Temperature dependence of the conductivity of an electronic crystal

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We study the temperature dependence of the conductivity of the two-dimensional (2D) electronic solid. In realistic samples, a domain structure forms in the solid and each domain randomly orients in the absence of the in-plane field. At a higher temperature, the electron transport is governed by thermal activation form of $\sigma_{xx}(T) \propto e^{-\Delta_0/k_BT}$. The impurities will localize the electron states along the edges of the crystal domains. At a sufficiently low temperature, another transport mechanism, called Mott's variable range hopping mechanism—similar to that in a disorder insulator—takes effect. We show that as the temperature decreases, a crossover from the fixed range hopping of the transport to the variable range hopping of transport in the 2D electron system may be experimentally observed.

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It was initially predicted by Wigner that two-dimensional (2D) electrons crystallize into a triangular lattice in the low-density limit where the electron-electron interactions dominate over the kinetic energy. In an ideally clean 2D system, the critical r_s was presented to be 37 ± 5 from quantum Monte Carlo simulations.[1] A strong magnetic field perpendicular to the 2D plane can effectively localize electron wave functions while keeping the kinetic energy controlled.[2] Since this lessens the otherwise severe low-density condition, it is believed that the Wigner crystal (WC) can be stabilized in a sufficiently strong magnetic field.[3–6] Approximate calculations[7] have shown that the WC becomes the lowest-energy state when the filling factor $\nu<1/6$ for a GaAs/AlGaAs electron system and around $\nu=1/3$ for the hole system.

As is well known, while the transport behavior of the WC is characterized by nonlinear current–voltage (I–V) curves, the temperature dependence of the conductivity of a WC is believed to be normal, namely, it has an ordinary thermal activation form.[8,9] Moreover, since the impurities pin the electron crystal, a domain structure forms in realistic samples.[10,11] While the electrons in a domain have an order as they are in the ideal crystal, the orientations of the domains are random. Another role played by the impurities is that they localize the electron states along the edges of the crystal domains. The electrons may hop between the edges of the domains. In this case, the transport behavior of the system is similar to that in a disordered insulator. In this work, we will show that the transport of the electrons in the 2D electronic solid obeys a generalized Mott's variable range hopping theory[12] for a low temperature $T\ll T_0$, while it obeys the ordinary thermal activation law for $T\geqslant T_0$. We find that T_0 is in the experimentally reachable regime if the sample parameters are properly chosen. This implies that one may experimentally observe a different temperature dependence of the conductivity at different temperature regimes. As T varies, the conductivity, according to our calculation, may have a crossover from $\sigma_{xx}(T) \propto e^{-A/T^{1/2}}$ for $T\ll T_0$ to a thermal activation form $\sigma_{xx}(T) \propto e^{-\Delta_0/k_BT}$.

Pinning of the WC by impurities as a result of breaking the translational invariance has been widely investigated.[5,13] In realistic samples, a domain structure is formed due to a finite impurity density. The electrons in each domain are ordered as they are in the crystal. Sherman[10] and Fil[11] had studied the angular pinning and the domain structure of the electronic crystal mediated by acoustic-phonon in III-V semiconductor. In the absence of the in-plane field, each domain orients randomly, just like the domains in ferromagnets. It can be shown that an in-plane field favors the domains to orient to the same direction.[14] Hence, the in-plane magnetic field may serve as a tunable means to probe the orientation of the crystal. An ideal electronic crystal is an insulator and the conductivity $\sigma_{xx} \propto e^{-\Delta_0/k_BT}$. This thermal activation form of the conductivity implies that the electrons are hopping with a fixed range mechanism. It has been confirmed by experiments with Δ_0 typically of the order of 1 K.[15] In a realistic domain structure, however, the localized electrons may hop between the edges of the randomly oriented domains. Since the experimentally reachable temperature is as low as 10 mK, the variable range hopping mechanism[12] may work in this temperature regime. In the following, we will calculate the electron conductivity according to the different electron hopping mechanisms and determine the characteristic temperature of the crossover region T_0.

In the usual Anderson localization, the envelope of the wave function falls off exponentially as $\phi\sim e^{-r/\xi}$, where ξ is the localization length, while in the magnetic field the electronic wave function of a perfect system is essentially a Gaussian as $\phi\sim e^{-r^2/2l_B^2}$, where l_B is the magnetic length. In a slightly disordered system, one can think that some of the states will be pinned at certain isolated impurity site. The mixing of these states due to quantum-mechanical tunneling leads to a simple exponential tail in the wave function.[9] In a strong magnetic field, the electrons condense into a crystal at lower filling factors. The electrons are Coulomb localized. When the temperature is high enough, the transport is of the thermal activation form which implies that the electrons are hopping with a fixed range mechanism.[8,9] The hopping range is determined by $R_0=\sqrt{1/\pi n_I}$, where n_I is the impurity density. However, localized states by impurities may exist along the edges of the domains of electronic crystal. When the

temperature is sufficiently low such that there is nearly no phonon with energy to assist the electron making the nearest hopping, Mott's variable hopping mechanism[12] allows the electrons to hop a larger distance $R > R_0$ to a state which has a smaller energy difference $\Delta(R)$. In turn, the hopping conduction is determined by the typical decay rate of the tails of the wave function. The hopping probability is then given by

$$p \propto \exp[-R/\xi - \Delta/k_B T], \tag{1}$$

where $R = |\mathbf{R}_i - \mathbf{R}_j|$ and Δ is the activation energy.

For noninteracting electrons, Mott hopping with an approximately constant density of states at the Fermi energy gives

$$\rho_{xx}(T) = \rho_0(T)\exp(r_c/\xi) = \rho_0(T)\exp(A_0/T)^{1/3}, \tag{2}$$

where r_c is a characteristic hopping length, which in this regime is equal to the Mott hopping length. However, this simple treatment does not work for the WC because of the strong Coulomb interaction between electrons.[16] The Coulomb gap depresses the density of states near the Fermi surface.[17–20] Efros *et al.*[21] had derived the density of states near the Fermi surface $N(E) \propto |\Delta E| = |E - E_F|$. The condition to find one state within a circle of radius R is given by

$$\pi R^2 N(E_F)\Delta(R) = 1. \tag{3}$$

Substitute $N(E_F)$ by the available states near the Fermi surface, i.e., $N(E_F) \to \overline{N}(E_F) = 1/\Delta E \int_0^{\Delta E} dE N(E)$, and note that $|\Delta E| \propto 1/R$, we get

$$\Delta(R) \sim \frac{\hbar v_F}{R}, \tag{4}$$

where v_F is the Fermi velocity. Put this R-dependent energy difference into formula (1) and maximizing p, one finds the optimal hopping range $R = \overline{R}$ and the maximum of the probability are given by

$$\overline{R^2} = \frac{\hbar v_F \xi}{k_B T}, \quad p \propto e^{-2\overline{R}/\xi} = e^{-A/T^{1/2}}, \tag{5}$$

with $A = [4\hbar v_F/k_B\xi]^{1/2}$. The conductivity in the variable range hopping is then[17,18]

$$\sigma_{xx} \propto p \propto e^{-A/T^{1/2}}. \tag{6}$$

The characteristic temperature T_0 above which the fixed range hopping dominates is determined by $\overline{R} = R_0$, namely

$$k_B T_0 = \pi n_I \hbar v_F \xi = \pi n_I \cdot \frac{\hbar^2 \xi}{m_b l_B}. \tag{7}$$

In a strong magnetic field, the decay length is comparable to the cyclotron radius $\xi \sim R_c$.[22,23] We find for sample with $n_I \sim 1.0 \times 10^8$ cm^{-2}, $T_0 \sim 40$ mK. This temperature is experimentally reachable. We anticipate that the different dependence of the conductivity in different temperature regimes can be observed in future experiments.

Now, we briefly discuss the effect of the tilted field. Consider an electron moving on a x-y plane under the influence of a strong magnetic field which is tilted an angle θ to the normal, with $\boldsymbol{B} = (B \tan\theta, 0, B)$. The electron is confined in a harmonic potential $V(z) = 1/2 m_b \Omega^2 z^2$ in the z direction, where m_b is the band mass of the electron and Ω the characteristic frequency. Such a quantum well has been chosen to deal with many quantum Hall systems[24–26] to substitute the realistic potential which is either triangular[27] or square.[28,29] We work in the "Landau gauge" by choosing the vector potential $A = \{0, xB_z - zB_x, 0\}$. The single particle wave function for the lowest L is:

$$\phi_X(\boldsymbol{r}) = \frac{1}{\sqrt{L_y}} e^{-iXy l_B^2} \Phi_0^{\omega_+}((x - X)\sin\tilde{\theta} + z\cos\tilde{\theta}) \times \Phi_0^{\omega_-}((x - X)\cos\tilde{\theta} - z\sin\tilde{\theta}), \tag{8}$$

where l_B is the magnetic length and X is an integer multiple of $2\pi l_B^2/L$. $\Phi_0^{\omega_\pm}$ is the harmonic oscillator wave function in the lowest-energy level corresponding to the frequencies $\omega_\pm$ and $\tan\tilde{\theta} = \omega_c^2/\omega_+^2 - \omega_c^2 \tan\theta$, with the cyclotron frequency $\omega_c = eB/m_b c$. The frequencies $\omega_\pm$ are given by[29]

$$\omega_\pm^2 = \frac{1}{2}\left(\Omega^2 + \frac{\omega_c^2}{\cos^2\theta}\right) \pm \sqrt{\frac{1}{4}\left(\Omega^2 - \frac{\omega_c^2}{\cos^2\theta}\right)^2 + \Omega^2\omega_c^2\tan^2\theta}. \tag{9}$$

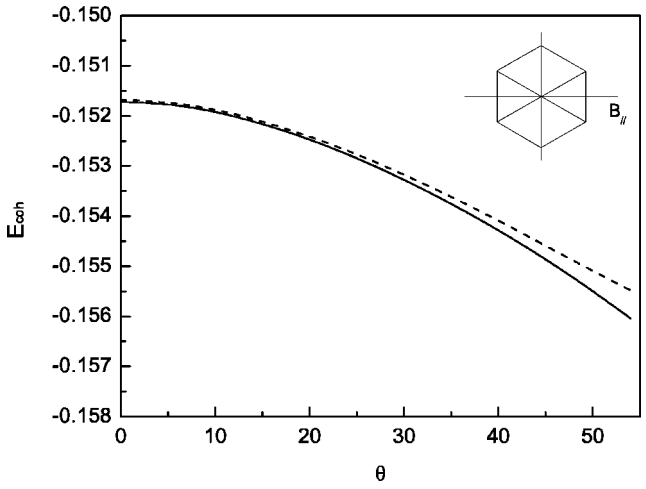

FIG. 1. The cohesive energy of the WC versus the tilting angle for two configurations of the crystal orientation. The real line: [110] parallel to the in-plane field (as shown in the inset); the broken line: [100] parallel to the in-plane field.

Obviously, the existence of an in-plane field deforms the electron wave function. However, this wave function deformation does not qualitatively change the electron hopping mechanism at a given temperature. The major effect of the tilted field would be on the variation of T_0. Figure 1 illustrates the cohesive energy for two typical configurations of the crystal orientation with respective to the in-plane field: The [100] or the [110] direction parallel to the in-plane field. The energy is always lower for the case of the [110] direction parallel to the in-plane field. The applied in-plane field lowers the cohesive energy of the electronic crystal and forces the domains to align in the same direction. Thus, the role of the in-plane field is to integrate the domains into

larger ones. In this way, the in-plane field causes some of impurities to be irrelevant and therefore reduces the effective impurity density. In determining T_0 from Eq. (7), only the relevant impurities should be counted in. Hence, one can replace n_I by an effective impurity density $n_I(B_\parallel)$. From Eq. (7), we see that T_0 is sensitive to $n_I(B_\parallel)$. Therefore, it is possible to observe the influence of the in-plane magnetic field on the characteristic temperature T_0 under proper parameters as the tilting angle varies.

In conclusion, the temperature dependence of conductivity is explored for the Wigner crystal in 2D electrons under a strong magnetic field. We argued that there are domains of electronic crystal in a realistic sample and predicted that the temperature dependence of the transport behavior may be different in different temperature regimes. We found that the conductivity experiences a crossover from the fixed range hopping to the variable range hopping mechanism as the temperature varied. The crossover temperature T_0 is reachable under present experimental technique. Finally, the possible effect on the crossover region by the in-plane magnetic field is discussed.

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