

Charge transport through open driven two-level systems with dissipation

Tobias Brandes,¹ Ramón Aguado,² and Gloria Platero²

¹*Department of Physics, University of Manchester Institute of Science and Technology (UMIST), P.O. Box 88, Manchester M60 1QD, United Kingdom*

²*Departamento de Teoría de la Materia Condensada, Instituto de Ciencia de Materiales de Madrid, CSIC, Cantoblanco 28049, Madrid, Spain*

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We derive a Floquet-like formalism to calculate the stationary average current through an ac driven double quantum dot in the presence of dissipation. The method allows us to take into account arbitrary coupling strengths both of a time-dependent field and a bosonic environment. We numerically evaluate a truncation scheme and compare with analytical, perturbative results such as the Tien-Gordon formula.

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I. INTRODUCTION

Coupled quantum systems with small effective Hilbert spaces are useful tools in order to study coherence, dissipation, and the interaction properties of few-particle systems. In an electronic context, an example are coupled quantum dots,^{1–5} where strong interactions between electrons^{6–8} define a Coulomb blockade regime with tunnel-split many-body ground states separated from the remaining excited states. The ultimate limit of two states defines a two-level system for the charge degree of freedom, with electrons of a fixed spin tunneling between two quantum dots. Studying transport and dissipation then leads to a nonequilibrium or “open” (pseudo)spin-boson problem, where the coupling to external reservoirs opens the path to investigate properties such as shot noise⁹ or decoherence in a controllable semiconductor environment.

Additional insight into the quantum dynamics of electrons can be gained by making the parameters of the problem time dependent. When the time dependence is slow, this can give rise to a variety of adiabatic phenomena such as charge pumping,^{10–19} adiabatic control of state vectors,^{20,21} or operations relevant for quantum information processing in a condensed-matter setting.^{22–28} Different physics occurs in the high-frequency regime where monochromatic time variation induces photoexcitations, such as for coupling of ac fields to quantum dots,^{29–34} which has been tested experimentally^{35–40} recently.

In general, ac driven systems^{41,42} and their application to various mesoscopic transport^{43–50} and tunneling^{51–63} regimes have quite a long history, although the inclusion of interactions and correlations is a relatively new area. In low-dimensional systems, investigations have concentrated on one-dimensional models,^{64–68} the modification of Kondo resonances by ac fields,^{69,70} mean-field-type approximations,⁷¹ or exact studies of driven few-electron systems.^{72,73}

In this paper, we combine ac driving with the *dissipative dynamics* of a two-level system (double quantum dot) under transport conditions, i.e., in a situation where electrons in the Coulomb blockade regime can tunnel from reservoirs into and off two tunnel-coupled quantum dots, with the possibility to absorb from or emit bosons into a heat bath while simultaneously interacting with a classical time-periodic electrical field. At first sight, combining such a multitude of

possible interactions within one and the same model might look unsuitable for a useful theoretical discussion. However, as we will demonstrate in this paper, it is possible to calculate experimentally relevant observables such as the time-averaged stationary current, with the help of the (heat bath) boson spectral density $J(\omega)$ as single, main input of the theory only. In particular, we show how within the polaron transformation approach and for a given $J(\omega)$, one can calculate the current for arbitrarily strong coupling to bosonic modes *and* an ac field.

The paper is organized as follows: in Sec. II, we describe the model Hamiltonian and derive a Floquet-like formalism for the stationary density operator. In Sec. III, we compare analytical results for limiting cases with numerical data, and conclude with a short discussion and an outlook in Sec. IV.

II. MASTER EQUATION FORMALISM FOR ac DRIVEN DOUBLE QUANTUM DOTS

In the following, we shall develop the general framework leading to explicit expressions for the stationary current through dissipative driven double quantum dots. Our approach is in part similar to the treatment of closed dissipative two-level systems with ac driving as reviewed by Grifoni and Hänggi.⁷⁴ Here, we generalize this approach to take into account tunneling between the dots and the leads. In the nondissipative case, this problem was treated by Gurvitz and Prager^{75,76} for nondriven double dots, and for coherently ac driven double dots by Stoof and Nazarov.³³

A. Model Hamiltonian

We assume that the driven two-level system is defined in a double quantum dot device.³ In the regime of strong Coulomb blockade, these can be tuned into a regime where the internal dynamics is governed by a time-dependent (pseudo) spin-boson model (dissipative two-level system⁷⁷), $\mathcal{H}_{SB}(t)$. The latter describes one additional “transport” electron which tunnels between a left (L) and a right (R) dot with *time-dependent* energy difference $\varepsilon(t)$ and interdot coupling $T_c(t)$, and is coupled to a dissipative bosonic bath ($\mathcal{H}_B = \sum_{\mathbf{Q}} \omega_{\mathbf{Q}} a_{\mathbf{Q}}^\dagger a_{\mathbf{Q}}$),

$$\mathcal{H}_{SB}(t) = \left[\frac{\varepsilon(t)}{2} + \sum_{\mathbf{Q}} \frac{g_{\mathbf{Q}}}{2} (a_{-\mathbf{Q}} + a_{\mathbf{Q}}^\dagger) \right] \hat{\sigma}_z + T_c(t) \hat{\sigma}_x + \mathcal{H}_B. \quad (1)$$

The effective Hilbert space of double dot (without any coupling to electron leads or bosons) then consists of two (many-body) states $|L\rangle = |N_L + 1, N_R\rangle$ and $|R\rangle = |N_L, N_R + 1\rangle$ and is defined by a pseudospin $\hat{\sigma}_z \equiv |L\rangle\langle L| - |R\rangle\langle R| \equiv \hat{n}_L - \hat{n}_R$ and $\hat{\sigma}_x \equiv |L\rangle\langle R| + |R\rangle\langle L| \equiv \hat{p} + \hat{p}^\dagger$.

The effects of the bosonic bath are fully described as usual by a spectral density

$$J(\omega) \equiv \sum_Q |g_Q|^2 \delta(\omega - \omega_Q), \quad (2)$$

where ω_Q are the frequencies of the bosons and the g_Q denote interaction constants. When showing particular results we will be using

$$J(\omega) = 2\alpha\omega e^{-\omega/\omega_c}, \quad (3)$$

corresponding to a generic Ohmic bath. More realistic forms can be easily incorporated into our formalism, but for simplicity in this work we restrict ourselves to the Ohmic case.

The coupling to external free electron reservoirs $\mathcal{H}_{res} = \sum_{k_\alpha} \epsilon_{k_\alpha} c_{k_\alpha}^\dagger c_{k_\alpha}$ is described by the usual tunnel Hamiltonian

$$\mathcal{H}_T = \sum_{k_\alpha} (V_k^\alpha c_{k_\alpha}^\dagger s_\alpha + \text{H.c.}), \quad (4)$$

$\hat{s}_\alpha = |0\rangle\langle\alpha|$ ($\alpha = L, R$). Here, a third state $|0\rangle = |N_L, N_R\rangle$ describes an ‘‘empty’’ double quantum dot. Its presence leads to strong modifications both in the mathematical description as well in the physics of this problem, as compared to the case of an isolated spin-boson Hamiltonian. Here, the reservoir-related parameters of \mathcal{H}_{res} and \mathcal{H}_T have been assumed to be time independent which again is an approximation which might not be always fulfilled in experiments. Again, we concentrate on the simplest possible case in this work and neglect the effect of, e.g., a time dependence in the external electrochemical potentials.

The full model as described by

$$\mathcal{H}(t) = \mathcal{H}_{SB}(t) + \mathcal{H}_{res} + \mathcal{H}_T, \quad (5)$$

now offers the possibility to study nonequilibrium properties of a time dependent, ‘‘open’’ dissipative two-level system. Note that in spite of the third, empty state $|0\rangle$ we continue to use the term ‘‘two-level system’’ here and in the following: although the presence of $|0\rangle$ leads to strong modifications of, e.g., the equations of motion of the density operator, it turns out that the internal dynamics of the system is still closely related to that of the dissipative spin-boson problem.

The time dependent spin-boson problem is in general characterized by the fact that *both* $\varepsilon(t)$ and $T_c(t)$ are time dependent. One can then investigate interesting effects such as adiabatic charge pumping, dissipative Landau-Zener tunneling,⁷⁸ or for the closed system (no coupling to the leads) the control of quantum superpositions.⁷⁹ Although this general time dependence offers the richest spectrum of possible physical phenomena, one is clearly strongly restricted by the fact that nearly no analytical solutions are available. In this paper, our goal is to develop a systematic theory for the stationary state of a somewhat simpler situation, i.e., the case where $T_c(t) \equiv T_c$ is constant, with the time dependence solely contained in the bias $\varepsilon(t)$.

B. Equations of motion

In the following, we treat the coupling to the reservoirs within the Born and Markov approximation with respect to \mathcal{H}_T ,^{33,80} such that higher-order effects like cotunneling or the Kondo effect are not considered. This Born-Markov approximation becomes exact in the limiting case of infinite source-drain voltage.⁷⁵ Specifically, one sets the Fermi distributions for the left (right) reservoir $f_L = 1$ ($f_R = 0$) where the chemical potentials of the leads no longer play any role. Furthermore, the tunnel rates which are given by

$$\Gamma_\alpha = 2\pi \sum_{k_\alpha} |V_k^\alpha|^2 \delta(\epsilon - \epsilon_{k_\alpha}), \quad \alpha = L/R, \quad (6)$$

are assumed to be independent of energy. We mention that the generalization to intermediate voltage regimes (finite bias) for double dots is a difficult and nontrivial problem even in the undriven case, which is why we only discuss the infinite-bias limit in this paper.

The derivation of the equations of motion for the dot observables is now very similar to the nondriven case.⁸⁰ The time dependence of the Hamiltonian enters via the replacement of the phase factors $e^{i\varepsilon(t-t')}$ in the free undriven time evolution of the dots, by $e^{i\int_{t'}^t ds \varepsilon(s)}$ for the driven case. Introducing the vectors $\mathbf{A} \equiv (\hat{n}_L, \hat{n}_R, \hat{p}, \hat{p}^\dagger)$, $\mathbf{\Gamma} = \Gamma_L \mathbf{e}_1$ ($\mathbf{e}_1, \dots, \mathbf{e}_4$ are unit vectors) and a time dependent matrix memory kernel M , the equations of motion (EOM) can be formally written as⁹ $[\langle \dots \rangle \equiv \text{Tr} \dots \rho(t)]$,

$$\langle \mathbf{A}(t) \rangle = \langle \mathbf{A}(0) \rangle + \int_0^t dt' \{M(t, t') \langle \mathbf{A}(t') \rangle + \mathbf{\Gamma}\}. \quad (7)$$

This formulation is a useful starting point for, e.g., the calculation of shot noise. Note that in contrast to the undriven case, the memory kernel M depends on both times t and t' because there is no time translation invariance in the presence of driving. Explicitly, the equations for the dot expectation values read

$$\begin{aligned} \frac{\partial}{\partial t} \langle n_L \rangle_t &= -iT_c \{ \langle p \rangle_t - \langle p^\dagger \rangle_t \} + \Gamma_L [1 - \langle n_L \rangle_t - \langle n_R \rangle_t], \\ \frac{\partial}{\partial t} \langle n_R \rangle_t &= iT_c \{ \langle p \rangle_t - \langle p^\dagger \rangle_t \} - \Gamma_R \langle n_R \rangle_t, \\ \langle p \rangle_t &= - \int_0^t dt' e^{i\int_{t'}^t ds \varepsilon(s)} \left[\left(\frac{\Gamma_R}{2} \langle p \rangle_{t'} + iT_c \langle n_L \rangle_{t'} \right) C(t-t') \right. \\ &\quad \left. - iT_c \langle n_R \rangle_{t'} C^*(t-t') \right], \\ \langle p^\dagger \rangle_t &= - \int_0^t dt' e^{-i\int_{t'}^t ds \varepsilon(s)} \left[\left(\frac{\Gamma_R}{2} \langle p^\dagger \rangle_{t'} - iT_c \langle n_L \rangle_{t'} \right) \right. \\ &\quad \left. \times C^*(t-t') + iT_c \langle n_R \rangle_{t'} C(t-t') \right]. \quad (8) \end{aligned}$$

Here, half the decay rate (tunnel rate $\Gamma_R/2$) of the system appears in the off-diagonal terms p and p^\dagger , acting as a source

of dephasing due to tunneling of an electron *out of* the right dot. Furthermore, the boson correlation function for a harmonic bath with spectral density $J(\omega)$, Eq. (2), and at equilibrium temperature $k_B T = 1/\beta$ enters,

$$C(t) \equiv e^{-Q(t)},$$

$$Q(t) \equiv \int_0^\infty d\omega \frac{J(\omega)}{\omega^2} \left[(1 - \cos \omega t) \coth\left(\frac{\beta\omega}{2}\right) + i \sin \omega t \right]. \quad (9)$$

In deriving the equations for the off-diagonal elements $\langle p^{(\dagger)} \rangle$, we used the polaron transformation (POL) and factorized the bosonic correlation functions from the dot operators in the equations of motions for the reduced density operator of the (pseudo) spin-boson system. This means that Eq. (8) is perturbative (though to infinite order) in the interdot coupling T_c .

Alternatively, one can perform a perturbation theory in the electron-boson coupling g_Q (weak coupling perturbative ‘‘PER’’ approach). In a calculation for an undriven double quantum dot, both approaches have been compared recently for the stationary current⁸¹ and the frequency-dependent current noise.⁹ For the spin-boson problem with $\Gamma_{R/L} = 0$, it is well known that POL is equivalent to a double-path integral ‘‘noninteracting blip approximation’’ (NIBA) that works well for zero bias $\varepsilon = 0$ but for $\varepsilon \neq 0$ does not coincide with PER at small couplings and very low temperatures. PER works in the correct bonding and antibonding eigenstate basis of the hybridized system, whereas the energy scale ε in POL is that of the two isolated dots ($T_c = 0$). This difference reflects the general dilemma of two-level-boson Hamiltonians: either one is in the correct basis of the hybridized two-level system and perturbative in g_Q , or one starts from the ‘‘shifted oscillator’’ polaron picture that becomes correct for $T_c = 0$. In fact, the polaron (NIBA) approach does not coincide with standard damping theory⁸² because it does not incorporate the square root, nonperturbative in T_c hybridization form of the level splitting $\Delta = \sqrt{\varepsilon^2 + 4T_c^2}$. However, for large $|\varepsilon| \gg T_c$, $\Delta \rightarrow |\varepsilon|$, and POL and PER turn out to agree very well for the undriven case.⁸¹

C. Stationary quantities

In a quantum system that is continuously driven by an external, time-dependent source, stationary quantities can be defined for expectation values approaching a fixed point or a quasistationary, periodic motion for large times t . In particular, we will be interested in quantities like the time averaged electronic current. It is then useful to split the time dependent part off $\varepsilon(t)$ as

$$\varepsilon(t) = \varepsilon + \tilde{\varepsilon}(t), \quad (10)$$

and to introduce the Laplace transform $\hat{f}(z) = \int_0^\infty dt e^{-zt} f(t)$ of a function $f(t)$. The time evolution of the isolated spin-boson system for $T_c = 0$ is governed by the correlation function $C(t) = C^*(-t)$. The Laplace transform of these,

$$\hat{C}_\varepsilon(z) \equiv \int_0^\infty dt e^{-zt} e^{i\varepsilon t} C(t),$$

$$\hat{C}_\varepsilon^*(z) \equiv \int_0^\infty dt e^{-zt} e^{-i\varepsilon t} C^*(t), \quad (11)$$

defines free propagators for quasiparticles in the uncoupled dots and in absence of coupling to electron reservoirs. In absence of electron-boson coupling, this simply describes the free time evolution of a particle described by the diagonal Hamiltonian $\varepsilon \hat{\sigma}_z$, whereas for nonzero boson coupling these become ‘‘dressed’’ polarons. In addition, the decay via the right reservoir at rate Γ_R leads to a finite quasiparticle lifetime and consequently a renormalization of the propagators as

$$\begin{aligned} \hat{D}_\varepsilon(z) &\equiv \frac{\hat{C}_\varepsilon(z)}{1 + \Gamma_R \hat{C}_\varepsilon(z)/2}, & \hat{E}_\varepsilon(z) &\equiv \frac{\hat{C}_{-\varepsilon}^*(z)}{1 + \Gamma_R \hat{C}_\varepsilon(z)/2}, \\ \hat{D}_\varepsilon^*(z) &\equiv \frac{\hat{C}_\varepsilon^*(z)}{1 + \Gamma_R \hat{C}_\varepsilon^*(z)/2}, & \hat{E}_\varepsilon^*(z) &\equiv \frac{\hat{C}_{-\varepsilon}(z)}{1 + \Gamma_R \hat{C}_\varepsilon^*(z)/2}. \end{aligned} \quad (12)$$

These expressions appear in the calculation in Appendix A, where Eq. (8) is solved for the coherences $\langle p \rangle$ and $\langle p^\dagger \rangle$ in order to obtain two closed equations for the occupancies $\langle n_{L/R} \rangle$,

$$\begin{aligned} z \hat{n}_L(z) - \langle n_L \rangle_0 &= - \int_0^\infty dt e^{-zt} [\langle n_L \rangle_t \hat{K}(z, t) - \langle n_R \rangle_t \hat{G}(z, t)] \\ &\quad + \Gamma_L \left[\frac{1}{z} - \hat{n}_L(z) - \hat{n}_R(z) \right], \\ z \hat{n}_R(z) - \langle n_R \rangle_0 &= \int_0^\infty dt e^{-zt} [\langle n_L \rangle_t \hat{K}(z, t) - \langle n_R \rangle_t \hat{G}(z, t)] \\ &\quad - \Gamma_R \hat{n}_R(z), \\ \hat{K}(z, t) &\equiv \int_0^\infty dt' e^{-zt'} [T_c(t+t') T_c^*(t) D_\varepsilon(t')] \\ &\quad + T_c^*(t+t') T_c(t) D_\varepsilon^*(t')], \\ \hat{G}(z, t) &\equiv \int_0^\infty dt' e^{-zt'} [T_c(t+t') T_c^*(t) E_\varepsilon(t')] \\ &\quad + T_c^*(t+t') T_c(t) E_\varepsilon^*(t')], \end{aligned} \quad (13)$$

where here and in the following we omit the $\langle \dots \rangle$ in the Laplace transformed expectation values to simplify the notation, and we defined

$$T_c(t) \equiv T_c e^{+i \int_0^t ds \tilde{\varepsilon}(s)}, \quad T_c^*(t) \equiv T_c e^{-i \int_0^t ds \tilde{\varepsilon}(s)}. \quad (14)$$

Up to here the transformations have been valid for an arbitrary time dependence in $\varepsilon(t)$. From now on, we specify to the time-periodic form

$$\varepsilon(t) = \varepsilon(t + 2\pi/\Omega), \quad (15)$$

where $2\pi/\Omega$ is the period of the time-dependent field (we further specify to a sinusoidal time dependence of $\varepsilon(t)$ below).

We expect the system to approach an asymptotic quasi-stationary state. Then, the time evolution of all quantities $f(t)$ can be decomposed into Fourier series

$$f(t) \rightarrow f^{as}(t) = \sum_n e^{-in\Omega t} f_n, \quad (16)$$

with multiples of the angular frequency Ω of the external field. Following Grifoni and Hänggi,⁷⁴ we decompose $\hat{K}(z, t)$ and $\hat{G}(z, t)$ into Fourier series,

$$\begin{aligned} \hat{K}(z, t) &= \sum_m K_m(z) e^{-im\Omega t}, \\ \hat{G}(z, t) &= \sum_m G_m(z) e^{-im\Omega t}. \end{aligned} \quad (17)$$

The corresponding Fourier expansions $\langle n_L \rangle_t^{asy} \equiv \sum_m \nu_m e^{-im\Omega t}$ and $\langle n_R \rangle_t^{asy} \equiv \sum_m \mu_m e^{-im\Omega t}$ of the asymptotic occupancies can then easily be Laplace transformed,

$$\hat{n}_L^{asy}(z) = \sum_m \frac{\nu_m}{z + im\Omega}, \quad \hat{n}_R^{asy}(z) = \sum_m \frac{\mu_m}{z + im\Omega} \quad (18)$$

and inserted back into Eq. (13). Comparing the complex poles at $z = -im\Omega$ in the two equations for $\hat{n}_L(z)$ and $\hat{n}_R(z)$ and assuming that $K_m(z)$ and $G_m(z)$ are regular there, one obtains an infinite system of linear equations for the Fourier coefficients ν_m and μ_m ,

$$\begin{aligned} -im\Omega \nu_M &= -\sum_n [\nu_n K_{M-n}(-im\Omega) - \mu_n G_{M-n}(-im\Omega)] \\ &\quad + \Gamma_L [\delta_{M,0} \nu_M - \mu_M], \\ [\Gamma_R - im\Omega] \mu_M &= \sum_n [\nu_n K_{M-n}(-im\Omega) \\ &\quad - \mu_n G_{M-n}(-im\Omega)]. \end{aligned} \quad (19)$$

Upon adding these two equations, one has

$$-\frac{\mu_M}{\nu_M} \equiv r_M \equiv \left[1 + \frac{\Gamma_R}{\Gamma_L - im\Omega} \right]^{-1}, \quad M \neq 0, \quad (20)$$

and Eq. (19) can be transformed into a single matrix equation for the coefficients ν_n ,

$$\sum_{n=-\infty}^{\infty} A_{mn} \nu_n = b_m, \quad (21)$$

where

$$\begin{aligned} A_{mn} &\equiv (\Gamma_L - in\Omega - r_n \Gamma_L) \delta_{m,n} + K_{m-n}(-im\Omega) \\ &\quad + r_n G_{m-n}(-im\Omega), \\ b_m &\equiv \frac{\Gamma_L \Gamma_R}{\Gamma_R + \Gamma_L} \delta_{m,0} + \frac{\Gamma_L}{\Gamma_R + \Gamma_L} G_m(-im\Omega). \end{aligned} \quad (22)$$

D. Charge current

In the Master equation approach, the expectation values of the electron current through the double dot is obtained in a fairly easy manner. One has to consider the average charge flowing through one of the three intersections, i.e., left lead/left dot, left dot/right dot, and right dot/right lead. This gives rise to the three corresponding electron currents $I_L(t)$, $I_R(t)$, and the interdot current $I_{LR}(t)$. From the equations of motion, Eq. (8), one recognizes that the temporal change of the occupancies $\langle n_{L/R} \rangle_t$ is due to the sum of an ‘‘interdot’’ current $\propto T_c$ and a ‘‘lead-tunneling’’ part. Specifically, the current from left to right through the left (right) tunnel barrier is

$$\begin{aligned} I_L(t) &= -e\Gamma_L \langle n_0 \rangle_t = -e\Gamma_L [1 - \langle n_L \rangle_t - \langle n_R \rangle_t], \\ I_R(t) &= -e\Gamma_R \langle n_R \rangle_t, \end{aligned} \quad (23)$$

and the interdot current is

$$\begin{aligned} I_{LR}(t) &= -ieT_c \{ \langle p \rangle_t - \langle p^\dagger \rangle_t \} = -e \frac{\partial}{\partial t} \langle n_R \rangle_t + I_R(t) \\ &= e \frac{\partial}{\partial t} \langle n_L \rangle_t + I_L(t). \end{aligned} \quad (24)$$

In the stationary case, all the three currents are the same: adding the two equations, Eq. (19), for $M=0$, we first obtain

$$\Gamma_R \mu_0 = \Gamma_L (1 - \mu_0 - \nu_0). \quad (25)$$

Using furthermore the Fourier expansion of $\langle n_{L/R} \rangle_t^{asy}$, we recognize from Eqs. (23) and (24) that

$$\bar{I} = \overline{I_L(t)} = \overline{I_{LR}(t)} = \overline{I_R(t)} = -e\Gamma_R \mu_0, \quad (26)$$

where the bar denotes the temporal average of the asymptotic quantities over one period $\tau \equiv 2\pi/\Omega$. This simple result means that the stationary current is determined by the Fourier component μ_0 only. Note, however, that μ_0 is part of the solution of an infinite set of the linear equations, Eq. (21). Using Eqs. (19) and (20), one can express μ_0 in terms of $\mu_{N \neq 0}$ for the alternative expression

$$\bar{I} = -e\Gamma_R \frac{K_0(0) - \sum_{n \neq 0} [K_{-n}(0)/r_n + G_{-n}(0)] \mu_n}{\Gamma_R + K_0(0)/r_0 + G_0(0)}. \quad (27)$$

This form is in particular useful for the discussion of the Tien-Gordon limit below.

E. Sinusoidal time dependence

In the following, we specify the time dependence of the bias $\varepsilon(t)$ to a monochromatic sinusoidal field

$$\varepsilon(t) = \varepsilon + \Delta \sin(\Omega t), \quad (28)$$

where the constant part is denoted by ε . We introduce the notation,

$$\begin{aligned} \hat{C}_\varepsilon(-i\omega) &= \hat{C}_{\varepsilon+\omega}(z=0^+) \equiv \hat{C}_{\varepsilon+\omega}, \\ \hat{C}_\varepsilon^*(-i\omega) &= \hat{C}_{\varepsilon-\omega}^*(z=0^+) \equiv \hat{C}_{\varepsilon-\omega}^*, \end{aligned} \quad (29)$$

and correspondingly for the propagators D , D^* , E , and E^* , Eq. (12). Then, invoking the decomposition of the phase factor into Bessel functions,

$$\begin{aligned} T_c(t+t')T_c^*(t') &= T_c^2 e^{i\int_{t'}^{t+t'} ds \Delta \sin(\Omega s)}, \\ &= T_c^2 \sum_{nn'} i^{n'-n} J_n\left(\frac{\Delta}{\Omega}\right) J_{n'}\left(\frac{\Delta}{\Omega}\right) \\ &\quad \times e^{-in\Omega t'} e^{-i(n-n')\Omega t}, \end{aligned} \quad (30)$$

and the definitions of the Fourier components $K_m(z)$ and $G_m(z)$, cf. Eqs. (13) and (17), one obtains

$$\begin{aligned} K_m(-im'\Omega) &= i^{-m} T_c^2 \sum_n \left[J_n\left(\frac{\Delta}{\Omega}\right) J_{n-m}\left(\frac{\Delta}{\Omega}\right) \hat{D}_{\varepsilon+(m'-n)\Omega} \right. \\ &\quad \left. + J_n\left(\frac{\Delta}{\Omega}\right) J_{n+m}\left(\frac{\Delta}{\Omega}\right) \hat{D}_{\varepsilon-(m'+n)\Omega}^* \right], \\ G_m(-im'\Omega) &= i^{-m} T_c^2 \sum_n \left[J_n\left(\frac{\Delta}{\Omega}\right) J_{n-m}\left(\frac{\Delta}{\Omega}\right) \hat{E}_{\varepsilon+(m'-n)\Omega} \right. \\ &\quad \left. + J_n\left(\frac{\Delta}{\Omega}\right) J_{n+m}\left(\frac{\Delta}{\Omega}\right) \hat{E}_{\varepsilon-(m'+n)\Omega}^* \right]. \end{aligned} \quad (31)$$

III. ANALYTICAL RESULTS

In the following, we first discuss the limits where analytical results for the stationary current \bar{I} can be obtained, and then turn to a comparison with numerical calculations.

A. Time-independent case

For $\Delta=0$, i.e., in absence of the time-dependent (driving) part in $\varepsilon(t)$, we recover previous results⁸⁰ for stationary transport in dissipative double quantum dots. One then has $\hat{K}(z,t) = \hat{K}(z)$ and $\hat{G}(z,t) = \hat{G}(z)$ such that $K_n(z) = G_n(z) = 0$ for $n \neq 0$. Using $K_0(0) = 2\text{Re}[T_c^2 \hat{C}_\varepsilon / (1 + \Gamma_R \hat{C}_\varepsilon / 2)]$, together with $G_0(0) = 2\text{Re}T_c^2 [\hat{C}_\varepsilon^* / (1 + \Gamma_R \hat{C}_\varepsilon / 2)]$, after some algebra we rederive the previous result⁸⁰ for the stationary current,

$$\bar{I} = -eT_c^2 \frac{2\text{Re}(\hat{C}_\varepsilon) + \Gamma_R |\hat{C}_\varepsilon|^2}{|1 + \Gamma_R \hat{C}_\varepsilon / 2|^2 + 2T_c^2 B_\varepsilon},$$

$$B_\varepsilon \equiv \text{Re} \left\{ \left(1 + \Gamma_R \hat{C}_\varepsilon / 2 \right) \left[\frac{\hat{C}_{-\varepsilon}}{\Gamma_R} + \frac{\hat{C}_\varepsilon^*}{\Gamma_L} \left(1 + \frac{\Gamma_L}{\Gamma_R} \right) \right] \right\} \quad (32)$$

(note the absence of the factor 2 in the definition of the rates here⁸⁰). The result, Eq. (32), which can be compared⁸¹ to an alternative derivation using perturbation theory in the boson coupling α , generalizes the case of elastic tunneling through double quantum dots to inelastic tunneling with coupling to an arbitrary bosonic heat bath. For $\alpha=0$, we rederive the Stoof-Nazarov expression for the stationary current without dissipation,³³

$$\bar{I}_{\alpha=0} = -e \frac{T_c^2 \Gamma_R}{\varepsilon^2 + \Gamma_R^2 / 4 + T_c^2 (2 + \Gamma_R / \Gamma_L)}. \quad (33)$$

B. Lowest order T_c^2 : Tien-Gordon result

In the time dependent case, we are able to derive analytical results by considering the limit of small interdot coupling T_c , or large frequencies Ω . These two limits do not yield identical results because apart from T_c and Ω , there are four other energy scales (bias ε , rates Γ_L , Γ_R , boson cutoff ω_c) in the problem.

Considering Eq. (33) for the undriven, nondissipative current, lowest order perturbation theory in T_c is valid for $T_c \sqrt{2 + \Gamma_R / \Gamma_L} \ll \Gamma_R, |\varepsilon|$. The additional energy scale Ω due to ac driving requires that this condition is generalized to

$$T_c \sqrt{2 + \frac{\Gamma_R}{\Gamma_L}} \ll \Omega, \Gamma_R, |\varepsilon + n\Omega|, \quad n = \pm 0, 1, 2, \dots, \quad (34)$$

which indicates that at the resonance points $\varepsilon = n\Omega$ such a perturbation theory must break down, as is corroborated by our numerical results discussed below.

Considering the expression for μ_M in Eq. (19), one recognizes that $\mu_M = O(T_c^2)$ because the Fourier components of the functions K and G are proportional to T_c^2 , cf. Eq. (13). Owing to the full expression, Eq. (27), the stationary current in lowest order of T_c is $\bar{I} = \bar{I}^{\text{TG}} + O(T_c^4)$ with

$$\bar{I}^{\text{TG}} \equiv -eK_0(0). \quad (35)$$

For a sinusoidal $\varepsilon(t) = \varepsilon + \Delta \sin(\Omega t)$, the explicit expression, Eq. (31), yields

$$\bar{I}^{\text{TG}} = -eT_c^2 \sum_n J_n^2\left(\frac{\Delta}{\Omega}\right) \text{Re} \left(\frac{2C_{\varepsilon+n\Omega}}{1 + \frac{\Gamma_R}{2} C_{\varepsilon+n\Omega}} \right). \quad (36)$$

Note that Eq. (36) is the Tien-Gordon formula. This can be easily demonstrated by expanding the nondriven stationary current, Eq. (32), to lowest order in T_c , namely, $\bar{I} = \bar{I}_0 + O(T_c^4)$, such that, for the driven case

$$\bar{I}^{\text{TG}} \equiv \sum_n J_n^2\left(\frac{\Delta}{\Omega}\right) \bar{I}_0|_{\varepsilon \rightarrow \varepsilon + n\Omega}. \quad (37)$$

To lowest order in T_c , the stationary current therefore is given by the Tien-Gordon formula: the current in the driven system is expressed by a sum over current contributions from sidebands $\varepsilon + n\Omega$, weighted with squares of Bessel functions. Note that the perturbative result $\bar{T}^{\text{TG}} \equiv -eK_0(0)$, Eq. (35), does not refer to any specific form of the periodic function $\varepsilon(t)$; it is valid for arbitrary periodic driving when the corresponding Fourier component $K_0(0)$ is used.

C. Nonadiabatic approximation

This approximation assumes that the frequency Ω is the largest energy scale in the problem,

$$\Omega \gg T_c, \varepsilon, \Gamma_R, \Gamma_L. \quad (38)$$

On the rhs of the integral equation, Eq. (13), for $\hat{n}_{L/R}(z)$, one then replaces the integral kernels $\hat{K}(z, t)$ and $\hat{G}(z, t)$ by their averages over one period of the ac field,

$$\hat{K}(z, t) \rightarrow \frac{\Omega}{2\pi} \int_0^{2\pi/\Omega} dt \hat{K}(z, t) \equiv K_0(z) \quad (39)$$

and similarly for $\hat{G}(z, t)$. The Fourier coefficients $K_n(z)$ and $G_n(z)$ with $n \neq 0$ then vanish and one obtains $\bar{T} \approx \bar{T}^{\text{fast}}$, where

$$\bar{T}^{\text{fast}} \equiv \frac{-e\Gamma_R K_0(0)}{\Gamma_R + G_0(0) + K_0(0)[1 + \Gamma_R/\Gamma_L]}. \quad (40)$$

We observe that within lowest order of the static tunneling T_c , Eq. (40) coincides with the Tien-Gordon expression, Eq. (35), which one obtains by setting $G_0(0) \propto T_c^2$ and $K_0(0) \propto T_c^2$ to zero in the denominator of Eq. (40). In fact, for the undriven case $\Delta = 0$ one can prove⁸¹ that the expression for the stationary current sums up an infinite number of terms $\propto T_c^2$, a fact that can be traced back to the integral equation structure of the underlying master equation. Here, Eq. (40) demonstrates that a similar summation effectively can be achieved in the ac driven case.

D. Higher-order corrections to Tien-Gordon

In order to systematically go beyond the Tien-Gordon approximation, Eq. (35), one has to perform an expansion of the current in powers of T_c^2 . This can be achieved by *truncating* the infinite set of linear equations, Eq. (22), in order to obtain approximations for the $n=0, \pm 1, \pm 2$ th sideband values of ν_n and μ_n . The simplest way to do this in practice is by a numerical solution of these equations as discussed below.

Barata and Wreszinski⁸³ have considered higher order corrections to dynamical localization in a *closed*, coherent two-level system, i.e., without coupling to external electron reservoirs or dissipation. They found that the next order in perturbation theory given a contribution different from zero was the third-order one, giving a contribution to a renormalization of the tunnel coupling T_c :

$$\delta T_c^{(3)} \equiv -\frac{2T_c^3}{\Omega^2} \times \sum_{n_1, n_2 \in \mathbb{Z}} \frac{J_{2n_1+1}\left(\frac{\Delta}{\Omega}\right) J_{2n_2+1}\left(\frac{\Delta}{\Omega}\right) J_{-2(n_1+n_2+1)}\left(\frac{\Delta}{\Omega}\right)}{(2n_1+1)(2n_2+1)}. \quad (41)$$

We now recall our expression

$$K_0(0) = \sum_n \left[T_c J_n\left(\frac{\Delta}{\Omega}\right) \right]^2 2\text{Re}D_{\varepsilon+n\Omega} \quad (42)$$

(and $G_0(0)$ correspondingly with $D_{\varepsilon+n\Omega}$ replaced by $E_{\varepsilon+n\Omega}$), cf. Eq. (31), which enters the Tien-Gordon result, Eq. (35), and the resummed nonadiabatic approximation Eq. (40). We use the renormalized T_c , Eq. (41), in order to define a renormalized function $K_0^{(3)}(0)$,

$$K_0^{(3)}(0) \equiv \sum_n \left[T_c J_n\left(\frac{\Delta}{\Omega}\right) + \delta T_c^{(3)} \right]^2 2\text{Re}D_{\varepsilon+n\Omega}, \quad (43)$$

and $G_0^{(3)}(0)$ correspondingly. This yields an expression for the current, renormalized up to third order in T_c , according to

$$\bar{T}^{(3)} \equiv \frac{-e\Gamma_R K_0^{(3)}(0)}{\Gamma_R + G_0^{(3)}(0) + K_0^{(3)}(0)[1 + \Gamma_R/\Gamma_L]}. \quad (44)$$

In the following, we discuss and compare our above results.

IV. DISCUSSION

A. Comparison of two numerical schemes

In order to numerically solve the integro-differential system, Eq. (8), it is convenient to write

$$\exp\left(i \int_{t'}^t ds \varepsilon(s)\right) \equiv e^{i\varphi_t} e^{-i\varphi_{t'}}, \quad (45)$$

with $\varphi_t \equiv \varepsilon t - (\Delta/\Omega)\cos\Omega t$, remembering our choice $\varepsilon(t) = \varepsilon + \Delta \sin\Omega t$. We then introduce the real and imaginary part of $\langle p \rangle$, use $e^{ix} = \cos x + i \sin x$, and specify to the Ohmic dissipation case for $C(t)$,

$$C(t) = |C(t)| e^{-i\Psi_t}, \quad \Psi_t = 2\alpha \arctan \omega_c t, \\ |C(t)| = [1 + (\omega_c t)^2]^{-\alpha} \left| \frac{\Gamma(1 + 1/\beta\omega_c + it/\beta)}{\Gamma(1 + 1/\beta\omega_c)} \right|^{4\alpha}. \quad (46)$$

We have solved Eqs. (8) numerically as a function of time, with the result for large times used to obtain the stationary current as a function of ε . For each value of ε , the time dependent equations have been solved up to a fixed final time t_f with a subsequent time average over the interval $[t_f - \Delta t, t_f]$. t_f has to be chosen sufficiently large, in particular, for larger values of α . Consequently, one then also has to increase the number of steps to achieve sufficient accuracy of

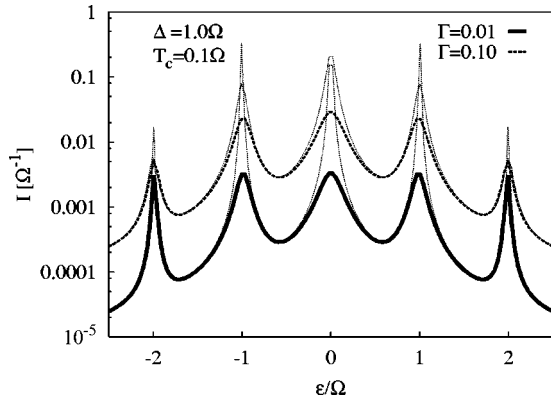


FIG. 1. Average current through double dot in Coulomb blockade regime with bias $\varepsilon + \Delta \sin \Omega t$. Coupling to left and right leads $\Gamma_L = \Gamma_R = \Gamma$. Dotted lines indicate Tien-Gordon result, Eq. (37).

the data. We have used these numerical results to check our method for the stationary quantities as obtained from truncating Eq. (22) at a finite photosideband number, and found good agreement between both methods. Whereas the direct integration of the equations of motion is somewhat slower than the truncation method, it has the advantage that it does not require analytic forms of the Laplace transform for the bosonic correlation functions \hat{C}_ε , Eqs. (11) and (29). The latter are required for the matrix scheme Eq. (22). In Appendix B we derive explicit expressions for zero temperature ($T=0$) and Ohmic dissipation. Note that in contrast to usual “ $P(E)$ ” theory, we require both the real part $\text{Re}[\hat{C}_\varepsilon(0)] = \pi P(\varepsilon)$ [where $P(\varepsilon)$ is the probability for inelastic tunneling with energy transfer ε ⁸²], and the imaginary part of \hat{C}_ε .

In the following, we show numerical results obtained with the truncation method.

B. Photosidebands (coherent case)

1. Comparison with Tien-Gordon approximation

In Fig. 1, we compare the exact numerical result for the average stationary current with the Tien-Gordon expression, Eq. (37), in the coherent case $\alpha=0$. One clearly recognizes the symmetric photoside peaks which, according to Eq. (37), appear at $\pm n\hbar\varepsilon$. The Tien-Gordon approximation overestimates the current close to these resonances, where terms of higher order in T_c become important due to the nonlinearity (in T_c) of the exact bonding and antibonding energies $\pm\sqrt{\varepsilon^2 + 4T_c^2}$ of the isolated two-level system. This again confirms that the Tien-Gordon result is perturbative in the tunneling T_c .

2. RWA and Bloch-Siegert shift

Close to the first side peak, Stoof and Nazarov have used a rotation wave approximation (RWA) to obtain analytical predictions for the first current side peak. In this approximation, one transforms into an interaction picture where the fast-rotating terms with angular frequency $\pm\Omega$ are transformed away, and terms with higher rotation frequencies (such as $\pm 2\Omega$) are neglected. The resulting expression for the current is³³

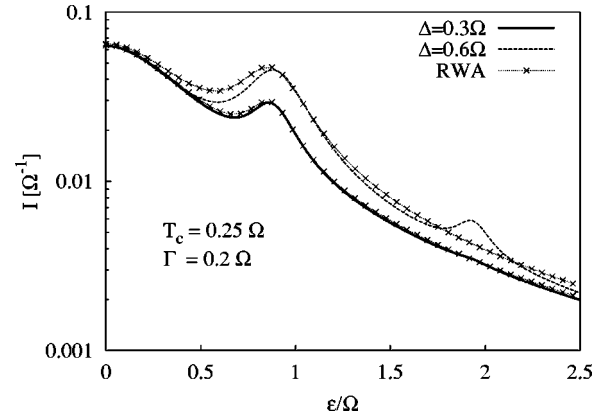


FIG. 2. Comparison between RWA, Eq. (47), and exact result for first current side peak.

$$I_{SN} = \frac{\Delta^2 \Gamma_R (a^2 - 4)}{c(c\Gamma_R^2 + b\Delta^2)} \frac{w^2}{w^2 + (\varepsilon - \varepsilon_r)^2}, \quad (47)$$

with the resonance point $\varepsilon_r \equiv \sqrt{\Omega^2 - 4T_c^2}$ and parameters $a = \Omega/T_c$, $b \equiv \Gamma_R/\Gamma_L + 2$, $c \equiv a^2 + b - 4$, and the half-width $w = (a/[2\sqrt{a^2 - 4}])\sqrt{\Gamma_R^2 + (b/c)\Delta^2}$. We compare I_{SN} with the exact result in Fig. 2.

For smaller driving amplitude Δ , the agreement is very good but becomes worse with increasing Δ . The position of the side-peak resonance point, which is independent of Δ in the Stoof-Nazarov approximation Eq. (47), starts to shift towards slightly larger values of the bias ε . In fact, for stronger ac driving the RWA is known to break down: in isolated two-level systems, the first corrections to the RWA lead to the well-known Bloch-Siegert shift⁸⁴ of the central resonance towards larger energies, which is consistent with the exact result in Fig. 2.

C. Dynamical localization and its lifting

In a quantum system driven by a periodic electric field, a phenomenon termed coherent destruction of tunneling (CDT) [also denoted dynamical localization (DL)] occurs under certain parameters of the external field.^{52,85} The periodicity of the external field allows to write the solutions of the Schrödinger equation as: $\psi(t) = \exp[-i\varepsilon t]\phi_j(t)$, where ε_j is called the quasienergy, and $\phi_j(t)$ is a function with the same period as the driving field: the Floquet state.

When two quasienergies approach degeneracy, the time scale for tunneling between the states diverges, producing the phenomenon of CDT.⁵² The time scale for localization is the inverse of the energy separation of the quasienergies.

In the case of an isolated two-level system driven by a monochromatic, sinusoidal field $\varepsilon(t) = \varepsilon + \Delta \sin(\Omega t)$, Eq. (28), CDT can be physically understood from the renormalization of the coupling T_c of the two levels,

$$T_c \rightarrow T_{c,\text{eff}} \equiv T_c J_0\left(\frac{\Delta}{\hbar\Omega}\right). \quad (48)$$

This expression is obtained from first-order perturbation theory in the tunneling T_c .⁸⁵ At the first zero of the Bessel

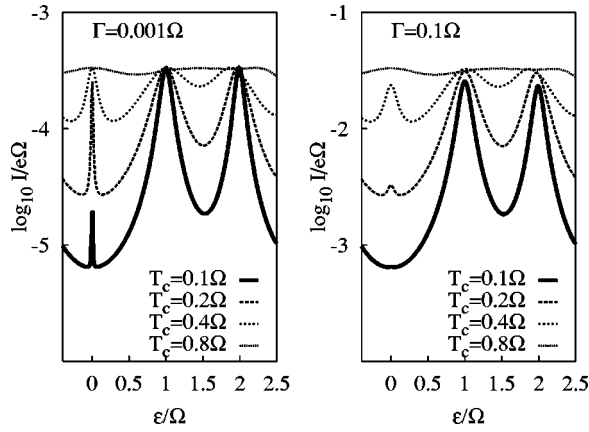


FIG. 3. Average current for ac driving amplitude $\Delta = z_0\Omega$ (z_0 first zero of Bessel function J_0) and various tunnel couplings T_c . Coupling to left and right leads $\Gamma_L = \Gamma_R = \Gamma$.

function J_0 , namely, when $\Delta/\hbar\Omega = 2.4048\dots$, the *effective tunnel splitting vanishes*, leading to a complete localization of the particle in the initial state.

In the following, we discuss how stronger tunnel amplitudes T_c , the coupling to the external leads, and dissipation modify this picture.

1. Current suppression

In Fig. 3, we show results for the average current and $\alpha = 0$ (no dissipation) in the DL regime. Here, we define this regime by $\Delta = z_0\Omega$, where $z_0 = 2.4048\dots$ is the first zero of the Bessel function J_0 . For this specific value of the ac driving Δ , to lowest order in T_c the average current is strongly suppressed for $|\varepsilon| \lesssim \Omega$ as compared with the undriven case $\Delta = 0$. For small T_c , this suppression is well described by the Tien-Gordon expression (not shown here): since at $\Delta = z_0\Omega$, the $n=0$ term in the sum, Eq. (37), is absent, the current is dominated by the shifted (undriven) current contributions at bias $\varepsilon + n\Omega$ with $|n| \geq 1$, which, however, are very small due to the resonance shape of the undriven current.

2. Central current peak and third-order result

Surprisingly, however, the coherent suppression of the current is *lifted* again very close to $\varepsilon = 0$, where a small and sharp peak appears. This peak becomes broader with increasing tunnel coupling T_c , but its height is suppressed for increasing reservoir coupling Γ , cf. Fig. 3 right. This feature is analyzed in Fig. 4, where we show results for the central current peak around $\varepsilon = 0$ in the DL regime for coherent ($\alpha = 0$, left) and incoherent ($\alpha > 0$, right) tunneling. As one recognizes, the Tien-Gordon description (which is perturbative in the tunnel coupling T_c) breaks down close to $\varepsilon = 0$ where higher order terms in T_c become important. As a matter of fact, for $\varepsilon = 0$ the only relevant energy scale of the isolated two-level systems is T_c itself. In contrast, the *third-order approximation*, Eq. (44), reproduces very well the additional peak at $\varepsilon = 0$, which indicates the importance of higher-order terms in that regime. At $\varepsilon = 0$, the charge be-

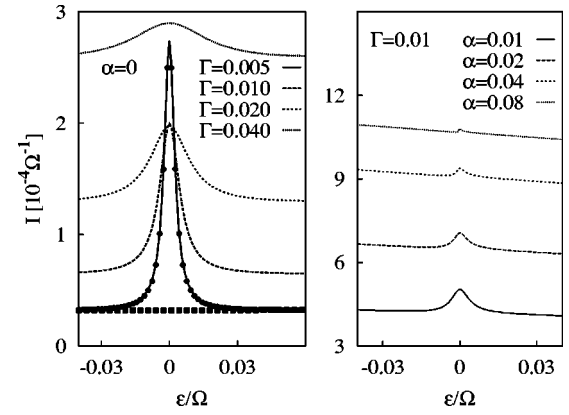


FIG. 4. Central peak of average current through ac driven double quantum dot. Parameters $T_c = 0.1$, $\Delta = z_0\Omega$ (all rates in units of Ω). Left: coherent case $\alpha = 0$ for different tunnel rates $\Gamma = \Gamma_L = \Gamma_R$, dots indicate third order results Eq. (44), squares indicate the Tien-Gordon result Eq. (37) for the case $\Gamma = 0.005$. Right: disappearance of central peak with increasing dissipation α .

tween the two dots is strongly delocalized in the undriven case, and this tunneling-induced quantum coherence persists into the strongly driven regime where its signature is a “lifting” of the DL close to $\varepsilon = 0$.

The width of the corresponding current peak is determined by the tunneling rate Γ . An increase of incoherent electron tunneling from the left lead therefore washes out the coherent lifting of the DL. This argument is emphasized in the right part of Fig. 4 which shows that the central peak in the DL regime vanishes for increasing dissipation strength α .

D. Dissipation and average current

1. Dissipative photosidebands

As mentioned above, for simplicity we restrict ourselves to an Ohmic dissipative bath at zero temperature ($T = 0$) in this paper, leaving the finite-temperature case or the case of more complicated spectral functions $J(\omega)$ for future work.

For $\Delta = 0$, we reproduce the analytical result, Eq. (32), and the corresponding inelastic current part for $\varepsilon > 0$ due to spontaneous boson emission.^{3,80} In Fig. 5, we show the stationary current as a function of bias ε for various Ohmic dissipation strengths α at zero temperature and finite ac driving amplitudes Δ . For $\Delta = \Omega$, apart from the central resonant tunneling peak, sidebands at $\varepsilon = n\Omega$ appear which reproduce the asymmetry of the central peak around $\varepsilon = 0$. This asymmetry is a clear signature of the coupling to the dissipative environment strongly modifying the current even at zero temperature.

The specific form of the inelastic current depends on the boson spectral density $J(\omega)$.⁸⁰ Note that in general, there is no monotonic dependence on the dissipation strength α since the boson correlation function \hat{C}_ε appears both in the denominator and the numerator of the expression for the current Eq. (32).

2. Dissipation and dynamical localization in the current

If the ac driving amplitude Δ is increased towards $z_0\Omega$ (z_0 is the first zero of the Bessel function J_0), one expects to

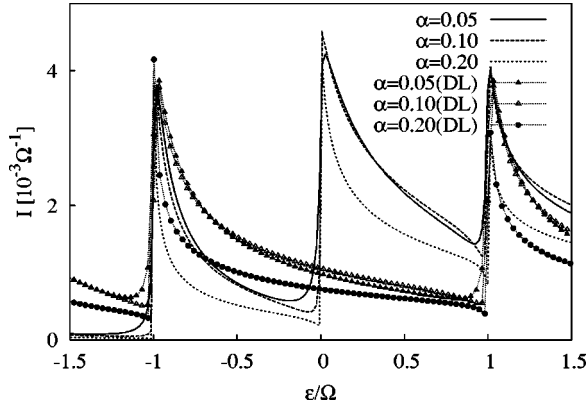


FIG. 5. Average current through double dot in Coulomb blockade regime with bias $\varepsilon + \Delta \sin \Omega t$ for various Ohmic dissipation strengths α at zero temperature. Driving amplitude $\Delta = \Omega$ for lines without symbols, $\Delta = z_0 \Omega$ (z_0 first zero of Bessel function J_0) for lines with symbols. Tunnel coupling between dots $T_c = 0.1\Omega$, bath cutoff $\omega_c = 500\Omega$, and lead tunnel rates $\Gamma_L = \Gamma_R = 0.01\Omega$.

enter the regime of dynamical localization and a strong suppression of the central current peak. In the coherent case $\alpha = 0$ (see above), resonant tunneling is usually strongly inhibited due to coherent suppression of tunneling.

For $\alpha > 0$, however, we find that the current suppression strongly depends on the static bias ε : we find suppression for $\varepsilon > 0$ and, in general, *larger* values of the current for $\varepsilon < 0$ as compared to the case of smaller ac amplitudes Δ . We explain this feature in the following: the dependence of the average current on the driving amplitude Δ for fixed α is clearly visible in Fig. 6. A small driving amplitude $\Delta \leq 0.2$ nearly does not change the current at all. However, the originally strongly asymmetric current curve becomes flattened out when Δ is tuned to larger values up to the dynamical localization value $\Delta = z_0 \Omega$. There, the ac field nearly completely destroys the strong asymmetry between the spontaneous emission ($\varepsilon > 0$) and the absorption side ($\varepsilon < 0$) of the current. The central $n=0$ photoband is completely suppressed and the dominant contribution to the current comes from the $n = \pm 1$ bands. For $\varepsilon < 0$, the current for $\Omega > |\varepsilon|$ is due to

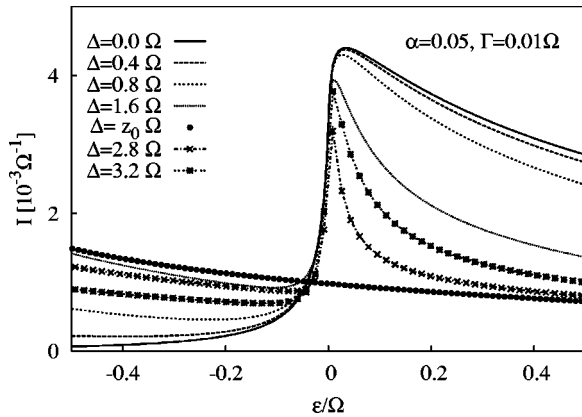


FIG. 6. Average current through driven double dot for various ac driving amplitudes Δ and fixed dissipation $\alpha = 0.05$, tunnel coupling $T_c = 0.1\Omega$.

photoexcitation of the electron into the first upper photosidebands and subsequent spontaneous emission of bosons of energy $E_1 \equiv \Omega - |\varepsilon|$ to the bath. In contrast, for $\Omega > \varepsilon > 0$, photon emission blocks the current because at $T=0$ there is no absorption of bosons from the bath. The remaining photon absorption channel then leads to boson emission at an energy $E_2 \equiv \Omega + \varepsilon$, which is larger as compared to the case for $\varepsilon < 0$, namely, $E_2 > E_1$, and therefore has a smaller probability $P(E) \propto E^{2\alpha-1} e^{-E/\omega_c}$, cf. Eq. (B9), leading to a smaller current. A similar argument can be used to explain why the current *increases* as one reduces ε , say from $\varepsilon/\Omega = 0$ to $\varepsilon/\Omega = -0.5$. In particular, the strongest effect of the dissipative bath occur near one-photon resonance conditions, i.e., when $\varepsilon/\Omega \approx \pm 1$, where the current is regulated by the spectral function of the bosonic bath at very low frequencies, either by absorption ($\varepsilon < 0$) or emission ($\varepsilon > 0$) of a photon. These processes appear in the current as nonanalytic cusps reflecting the power-law behavior of $P(E)$. This has to be compared with the Lorentzian shape of the photosidebands in the absence of dissipation (Fig. 1). If one tunes to even larger values of $\Delta > z_0 \Omega$, the central $n=0$ photoband reappears and the original strong asymmetry of the current curve is restored.

V. CONCLUSION AND OUTLOOK

Our results suggest that the combination of ac fields and dissipation in double quantum dots leads to a rich variety of nontrivial effects. In particular, we have shown that a time dependent monochromatic field drastically modifies the dissipative inelastic stationary current, in particular, for stronger ac driving in the dynamical localization regime. Corrections to the Tien-Gordon formula appear at larger tunnel coupling between the dots and become extremely important near zero bias in the DL regime, also in the nondissipative case.

The method presented in this work has the benefit of accounting for an arbitrary dissipative environment via the correlation function \hat{C}_ε . In the generic case, explicit analytical forms for this function are difficult to obtain and it might be easier to integrate the original equations of motion directly. Alternatively, one can numerically evaluate \hat{C}_ε and use it as an input into our Floquet-like formalism. We also mention that the entire approach is based on the decoupling of the bosonic degrees of freedom in the polaron transformed master equation. One is therefore always restricted to the range of validity of the NIBA of the original spin-boson problem.^{74,82} Discussing larger temperatures T should thus lead to more reliable results as compared to the “test models” C_ε which were discussed here for $T=0$.

A future extension of our approach should therefore be the derivation of a systematic perturbation theory in the electron-boson coupling, starting from the bonding-antibonding basis of the double dots. In a calculation for an undriven double quantum dot, such an approach has been successfully used recently to extract dephasing and relaxation times from the frequency-dependent noise spectrum.⁹

Even for the coherent case $\alpha = 0$, our results have shown that there are nontrivial effects due to the combined quantum coherence inherent in the double dot, and the coherence in-

duced by the external driving field. In particular, we found systematic corrections to standard approximation such as the Tien-Gordon formula or the rotating wave approximation. The constituting quantities K_m and G_m of our theory, cf. Eqs. (27) and (31), describe dissipative tunneling $\propto T_c^2$ of one additional quasiparticle between the two dots under the influence of the ac field, which again indicates that our approach is essentially perturbative in T_c , although to infinite order and exact for $\alpha=0$. We showed that partial resummations beyond the Tien-Gordon result are justified in a nonadiabatic, high-frequency approximation, but for the general case one has to rely on a systematic evaluation of Eq. (27).

ACKNOWLEDGMENTS

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APPENDIX A: DOT OCCUPANCIES IN LAPLACE SPACE

Here, we derive Eq. (13) for the occupancies $\langle n_{L/R} \rangle$. We define

$$q(t) \equiv \langle p \rangle_t e^{-i \int_0^t ds \tilde{\varepsilon}(s)}, \quad q^\dagger(t) \equiv \langle p^\dagger \rangle_t e^{+i \int_0^t ds \tilde{\varepsilon}(s)}. \quad (\text{A1})$$

This is inserted into the equations of motion in the time domain, Eq. (8), which upon Laplace transformation becomes

$$\begin{aligned} z \hat{n}_L(z) - \langle n_L \rangle_0 &= -i \int_0^\infty dt e^{-zt} \{ T_c(t) q(t) - T_c^*(t) q^\dagger(t) \} \\ &\quad + \Gamma_L \left[\frac{1}{z} - \hat{n}_L(z) - \hat{n}_R(z) \right], \\ z \hat{n}_R(z) - \langle n_R \rangle_0 &= i \int_0^\infty dt e^{-zt} \{ T_c(t) q(t) - T_c^*(t) q^\dagger(t) \} \\ &\quad - \Gamma_R \hat{n}_R(z), \\ \hat{q}(z) &= -\frac{\Gamma_0}{2} \hat{q}(z) \hat{C}_\varepsilon(z) - i \left[\int_0^\infty dt' e^{-zt'} T_c^*(t') [\langle n_L \rangle_{t'} \hat{C}_\varepsilon(z) \right. \\ &\quad \left. - \langle n_R \rangle_{t'} \hat{C}_{-\varepsilon}^*(z) \right], \\ \hat{q}^\dagger(z) &= -\frac{\Gamma_0}{2} \hat{q}^\dagger(z) \hat{C}_\varepsilon^*(z) + i \left[\int_0^\infty dt' e^{-zt'} T_c(t') \right. \\ &\quad \left. \times [\langle n_L \rangle_{t'} \hat{C}_\varepsilon^*(z) - \langle n_R \rangle_{t'} \hat{C}_{-\varepsilon}(z)] \right], \end{aligned} \quad (\text{A2})$$

where we used the convolution theorem in the equations for $\hat{q}(z)$ and $\hat{q}^\dagger(z)$ and the definitions Eq. (11). Using the defi-

nitions for the propagators D and E , Eq. (12), we obtain upon solving for $\hat{q}^{(\dagger)}(z)$ and Laplace back transforming,

$$\begin{aligned} q(t) &= -i \int_0^t dt' T_c^*(t') [\langle n_L \rangle_{t'} D_\varepsilon(t-t') - \langle n_R \rangle_{t'} E_\varepsilon(t-t')], \\ q^\dagger(t) &= i \int_0^t dt' T_c(t') [\langle n_L \rangle_{t'} D_\varepsilon^*(t-t') - \langle n_R \rangle_{t'} E_\varepsilon^*(t-t')], \end{aligned} \quad (\text{A3})$$

involving the propagators in the time domain. Insertion into Eq. (A2) yields

$$\begin{aligned} z \hat{n}_L(z) - \langle n_L \rangle_0 &= - \int_0^\infty dt e^{-zt} \int_0^t dt' \langle n_L \rangle_{t'} [T_c(t) T_c^*(t') \\ &\quad \times D_\varepsilon(t-t') + T_c^*(t) T_c(t') D_\varepsilon^*(t-t')] \\ &\quad + \int_0^\infty dt e^{-zt} \int_0^t dt' \langle n_R \rangle_{t'} [T_c(t) T_c^*(t') \\ &\quad \times E_\varepsilon(t-t') + T_c^*(t) T_c(t') E_\varepsilon^*(t-t')] \\ &\quad + \Gamma_L \left(\frac{1}{z} - \hat{n}_L(z) - \hat{n}_R(z) \right). \end{aligned} \quad (\text{A4})$$

At this point, it is useful to use a relation for a generalized convolution of a function $K(t, t')$ and $f(t')$,

$$\begin{aligned} \int_0^\infty dt e^{-zt} \int_0^t dt' K(t, t') f(t') \\ = \int_0^\infty dt e^{-zt} f(t) \int_0^\infty dt' e^{-zt'} K(t+t', t), \end{aligned} \quad (\text{A5})$$

which can be easily proven by substitutions. Note that the usual Laplace convolution theorem is recovered from Eq. (A5) if $K(t, t') = K(t-t')$ is only a function of the difference of its two arguments. Eq. (A4) and a similar equation for $\hat{n}_R(z)$ then lead to Eq. (13).

APPENDIX B: CALCULATION OF THE BOSON CORRELATION FUNCTION

Explicit expressions for the bosonic correlation functions \hat{C}_ε , Eqs. (11) and (29), which can be obtained in the zero temperature ($T=0$) case for Ohmic dissipation. In this case,

$$J(\omega) = 2\alpha\omega \exp(-\omega/\omega_c),$$

$$C(t) = (1 + i\omega_c t)^{-2\alpha}, \quad g \equiv 2\alpha. \quad (\text{B1})$$

We have

$$\begin{aligned} \hat{C}(z) &\equiv \int_0^\infty dt e^{-zt} (1 + i\omega_c t)^{-2\alpha} \\ &= (i\omega_c)^{-2\alpha} z^{2\alpha-1} e^{-iz/\omega_c} \Gamma(1-2\alpha, -iz/\omega_c), \end{aligned} \quad (\text{B2})$$

where we used Gradstein-Ryshik 3.3824 and Γ denotes the incomplete Gamma function. We set $\omega_c=1$ for a moment to simplify notations and obtain

$$\hat{C}(-i\varepsilon) = -i(-\varepsilon)^{2\alpha-1}e^{-\varepsilon}\Gamma(1-2\alpha, -\varepsilon). \quad (\text{B3})$$

Note that ε must have a small positive imaginary part here ($\text{Re}z>0$ in the definition of the Laplace transformation): the incomplete Gamma function $\Gamma(1-2\alpha, z)$ has a branch point at $z=0$. However, we can use the series expansion

$$\Gamma(1-2\alpha, x) = \Gamma(1-2\alpha) - \sum_{n=0}^{\infty} \frac{(-1)^n x^{1-2\alpha+n}}{n!(1-2\alpha+n)},$$

$$1-2\alpha \neq 0, -1, -2, \dots, \quad (\text{B4})$$

to obtain

$$\hat{C}(-i\varepsilon) = -i(-\varepsilon)^{2\alpha-1}e^{-\varepsilon}\Gamma(1-2\alpha)$$

$$+ ie^{-\varepsilon} \sum_{n=0}^{\infty} \frac{\varepsilon^n}{n!(1-2\alpha+n)}, \quad 2\alpha \neq 1, 2, 3, \dots$$

$$(\text{B5})$$

The second term is an analytic function of ε .

Now we write

$$-i(-\varepsilon)^{2\alpha-1} = \begin{cases} -i|\varepsilon|^{2\alpha-1}, & \varepsilon < 0 \\ \varepsilon^{2\alpha-1}e^{-\pi i(1/2+2\alpha-1)}, & \varepsilon > 0. \end{cases}$$

$$= \begin{cases} -i|\varepsilon|^{2\alpha-1}, & \varepsilon < 0 \\ \varepsilon^{2\alpha-1}(\sin 2\pi\alpha + i \cos 2\pi\alpha), & \varepsilon > 0. \end{cases}$$

$$(\text{B6})$$

Recall the reflection formula for the Gamma function,

$$\Gamma(1-z) = \frac{\pi}{\Gamma(z)\sin \pi z}. \quad (\text{B7})$$

This yields

$$\varepsilon > 0: \quad \hat{C}(-i\varepsilon) = \frac{\pi}{\Gamma(2\alpha)} \varepsilon^{2\alpha-1} e^{-\varepsilon}$$

$$+ i \left[\frac{\pi}{\Gamma(2\alpha)} \varepsilon^{2\alpha-1} e^{-\varepsilon} \cot 2\pi\alpha \right.$$

$$\left. + e^{-\varepsilon} \sum_{n=0}^{\infty} \frac{\varepsilon^n}{n!(1-2\alpha+n)} \right].$$

$$\varepsilon < 0: \quad \hat{C}(-i\varepsilon) = ie^{-\varepsilon} \left[-\frac{\pi}{\Gamma(2\alpha)\sin 2\pi\alpha} |\varepsilon|^{2\alpha-1} \right.$$

$$\left. + \sum_{n=0}^{\infty} \frac{\varepsilon^n}{n!(1-2\alpha+n)} \right]. \quad (\text{B8})$$

From this, we can read off the real and the imaginary part of $\hat{C}(-i\varepsilon)$. The real part is

$$\text{Re}\hat{C}(-i\varepsilon) \equiv \pi P(\varepsilon) = \frac{\pi}{\Gamma(2\alpha)} \varepsilon^{2\alpha-1} e^{-\varepsilon} \theta(\varepsilon). \quad (\text{B9})$$

The imaginary part is

$$\text{Im}\hat{C}(-i\varepsilon) \equiv e^{-\varepsilon} \left[\sum_{n=0}^{\infty} \frac{\varepsilon^n}{n!(1-2\alpha+n)} \right.$$

$$\left. + \frac{\pi|\varepsilon|^{2\alpha-1}}{\Gamma(2\alpha)\sin 2\pi\alpha} \begin{cases} -1, & \varepsilon < 0 \\ \cos 2\pi\alpha, & \varepsilon > 0 \end{cases} \right].$$

$$(\text{B10})$$

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