Theoretical investigation of a protected quantum bit in a small Josephson junction array with tetrahedral symmetry

R. A. Usmanov and L. B. Ioffe
Department of Physics and Astronomy, Rutgers University, Piscataway, New Jersey 08854, USA and Landau Institute for Theoretical Physics, 117940 Moscow, Russia
(Received 15 December 2003; published 21 June 2004)

We consider a Josephson junction device which has a symmetry of a tetrahedron; it can be visualized as a tetrahedron that contains two Josephson junctions at each edge. We find the conditions for which the ground state of the system is degenerate or almost degenerate; in this case, the low-energy degrees of freedom can be mapped on a quantum spin $1/2$. We evaluate the effect of the physical perturbations and imperfections on the level splitting in this system and find that they are small for most perturbations. We argue that this system can provide a possible physical implementation of a protected quantum bit with a built-in error correction. We propose a way of manipulating the effective quantum spin by means of electrical potentials and an experimental scheme to read the information that it contains.

DOI: 10.1103/PhysRevB.69.214513 PACS number(s): 74.81.Fa, 85.25.−j

I. INTRODUCTION

A successful implementation of a quantum bit, a building block of a putative quantum computer should meet the conflicting requirements of extremely low phase decoherence and scalabilility. Quantitatively, to avoid a huge overhead imposed by error-correction schemes one should achieve a decoherence rate of about 10^{-8}–10^{-10} per unit time (time of one operation).[1] Many qubit implementations were proposed but none is fully satisfactory. Basically, there are two approaches: we can either start from one of the atomic size systems which has naturally low decoherence rates and try to build a larger system from it, or we can start from a mesoscopic system (which is much easier to scale) and try to protect it from the environment. For instance, the atomic levels of the trapped atoms (controlled by laser pulses),[2,3] photons in the cavities,[4] and nuclear spin controlled by NMR techniques,[5] have very low decoherence rates but are very difficult to combine to form a large system. Examples of the alternative approach are solid-state qubits, i.e., quantum dots,[6] and Josephson junction devices.[7–14] They are much easier to scale but difficult to protect from the noise. Recently, however, it was shown that even a simple Josephson junction device ("Cooper box") can be operated in such a regime that its decoherence rate reaches 10^{-4} in natural units.[14] This important result shows that Josephson junction devices are very promising candidates for scalable solid-state qubits. However, a relatively simple device design employed in Ref. 14 suffers from a serious problem, namely, the spontaneous phonon emission, that makes its improvement highly unlikely. This problem appears in all qubits based on devices where the two lowest quantum levels are separated by a large gap but can be resolved by the designs where the two quantum levels are degenerate or almost degenerate.[15]

Thus, the natural avenue of research is to study somewhat more complicated Josephson junction devices (consisting of 6–20 Josephson junctions) that are better decoupled from the environment, rather than a simple (two junctions) device and have a degenerate or almost degenerate ground state, and to use these two low-energy states as a logical bit. This device should satisfy the following requirements: its ground state should be degenerate or almost degenerate and separated from other states by a significant gap, ΔE. This gap sets the scale for the time of the operations because an attempt to change the state of the system faster than by $1/\Delta E$ would excite higher levels. Further, the physical noises (magnetic flux, rf noise, etc.) should have little effect on the level splitting in this system.

The basic idea of some of the recent suggestions for solid-state qubits is to use a small but highly symmetric Josephson junction.[16,17] The essential observation is that a structure with a non-Abelian symmetry group naturally has degenerate states that correspond to higher dimensional representations of the group. The simplest system with a non-Abelian symmetry group is the tetrahedral qubit similar to the one shown in Fig. 1. The paper[16] has considered the simplest structure of this type, which contained six junctions, one for each edge. It was found that the ground state of this system can be indeed made degenerate by an appropriate tuning of the parameters, and that this degeneracy is very insensitive to the noise in the electric potential and magnetic flux: the coupling in the linear order is absent. The effect of the noise in the Josephson junction energy affects it linearly and thus this

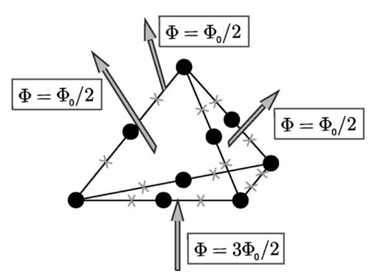

FIG. 1. The superconducting tetrahedron. Each edge consists of a superconducting wire with two Josephson junctions. The tetrahedron is placed in a uniform magnetic field so that the fluxes through the vertical faces are $\Phi_0/2$ each, while the flux through the horizontal face is $3\Phi_0/2$.

effect is stronger. The physical reason for this is that classically the state of the six-junction tetrahedron is continuously degenerate and thus a small deviation of the junction strength from its ideal value has a relatively big effect. Here we study a modified version of this array in which each edge contains two junctions. This eliminates the classical degeneracy mentioned above and leads to a quantitatively smaller effect of the noise in the Josephson junction strength, preserving the main attractive features of the array in:[16] absence of linear coupling to flux and to rf noise.

In Sec. II we describe the physical system, identify the relevant degrees of freedom, and derive the energy level structure in the quasiclassical limit. In Sec. III we study the effects of physical perturbations: the effect of magnetic noise, the potentials and the Josephson junction strength, and the thermal effects. In Sec. IV we discuss the manipulations and reading schemes. Section V is the conclusion.

II. THE SYSTEM

The symmetries of the system become transparent when it is viewed as a tetrahedron made of superconducting wires with two Josephson junctions at each edge as shown in Fig. 1. Further, this tetrahedron is placed in a magnetic field, so that the magnetic flux through each lateral face equals $\Phi_0/2$ and the magnetic flux through the base equals $3\Phi_0/2$. Note that fluxes differing by flux quanta are physically indistinguishable, so all faces of the tetrahedron are equivalent.

Each junction is characterized by its Josephson energy, $E_J=(\hbar/2e)I_c$, and by its charging energy, $E_C=(e^2/2C)$, while the system as a whole is characterized by the capacitance matrix of the superconducting wires. In principle, the capacitance matrix also contains the contributions from the self-capacitances of individual islands, but in a typical physical implementation (see below) these capacitances are much smaller than those of the junctions and for the rest of the paper we will neglect them. The whole system is described by the Lagrangian

$$\mathcal{L}=\sum_{i=1}^{12}\frac{1}{16E_c}\dot{\phi}_i^2+E_J\cos(\phi_i-a_i),$$

where ϕ_i are the phase differences on the Josephson junctions and a_i are chosen to produce the correct magnetic fluxes.

It will be more convenient for the following analysis to consider not the three-dimensional (3D) tetrahedron, but the equivalent projection shown in Fig. 2, which is also, of course, much easier to realize. To preserve the frustration induced by a magnetic field, we need to place the system in a uniform field so that the flux through each small triangle is $\Phi_0/2$.

It is not difficult to see that the symmetry of the system is the permutation group S_4. This group has two-dimensional (2D) representations and thus some of its levels are doubly degenerate. In order to implement the qubit, one of these doublets should correspond to the ground state. In order to find the level structure and establish the parameter range when it is indeed the case, we consider the quasiclassical limit, $E_J \gg E_C$. The numerical diagonalization that we carried over for this and other systems (see, e.g., Ref. 16) shows that the energy structure obtained in this limit survives down to very small E_J/E_C ratios.

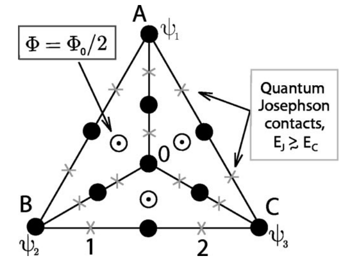

FIG. 2. The equivalent system obtained by tetrahedron projection onto its base. There are two Josephson junctions at each edge. This planar system is placed in the perpendicular magnetic field so that the flux through each of the smaller triangles is half-fiux quanta, $\Phi_0/2$.

In the quasiclassical limit we have to consider the potential relief first. As a function of the gauge invariant phase differences between the four vertices of the big triangle, the potential energy of the system has six classical minima:

$$V_1:\ \left(\psi_1=0,\psi_2=-\frac{\pi}{2},\psi_3=\frac{\pi}{2}\right),$$

$$V_2:\ \left(\psi_1=-\frac{\pi}{2},\psi_2=\frac{\pi}{2},\psi_3=0\right),$$

$$V_3:\ \left(\psi_1=-\frac{\pi}{2},\psi_2=0,\psi_3=-\frac{\pi}{2}\right),$$

$$W_1:\ \left(\psi_1=0,\psi_2=\frac{\pi}{2},\psi_3=-\frac{\pi}{2}\right),$$

$$W_2:\ \left(\psi_1=\frac{\pi}{2},\psi_2=-\frac{\pi}{2},\psi_3=0\right),$$

$$W_3:\ \left(\psi_1=-\frac{\pi}{2},\psi_2=0,\psi_3=\frac{\pi}{2}\right).$$

These minima are mapped one onto another by symmetry transformations.

In the quasiclassical limit $E_J \gg E_C$ one can restrict oneself only to the lowest quantum state in each minima, which yields six degenerate states in the classical limit. Quantum fluctuations lead to the transitions between these states, removing this degeneracy. Thus, in the leading quasiclassical approximation the system is described by the Hamiltonian

$$H=\begin{bmatrix} 0 & a & a & b & c & c \\ a & 0 & a & c & b & c \\ a & a & 0 & c & c & b \\ b & c & c & 0 & a & a \\ c & b & c & a & 0 & a \\ c & c & b & a & a & 0 \end{bmatrix},$$

where a, b, c are tunneling amplitudes. Clearly, the absolute values of a and c should be equal because these amplitudes describe the equivalent transitions corresponding to the rotations of the classical state around different altitudes of the tetrahedron. However, their signs can be opposite due to half-integer magnetic fluxes through each small triangle. As a result, there are two possibilities: $a=c$ and $a=-c$. The amplitude b corresponds to the inversion of all currents.

We diagonalize the Hamiltonian and obtain the energy eigenvalues and the corresponding eigenvectors,

$$E_1=-a+b-c,$$

$$\psi_1=\frac{1}{2\sqrt{3}}(2V_1-V_2-V_3+2W_1-W_2-W_3),$$

$$\psi_2=\frac{1}{2}(V_2-V_3+W_2-W_3);$$

$$E_2=-a-b+c,$$

$$\chi_1=\frac{1}{2\sqrt{3}}(2V_1-V_2-V_3-2W_1+W_2+W_3),$$

$$\chi_2=\frac{1}{2}(V_2-V_3-W_2+W_3);$$

$$E_3=2a+b+2c,$$

$$\eta=\frac{1}{\sqrt{6}}(V_1+V_2+V_3+W_1+W_2+W_3);$$

$$E_4=2a-b-2c,$$

$$\phi=\frac{1}{\sqrt{6}}(V_1+V_2+V_3-W_1-W_2-W_3).$$

We see that if $c=a$ then the ground state is a singlet, the first excited state is a triplet, and the second excited state is a doublet. But if $c=-a$ and $a>0$ then the ground state is a doublet, which is what we need (See Fig. 3). Therefore, we want to determine under which physical conditions $c=-a$ and $a>0$. Of course, these physical conditions should preserve the tetrahedron symmetry.

Since phases and charges are conjugate variables, the amplitude t of the process in which the ith phase changes by $\Delta\phi_i$ is given by $t=-|t|\exp(iq_i\Delta\phi_i)$. Here, q_i is the charge of the ith island (expressed in terms of the Cooper pairs number). Therefore, the sign of an amplitude is defined by the exponent $\exp(i\Sigma q_i\Delta\phi_i)$, that is, by the charges of all islands,

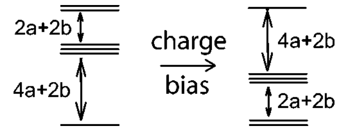

FIG. 3. The energy spectrum of the ideal array.

$$a=-|a|\exp(i\sum q_i\Delta\phi_i),$$

$$b=-|b|\exp(i\sum q_i\Delta\phi_i'),$$

$$c=-|c|\exp(i\sum q_i\Delta\phi_i'').$$

The overall sign $(-)$ is fixed by the condition that in the absence of charge frustration the tunneling decreases the energy of the symmetric wave function, thus in this case, all tunneling amplitudes should be negative. For a to be positive the charges of the four tetrahedron vertices should be equal to $1/2$. The charges of all other islands should be equal to zero. If this is the case, b is positive while c is negative. The ground state is a doublet given by $E_0=-2a-b$:

$$\chi_1=\frac{1}{2\sqrt{3}}(2V_1-V_2-V_3-2W_1+W_2+W_3),$$

$$\chi_2=\frac{1}{2}(V_2-V_3-W_2+W_3), \tag{1}$$

while the first excited level is a triplet and the second excited level is a singlet. The tetrahedron symmetry is valid in this case because the total charge is an integer.

To find the absolute values $|a|$, $|b|$ of the tunneling amplitudes we need to find the saddle trajectories that correspond to these processes; as usual, these trajectories can be viewed as a classical motion in the inverted potential. We solve these equations of motion numerically, use the results to calculate the action of these processes, and obtain

$$|a|\approx 2E_J^{3/4}E_C^{1/4}\exp(-1.58\sqrt{E_J/E_C}),$$

$$|b|\approx 8E_J^{3/4}E_C^{1/4}\exp(-3.08\sqrt{E_J/E_C}), \tag{2}$$

that is, $|a|\gg|b|$ in the quasiclassical limit.

III. EFFECTS OF PERTURBATIONS

The discussion above assumed the tetrahedron to be completely symmetric. We now discuss the effect of physical imperfections that violate the symmetry of the tetrahedron. Generally, a physical perturbation that reduces the symmetry of the tetrahedron to the one of the Abelian group or further splits the degenerate doublets. However, the perturbation that is applied only to one vertex of the tetrahedron reduces the symmetry to the one of the triangle; this symmetry group is non-Abelian (because it includes reflections) and thus such a perturbation does not split the degeneracy. In particular, electrostatic potential applied to one island that is a vertex of the tetrahedron does not affect the degeneracy. For small deviations it means that the effect of the electrostatic potential appears only in the second order of the perturbation theory which contains the products of potentials on different islands.

Consider now the effect of magnetic flux deviations. Because the total magnetic flux through the 3D tetrahedron is zero, the increase of magnetic flux through one face of the tetrahedron should be accompanied by its increase through the other three faces. Such a perturbation reduces the symmetry to the symmetry of the group of triangle rotations, Z_3, which is Abelian. This symmetry by itself would not be sufficient to preserve the doublet splitting. However, in the case of the integer total charge, the Hamiltonian has additional symmetry, namely, the one of time reversal. This symmetry ensures that complex one-dimensional (1D) representations of Z_3 have the same energies. The inspection of the ground states (1) shows that they correspond to the symmetric and antisymmetric combinations of 1D irreducible representations and thus their degeneracy is not affected by the flux through one face. In more physical terms, it means that all matrix elements of the current operator between the ground states are zero and thus magnetic field does not split the degeneracy in the linear order. Generally, we expect that for small deviations $\delta\Phi$ ($\delta\Phi \ll \Phi_0$), the splitting is given by

$$\varepsilon \sim \left(\frac{\delta\Phi}{\Phi_0}\right)^2 E_J,$$

because in the quasiclassical approximation the tunneling amplitudes a and b do not depend on $\delta\Phi$, but the energies of the six-potential energy minima change in different ways. The coefficient in the last formula has to be determined numerically. For example, in the case of an additional flux $\delta\Phi$ through one face and $-\delta\Phi$ through another, we get

$$\varepsilon \approx 40\left(\frac{\delta\Phi}{\Phi_0}\right)^2 E_J.$$

The situation is different with the charge noise: it has no effect on the classical minima but changes significantly the tunneling amplitudes between them. Suppose we have an additional charge δq on the island A (Fig. 2). There are no linear (in charge noise δq) terms in splitting because of the time-reversal symmetry. In the appropriate basis, the perturbation is given by

$$h_1 = \varepsilon \hat{\sigma}_z, \tag{3}$$

where $\epsilon \sim |a| \delta q^2$, with the proportionality coefficient being of the order of unity. The basis of this perturbation can be found using the representation theory. The applied perturbation preserves the symmetry between islands B and C. This means that the eigenstates of the Hamiltonian should be the eigenstates of the transformation $B \to C$; this transformation maps classical minima $V_i \to W_i$. The vectors (χ_1, χ_2) from Eq. (1) are eigenvectors of this transformation, thus they also give the eigenstates of the perturbed Hamiltonian. We conclude that this perturbation has a form (3) in the basis (χ_1, χ_2) from Eq. (1). This can be also verified by the direct calculation of the eigenvectors of the perturbed Hamiltonian. If we put additional charge on another island (B or C in Fig. 2) the perturbation in the basis (χ_1, χ_2) becomes

$$h_{2,3} = \varepsilon\left(-\frac{1}{2}\hat{\sigma}_z \pm \frac{\sqrt{3}}{2}\hat{\sigma}_x\right),$$

where the plus or minus sign corresponds to islands B and C. That is, one can rotate the quantum state in different directions by inducing charges on different islands. This fact can be used to manipulate the system, and indeed, the application of the external potential to islands, shown in Fig. 2, induces the charges which create the effective fields acting on the doublet. Thus, using the appropriate combination of these potentials one can create an arbitrary field in the $(x-z)$ plane of the doublet which is sufficient to rotate it in arbitrary direction. Note that since the linear component of the potential has no effect on the energy, one can use a periodic low frequency potential to rotate the effective spin.

Now, let us consider the situation when all charges and magnetic fluxes are equal to their ideal values but the junction energies are slightly different. In the general case, the splitting depends linearly on the inaccuracy in the junction preparation δE_J. The splitting ε can be calculated numerically. For example, in the case when the Josephson energies of only the junctions 1 and 2 in Fig. 2 are different from E_J, the splitting ε is given by $\varepsilon \approx 0.4 \delta E_J$. It is clear from the above discussion that this linear dependence can be compensated by tuning the magnetic fluxes through the small triangles forming the system.

Finally, we need to take into account thermal effects. At low temperatures $T \ll \Delta_s$ (where Δ_s is the superconducting gap) one can neglect the effect of the quasiparticles. To be more precise, in order to ignore the quasiparticles one needs their number in each wire to be much smaller than one, i.e., $W\nu T \exp(-\Delta_s/T) \ll 1$, where W is the volume of the superconductor, $\nu \sim n_e/\epsilon_F$ is the density of states, and n_e is the electron density. For a typical Al wire of 0.01 $\mu\mathrm{m}^3$ volume this is satisfied for $T \leqslant 0.1$K. Note, however, that if this condition is not satisfied, the thermally excited quasiparticles would lead to the random and fluctuating charge of each wire, which would affect the signs of the transition amplitudes and destroy the quantum coherence of the states (II). In the following we shall therefore assume that this condition is satisfied, and that there are no BCS quasiparticles in the whole system.

In this case the only dangerous modes that can excite the quantum Josephson system are phonons and photons. In a typical setup both phonons and photons are gapless (athough one can eliminate the photons placing the whole system in the resonator), but the interaction with photons is extremely small because a typical energy of the excitation is less than 1 K, which corresponds to the photon wavelength of the order of $\lambda \sim 1$ cm or more. The dipole matrix element for the emission or absorption of the photon contains $(L/\lambda)^4$, where L is the typical linear size of the system and thus is vanishingly small. The situation is different in the case of phonons,[15] here the wavelength, λ_s, corresponding to such

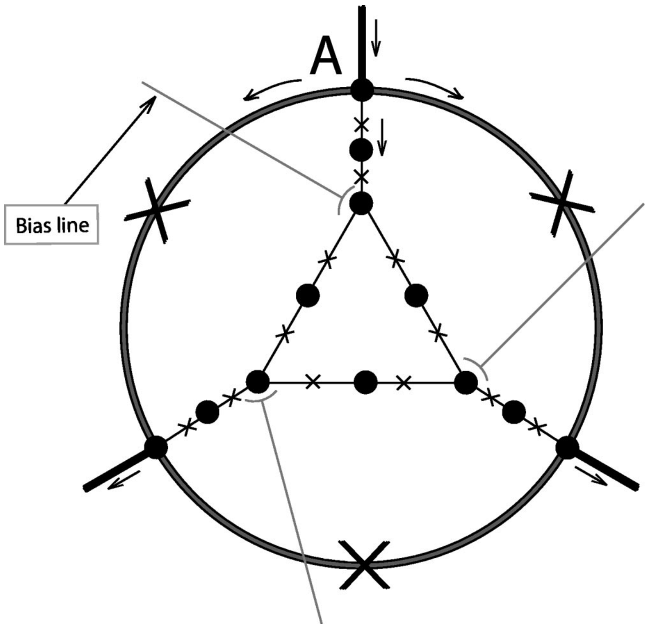

FIG. 4. Schematics of the measurement.

excitations is about the size of the system and so these processes are not rare. Note, however, that the transitions between the two states, which are the result of splitting of the ground state, involve a very small energy transfer, so they are suppressed by a factor similar to the one for the case of photon excitations $(L/\lambda_s)^4$. So, we conclude that the main effect of the temperature is the phonon-mediated excitation of the higher energy levels. The probability of these transitions is determined by the Boltzmann exponent

$$P = \frac{1}{Z}\exp(-\Delta E/T),$$

where Z is the partition function and ΔE is the difference of the initial and the final energies. Taking the energy level structure into account, we find that the transitions from the doublet into the triplet play the most important role. In this case, $\Delta E = 2(a+b) \approx 2a$ (since $|b| \ll |a|$) and the temperature T should be much less than $|a|$ $(T \ll |a|)$.

IV. READING SCHEME

Let us suppose that, after some manipulations, the system is in a quantum state $\alpha|0\rangle + \beta|1\rangle$, where the states $|0\rangle$ and $|1\rangle$ coincide with the states χ_1 and χ_2 from Eq. (1). The coefficients α and β are unknown and we want to deduce them from the results of some experiment. We follow the ideas proposed in Refs. 14 and 18, which used a similar experimental scheme for a two-junction system. The schematics are shown in Fig. 4. The central point of the triangle is fabricated in the form of three islands with each pair of the islands being connected by a very big Josephson junction. If the external currents across these "big" Josephson junctions are zeros, the device shown in Fig. 4 is equivalent to the one in Fig. 2.

The measurement starts with a state in which these external currents are zeros. Then we apply nonzero current to one of the outside wires (e.g., the top wire in Fig. 4). At point A in Fig. 4 the current splits. Most of it flows through the two big junctions but a small part of it flows through the tetrahedron. If the total outside current is rather big, the value and sign of this small current depend on the state of the tetrahedron. A value of the total outside current corresponds to the moment when the big junctions fall into the oscillating regime depends on this small current flowing through the tetrahedron. It is possible to distinguish three different situations: (1) the total current I_1 for the states V_1, W_1, (2) the total current I_2 for the states V_2, V_3, and (3) the total current I_3 for the states W_2, W_3. Therefore, the total current through the outside wire is different for various quantum states. Repeating this experiment many times allows us to determine the absolute values of the coefficients α and β.

We emphasize that the described effect does not contradict the statement of the last section, that the effect of the field deviations appears only in the second order, because here we have a nonlinear effect: when the current through the big junctions is close to the critical one, it induces the additional phase $\pi/2$ on the corresponding tetrahedron edge. This deforms the quasiclassical states of the tetrahedron and induces significant average currents through each edge of the tetrahedron.

To convert the measured probabilities p_1, p_2, p_3 for the critical current to have values I_1, I_2, I_3 into the amplitudes α, β, we can use the following formulas:

$$I_1 \to p_1 = \frac{2}{3}\alpha_2,$$

$$I_2 \to p_2 = 2\left(\frac{\beta}{2} - \frac{\alpha}{2\sqrt{3}}\right)^2,$$

$$I_3 \to p_3 = 2\left(\frac{\beta}{2} + \frac{\alpha}{2\sqrt{3}}\right)^2.$$

This experiment has two potential pitfalls. First, the additional phases induced by an external current on the big Josephson junctions disturb the quantum state of the tetrahedron and might smear the differences between the currents (I_1, I_2, I_3) in different states. We have solved the classical equations for the minima in the presence of these phases and verified that the quantum states evolve smoothly with the application of the external current characterized by similar values $(\tilde{I}_1, \tilde{I}_2, \tilde{I}_3)$. We did not find any evidence for an abrupt transition from one state into another. Second, the current operator $\hat{J}$ generally has nondiagonal matrix elements in the basis of the Hamiltonian eigenstates. If these matrix elements are not small compared to the diagonal elements, the measurement process itself would excite higher states and would show not the slighly modified values of the currents, $(\tilde{I}_1, \tilde{I}_2, \tilde{I}_3)$ corresponding to the states (1–6), but the values of the current corresponding to the transition to higher states. In order to estimate this effect, we calculated numerically the current matrix elements between all low-energy states. We have found that all nondiagonal elements are small in the quasiclassical limit and thus the described measurement should be able to distinguish these states with a probability close to 1.

V. CONCLUSION

We have proposed and studied a small Josephson junction device that has two degenerate quantum ground states well protected from the most common sources of the physical noise. We have shown that electric and magnetic fields split the ground-state degeneracy very weakly because the linear coupling to these fields is completely absent. The only known source of physical noise that remains linearly coupled to the ground-state doublet is the noise in the value of the Josephson coupling itself, but this effect is usually believed to become very small at low temperatures. We have further shown that it is possible to manipulate the quantum states appearing in this system by applying appropriate potentials to the different islands, and that it is possible to read out the final state measuring the critical current through the external classical Josephson junctions. All this makes this system very attractive for the implementation of the qubit with a very long phase coherence time. However, this system does not resolve all problems. Ideally, one would like to find the system in which all three types of noises (electrical, magnetic, and Josephson coupling) are suppressed in the linear order. Further, one would like to be able to precisely perform most manipulations, for example, rotations by π in different directions. Big Josephson junction arrays that satisfy these criteria are known.[17] In these big arrays the couplings to all noises are suppressed exponentially and all operations needed for quantum computations can be performed exactly, but unfortunately, these arrays are too complex for today's technology. The challenging problem is to identify a simpler system (consisting of less than 20 junctions) that satisfies less stringent criteria: the absence of linear coupling for three sources of noise, both in an idle state and under most common manipulations.[19,20]

ACKNOWLEDGMENTS

We acknowledge useful discussions with G. Blatter, V. Geshkenbein, M. Feigelman, D. Ivanov, A. Dyugaev, and M. Lashkevich. R.U. is grateful to S. Frolov for his helpful advice in the preparation of the text of this paper. The work was supported by the NSF, under Grant No. DMR 0210575.

[1] J. Preskill, Proc. R. Soc. London, Ser. A **454**, 385 (1998).
[2] J. I. Cirac and P. Zoller, Phys. Rev. Lett. **74**, 4091 (1995).
[3] C. Monroe, D. Meekhof, B. King, W. Itano, and D. Wineland, Phys. Rev. Lett. **75**, 4714 (1995).
[4] Q. Turchette, C. Hood, W. Lange, H. Mabushi, and H. J. Kimble, Phys. Rev. Lett. **75**, 4710 (1995).
[5] N. A. Gershenfeld and I. L. Chuang, Science **275**, 350 (1997).
[6] D. Loss and D. P. DiVincenzo, Phys. Rev. A **57**, 120 (1998).
[7] B. E. Kane, Nature (London) **393**, 133 (1998).
[8] A. Shnirman, G. Schön, and Z. Hermon, Phys. Rev. Lett. **79**, 2371 (1997).
[9] D. V. Averin, Solid State Commun. **105**, 659 (1998).
[10] Y. Makhlin, G. Schön, and A. Shnirman, Nature (London) **398**, 305 (1999).
[11] M. F. Bocko, A. M. Herr, and M. J. Feldman, IEEE Trans. Appl. Supercond. **7**, 3638 (1997).
[12] L. B. Ioffe, V. B. Geshkenbein, M. V. Feigel'man, A. L. Fauchère, and G. Blatter, Nature (London) **398**, 679 (1999).
[13] J. E. Mooij, T. P. Orlando, L. S. Levitov, L. Tian, C. H. van der Wal, and S. Lloyd, Science **285**, 1036 (1999).
[14] D. Vion, A. Aassime, A. Cottet, P. Joyez, H. Pothier, C. Urbina, D. Esteve, and M. Devoret, Science **296**, 886–889 (2002).
[15] L. B. loffe, V. B. Geshkenbein, Ch. Helm, and G. Blatter (unpublished).
[16] M. V. Feigelman, L. Ioffe, G. Blatter, and D. Geshkenbein (unpublished).
[17] B. Doucot, L. B. Ioffe, and G. Vidal, cond-mat/0302104 (unpublished).
[18] C.H. van der Wal, F. K. Wilhelm, C. J. P. Harmans, and J. E. Mooij, cond-mat/0211664, 2002 (unpublished).
[19] L. B. Ioffe and M. V. Feigelman, Phys. Rev. B **66**, 224503 (2002).
[20] B. Doucot, M. V. Feigelman, and L. B. Ioffe, Phys. Rev. Lett. **90**, 107003 (2003).