

# Interaction-induced dephasing of Aharonov-Bohm oscillations

T. Ludwig<sup>1</sup> and A. D. Mirlin<sup>1,2,\*</sup>

<sup>1</sup>*Institut für Nanotechnologie, Forschungszentrum Karlsruhe, 76021 Karlsruhe, Germany*

<sup>2</sup>*Institut für Theorie der Kondensierten Materie, Universität Karlsruhe, 76128 Karlsruhe, Germany*

(Received 12 December 2003; revised manuscript received 10 February 2004; published 25 May 2004)

We study the effect of the electron-electron interaction on the amplitude of mesoscopic Aharonov-Bohm oscillations in quasi-one-dimensional (Q1D) diffusive rings. We show that the dephasing length  $L_\phi^{\text{AB}}$  governing the damping factor  $\exp(-2\pi R/L_\phi^{\text{AB}})$  of the oscillations is parametrically different from the common dephasing length for the Q1D geometry. This is due to the fact that the dephasing is governed by energy transfers determined by the ring circumference  $2\pi R$ , making  $L_\phi^{\text{AB}}$   $R$ -dependent.

DOI: 10.1103/PhysRevB.69.193306

PACS number(s): 73.23.-b, 73.63.Nm, 73.20.Fz

The Aharonov-Bohm (AB) oscillations of conductance are one of the most remarkable manifestations of electron phase coherence in mesoscopic samples. Quantum interference of contributions of different electron paths in a ring threaded by a magnetic flux  $\Phi$  makes the conductance  $g$  an oscillatory function of  $\Phi$ , with a period of the flux quantum  $\Phi_0 = hc/e$ ; see Refs. 1–3 for reviews. In a diffusive ring these  $\Phi_0$ -periodic conductance oscillations are sample specific (and would vanish upon the ensemble averaging), due to a random phase associated with diffusive paths. In this respect, the  $\Phi_0$ -periodic AB effect is a close relative of mesoscopic conductance fluctuations.

Another type of the AB effect is induced by interference of time-reversed paths encircling the ring and is intimately related to the weak localization (WL) correction.<sup>2</sup> Its principal period is  $\Phi_0/2$ . It survives the ensemble averaging but is suppressed by a magnetic field penetrating the sample. Below we concentrate on the first (mesoscopic or  $\Phi_0$  periodic) AB effect. Our results are, however, applicable to the second (weak localization or  $\Phi_0/2$ -periodic) type of AB oscillations as well, as we discuss at the end of the paper.

In the present paper we will investigate the effect of the electron-electron interaction on the amplitude of the AB oscillations. Interaction-induced inelastic processes lead to dephasing of electrons, and thus to a damping of interference phenomena, in particular, of AB oscillations. The mesoscopic AB oscillations can thus serve as a “measuring device” for the electron decoherence. This idea was, in particular, implemented in recent experiments,<sup>4,5</sup> where the low-temperature behavior of the dephasing time  $\tau_\phi$  was studied, and two mechanisms of decoherence were identified: scattering off magnetic impurities<sup>6</sup> and electron-electron scattering.

Let  $g(\Phi)$  be the dimensionless (measured in units of  $e^2/h$ ) conductance of the ring. It is convenient to expand the conductance fluctuations  $\delta g(\Phi)$  as a Fourier series

$$\delta g(\Phi) = \delta g_0 + 2 \sum_{n=1}^{\infty} \delta g_n \cos(2\pi n \Phi / \Phi_0 + \theta_n). \quad (1)$$

Within the conventional approach, when the dephasing time  $1/\tau_\phi$  is introduced as a mass of the diffuson and cooperon propagators,  $\mathcal{P}_{D,C}(q, \omega) \sim 1/(Dq^2 - i\omega + 1/\tau_\phi)$ , the variance of the  $n$ th harmonic of the oscillations is suppressed by the factor<sup>2</sup>

$$\langle \delta g_n^2 \rangle \sim \frac{L_T^2 L_\phi}{R^3} e^{-2\pi R n / L_\phi}, \quad (2)$$

where  $L_\phi = (D\tau_\phi)^{1/2}$  is the dephasing length,  $L_T = (D/T)^{1/2}$  the thermal length,  $D$  the diffusion constant,  $T$  the temperature,  $R$  the radius of the ring, and we set  $\hbar = 1$ . [It is assumed in Eq. (2) that  $L_\phi, L_T \ll 2\pi R$ .] For a thin ring,  $L_\phi$  is then identified with the dephasing length governing the WL correction in the quasi-one-dimensional (Q1D) geometry, which was found by Altshuler, Aronov, and Khmelnitskii<sup>7,8</sup> to be

$$L_\phi = (D\tau_\phi)^{1/2}, \quad \tau_\phi^{-1} \sim \left( \frac{T}{\nu D^{1/2}} \right)^{2/3}. \quad (3)$$

In fact, Aleiner and Blanter showed recently<sup>9</sup> that  $\tau_\phi$  relevant to the mesoscopic conductance fluctuations in wires has indeed the same form, Eq. (3), as the WL dephasing time. This seems to support the assumption that the dephasing times governing different mesoscopic phenomena are identical. Equations (2) and (3) are commonly used for the analysis of experimental data.

We will show below, however, that contrary to the naive expectations the formulas (2) and (3) do not describe correctly the dephasing of AB oscillations. Specifically, if the interaction-induced exponential damping factor of AB oscillations is presented in the form  $\langle \delta g_n^2 \rangle \sim \exp(-2\pi R n / L_\phi^{\text{AB}})$ , the corresponding length  $L_\phi^{\text{AB}}$  is parametrically different from Eq. (3). Moreover,  $L_\phi^{\text{AB}}$  depends on the system size  $R$ .

We consider a ring coupled symmetrically by two leads to the bulk electrodes. We will assume the ring and the leads to be of a Q1D geometry, i.e., the widths of the wires are much larger than the Fermi wavelength but much smaller than  $R$  and  $L_\phi$ . The only geometric parameter characterizing the problem is then the ratio  $\gamma$  of the resistance of the ring itself to the total resistance of the ring with the leads. By definition,  $0 < \gamma < 1$ .

Following Refs. 7–10, the electron-electron interaction can be represented by external time-dependent random fields  $\varphi^\alpha(\mathbf{r}, t)$ , with the correlation function  $\langle \varphi^\alpha(\mathbf{r}, t) \varphi^\beta(\mathbf{r}', t') \rangle$  determined from the fluctuation-dissipation theorem,

$$\langle \varphi^\alpha(\mathbf{r}) \varphi^\beta(\mathbf{r}') \rangle_\omega = -\text{Im} U(\mathbf{r}, \mathbf{r}'; \omega) \delta_{\alpha\beta} \coth \frac{\omega}{2T}. \quad (4)$$

The conventional form for the dynamically screened Coulomb interaction in a diffusive system is<sup>8</sup>

$$U(q, \omega) = \frac{1}{U_0^{-1}(q) + \Pi(q, \omega)} \simeq \Pi^{-1}(q, \omega), \quad (5)$$

where  $U_0(q)$  is the bare Coulomb interaction,  $\Pi(q, \omega) = \nu D q^2 / (D q^2 - i\omega)$  is the polarization operator, and  $\nu$  is the density of states. For a diffusive system, the bare Coulomb interaction may be neglected compared to the polarization since the inverse screening length, which is of the order of the Fermi wave number, is much larger than typical momenta  $q$ . As we will see below, the characteristic momenta  $q$  for our problem are of the order of the inverse system size  $R^{-1}$ . In view of the nontrivial geometry of our system, it is thus more appropriate to work in the coordinate representation. A corresponding generalization of Eq. (5) can be readily obtained, yielding

$$\text{Im } U(\mathbf{r}, \mathbf{r}'; \omega) \simeq \text{Im } \Pi^{-1}(\mathbf{r}, \mathbf{r}'; \omega) = -\frac{\omega}{\nu D} \mathcal{D}(\mathbf{r}, \mathbf{r}'), \quad (6)$$

where  $\mathcal{D}$  is the propagator for the Laplace equation,  $-\nabla^2 \mathcal{D}(\mathbf{r}, \mathbf{r}') = \delta(\mathbf{r} - \mathbf{r}')$  with zero boundary conditions at the contacts with bulk electrodes. Substituting Eq. (6) in Eq. (4), we get, for relevant frequencies  $\omega \ll T$ ,

$$\langle \varphi^\alpha(\mathbf{r}, t) \varphi^\beta(\mathbf{r}', t') \rangle = \frac{2T}{\nu D} \mathcal{D}(\mathbf{r}, \mathbf{r}') \delta_{\alpha\beta} \delta(t - t'). \quad (7)$$

Since the ring we are considering consists of Q1D wires, the corresponding diffusion propagator satisfies the one-dimensional diffusion equation

$$\begin{aligned} \{ \partial_t - D \partial_x^2 + i[\varphi^\alpha(x, t) - \varphi^\beta(x, t)] \} \mathcal{P}_{\delta\Phi}^{\alpha\beta}(x, t; x', t') \\ = \delta(x - x') \delta(t - t') \end{aligned} \quad (8)$$

supplemented by appropriate matching conditions at junctions of the ring and leads. Here  $\delta\Phi = \Phi_1 - \Phi_2$  is difference in the AB flux between the two measurements, which is incorporated in the matching conditions. The conductance correlation function is given by the conventional two-diffuson diagrams,<sup>11-13</sup> yielding (we drop the prefactor of order unity)

$$\begin{aligned} \langle \delta g(\Phi_1) \delta g(\Phi_2) \rangle \sim \frac{D^2}{TR^4} \int dx_1 dx_2 \int dt dt' \tilde{\delta}(t - t') \\ \times \langle \mathcal{P}_{\delta\Phi}^{12}(x_1, x_2, t) \mathcal{P}_{\delta\Phi}^{21}(x_2, x_1, t') \rangle, \end{aligned} \quad (9)$$

where angular brackets denote averaging over the external fields,  $\tilde{\delta}(t)$  is given by

$$\begin{aligned} \tilde{\delta}(t) = 12\pi T \int \frac{d\epsilon_1}{2\pi} \frac{d\epsilon_2}{2\pi} f'(\epsilon_1) f'(\epsilon_2) e^{i(\epsilon_1 - \epsilon_2)t} \\ = 3\pi T^3 t^2 \sinh^{-2}(\pi T t), \end{aligned} \quad (10)$$

and  $f(\epsilon)$  is the Fermi distribution function. The function  $\tilde{\delta}(t)$  is peaked at  $t=0$  with a width  $T^{-1}$ . We will replace it

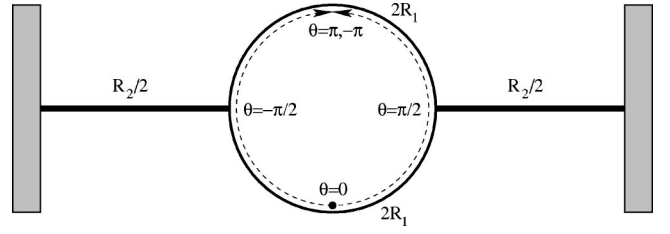


FIG. 1. The sample geometry and the angular coordinate  $\theta$  introduced above. The paths representing the saddle-point solution are shown. The geometric parameter  $\gamma$  is defined as the ratio of the resistance of the ring without the leads to the resistance of the total sample,  $\gamma = R_1 / (R_1 + R_2)$ .

below by the delta function  $\delta(t)$ . This is justified if the dephasing effect during the time  $t - t' \sim T^{-1}$  is negligible, i.e.,  $\langle \phi^\alpha(x, t) \phi^\alpha(x, t') \rangle T^{-1} \ll 1$ . Using Eq. (7), we find that the latter condition is equivalent to the requirement that the conductance of the sample is much larger than the conductance quantum  $e^2/h \simeq (25\text{k}\Omega)^{-1}$ . This condition is well satisfied in typical experiments with metallic rings, thus justifying the replacement  $\tilde{\delta}(t) \rightarrow \delta(t)$ .

We now express the diffusion propagators in Eq. (9) as path integrals. We are interested in the regime of strong dephasing, when the relevant paths propagate only inside the ring and do not extend into the leads (see below). It is convenient to introduce the angular coordinate  $\theta$  on the ring ( $-\pi \leq \theta \leq \pi$ ), with  $\theta = \pm \pi/2$  corresponding to the junctions with the leads (Fig. 1).

Expanding the conductance fluctuations in Fourier harmonics with respect to the flux,  $\delta g(\Phi) \rightarrow \delta g_n$ , we then get

$$\begin{aligned} \langle \delta g_n^2 \rangle \sim \frac{D^2}{TR^4} \int d\Theta_1 d\Theta_2 \int dt \int_{\Theta_2}^{\Theta_1} D \theta_1(t) \int_{\Theta_2}^{\Theta_1} D \theta_2(t) \\ \times \exp \left\{ - \int_0^t dt' \left[ \frac{R^2 \dot{\theta}_1^2}{4D} + \frac{R^2 \dot{\theta}_2^2}{4D} + V(\theta_1, \theta_2) \right] \right\}, \end{aligned} \quad (11)$$

where the path integral goes over pairs of paths  $\theta_1(t), \theta_2(t)$  which have a relative winding number  $n$ . The “potential”  $V(\theta_1, \theta_2)$  in Eq. (11) is given by  $V(\theta_1, \theta_2) = \langle [\phi^\alpha(\theta_1) - \phi^\alpha(\theta_2)]^2 \rangle$ ; its explicit form can be straightforwardly obtained according to Eq. (7) by solving the diffusion equation in the ring with leads.<sup>14</sup> We will only need below the form of  $V(\theta_1, \theta_2)$  for both coordinates being in the same arm of the ring. For  $|\theta_i| \leq \pi/2$  we find

$$V(\theta_1, \theta_2) = \frac{2TR}{\nu D} \left[ |\theta_1 - \theta_2| - \frac{\gamma+1}{2\pi} (\theta_1 - \theta_2)^2 \right]; \quad (12)$$

the expression for  $|\theta_i| > \pi/2$  follows from symmetry considerations.

We consider first the fundamental harmonic ( $n=1$ ) of the AB oscillations; a generalization to higher harmonics,  $n=2, 3, \dots$  will be done in the end. For  $n=1$  the relevant pairs of paths interfere after half encircling the ring in the opposite directions. We are interested in the regime of a rela-

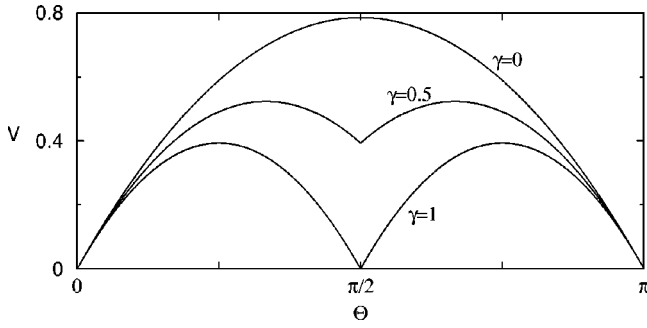


FIG. 2. The potential  $V(\theta)$  of the tunneling problem, Eq. (13), plotted in units of  $4TR/\nu D$  for different values of  $\gamma$ .

tively high temperature, when the dephasing effect is strong. In this case, the path integral in Eq. (11) can be evaluated via the saddle-point method. As has been mentioned above, the paths representing the saddle-point solution (instanton) do not extend into the leads. Indeed, exploring a part of a lead and returning back into the ring would only increase the action of the path. It is clear from the symmetry considerations that the optimal paths satisfy  $\theta_1(t) = -\theta_2(t)$ . Furthermore, it is easy to see that the optimal initial and final points are located at maximum distance from the leads, i.e.,  $\Theta_1 = 0$  and  $\Theta_2 = \pi$  (or vice versa). To within exponential accuracy, the problem is then reduced to that of a particle of mass  $R^2/D$  tunneling with zero energy in the potential (Fig. 2)

$$V(\theta) = \frac{4TR}{\nu D} \times \begin{cases} \left[ \theta - \frac{\gamma+1}{\pi} \theta^2 \right], & 0 \leq \theta \leq \frac{\pi}{2} \\ \left[ (\pi - \theta) - \frac{\gamma+1}{\pi} (\pi - \theta)^2 \right], & \frac{\pi}{2} \leq \theta \leq \pi \end{cases} \quad (13)$$

from  $\theta=0$  to  $\theta=\pi$ . Since the potential is composed of quadratic parts, the corresponding instanton action is easily calculated, yielding  $\langle \delta g_1^2 \rangle \propto e^{-S}$  with

$$S = C_\gamma \frac{T^{1/2} R^{3/2}}{\nu^{1/2} D}. \quad (14)$$

Here  $C_\gamma$  is a coefficient of order unity depending on the geometrical factor  $\gamma$ ,

$$C_\gamma = \left[ \frac{\pi}{2(\gamma+1)} \right]^{3/2} [2\gamma(1-\gamma^2)^{1/2} + \pi + 2 \arcsin \gamma]; \quad (15)$$

$C_\gamma$  is equal to  $\pi^{5/2}/2^{3/2}$  in the limit of long leads ( $\gamma \rightarrow 0$ ) and to  $\pi^{5/2}/4$  in the limit of short leads ( $\gamma \rightarrow 1$ ).

The above calculation can be straightforwardly generalized to higher harmonics of the AB oscillations,  $n = 2, 3, \dots$ . The optimal paths still begin at  $\Theta = 0$  or  $\pi$  but now perform  $n/2$  windings in the opposite directions. Therefore, the corresponding action is  $S_n = nS$ .

To calculate the preexponential factor, we have to take into account small fluctuations of the initial and final points  $\Theta_1, \Theta_2$  around their optimal values, as well as fluctuations of the paths  $\theta_1(t), \theta_2(t)$  around the instanton solution. The cor-

responding calculation is sketched in the Appendix. The final result for the variance of the harmonics of the mesoscopic AB oscillations reads (up to a numerical prefactor)

$$\langle \delta g_n^2 \rangle \sim \left( \frac{L_T}{R} \right)^{7/2} \left( \frac{\nu D}{R} \right)^{3/4} e^{-nS}, \quad (16)$$

where  $n = 1, 2, \dots$ , and the action  $S$  is given by Eq. (14).

Let us discuss the obtained result (16), (14). First of all, it is essentially different from what one would obtain by using the formulas (2), (3). Indeed, the exponent in Eq. (16) scales in a different way with the temperature and with the system size, as compared to Eqs. (2) and (3). It is instructive to rewrite Eq. (16) in a form analogous to Eq. (2),

$$\langle \delta g_n^2 \rangle \sim \left( \frac{L_T}{R} \right)^2 \left( \frac{L_\phi^{\text{AB}}}{R} \right)^{3/2} e^{-2\pi n R / L_\phi^{\text{AB}}}, \quad (17)$$

thus defining the Aharonov-Bohm dephasing length  $L_\phi^{\text{AB}}$ ,

$$L_\phi^{\text{AB}} = \frac{2\pi}{C_\gamma} \frac{\nu^{1/2} D}{T^{1/2} R^{1/2}}, \quad (18)$$

which is much shorter than  $L_\phi$  in the strong-dephasing regime  $L_\phi^{\text{AB}} \ll 2\pi R$ . The corresponding dephasing rate  $1/\tau_\phi^{\text{AB}} = D/(L_\phi^{\text{AB}})^2$  is thus given by

$$\frac{1}{\tau_\phi^{\text{AB}}} = \left( \frac{C_\gamma}{2\pi} \right)^2 \frac{TR}{\nu D}. \quad (19)$$

To shed more light on the physical reason for the difference between the conventional Q1D formula (3) and our result (18), (19), the following qualitative argument is instructive. Calculating perturbatively the dephasing rate using the formula (5) for the screened Coulomb interaction in a diffusive system, one gets

$$\tau_\phi^{-1} = \int \frac{dq}{2\pi} \frac{T}{\nu D q^2}. \quad (20)$$

In the calculation of the dephasing rate in a wire,<sup>8,10,9</sup> the arising infrared divergence is cut off self-consistently, since only processes with energy transfers  $\omega \gtrsim \tau_\phi^{-1}$  contribute. As a result, the lower limit of integration in Eq. (20) is  $\sim L_\phi^{-1}$ , yielding the result (3). On the other hand, in the case of the Aharonov-Bohm dephasing rate, the relevant paths have to encircle the ring. Therefore, despite the fact that  $L_\phi^{\text{AB}} \ll 2\pi R$ , the low-momentum cutoff in Eq. (20) is set by the inverse system size  $(2\pi R)^{-1}$ . This yields  $1/\tau_\phi^{\text{AB}} \sim TR/\nu D$ , reproducing (up to a numerical coefficient) the result (19).

It is worth emphasizing that our result (19) for the dephasing rate depends also on the geometry of the leads through the coefficient  $C_\gamma$ . We note a certain similarity to the dependence of the dephasing rate in a ballistic AB ring on the probe configuration recently found in Ref. 15.

As has been mentioned in the introduction, our results are also applicable to the WL ( $h/2e$  periodic) AB oscillations. Their  $n$ th harmonic  $\delta g_n^{\text{WL}}$  is determined by cooperon paths with winding number  $n$ . Assuming that the magnetic flux

penetrating the sample is negligible and comparing the path-integral representations for  $\langle \delta g_n^2 \rangle$  and  $\delta g_n^{\text{WL}}$  we find

$$\langle \delta g_n^2 \rangle = \frac{e^2 D}{3TL^2} \delta g_n^{\text{WL}}, \quad (21)$$

where  $L = \pi R / \gamma$ . This implies, in particular, that the dephasing length  $L_\phi^{\text{AB}}$ , Eq. (18), is the same for both types of the AB effect. Equation (21) is a generalization of the relation<sup>9</sup> between the WL correction and conductance fluctuations for single-connected geometries.

To summarize, we have studied how the Aharonov-Bohm oscillations are suppressed by dephasing caused by the electron-electron interaction. Using the path-integral formalism and the instanton method, we have obtained the result (16), (14) which is parametrically different from the naive expectation (2), (3). This demonstrates that the AB dephasing rate  $1/\tau_\phi^{\text{AB}}$ , Eq. (19), is parametrically different from the dephasing rate  $1/\tau_\phi$ , Eq. (3), corresponding to a single-connected geometry. Physically, the difference can be traced back to the fact that  $1/\tau_\phi$  is determined self-consistently by the processes with energy transfers of the order of  $1/\tau_\phi$  itself (or equivalently with momentum transfers  $\sim 1/L_\phi$ ), while the characteristic energy and momentum transfers governing  $1/\tau_\phi^{\text{AB}}$  are determined by the system size. For this reason, the Aharonov-Bohm dephasing rate  $1/\tau_\phi^{\text{AB}}$  depends on the ring radius  $R$ , diverging in the limit  $R \rightarrow \infty$ .

Valuable discussions with I. L. Aleiner, B. L. Altshuler, N. O. Birge, H. Bouchiat, M. Büttiker, V. I. Fal'ko, and I. V. Gornyi are gratefully acknowledged.

## APPENDIX

To find the preexponential factor in the formula (16) for the variance of the amplitude of the AB oscillations, we have

to perform several Gaussian integrations over deviations from the instanton solution in the path integral (11). We will only calculate the parametric dependence of the prefactor, neglecting numerical factors of order unity.

First, let us consider small offsets of the initial and final points of the paths from their optimal position. The second-order variation of the action  $\delta^2 S$  will be a quadratic form of the offsets,  $\delta^2 S = u_{ij} \delta \Theta_i \delta \Theta_j$ , where  $i = 1, 2$ . Using that  $\delta^2 S \sim 1$  for  $\delta \Theta_i \sim 1$ , we get

$$(\det u_{ij})^{-1/2} \sim S^{-1}. \quad (A1)$$

Second, we have to account for small deviations of the paths from the instanton solution. The corresponding factor can be identified as the propagator for a harmonic oscillator with the parameters  $m \sim R^2/D$  and  $m\omega^2 \sim RT/D\nu$ . There are two such factors (one for each of the paths), yielding together

$$[(m\omega)^{1/2}]^2 \sim \frac{T^{1/2} R^{3/2}}{\nu^{1/2} D}. \quad (A2)$$

Finally, there is a Gaussian integration over the deviations of the time  $t$  spent on the path from its optimal value  $t_{\text{opt}} \sim (\nu R/T)^{1/2}$ . The corresponding factor can be estimated as

$$\left( \frac{\partial^2 S}{\partial t^2} \right)_{t=t_{\text{opt}}}^{-1/2} \sim \left( \frac{S}{t_{\text{opt}}^2} \right)^{-1/2} \sim \frac{\nu^{3/4} D^{1/2}}{T^{3/4} R^{1/4}}. \quad (A3)$$

Combining Eqs. (11), (A1), (A2), and (A3), we obtain the total preexponential factor as given in Eq. (16).

\*Also at Petersburg Nuclear Physics Institute, 188350 St. Petersburg, Russia.

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<sup>13</sup>We discard the cooperon contribution to the correlation function, which does not influence the variance  $\langle g_n^2 \rangle$  of harmonics of the AB oscillations but only the statistics of the oscillation phase. If the flux threading the metal is much less than one flux quantum, the phases  $\theta_n$  in Eq. (1) will be 0 or  $\pi$ , while in the opposite case they are randomized.

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