Entanglement between two superconducting qubits via interaction with nonclassical radiation

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We propose a scheme to physically interface superconducting nanocircuits and quantum optics. We address the transfer of quantum information between systems having different physical natures and defined in Hilbert spaces of different dimensions. In particular, we investigate the transfer of the entanglement initially in a nonclassical state of an infinite dimensional system to a pair of superconducting charge qubits. This setup is able to drive an initially separable state of the qubits into an almost pure, highly entangled state suitable for quantum information processing.

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I. INTRODUCTION

Control of the dynamics of a complex quantum system requires a trade-off between tunability and protection against noise. To this end one can be interested in processes where some physical properties of a subsystem are reliably transferred onto the state of a second one (of perhaps different nature) where information can be manipulated. The connection between the two subsystems is effectively realized via a physical *interface*. An interface is a communication channel used to connect the elements of a quantum register to perform quantum information processing or a physical mechanism that gives full access to the system under investigation and allows to manipulate it.

To investigate this problem, in this paper we describe the coupling between a nanoelectronic circuit implementing a pair of quantum bits and a two-mode electromagnetic field. We discuss a mechanism for the transfer of entanglement from a two-mode squeezed state to the pair of qubits. Here, the information sheltered in the electromagnetic medium may be manipulated, using just single-qubit operations, when transferred to the solid-state subsystem. This may offer advantages with respect to integrability and scalability. In particular, we consider the field modes to interact with a pair of (initially independent) superconducting quantum interference devices (SQUIDs) that embody two charge qubits.[1] Direct experimental evidence of the use of these systems as controllable coherent two-level systems has already been provided.[4,5] We find that a nearly maximally entangled state of two qubits can be tailored, with our interaction model, via an effective process of transfer of quantum correlations. The entanglement *poured* into the joint state of the qubits can be regulated controlling the interaction times between qubits and field modes. At the interaction time corresponding to the maximum of the transferred entanglement, the qubits are in an almost pure state that may be used for efficient quantum information processing.

This work is organized as follows. In Sec. II we introduce the system we consider and derive the effective model for the coupling between a superconducting charge qubit and a field mode. Section III is devoted to the study of the process of transfer of quantum correlations from the two-mode field to the qubits. The joint state of these latter, once the field modes are traced out, turns out to be entangled. We quantify the amount of entanglement between the qubits and find the corresponding degree of mixedness of the state. Finally, in Sec. IV, we investigate about the variations in the amount of transferred entanglement as the initial preparation of the qubits is changed. We find that the transfer process is optimized if the qubits are initially in their computational ground state.

II. THE HAMILTONIAN

A scheme of principle of the system we consider is sketched in Fig. 1(a). In details, a pair of SQUIDs is exposed to a two-mode correlated state of radiation. In order to capture the essence of this interaction, each field mode is modeled as an LC circuit coupled to the corresponding SQUID via the capacitance C_c, as shown in Fig. 1(b). The SQUIDs can be individually addressed by gate voltages V_g whereas an external magnetic flux ϕ_{ext} allows to change the Josephson coupling $E_J(\phi_{ext})$ (Ref. 1) and to modulate the interaction among the subsystems.[2,3] We first analyze a single SQUID plus LC oscillator. We introduce the phase drop across the SQUID (φ_a) and across the LC circuit (φ_b). The conjugated variables are the excess charge on the SQUID island (Q_a) and the charge on the oscillator's capacitance (P_b). The Hamiltonian describing the system is

$$H = H_{\mathrm{SQUID}} + H_{em} + H_c = \frac{(Q_a - C_g V_g)^2}{2C} - E_J(\phi_{ext})\cos\left(\frac{2e}{\hbar}\varphi_a\right) + \frac{P_b^2}{2C_2} + \frac{\varphi_b^2}{2L_o} + \frac{P_b(Q_a - C_g V_g)}{C_1}. \tag{1}$$

where

$$C = \mathcal{D}/(C_0 + C_c),$$

$$C_1 = \mathcal{D}/(C_g + 2C_{J_0} + C_c), \quad C_2 = \mathcal{D}/C_c,$$

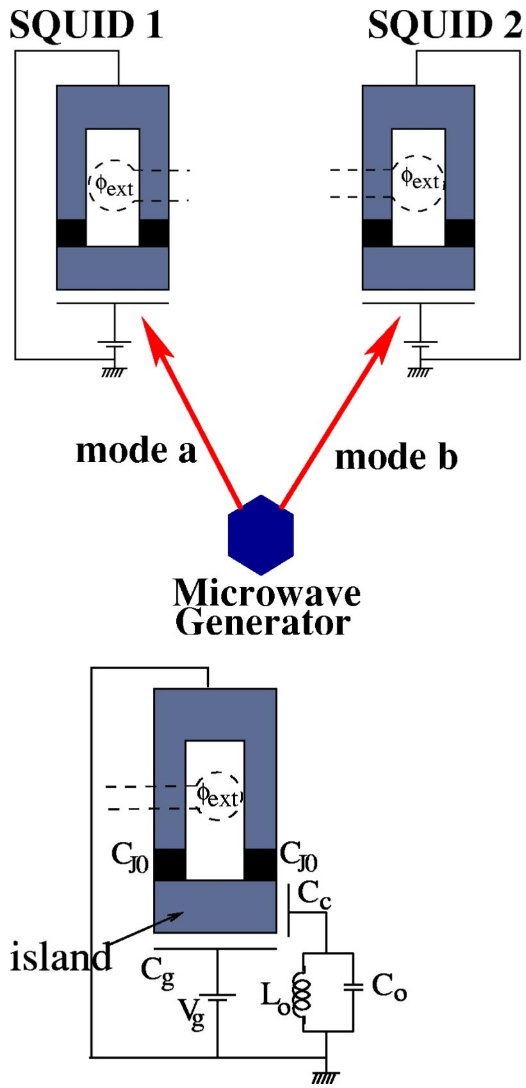

FIG. 1. (Color online) (a) Setup for an entanglement transfer process via the interface between quantum correlated field modes and a pair of charge qubits. Each SQUID is threaded by an external magnetic flux to modulate $E_J(\phi_{ext})$. (b) Equivalent circuit for the single SQUID capacitively coupled to a single field mode, modeled as a LC oscillator.

$$\mathcal{D} = (C_o + C_c)(C_g + 2C_{J_0}) + C_c C_o,$$

and

$$E_J(\phi_{ext}) = 2E_J^0 \cos(2e\phi_{ext}/\hbar),$$

with E_J^0 the maximum Josephson energy. Here, C_{J_0} is the single junction capacitance. The SQUID Hamiltonian H_{SQUID} can be tuned by V_g and ϕ_{ext}. The field mode, described by the oscillator $H_{em}=P_b^2/2C_2+\varphi_b^2/2L_o$, has effective frequency $\omega=(L_oC_2)^{-1/2}$ which comes from the inductance L_o and the total capacitance C_2 seen by the charge P_b. The coupling Hamiltonian $H_c=P_b(Q_a-C_gV_g)/C_1$ describes the Coulomb interaction between the charges Q_a and P_b.

We assume large charging energy, $e^2/2C \gg E_J(\phi_{ext})$, and low temperatures $T \ll e^2/2C$. In this regime the SQUID can be described by the states $|m\rangle_s$ $(m=0,1)$ representing m Cooper pairs in excess in the island, and implements a charge Josephson qubit.[1] Typical values of $C_{J_0}\simeq 10^{-15}$ F and $C_g \simeq 10^{-17}$ F guarantee $e^2/2C \sim 1\ K \gg E_J^0 \sim 100$ mK. In this system, the main sources of decoherence are noise of electrostatic origin, voltage fluctuations of the circuit[1,6] or stray polarization due to charged impurities located close to the device.[7] If we set $C_gV_g=e$, $|0\rangle_s$ and $|1\rangle_s$ have the same electrostatic energy and the SQUID is not affected, at first order, by this charge noise.[2,5] At this working point, $\hat{H}_{\mathrm{SQUID}} = 1/2E_J(\phi_{ext})\hat{\sigma}_{z,s}$ with a computational basis $\{|+\rangle,|-\rangle\}_s$, where $|\pm\rangle_s=(1/\sqrt{2})(|1\rangle\pm|0\rangle)_s$ are eigenstates of $\hat{\sigma}_{z,s}$, the z-Pauli matrix, splitted by $E_J/\hbar \sim 10$ GHz. We introduce the operators $\hat{a}$ and $\hat{a}^\dagger$ $([\hat{a},\hat{a}^\dagger]=1)$ via $\hat{P}_b=(\hbar\omega C_2/2)^{1/2}(\hat{a}+\hat{a}^\dagger), \hat{\varphi}_b=i(\hbar/2\omega C_2)^{1/2}(\hat{a}-\hat{a}^\dagger)$ and we get $H_{em}=\hbar\omega(\hat{a}^\dagger\hat{a}+1/2)$. Taking $C_2\simeq 1$ pF and $L_o\simeq 10$ nH, achievable by present day technology, we have $\omega\simeq 10$ GHz. The coupling between the SQUID and the field mode can be tuned on and off resonance by modulating the energy splitting of the qubit via ϕ_{ext}. If $E_J(\phi_{ext})/\hbar$ is set to be much different from ω, the coupling is effectively turned off and the qubit evolves independently from the field mode. On the other hand, for the quasiresonant condition $E_J(\phi_{ext})\simeq\hbar\omega$, we use $\hat{Q}_a=2e\hat{\sigma}_{x,s}$, with $\hat{\sigma}_{x,s}=(|+\rangle_s\langle-|+|-\rangle_s\langle+|)$ the x-Pauli matrix, so that

$$\hat{H}_c = \hbar\Omega[(\hat{a}\hat{\sigma}_{+,s} + \hat{a}^\dagger\hat{\sigma}_{-,s}) + (\hat{a}^\dagger\hat{\sigma}_{+,s} + \hat{a}\hat{\sigma}_{-,s})], \qquad (2)$$

where $\Omega=e\sqrt{2\omega C_2/\hbar C_1^2}$ is the Rabi frequency of the interaction and $\hat{\sigma}_{+,s}=\hat{\sigma}^\dagger_{-,s}=|+\rangle_s\langle-|$. The Hamiltonian Eq. (2) is frequently found in quantum optics problems. The first and second term preserve the total number of excitations in the system and allow for the restriction of the computational basis to $\{|-,n\rangle,|+,n-1\rangle\}_{s,em}$, where $|n\rangle_{em}$ is an n-photon Fock state.[8] The other (counter-rotating) terms induce leakage from this subspace. They can be neglected in the rotating wave approximation. This can be used when $\Omega \ll \omega, E_J(\phi_{ext})/\hbar$, achieved if we take $C_c\simeq 10^{-17}$ F (weakly coupled subsystems) so that $C_1\simeq 10^{-11}$ F and $\Omega \simeq 0.1$ GHz. In this regime, the eigenstates of the SQUID plus field mode system are entangled states forming a series of doublets splitted by $\hbar\Omega\sqrt{n}$. It is worth stressing that, at the working point $V_g=e/C_g$, intradoublet transitions are forbidden.[2] The system is thus protected, to a certain extent, from decoherence.

III. TRANSFER OF ENTANGLEMENT TO THE QUBITS

We now consider both the SQUIDs of our system and describe their interaction with a two-mode nonclassical state of radiation. We consider the two-mode squeezed state[8] $|S(r)\rangle_{ab}=\Sigma_{n=0}^{\infty}\eta_n(r)|n,n\rangle_{ab}$, where r is the squeezing parameter and $\eta_n(r)=(\tanh r)^n/\cosh r$. This is a quantum-correlated state of modes a and b and the above expression is the Schmidt representation of their joint state. The entanglement between the modes is a function of r. Squeezed microwaves can be generated *off-line* using Josephson parametric oscillators in nondegenerate configurations[9] and then used for our protocol. The SQUIDs can be integrated in the waveguides used for the transmission of the signal,[9] with the gate-plates orthogonal to the direction of propagation of the fields. Quality factors $\sim 10^4$ for a superconducting transmission line are within the state of the art. For $\omega \sim 10$ GHz, this gives photon lifetimes ~ 1 μsec, allowing for a coherent dynamics. The SQUIDs are prepared in a pure separable state $\rho_{12}(0)=\rho_1(0)\otimes\rho_2(0)$. The interaction between each SQUID and a field mode is driven by the corotating part of Eq. (2). The joint time-evolution operator is the tensorial product $\hat{U}(t)=\hat{U}_{a1}(t)\otimes\hat{U}_{b2}(t)$ where, in the single-qubit basis $\{|-\rangle, |+\rangle\}_a$, the unitary operator $\hat{U}_{a1}(t)$ is given by[10]

$$\hat{U}_{a1}(t)=\begin{pmatrix} \cos(\Omega\sqrt{\hat{a}^\dagger\hat{a}}t) & -i\hat{a}^\dagger\dfrac{\sin(\Omega\sqrt{\hat{a}\hat{a}^\dagger}t)}{\sqrt{\hat{a}\hat{a}^\dagger}} \\ -i\hat{a}\dfrac{\sin(\Omega\sqrt{\hat{a}^\dagger\hat{a}}t)}{\sqrt{\hat{a}^\dagger\hat{a}}} & \cos(\Omega\sqrt{\hat{a}\hat{a}^\dagger}t) \end{pmatrix}. \quad (3)$$

An analogous expression can be written for $\hat{U}_{b2}(t)$. However, the reduced state of the SQUIDs, $\rho_{12}(t)$, is in general inseparable because the joint evolution with the field modes could have transferred quantum correlations to the qubits. To give a complete picture of the dynamics of the qubits, we derive the operator-sum representation of the SQUIDs evolution $\$(\rho_{12}):\rho_{12}(0)\to\rho_{12}(t)=\Sigma_\mu\hat{K}_\mu\rho_{12}(0)\hat{K}_\mu^\dagger$, with $\$$ the superoperator that takes the density matrix $\rho_{12}(0)$ to $\rho_{12}(t)$ and $\{\hat{K}_\mu\}$ the set of Kraus operators corresponding to $\hat{U}(t)$. Each Kraus operator projects the state of the SQUIDs into a pure state. If there are two or more terms involved in this representation, the initially pure state evolves into a mixed state. We have

$$\begin{aligned}\rho_{12}(t) &= \mathrm{Tr}_{ab}\{\hat{U}(t)\rho_{12}(0)\otimes\rho_{ab}(0)\hat{U}^\dagger(t)\} \\ &= \sum_{m,p=0}^{\infty}\left(\sum_{n=0}^{\infty}\eta_n(r)_{ab}\langle m,p|\hat{U}(t)|n,n\rangle_{ab}\right)\rho_{12}(0) \\ &\quad\times\left(\sum_{l=0}^{\infty}\eta_l(r)_{ab}\langle l,l|\hat{U}^\dagger(t)|m,p\rangle_{ab}\right). \end{aligned} \quad (4)$$

Calculating the matrix elements of $\hat{U}(t)$ over the Fock states of the field modes, a set of five Kraus operators is found. If the initial state of the two SQUIDs is specified, a simplification is possible and the number of Kraus operators is reduced. We assume $\rho_{12}(0)=|-,-\rangle_{12}\langle -,-|$, that can be prepared using standard techniques.[1] We get the effective representation $\rho_{12}(t)=\Sigma_{\mu=1}^3\Sigma_{m=0}^\infty\hat{K}_\mu^m|-,-\rangle_{12}\langle -,-|\hat{K}_\mu^{m\dagger}$, where

$$\begin{aligned}\hat{K}_1^m &= \eta_m\cos^2(\Omega\sqrt{m}t)|-,-\rangle_{12}\langle -,-| - \eta_{m+1} \\ &\quad\times\sin^2(\Omega\sqrt{m+1}t)|+,+\rangle_{12}\langle -,-|,\end{aligned}$$

$$\hat{K}_2^m = \eta_m\cos(\Omega\sqrt{m}t)\sin(\Omega\sqrt{m}t)|-,+\rangle_{12}\langle -,-|,$$

$$\hat{K}_3^m = \eta_m\cos(\Omega\sqrt{m}t)\sin(\Omega\sqrt{m}t)|+,-\rangle_{12}\langle -,-|. \quad (5)$$

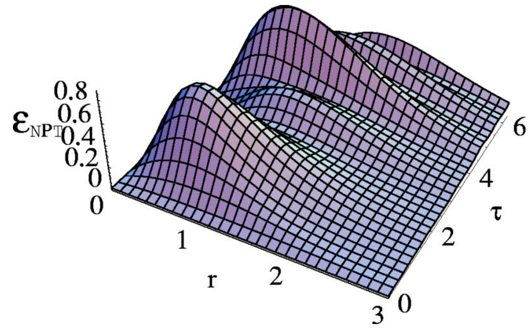

FIG. 2. (Color online) $\mathcal{E}_{NPT}$ vs $\tau=\Omega t$ and r. Iff $\mathcal{E}_{NPT}>0$, there is entanglement between the superconducting qubits. A local maximum $\mathcal{E}_{NPT}^{max}\simeq 0.87$ is achieved for $\tilde{r}=0.86$ and $\tilde{\tau}\simeq 3\pi/2$.

$\hat{K}_1^m$ is responsible for zero and two-photon processes that leave the two field modes with the same number of photons. $\hat{K}_2^m$ and $\hat{K}_3^m$ describe single-photon processes in which one of the SQUIDs absorbs an incoming photon. Using Eqs. (5), the density matrix of the SQUIDs, in the ordered basis $\{|-,-\rangle,|-,+\rangle,|+,-\rangle,|+,+\rangle\}_{12}$, takes the form

$$\rho_{12}(r,t)=\begin{pmatrix} A(r,t) & 0 & 0 & -D(r,t) \\ 0 & B(r,t) & 0 & 0 \\ 0 & 0 & B(r,t) & 0 \\ -D(r,t) & 0 & 0 & C(r,t)\end{pmatrix}, \quad (6)$$

where

$$A(r,t)=\sum_{n,0}^{\infty}\chi_{nn}(r)\cos^4(\Omega\sqrt{n}t),$$

$$B(r,t)=\sum_{n,0}^{\infty}\chi_{nn}(r)\sin^2(\Omega\sqrt{n}t)\cos^2(\Omega\sqrt{n}t),$$

$$D(r,t)=\sum_{n,0}^{\infty}\chi_{nn+1}(r)\sin^2(\Omega\sqrt{n+1}t)\cos^2(\Omega\sqrt{n}t),$$

$$C(r,t)=1-2B(r,t)-A(r,t). \quad (7)$$

Here, $\chi_{nm}(r)=\eta_n(r)\eta_m(r)$. To quantify the entanglement between the qubits, we choose the negativity of partial transposition (NPT). NPT is a necessary and sufficient condition for entanglement of any bipartite qubit state.[11] The corresponding entanglement measure is defined as $\mathcal{E}_{NPT}=-2\lambda^-(r,t)$, where $\lambda^-(r,t)$ is the unique negative eigenvalue of the two-qubit partially transposed density matrix[11] $\rho_{12}^{T_2}$ (the transposition is with respect to qubit 2). In our case, just $\lambda^-(r,t)=B(r,t)-D(r,t)$ can be negative for some value of r and t and it is used to compute the entanglement. $\mathcal{E}_{NPT}$ is shown in Fig. 2 as a function of the degree of squeezing r and the rescaled interaction time $\tau=\Omega t$. It turns out that $\mathcal{E}_{NPT}$ never becomes negative and only asymptotically goes to zero as r is increased. Once the interaction starts, the entanglement is transferred to the qubits, *collapsing* and *reviving* as the interaction time increases. The maximum of the trans-

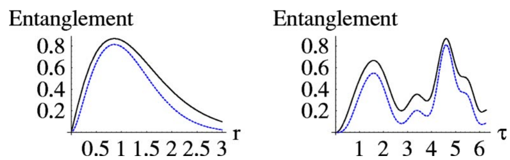

FIG. 3. (Color online) Comparison between EoF (dashed line) and $\mathcal{E}_{NPT}$ (solid line). In (a) we plot the behavior of the two entanglement measures against the squeezing parameter r. We have taken $\tau=3\pi/2$. In (b), the entanglement functions are plotted vs τ, for $r=0.86$.

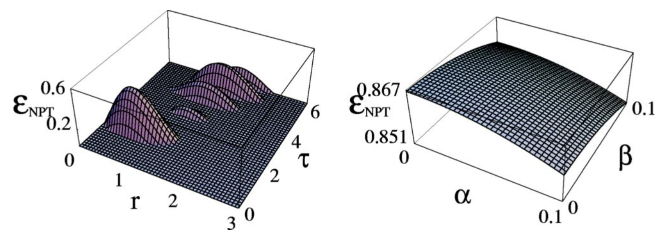

FIG. 4. (Color online) (a) Transferred entanglement averaged over the possible preparations of the SQUIDs. The maximum entanglement is reduced with respect to the case of $|-,-\rangle_{12}$. This is due to the contribution from $|+,+\rangle_{12}$, that is separable for wide ranges of τ and r and spoils the average entanglement. (b) The amount of entanglement between the qubits as a function of the preparation of the initial states when $r=\tilde{r}$ and $\tau=\tilde{\tau}$. The state $|-,-\rangle_{12}$, obtained for $\alpha=\beta=0$, corresponds to the maximum of the transferred entanglement.

ferred entanglement is $\mathcal{E}_{NPT}^{max}=0.87$, obtained for $\tilde{\tau}\simeq 3\pi/2$ and $\tilde{r}=0.86$. As long as correlations are present between the two modes, entanglement is set in the state of the qubits. Furthermore, we have checked that mixedness of the state of radiation due to imperfections in the generation process (squeezing thermofields instead of vacuum, for example) does not affect the entanglement transfer. $\mathcal{E}_{NPT}$ is not a monotone function of r as can be seen in Fig. 3(a). It is known that the correlations in $|S(r)\rangle_{ab}$ approach those of the maximally entangled Einstein-Podolski-Rosen (EPR) state when $r\rightarrow\infty$.[12] Increasing r, the contribution by higher photon-number terms in the squeezed state becomes more relevant. The interaction of each qubit with the elements of this distribution of Fock states results in induced Rabi floppings (characterized by frequencies $\Omega\sqrt{n}$) that mutually interfere, spoiling the degree of entanglement between the SQUIDs. This shows that a perfectly correlated state defined in an infinite dimensional Hilbert space can not be mapped onto a maximally entangled state of two qubits. On theother hand, the discreteness of this distribution induces the entanglement to collapse and revive as time goes by. This analysis is confirmed by considering the entanglement of formation (EoF). This entanglement measure quantifies the resources, in terms of number of $|\mathrm{EPR}\rangle$ singlets, needed to create a given entangled state using only classically coordinated local operations.[13] For a bipartite system, EoF can be calculated as[14] $\mathrm{EoF}(\rho_{12})=-x\log_2 x-(1-x)\log_2(1-x)$, where $x=[1+\sqrt{1-\mathcal{C}^2(\rho_{12})}]/2$ and $\mathcal{C}(\rho_{12})=\max\{0,\alpha_1-\alpha_2-\alpha_3-\alpha_4\}$ is the *concurrence*.[14] Here, $\{\alpha_i\}$ $(i=1,\ldots,4)$ are the square roots of the eigenvalues (in nonincreasing order) of the non-Hermitian operator $\overline{\rho_{12}}=\rho_{12}(\sigma_y\otimes\sigma_y)\rho_{12}^*(\sigma_y\otimes\sigma_y)$. In this expression, σ_y is the y-Pauli matrix and ρ_{12}^* is the complex conjugate of ρ_{12}, in the computational basis. Despite $\overline{\rho_{12}}$ is non-Hermitian, each α_i is real and non-negative.[14]

For a pure state of two qubits, EoF is a monotonous function of NPT since $\mathcal{E}_{NPT}$ is equivalent to the concurrence. In Fig. 3 the two entanglement measures are compared, as functions of both r and τ. From the behavior of EoF, we argue that almost one EPR singlet is required to prepare $\rho_{12}(\tilde{r},\tilde{\tau})$. A further analysis of $\rho_{12}(\tilde{r},\tilde{\tau})$ shows that $|B(\tilde{r},\tilde{\tau})|\ll|A(\tilde{r},\tilde{\tau})|,|C(\tilde{r},\tilde{\tau})|,|D(\tilde{r},\tilde{\tau})|$. If, in zero-order approximation, we neglect $B(\tilde{r},\tilde{t})$ in $\rho_{12}(\tilde{r},\tilde{t})$, we get a density matrix close to that of the pure, nonmaximally entangled state $(\sqrt{A(r,t)}|--\rangle-\sqrt{C(r,t)}|++\rangle)_{12}$. In general, $D(r,t)\neq\sqrt{A(r,t)C(r,t)}$, so that the state is mixed. The degree of mixedness in this *purified* version of $\rho_{12}(r,t)$ is quantified using the linearized entropy $S_l[\rho_s(r,t)]=4/3\{1-\mathrm{Tr}[\rho_{12}^2(r,t)]\}$, that ranges from 0 (pure states) to 1 (maximally mixed ones). We get $S_l[\rho_{12}(\tilde{r},\tilde{\tau})]\simeq 0.01$ showing that, for these parameters, the two SQUIDs are in a nearly pure state. This result is interesting: it has been proved, for example, that a bipartite mixed state becomes useless for quantum teleportation whenever its linearized entropy exceeds $1-(2/[N(N+1)])$, with N the dimension of each subsystem.[15] For qubits, the threshold is $2/3\geqslant S_l[\rho_{12}(\tilde{r},\tilde{\tau})]$ and the state of the entangled SQUIDs could be used, in general, as a quantum channel in efficient protocols for distributed quantum computation. We have calculated the purity of the state when $B(\tilde{r},\tilde{\tau})$ is included, finding the same order of magnitude of the previous result.

IV. AVERAGE ENTANGLEMENT AND OPTIMAL PREPARATION

We now consider the average value of the transferred entanglement as the preparation of the initial state of the SQUIDs is varied. This allows us to investigate about the dynamics of the superconducting qubits once different separable states as $(\cos\alpha|-\rangle+e^{i\varphi}\sin\alpha|+\rangle)_1\otimes(\cos\beta|-\rangle+e^{i\psi}\sin\beta|+\rangle)_2$ are considered. For simplicity, we take $\varphi=\psi=0$ and we use $\mathcal{E}_{NPT}$ to calculate the average value of the entanglement. The evolution of the SQUIDs involves the complete set of Kraus operators. However, the density matrix $\rho_{12}(r,t)$, averaged over an uniform distribution for α,β, still keeps the form in Eq. (6) but with more complicated matrix elements. The results are shown in Fig. 4(a). The amount of transferred entanglement is reduced and the peak at $\tau=\tilde{\tau}$, $r=\tilde{r}$ is shrunk to $\simeq 0.4$. This can be understood considering the behavior of $\mathcal{E}_{NPT}$ for $|+,+\rangle_{12}$ as initial state. In this case, $\mathcal{E}_{NPT}$ remains negative for a wide range of values of r and τ and has a small positive bump for $r\simeq 0.6$ and $\tau\simeq 1.7$ (that corresponds to the first peak in Fig. 2). This suggests that $|-,-\rangle_{12}$ plays a privileged role in the process of entanglement transfer. To support this idea, we look for the optimal preparation of the SQUIDs.

We assume $\alpha,\beta\ll 1$ and, after some lengthy calculations, we find the explicit expression for the qubits density matrix

$$\rho_{12}(r,t) \simeq \begin{pmatrix} a & \beta b + \alpha d & \beta d + \alpha b & -c \\ \beta b + \alpha d & a' & 0 & \alpha f - \beta d \\ \alpha b + \beta d & 0 & a' & \beta f - \alpha d \\ -c & \alpha f - \beta d & \beta f - \alpha d & a'' \end{pmatrix}, \tag{8}$$

with

$$a = \sum_{n,0}^{\infty} \chi_{nn}(r)\cos^4(\sqrt{n}\tau),$$

$$a' = \sum_{n,0}^{\infty} \chi_{nn}(r)\cos^2(\sqrt{n}\tau)\sin^2(\sqrt{n}\tau),$$

$$b = \sum_{n,0}^{\infty} \chi_{nn}(r)\cos^3(\sqrt{n}\tau)\cos(\sqrt{n+1}\tau),$$

$$c = \sum_{n,0}^{\infty} \chi_{nn+1}(r)\cos^2(\sqrt{n}\tau)\sin^2(\sqrt{n+1}\tau),$$

$$d = \sum_{n,0}^{\infty} \chi_{nn+1}(r)\cos(\sqrt{n+1}\tau)\cos(\sqrt{n}\tau)\sin^2(\sqrt{n+1}\tau),$$

$$f = \sum_{n,0}^{\infty} \chi_{nn}(r)\cos(\sqrt{n}\tau)\cos(\sqrt{n+1}\tau)\sin^2(\sqrt{n}\tau), \tag{9}$$

and $a''=1-a-2a'$. This time, it is hard to obtain an analytic expression for the eigenvalues of $\rho_{12}^{T_2}$. However, some insight can be gained by specifying the values of both r and τ. In Fig. 4(b) we plot $\mathcal{E}_{NPT}$ versus α and β for $r=\tilde{r}$ and $\tau=\tilde{\tau}$. The transferred entanglement has a maximum equal to 0.87 for $\alpha=\beta=0$ and slowly decays. This could be important, experimentally, because small errors in the preparation of the initial state do not dramatically spoil the amount of entanglement transferred to the qubits. The same qualitative behavior is found for other values of r and t. Thus, the initial preparation $|-,-\rangle_{12}$ provides the maximum achievable entanglement transfer. The entanglement of the SQUIDs, after the interaction, can be revealed by detecting the population $B(r,\tau)$ and the coherence $D(r,\tau)$ of the density matrix using local resonant pulses on the SQUIDs, along the same lines depicted in Ref. 16.

V. CONCLUSIONS

We have proposed a physical interface between quantum optics and a solid-state system based on resonant effective interactions between field modes and charge qubits. Our study is directed to the design of efficient interaction models to couple elements of a device for quantum information processing embodied by physical systems having different nature. We have explicitly examined the problem of the transfer of entanglement between such subsystems. When a quantum-correlated state of light is considered, entanglement-transfer from the fields to the qubits can be efficiently tailored controlling the interaction times between fields and qubits. We have characterized the state of the qubits in terms of entanglement and purity, finding that the ground state of the qubits is the initial preparation that guarantees the optimal entanglement transfer. The process we describe in this paper turns out to be a reliable mechanism for the engineering of the entanglement between two solid-state qubits. Our scheme is thus able to combine two key requirements of a device for distributed quantum information: the long-haul transmission of information typical of a photonic channel and the manipulability of a qubit subsystem.

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