Finite-size scaling of power-law bond-disordered Anderson models

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We investigate numerically the nature of energy eigenstates in one-dimensional bond-disordered Anderson models with hopping amplitudes decreasing as $H_{ij} \propto 1/|i-j|^{\alpha}$. The eigenstates become delocalized whenever the hopping amplitudes decay slower than $1/r$. By performing an exact diagonalization scheme on finite chains, we compute the participation ratio of all energy eigenstates. Employing a finite-size scaling analysis, we report on the relevant scaling exponents characterizing this delocalization transition as well as the level-spacing distribution at the critical point $\alpha=1$. The random hopping amplitudes are taken from both uniform and random sign distributions. We show that these models display similar critical behavior in the vicinity of $\alpha=1$. However, the random sign model exhibits an asymptotic delocalization in the limit of $\alpha \to \infty$ and the universal scaling behavior in this regime is also reported.

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I. INTRODUCTION

Disordered electronic systems can undergo an Anderson transition, as the strength of disorder increases, from a phase of extended to localized states.[1] However, when only short-range couplings are considered, scaling arguments supported by numerical and analytical results restrict its occurrence to systems with spatial dimensionality $d>2$.[2] An exception is the occurrence of extended states in two-dimensional models with broken time-reversal symmetry induced by a strong magnetic field, an important realization being the Anderson critical point associated with the quantum Hall plateau transition.[3] Another exception is the existence of a metallic phase in two-dimensional systems with preserved time-reversal symmetry but with broken spin-rotation symmetry due to the presence of a strong spin-orbit coupling.[4] In one-dimensional systems, all one-electron eigenstates remain exponentially localized for any amount of disorder even with broken time-reversal and spin-rotation symmetries. Localization induced by disorder is a feature shared by general physical systems exhibiting collective excitations such as magnetic[5–7] and vibrational[8,9] modes.

Disorder can be introduced through a random distribution of on-site potentials and/or off-diagonal hopping amplitudes. Pure off-diagonal disorder is known to be less effective to induce localization than diagonal disorder. In two dimensions, for example, the states are not exponentially localized. Instead, they present power-law tails characteristic of critical states.[10–12] In $d=1$, the states remain exponentially localized, except the one at the band center which exhibits a stretched exponential envelope.[13,14] One-dimensional systems may support extended states when correlations are introduced in the disorder distribution. However, short-range correlations can stabilize extended states with no backscattering only at discrete resonance energies.[15,16] On the other hand, long-range correlations can delocalize a finite fraction of the collective modes.[17,18] In such a case, delocalization is also more effective in the presence of pure off-diagonal disorder which requires weaker correlations to support extended modes.[19]

Delocalization of collective models in low-dimensional disordered systems can also be induced by the presence of long-range couplings.[20–23] Recently, Mirlin *et al.* introduced the power-law random band matrix (PRBM) model[20,21] describing one-dimensional electronic systems with random long-range hopping amplitudes with standard deviation decaying as $1/r^{\alpha}$ for sites at a distance $r \gg b$, where b is a typical bandwidth. It was shown that at $\alpha=1$ this model presents an Anderson-like transition with all states being localized for $\alpha>1$ and extended for $\alpha<1$. The PRBM model effectively describes some features of a series of physical systems such as the quantum Fermi accelerator,[25] two interacting particles in a one-dimensional(1D) random potential[26] and the Luttinger liquid at finite temperatures,[27] among others.[20,21] Further, its simplicity allows for analytical and extensive numerical studies of the universal scaling behavior at the vicinity of the Anderson transition. In particular at the critical point $\alpha=1$, the inverse participation ratio distribution, the wave-functions multifractal spectra, and the level statistics have been investigated both analytically and numerically.[21,24] However, the scaling behavior governing the approach to the critical point has not yet been clearly settled. In the limit of very large bandwidth $b \gg 1$, a perturbative approach associated with a renormalization group treatment has predicted that the characteristic length scale should diverge as $\ln \xi \propto (\alpha-1)$ as α approaches the critical point from above.[20] A numerical analysis for the case of intermediate bandwidth provided some support to this prediction but indicated that the one-parameter scaling hypothesis may be violated with a distinct exponent governing the approach to the critical point from below.[28] Within the same spirit of the PRBM, a model for noninteracting electrons in a 2D lattice with random on-site potentials and random power-law decaying transfer terms was numerically investigated by exploring the finite-size scaling properties of the fluctuations in the mean level spacing.[29] It was found that the one-electron eigenstates become extended for transfer terms decaying slower than $1/r^2$. The correlation length exponent governing this transition was estimated to be $\nu=2.60(15)$ and the same on both sides of the transition. Finally, a model for noninteracting electrons in a 1D chain with nonrandom power-law decaying hopping terms and random on-site po-

tentials has been shown to support extended states at the top of the band.[30,31]

In this work, we numerically investigate the universal scaling behavior of the one-dimensional Anderson model with power-law pure off-diagonal disorder. The hopping amplitudes will be chosen from two distinct distributions, namely the uniform and the bimodal (random signs) ones. Through direct diagonalization of the Anderson Hamiltonian on finite chains, we compute the critical level-spacing statistics and the average participation number as a function of the power-law decay exponent α. Using standard finite-size scaling analysis, the participation number data will be shown to follow a universal scaling behavior in the vicinity of $\alpha=1$ with the same exponent on both sides of the critical point, thus supporting the one-parameter scaling hypothesis. Furthermore, the random sign model will be shown to exhibit a second delocalization transition as $\alpha\to\infty$, whose scaling behavior will also be reported.

II. THE POWER-LAW PURE OFF-DIAGONAL 1D ANDERSON MODEL

We consider a single electron in a one-dimensional chain with open boundaries, described by the Anderson Hamiltonian

$$H=\sum_{i\neq j}^{N} t_{ij}|i\rangle\langle j|, \tag{1}$$

where $|i\rangle$ represents the state with the electron localized at site *i*. In the present pure random bond Anderson model, the on-site potentials ϵ_i are site independent and in Eq. (1) were taken to be $\epsilon_i=0$ without any loss of generality. Disorder is introduced by assuming the hopping amplitudes t_{ij} to be randomly distributed. For uncorrelated and nearest-neighbors pure off-diagonal disorder all one-electron eigenstates are exponentially localized, except the state at the band center which has a stretched exponential envelope with the density of states presenting a logarithmic singularity at the band center. These features are due to the particle-hole symmetry presented by bipartite lattices.[32] Power-law decaying hopping amplitudes, however, may stabilize truly extended states. We will, hereafter, consider

$$t_{ij}=W_{ij}/r_{ij}^{\alpha}, \tag{2}$$

where r_{ij} is the distance between the sites *i* and *j* and W_{ij} is a random variable.

Recently, a particular case of the above Hamiltonian was numerically investigated. The particle-hole symmetry was preserved by just allowing hopping between sites separated by an odd number of lattice constants.[33] It was found that the density of states singularity is gradually weakened by long-range hopping and that the wave functions present power-law decaying tails. Also a critical phase is anticipated for $1<\alpha<2$ with continuously varying wave-functions correlation exponents. The numerical results further suggest the emergence of an ordered phase for $\alpha<1$.

Here, we allow for transfer terms between any pair of lattice sites and therefore the present model has a broken particle-hole symmetry. Two particular distributions of W_{ij} will be used. The first one is the usual uniform distribution with W_{ij} being chosen randomly in the interval $[-W, +W]$. The second one is the bimodal (random signs) distribution where $W_{ij}=\pm W$ with the signs being chosen at random to give $\langle W_{ij}\rangle=0$. As a function of the exponent α characterizing the decay of the hopping amplitudes, both models display a localization-delocalization transition at $\alpha=1$. For off-diagonal terms decaying slower than $1/r_{ij}$, i.e., for $\alpha<1$, all states become delocalized. In the limit of $\alpha\to\infty$ one recovers the 1D Anderson model with just first-neighbors random hopping amplitudes. For the case of uniformly distributed couplings, the states remain localized. However, when randomness is introduced only in the signs of the transfer terms, the model with first-neighbors couplings exhibits just extended Bloch-like eigenstates with the wave-function amplitudes exhibiting random signs. In this case, localization can be induced only by the inclusion of next-neighbors couplings. Therefore, the random signs model with power-law decaying transfer terms shall present a second delocalization transition as $\alpha\to\infty$.

In the following sections, we are going to provide an extensive numerical study of the nature of the one-electron eigenstates on these two models. The present hopping decay law is similar to the limit of very small bandwidths of the PRBM model, a regime where the universal scaling behavior governing the approach to the critical point has not been investigated yet. We would like to stress that the present model also differs from the PRBM model by the absence of diagonal disorder. Models with pure off-diagonal disorder are known to exhibit new features not present in models with diagonal disorder such as a stretched exponential wave function in the center of the band in 1D (Refs. 13 and 14) and power-law scaling of the participation number in 2D.[10–12] We will be particularly interested in employing a finite-size scaling analysis of the average participation number to obtain the universal critical exponents governing the Anderson transition at the vicinity of $\alpha=1$ as well as the asymptotic delocalization of the random signs model as $\alpha\to\infty$.

III. DENSITY OF STATES AND CRITICAL LEVEL STATISTICS

We used an exact diagonalization scheme to obtain the energy spectrum of finite chains with sizes ranging from $L=200$ up to $L=1600$ sites. In order to average over distinct disorder configurations, data from several chains were joined making up a total number of 32×10^3 states for each chain size and power-law exponent α for both uniform and random sign distributions of hopping amplitudes. The bandwidth diverges with increasing system size for $\alpha\leqslant1/2$,[33] with a logarithmic divergence at $\alpha=1/2$. Above this point, the thermodynamic limit is well defined.

In Fig. 1 we illustrate the main dependence of the density of states (DOS) on the power-law decay exponent α for the case of a uniform hopping amplitude distribution. For large values of α the DOS converges to the one expected for the 1D Anderson model with nearest-neighbors off-diagonal disorder. It diverges at the band center where the eigenstate has

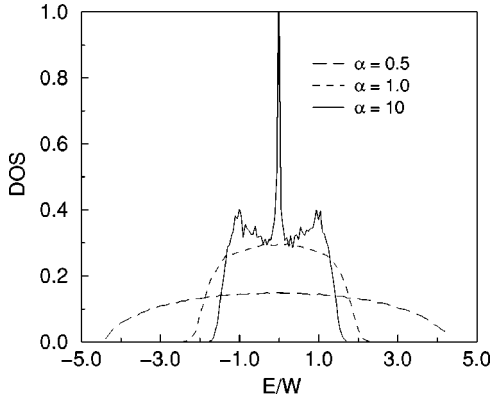

FIG. 1. Density of states for the Anderson model with power-law decaying random hopping amplitudes following a uniform distribution, as obtained from exact diagonalization of chains with $L=1600$. Typical values of the decay exponent α are represented. For large values of α one recovers the DOS of the usual 1D Anderson model with off-diagonal disorder. The smoothness at $\alpha<1$ is a signature of delocalization.

a stretched exponential envelope.[14] Large fluctuations on the DOS reflect the localized nature of the finite-energy eigenstates. The band edges present exponential tails due to the random nature of the underlying Hamiltonian. The inclusion of long-range hopping amplitudes at finite values of α makes the central singularity to disappear due to the lack of particle-hole symmetry. At the critical value $\alpha=1$ there is no sign of the original central peak. The smoothness of the DOS at smaller values of α is connected with the extended nature of the energy eigenstates in this regime.

The DOS of the random signs model is depicted in Fig. 2. Since the bimodal distribution has a larger variance, the bandwidth is larger in this case than in the model with a uniform distribution. In the regime $\alpha\gg1$, it converges to the density of states of the 1D tight-binding model with no disorder, displaying the characteristic parabolic shape with divergences at the band edges $E/W=\pm2$. Disorder becomes effective in this model through the presence of long-range off-diagonal terms. The band-edge singularities become rounded-off and the localization of eigenstates introduces fluctuations in the DOS. As this model also presents a delocalization transition at $\alpha=1$, fluctuations in the DOS become vanishingly small below this critical point.

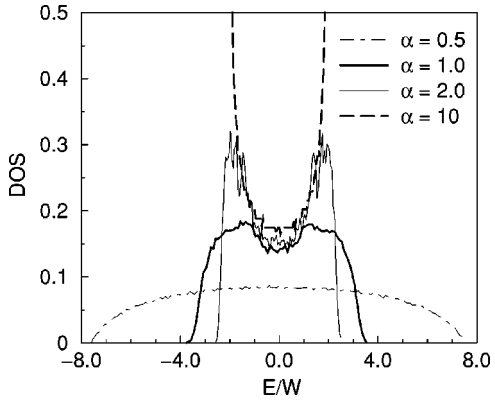

FIG. 2. Density of states for the Anderson model with power-law decaying random hopping amplitudes with random signs, as obtained from exact diagonalization of chains with $L=1600$. Typical values of the decay exponent α are represented. For large values of α one recovers the DOS of the pure 1D tight-binding Hamiltonian. At small α the DOS becomes similar to that obtained for the model with uniformly distributed disorder.

The eigenstates at the delocalization transition point $\alpha=1$ are critical for any of the above two models. This feature makes them ideally suited for the analysis of the critical level-spacing statistics. Localized states are distributed in energy following a Poisson law $P(s)=\exp(-s)$, where s is the level spacing measured in units of the mean spacing. It displays a standard deviation $\Delta s=\sqrt{\langle s^2\rangle-\langle s\rangle^2}=1$. Delocalized eigenfunctions repel each other and the level spacing in this phase obeys the Wigner surmise, which in the present case takes the form $P(s)=(\pi/2)s\exp[-(\pi/4)s^2]$,[34] with a smaller standard deviation $\Delta s=0.522$. At the Anderson transition a new universal critical statistics intermediate between Wigner and Poisson has been suggested as a consequence of the multifractality of critical wave functions.[35–40] Previous numerical results have shown that a reasonable overall fit to the form[37]

$$P(s)=Bs\exp(-As^{\gamma}) \tag{3}$$

could be found at the vicinity of the Anderson transition in cubic lattices when time-reversal and spin-rotation symmetries are preserved. The constants A and B are chosen to keep $P(s)$ normalized and $\langle s\rangle=1$. The above form interpolates between a linear distribution at small level spacings and a stretched exponential distribution at large s. The linear start of the critical level-spacing distribution is generally accepted. However, the large s stretched exponential tail is not generally accepted and some works have even suggested it to be indeed Poissonian but with an exponential coefficient above unity.[40–42] In particular, analytical expressions for the level-spacing distribution were derived from an ensemble of random matrices with $(1/a)(\ln x)^2$ potentials, which reproduce accurately numerical data for the level-spacing distribution at the Anderson transition in three-dimensional lattices,[40] with $a=2.95$ for an ensemble of real symmetric matrices. The random matrix theory predicts a linear behavior at small s and an exponential decay for $s\gg1/a$. In Fig. 3 we show the critical level-spacing distribution as obtained from the present model with uniformly distributed hopping amplitudes. Short- and long-dashed lines correspond to the Poisson and Wigner distributions, respectively. Within our numerical accuracy, the critical distributions of the uniform and random signs models coincide. To obtain the level-spacing distribution, we used an energy window near the band center corresponding to a fraction of 40% of all eigenstates. A spectral unfolding procedure was employed to keep the average level spacing equal to unity in each segment of the energy window.[37] At short level distances, the distribution depicts a linear behavior. In the inset, the asymptotic behavior at large level distances is displayed. It is intermediate between the standard exponential with unitary coefficient and Gaussian decays. The present data do not have

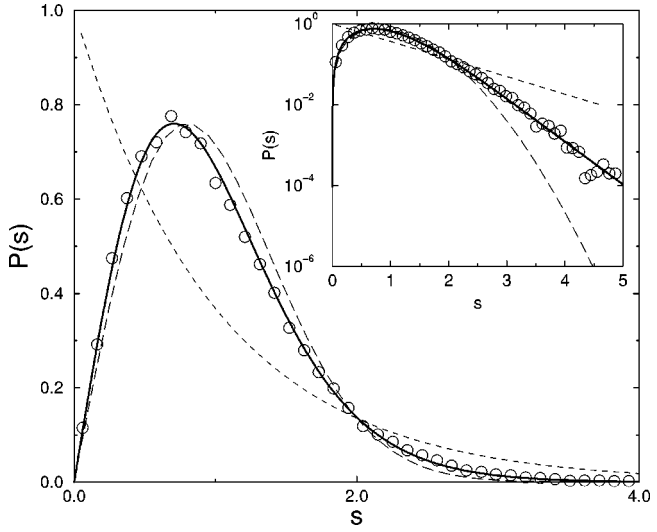

FIG. 3. The critical level-spacing distribution function for the model with uniformly distributed disorder as obtained from direct diagonalization of chains with $L=1600$ sites after unfolding the energy spectrum. The short- and long-dashed lines correspond to the Poisson and Wigner distributions, respectively. The solid line is the best fit to the analytic expression based on the random matrix theory with $a=2.2$ (see main text). The inset shows that the asymptotic behavior of the critical distribution is intermediate between Poisson and Wigner forms.

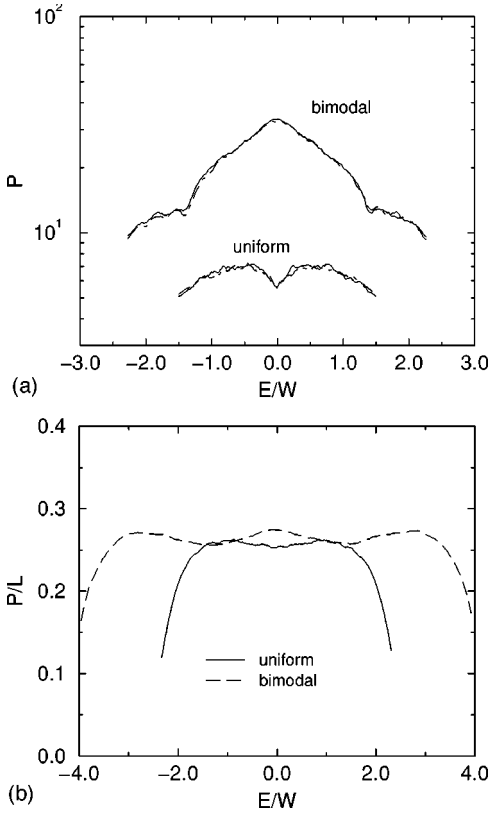

FIG. 4. (a) The average participation ratio as a function of energy for $\alpha=2$. Results for both uniform and bimodal distributions are represented. The superposed lines correspond to the results from chains with $L=800$ (dashed) and $L=1600$ (solid) sites. States are substantially more localized in the model with uniformly distributed disorder; (b) the normalized average participation ratio vs energy for $\alpha=0.5$ and $L=1600$. For both disorder distributions P/L is of the order of 1/4 except near the band edges where a few states may remain localized due to finite-size effects.

enough accuracy to decide between a fast Poissonian and a stretched exponential decay by just analyzing its asymptotic decay. Both laws provide reasonable fits of the distribution tail. However, the simple form of Eq. (3) does not provide a good overall fit. On the other hand, the analytic expression provided by the random matrix theory[40] gives an excellent fit of the entire curve, with the single fitting parameter $a=2.2$. This result brings further numerical support in favor of the random matrix prediction for the actual form of the critical level spacing. The critical level-spacing distribution has a standard deviation $\Delta s=0.595$ which is very close to the value reported for the PRBM model with both diagonal and off-diagonal disorder in the regime of intermediate bandwidth.[43]

IV. SCALING ANALYSIS OF THE PARTICIPATION NUMBER

All energy eigenfunctions $|\phi_k\rangle$ were also computed during the diagonalization procedure. In order to investigate the universal scaling behavior governing the delocalization transition, we evaluated the average participation number defined as

$$P(E)=\left\langle \frac{1}{\sum_{n=1}^{N} |\langle n|\phi_k\rangle|^4} \right\rangle , \tag{4}$$

where the external brackets represent an average over all eigenstates $|\phi_k\rangle$ with energy inside a small window around E (typically containing 200 states). Figure 4 shows the main features related to $P(E)$ for the two disorder distributions studied. For $\alpha>1$ all states are exponentially localized and the participation number is size independent within the entire energy band. In Fig. 4(a), we represent the results for $\alpha=2$. As uniformly distributed disorder promotes a more effective localization, the participation ratio is substantially smaller in this case as compared with the bimodal distribution. For $\alpha<1$ the states become delocalized and the participation number scales linearly with the chain size. In this regime, represented in Fig. 4(b) for $\alpha=0.5$, the participation ratio $P(E)/L$ stays around 1/4 irrespective to the disorder distribution, except near the band edges where the energy eigenstates still remain localized due to finite-size effects which are more pronounced for this limiting case.

The universal behavior at the vicinity of the delocalization transition can be obtained using finite-size scaling arguments. Exploiting the fact that almost all states within the energy band have the same nature, we computed the average value $P(\alpha,L)=\langle P(E,\alpha,L)\rangle_E$. Assuming a one-parameter scaling hypothesis, the average participation number scales near the transition point $\alpha=1$ as

$$P(\alpha,L)=L^{D_2}\mathcal{F}[(\alpha-1)L^{1/\nu}], \tag{5}$$

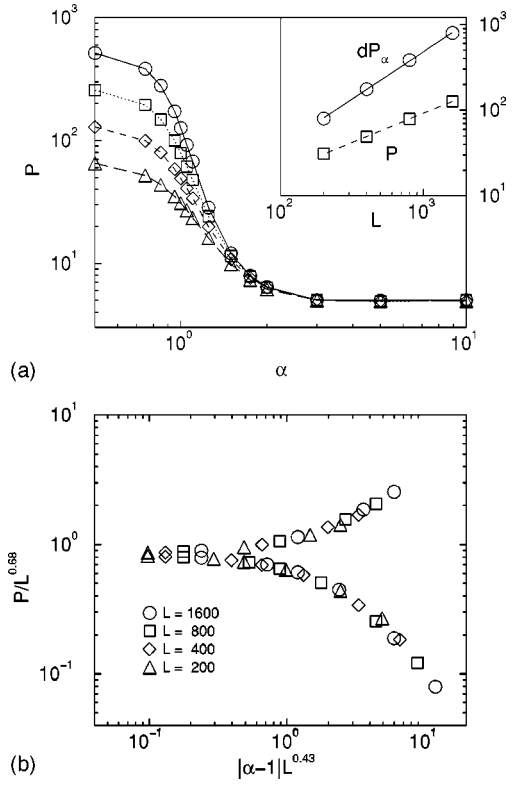

FIG. 5. Finite-size scaling analysis for the model with uniformly distributed disorder. (a) The participation number averaged over all states as a function of the decay exponent α for several chain sizes. The inset shows the power-law size dependence of P and dP_α from which we estimate $D_2=0.68(1)$ and $\nu=2.34(7)$. (b) The collapse of data from the critical region using the above estimated exponents. The collapse of both branches supports the one-parameter scaling hypothesis.

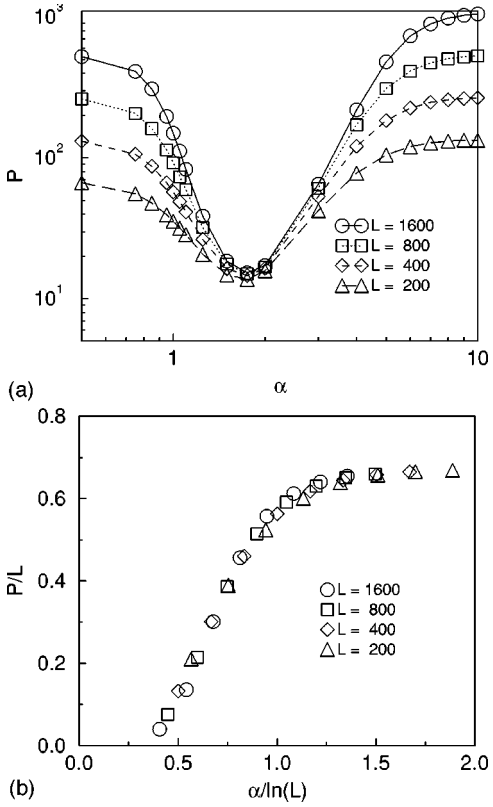

FIG. 6. Finite-size scaling analysis for the model with bimodal distributed disorder (random signs hopping amplitudes). (a) The participation number averaged over all states as a function of the decay exponent α for several chain sizes. The transition at $\alpha=1$ is similar to the one reported for the model with uniformly distributed disorder. In addition, the states become asymptotically delocalized as $\alpha\to\infty$; (b) the collapse of data from the asymptotic region. The collapse supports the scaling relation proposed in Eq. (6).

where D_2 governs the power-law divergence of the average participation number at $\alpha=1$ and ν governs the divergence of the relevant length scale as one approaches the transition. The correlation exponent can be measured by noticing that $dP_\alpha \equiv dP/d\alpha|_{\alpha=1} \propto L^{D_2+1/\nu}$.

In Fig. 5 we report our main results concerning the above finite-size scaling analysis for the model with uniformly distributed disorder. The average participation number presents a single transition at $\alpha=1$ [see Fig. 5(a)], represented by the divergence of the slope at $\alpha=1$ with increasing system sizes. The inset shows the critical power-law size dependence of P and dP_α from which we estimate $D_2=0.68(1)$ and $\nu=2.34(7)$. Notice that the data follow well-defined power laws with negligible fluctuations and no evident corrections to scaling for the systems size considered. Small fluctuations are a result of the large statistics obtained by computing the average participation number of all eigenstates. The absence of strong corrections to scaling is a less expected feature, once it is usually needed to consider very large system sizes to capture the long-range character of general systems with slowly decaying terms. The peculiar property of the present model with random power-law decaying hopping amplitudes is that the thermodynamic limit becomes ill defined only for $\alpha<1/2$, below which the bandwidth increases with system size.[33] The Anderson transition is sufficiently far from this point, in a region where the thermodynamic limit is well defined, and the asymptotic scaling regime sets up for relatively small chain sizes.

All data in the critical region belonging to distinct chain sizes were collapsed into a universal curve as shown in Fig. 5(b). The nice data collapse from both sides of the transition corroborates the single-parameter scaling hypothesis. We would like to stress that the presently reported value of ν is quite distinct from the one predicted for the PRBM model in the limit of large bandwidth[20] and it is somewhat below the one reported for the 2D model with random on-site and random power-law decaying hopping amplitudes.[29] However, it is intriguingly close to the best estimated value for the Anderson transition in 2D with broken time-reversal symmetry.[3]

For the model with bimodal off-diagonal disorder distribution, a similar finite-size scaling behavior is obtained at the vicinity of $\alpha=1$ [see Fig. 6(a)] with the resulting exponents being the same (within our numerical accuracy) as the above reported ones. This result indicates that the scaling behavior of the Anderson transition in this model is universal with respect to the disorder distribution form. However, for the bimodal disorder distribution, the states become asymp-

totically delocalized as $\alpha \to \infty$. This asymptotic delocalization can also be represented in a universal scaling form as depicted in Fig. 6(b). The data collapse implies that in this regime

$$P(\alpha, L) = L\mathcal{G}(\alpha/\ln L), \tag{6}$$

in such a way that the characteristic length scale exhibits an asymptotic logarithmic divergence.

V. CONCLUSION

We investigated in detail the nature of one-electron eigenstates in the one-dimensional Anderson model with off-diagonal disorder and matrix elements H_{ij} decaying as $1/|i-j|^{\alpha}$. The disorder was considered to follow either a uniform or a bimodal (random signs) distribution. Both models present features similar to the power-law random band model introduced by Mirlin *et al.*[20,21] with all states being localized for $\alpha > 1$ and delocalized for $\alpha < 1$. Following an exact diagonalization procedure, we computed the energy spectrum and participation number to characterize the universal scaling properties in the vicinity of the delocalization transition occurring at $\alpha = 1$. The critical level-spacing distribution was computed and found to exhibit a linear behavior at small level spacing, crossing over to an asymptotic decay at large level spacing between the Poisson and Wigner forms, well fitted by the random matrix theory prediction for an ensemble of real symmetric matrices with squared logarithmic potentials.[40] A finite-size scaling analysis of the participation number allowed us to estimate the power-law exponent of its size dependence at criticality $D_2 = 0.68(1)$ as well as the correlation length exponent $\nu = 2.34(7)$. These exponents are the same for both uniform and random sign disorder distributions. The energy eigenstates of the random signs model become asymptotically delocalized for $\alpha \to \infty$ with a logarithmic diverging characteristic length scale. The reported correlation length exponent is very close to the best estimated value for the Anderson transition in 2D with broken time-reversal symmetry, such as the quantum Hall plateau transition.[3] The similarity between the 1D Anderson model with random long-range hopping amplitudes and theories for the quantum Hall critical point have already been pointed out.[3,21] It would be of great value to have in future contributions further arguments aiming to support the present evidence that the Anderson transition in the 1D models investigated here indeed has the same correlation length critical exponent of the quantum Hall plateau transition in 2D systems under a strong magnetic field.

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