tag{21}$$

where we have replaced $\mathrm{Im}\,\Sigma_a^R(\epsilon)$, which appears in the exponent after integration, by its zero-field value $-\hbar/2\tau_0$. The oscillatory term in the above expression is slightly different from the usual expression, which is periodic in $\hbar\omega_c$, due to the presence of the Rashba and Zeeman splittings. In what follows we will ignore this modification of the density of states, setting $\lambda = 0$ and $\Delta_z = 0$ in the oscillatory term. This does not mean, of course, that we are ignoring SO interactions altogether, since they still enter in the matrix elements

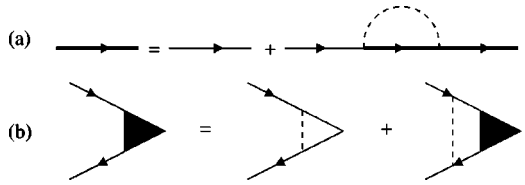

FIG. 1. (a) SCBA equation for the disorder-averaged Green's function. (b) Equation for the vertex function D. Dashed lines denote impurity potential correlator $\langle V(\mathbf{r})V(\mathbf{r}')\rangle$.

of Γ. Then the density of states becomes independent of the index a and is given by the following simple expression:

$$\varrho(\epsilon) = \varrho_0\left[1 + 2\sum_{p=1}^{\infty} e^{-\pi p/\omega_c\tau_0}\cos\left(\frac{2\pi p\epsilon}{\hbar\omega_c} - \pi p\right)\right], \tag{22}$$

where $\varrho_1(\epsilon) = \varrho_2(\epsilon) = \varrho(\epsilon)/2$ and $\varrho_0 = m/\hbar^2\pi$ is the zero-field density of states. If the magnetic field is not too large, i.e., if $\omega_c\tau \lesssim 1$, only the first term in the sum over p in Eq. (22) needs to be retained. Then the oscillatory term in the density of states is purely sinusoidal. Substituting Eq. (22) in Eq. (16) we find that SCBA self-energy is also independent of the index a and is given by

$$\mathrm{Im}\,\Sigma^R(\epsilon) \equiv -\frac{\hbar}{2\tau(\epsilon)} = -\frac{\pi\hbar^3}{2m\tau_0}\varrho(\epsilon). \tag{23}$$

The above disorder self-energy reduces to the usual expression $-\hbar/2\tau_0$ in the zero-field limit. At finite fields, the oscillatory term in the density of states in Eq. (23) is at the origin of a number of magnetoquantum oscillation phenomena, including the SdH oscillations of the electrical resistivity. As will be shown later, it also leads to similar magnetoquantum oscillation effects in the spin relaxation rate.

III. SPIN DENSITY RESPONSE FUNCTION

We can now calculate spin-density response function of a disordered 2DEG system with the Rashba SO interactions in a perpendicular magnetic field. Transverse and longitudinal spin relaxation rates can be found from the poles of the corresponding spin-density response functions. The calculation is similar to the analogous calculation that can be done at zero field.[22] We will use $\hbar = 1$ units in the rest of this section, except for the final results.

We start from the general density matrix response function:

$$\chi_{\sigma_1\sigma_2,\sigma_3\sigma_4}(\mathbf{r} - \mathbf{r}', t - t') = -i\theta(t - t')\langle[\hat{\varrho}^{\dagger}_{\sigma_1\sigma_2}(\mathbf{r}, t), \hat{\varrho}_{\sigma_3\sigma_4}(\mathbf{r}', t')]\rangle, \tag{24}$$

where $\hat{\varrho}_{\sigma_1\sigma_2}(\mathbf{r}, t) = \Psi^{\dagger}_{\sigma_2}(\mathbf{r}, t)\Psi_{\sigma_1}(\mathbf{r}, t)$ is the generalized density operator, whose expectation value is the density matrix. As usual, the calculation is most conveniently done by analytical continuation of the imaginary time response function.[23] Fourier transformed imaginary time response function can be written as

$$\chi_{\sigma_1\sigma_2,\sigma_3\sigma_4}(\mathbf{q},i\Omega)=\frac{1}{\beta}\sum_{i\omega}P_{\sigma_1\sigma_2,\sigma_3\sigma_4}(\mathbf{q},i\omega,i\omega+i\Omega), \quad (25)$$

where

$$P_{\sigma_1\sigma_2,\sigma_3\sigma_4}(\mathbf{r}-\mathbf{r}',i\omega,i\omega+i\Omega) = \langle \mathcal{G}_{\sigma_3\sigma_1}(\mathbf{r}',\mathbf{r},i\omega)\mathcal{G}_{\sigma_2\sigma_4}(\mathbf{r},\mathbf{r}',i\omega+i\Omega)\rangle \quad (26)$$

is the polarization bubble diagram and $\mathcal{G}$ are imaginary time Green's functions. P can be calculated by summing all the SCBA diagrams for the disorder self-energy and all the ladder vertex corrections to the polarization bubble, which constitutes a conserving approximation for the density matrix response function. The result can be written in matrix notation as $P=P^0D$, where P^0 is the "bare" polarization bubble with only the self-energy corrections included:

$$P^0_{\sigma_1\sigma_2,\sigma_3\sigma_4}(\mathbf{r}-\mathbf{r}',i\omega,i\omega+i\Omega) = \mathcal{G}_{\sigma_3\sigma_1}(\mathbf{r}',\mathbf{r},i\omega)\mathcal{G}_{\sigma_2\sigma_4}(\mathbf{r},\mathbf{r}',i\omega+i\Omega), \quad (27)$$

with $\mathcal{G}$ here being the disorder-averaged Green's function. The vertex part, or, as it is often called, diffusion propagator, D satisfies the following matrix equation:

$$D=\mathbf{1}+\gamma P^0 D, \quad (28)$$

where $\gamma\equiv\langle V^2(\mathbf{r})\rangle$. The solution of this equation is

$$D=[\mathbf{1}-\gamma P^0]^{-1}. \quad (29)$$

We can now calculate the retarded real time response function by analytically continuing Eq. (25) to real frequencies. The result can be written as an integral along the branch cut of the SCBA Green's function:

$$\chi(\mathbf{q},\Omega)=\int_{-\infty}^{\infty}\frac{d\epsilon}{2\pi i}n_F(\epsilon)[P(\mathbf{q},\epsilon+i\eta,\epsilon+\Omega+i\eta) - P(\mathbf{q},\epsilon-i\eta,\epsilon+\Omega+i\eta)+P(\mathbf{q},\epsilon-\Omega-i\eta,\epsilon+i\eta) - P(\mathbf{q},\epsilon-\Omega-i\eta,\epsilon-i\eta)]. \quad (30)$$

At low frequencies and low temperatures Eq. (30) can be simplified to

$$\chi(\mathbf{q},\Omega)=\frac{i\Omega}{2\pi}P(\mathbf{q},\epsilon_F-i\eta,\epsilon_F+\Omega+i\eta)+\frac{\varrho(\epsilon_F)}{2}. \quad (31)$$

Thus, the problem reduces to calculating the matrix $P^0(\mathbf{q},\epsilon_F-i\eta,\epsilon_F+\Omega+i\eta)$. We will in fact be interested only in the relaxation of uniform spin polarization and thus will set $\mathbf{q}=\mathbf{0}$ henceforth. The calculation of P^0 in our case closely resembles the analogous calculation in the zero-field problem.[22]

We start from the definition of P^0 Eq. (27), using the disorder-averaged Green's functions calculated in Sec. II:

$$\begin{aligned}P^0_{\sigma_1\sigma_2,\sigma_3\sigma_4}(\epsilon_F-i\eta,\epsilon_F+\Omega+i\eta) &= \frac{1}{L^2}\int d\mathbf{r}d\mathbf{r}' G^A_{\sigma_3\sigma_1}(\mathbf{r}',\mathbf{r},\epsilon_F)G^R_{\sigma_2\sigma_4}(\mathbf{r},\mathbf{r}',\epsilon_F+\Omega)\\ &= \frac{1}{L^2}\int d\mathbf{r}d\mathbf{r}' \sum_{nak,n'a'k'}\frac{\Psi_{nak}(\mathbf{r}',\sigma_3)\Psi^*_{nak}(\mathbf{r},\sigma_1)}{\epsilon_F-\epsilon_{na}-i/2\tau} \times\frac{\Psi_{n'a'k'}(\mathbf{r},\sigma_2)\Psi^*_{n'a'k'}(\mathbf{r}',\sigma_4)}{\epsilon_F+\Omega-\epsilon_{n'a'}+i/2\tau}, \end{aligned} \quad (32)$$

where $\tau\equiv\tau(\epsilon_F)$ and the wave functions are given by Eq. (7). The spatial integrals in the above equation can be easily evaluated using the orthonormality of the Landau gauge eigenfuctions and are given by

$$\begin{aligned}&\int d\mathbf{r}\Psi^*_{nak}(\mathbf{r},\uparrow)\Psi_{n'a'k'}(\mathbf{r},\uparrow)=v^*_a v_{a'}\delta_{n,n'}\delta_{k,k'},\\ &\int d\mathbf{r}\Psi^*_{nak}(\mathbf{r},\downarrow)\Psi_{n'a'k'}(\mathbf{r},\downarrow)=u^*_a u_{a'}\delta_{n,n'}\delta_{k,k'},\\ &\int d\mathbf{r}\Psi^*_{nak}(\mathbf{r},\uparrow)\Psi_{n'a'k'}(\mathbf{r},\downarrow)=v^*_a u_{a'}\delta_{n,n'+1}\delta_{k,k'},\\ &\int d\mathbf{r}\Psi^*_{nak}(\mathbf{r},\downarrow)\Psi_{n'a'k'}(\mathbf{r},\uparrow)=u^*_a v_{a'}\delta_{n,n'-1}\delta_{k,k'}. \end{aligned} \quad (33)$$

The sums over Landau-level indices in Eq. (32) can be done as follows. We first decouple the product of two Green's functions appearing in (32) as

$$\begin{aligned}&\frac{1}{(\epsilon_F-\epsilon_{na}-i/2\tau)(\epsilon_F+\Omega-\epsilon_{n'a'}+i/2\tau)}\\ &=\frac{1}{\Omega+(-1)^a\Delta-(-1)^{a'}\Delta+i/\tau}\left(\frac{1}{\epsilon_F-\epsilon_{na}-i/2\tau}-\frac{1}{\epsilon_F+\Omega-\epsilon_{n'a'}+i/2\tau}\right). \end{aligned} \quad (34)$$

The real parts of the decoupled sums over Landau-level indices can then be neglected by our assumption that $\epsilon_F \gg \hbar\omega_c,\lambda p_F,\hbar\Omega$. The imaginary parts are simply proportional to the density of states at Fermi energy $\varrho(\epsilon_F)$. Thus we obtain the following expressions for the matrix elements of P^0:

$$\gamma P^0_{\uparrow\uparrow,\uparrow\uparrow}=\gamma P^0_{\downarrow\downarrow,\downarrow\downarrow}=\left(1-\frac{1}{2}\sin^2(2\vartheta)\right)f_0(\Omega)+\frac{\sin^2(2\vartheta)}{4}[f_+(\Omega)+f_-(\Omega)],$$

$$\gamma P^0_{\uparrow\uparrow,\downarrow\downarrow}=\gamma P^0_{\downarrow\downarrow,\uparrow\uparrow}=\frac{\sin^2(2\vartheta)}{4}[2f_0(\Omega)-f_+(\Omega)-f_-(\Omega)],$$

$$\gamma P^0_{\uparrow\downarrow,\uparrow\downarrow}=\frac{\sin^2(2\vartheta)}{2}f_0(\Omega+\omega_c)+\sin^4(\vartheta)f_+(\Omega+\omega_c)+\cos^4(\vartheta)f_-(\Omega+\omega_c),$$

$$\gamma P^0_{\downarrow\uparrow,\downarrow\uparrow} = \frac{\sin^2(2\vartheta)}{2} f_0(\Omega - \omega_c) + \sin^4(\vartheta) f_-(\Omega - \omega_c) + \cos^4(\vartheta) f_+(\Omega - \omega_c), \tag{35}$$

where functions f_0 and $f_\pm$ are given by

$$f_0(\Omega) = \frac{1}{1 - i\Omega\tau},$$

$$f_\pm(\Omega) = \frac{1}{1 - i\Omega\tau \pm i\Delta\tau}. \tag{36}$$

All other matrix elements of P^0 vanish.

To calculate the transverse and longitudinal spin relaxation rates we need to find the poles of the spin-density response function, or, equivalently, zeroth of the inverse diffusion propagator D^{-1}. It is most easily done by transforming D^{-1} to the physical space of charge and spin-density components, which is accomplished by multiplying it on both sides with Pauli matrices:

$$D^{-1}_{\alpha\beta} = \frac{1}{2} \tau^\alpha_{\sigma_1\sigma_2} D^{-1}_{\sigma_1\sigma_2,\sigma_3\sigma_4} \tau^\beta_{\sigma_4\sigma_3}, \tag{37}$$

where $\alpha,\beta = c,x,y,z$ and τ^c is the identity matrix. Thus we obtain

$$D^{-1}_{cc}(\Omega) = 1 - f_0(\Omega),$$

$$D^{-1}_{zz}(\Omega) = 1 - f_0(\Omega) + \frac{\sin^2(2\vartheta)}{2}[2f_0(\Omega) - f_+(\Omega) - f_-(\Omega)],$$

$$D^{-1}_{+-}(\Omega) = 1 - \frac{\sin^2(2\vartheta)}{2} f_0(\Omega + \omega_c) - \sin^4(\vartheta) f_+(\Omega + \omega_c) - \cos^4(\vartheta) f_-(\Omega + \omega_c). \tag{38}$$

Clearly $D^{-1}_{cc}(0) = 0$, which is a consequence of charge conservation. For the spin part of the inverse diffusion propagator, to leading order in $\lambda p_F \tau/\hbar$ and $\Omega\tau$, and assuming that $\Delta_z \ll \hbar\omega_c$, we obtain

$$\tau^{-1} D^{-1}_{zz}(\Omega) = -i\Omega + \frac{1}{\tau_z},$$

$$\tau^{-1} D^{-1}_{+-}(\Omega) = -i\Omega + i\hbar^{-1}\Delta_z + \frac{1}{\tau_\perp}, \tag{39}$$

where the longitudinal and transverse spin relaxation rates are given by

$$\frac{1}{\tau_z} = \frac{8\lambda^2 m \epsilon_F \tau/\hbar^2}{1 + (\omega_c\tau)^2}, \quad \frac{1}{\tau_\perp} = \frac{1}{2\tau_z}. \tag{40}$$

Equation (40) is our main result. The form of these expressions is rather similar to the well-known Dyakonov-Kachorovskii expressions[2] for the spin-relaxation rates in the zero-field case, except for the factor $1/[1+(\omega_c\tau)^2]$, which describes the suppression of spin relaxation by the perpendicular field. The mechanism of this suppression can be understood in quasiclassical terms by imagining electrons moving along classical cyclotron orbits and undergoing impurity scattering and the Rashba spin precession. Orbital motion affects the spin precession, since the direction of the Rashba field is always transverse to the direction of electron's velocity, and thus changes as the electron moves along a circular cyclotron orbit. However, if $\omega_c\tau \ll 1$, impurity scattering randomizes the direction of electron's motion and the influence of the magnetic field on the orbital motion is then negligible. On the other hand, when $\omega_c\tau \gg 1$, electrons can move around cyclotron orbits almost freely and thus the Rashba spin precession is averaged out. This picture of the suppression of the DP spin relaxation by a perpendicular magnetic field has been discussed before by several authors.[6–8] However, we are not aware of a rigorous derivation of a simple analytical expression for the spin-relaxation rate, such as Eq. (40).

Probably the most noteworthy feature of our result is the oscillatory dependence on the magnetic field, which appears due to the oscillatory term in the scattering rate $1/\tau$ in Eqs. (22) and (23). If $\omega_c\tau < 1$, we may neglect the $(\omega_c\tau)^2$ term in the denominator in Eq. (40) and leave only the first term in the sum over p in Eq. (22). In this case spin relaxation rates will exhibit purely sinusoidal SdH-like oscillations. Similar effects have been discussed earlier in the context of nuclear spin-lattice relaxation in quantum Hall systems.[24–26] However, the possibility of such magnetoquantum oscillations in the SO-induced spin relaxation rates has never been discussed before. Such oscillations have in fact been observed in recent experiments on spin relaxation in InGaAs quantum wells.[27]

In conclusion, we have considered the problem of spin relaxation in a 2DEG with the Rashba SO interactions in a perpendicular magnetic field. We have given a rigorous microscopic derivation of the transverse and longitudinal spin relaxation rates from the poles of the spin-density response function in the limit $\epsilon_F \gg \hbar\omega_c$. It is possible to extend our calculation to other regimes, including the quantum Hall regime $\epsilon_F \sim \hbar\omega_c$. Our results demonstrate that magnetoquantum oscillation effects should be observed in spin-relaxation phenomena in 2DEGs. One of the advantages of our approach is that it can be easily extended to study another very interesting and still unexplored question of what is the effect of Coulomb interactions on the spin relaxation in semiconductor 2DEGs. This question will be addressed in a future publication.

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