Controlled rotation in a double quantum dot structure

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The coherent manipulation of a double quantum dot system by an external driving field is analyzed. Using a controlled rotation method a general superposition state of the lower states is formed. Several interaction schemes are discussed and analytic results are presented. These are found to agree very well with the results of numerical simulations.

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I. INTRODUCTION

The interaction of electrons confined in symmetric double quantum dot structures with external oscillatory (ac, microwave, or laser) driving fields has attracted much attention in the past decade. Several interesting phenomena have been recognized when the quantum dot structure contains one or two electrons. Examples include one[1–12] and two-electron localization[13–18] in one of the wells under the interaction of the quantum dots with electromagnetic fields that obey certain conditions, controlled transfer of electrons between the two quantum dots,[19–26] and even creation of maximally entangled states in two-electron quantum dot systems.[27,28] These effects are in the central interest of the topical area of quantum computation,[29] as quantum dots are artificially created structures with potential scalability.[30]

In many from the studies outlined above a two-level model interacting with an oscillatory field is adequate for capturing the system's dynamics.[5–12,14–19,23,25,28] Several three-level population-transfer schemes[31,32] have also been applied successfully in this area.[20–22,26] In particular, Openov[20] used a system of a double quantum dot structure interacting with a pulsed field for the electron transfer between the two dots and applied his findings for the implementation of a NOT quantum gate. In addition, Brandes and Renzoni[21] have used coherent population trapping[31] for the realization of a very sensitive optically controlled current switch, and Brandes *et al.*[22] have proposed to use the stimulated Raman adiabatic passage technique[32] for the determination of dephasing rates in double quantum dots interacting with microwave pulses. Finally, Borzi *et al.*[26] have proposed an optimal control method for electron transfer in a three-level system with application in semiconductor nanostructures.

In this paper we study the potential of coherent manipulation of a double quantum dot structure similar to the one studied by Openov.[20] We present a general method based on controlled rotation[33] which exploits the coupled and uncoupled states of the system[31] and show that there is a large degree of control of the system's dynamics. Moreover, we analyze several interaction schemes that could yield the required dynamics and present numerical results that verify our analytic findings.

The paper is organized as follows: In Sec. II we introduce the physical model of the double quantum dot structure. We also discuss several approximations that symplify the model considerably. In Sec. III we present the controlled rotation scheme and discuss several excitation schemes that realize the scheme under consideration. We also show the results of the numerical simulations. Finally, we summarize our findings in Sec. IV.

II. MODEL SYSTEM

We consider two identical quantum dots, each one having only two bound-state energy levels. The quantum dots are taken to be widely separated. Thus, the lower pair of energy levels $|A1\rangle$, $|B1\rangle$ are essentially degenerate as the tunneling of an electron through the potential barrier between these energy levels is very improbable. The upper energy levels $|A2\rangle$, $|B2\rangle$ are taken to be near the edge of the potential barrier, therefore the tunneling probability of an electron between these two levels is significantly high. The system interacts with an external electromagnetic field $\mathbf{E}(t)$. The Hamiltonian of the system is given by[20]

$$\hat{H}=\varepsilon_1(|A1\rangle\langle A1|+|B1\rangle\langle B1|)+\varepsilon_2(|A2\rangle\langle A2|+|B2\rangle\langle B2|) -\hbar U(|A2\rangle\langle B2|+|B2\rangle\langle A2|)+\mathbf{E}(t)\boldsymbol{\mu}(|A1\rangle\langle A2| +|B1\rangle\langle B2|+\text{H.c.}), \quad (1)$$

where $\boldsymbol{\mu}$ is the electric dipole moment for the transition $|\alpha 1\rangle\leftrightarrow|\alpha 2\rangle$ $(\alpha=A,B)$, ε_1 is the energy of level $|\alpha 1\rangle$, ε_2 is the energy of level $|\alpha 2\rangle$, and U is the electron hopping frequency between the two dots for the excited electronic states, defined later in this section. We transform the Hamiltonian of Eq. (1) to the interaction picture with the unitary transformation

$$\hat{V}(t)=\exp\left[-i\left(\frac{\varepsilon_1}{\hbar}(|A1\rangle\langle A1|+|B1\rangle\langle B1|) +\frac{\varepsilon_2}{\hbar}(|A2\rangle\langle A2|+|B2\rangle\langle B2|)\right)t\right]. \quad (2)$$

This transformation eliminates the diagonal terms of the Hamiltonian (1). The upper bare energy levels are coupled through tunneling and give rise to a pair of symmetric and antisymmetric levels. We denote $|1\rangle$ $(|2\rangle)$ the lower state

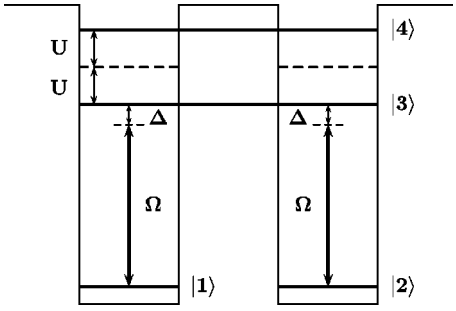

FIG. 1. Schematic diagram of the coupled quantum dot structure studied. The system possess two degenerate lower levels ($|1\rangle$ and $|2\rangle$), and two separated upper levels ($|3\rangle$ and $|4\rangle$). The lower states are coupled near resonantly to the excited state $|3\rangle$ by an external electromagnetic field.

$|\alpha 1\rangle$ in the $\alpha=A$ ($\alpha=B$) quantum dot and $|3\rangle=(|A2\rangle+|B2\rangle)/\sqrt{2}$ [$|4\rangle=(|A2\rangle-|B2\rangle)/\sqrt{2}$] the symmetric (antisymmetric) superposition of the upper states, respectively. The electron hopping frequency U/π between the excited bare states can be expressed as the energy difference of the states $|3\rangle$ and $|4\rangle$,

$$\varepsilon_- - \varepsilon_+ = 2\hbar U, \quad \text{where} \quad 2U = \frac{\pi}{\text{tunneling time}}, \tag{3}$$

with $\varepsilon_+ = \varepsilon_2 - \hbar U$ ($\varepsilon_- = \varepsilon_2 + \hbar U$) being the energy of the symmetric (antisymmetric) upper states, respectively. The scheme of the system using the basis $\{|1\rangle,|2\rangle,|3\rangle,|4\rangle\}$ is shown in Fig. 1. The electromagnetic field $\mathbf{E}(t)$ is supposed to oscillate at a frequency ω_L,

$$\mathbf{E}(t) = \mathbf{E}_0(t)\cos(\omega_L t), \tag{4}$$

with $\mathbf{E}_0(t)$ being the slowly varying envelope of the field. If the frequency ω_L is much larger than the field-matter interaction-strength $\mathbf{E}_0(t)\boldsymbol{\mu}$ [that is called the Rabi frequency and denoted by $\Omega'(t)=\mathbf{E}_0(t)\boldsymbol{\mu}/\hbar$ below], then the rotating-wave approximation can be applied.[34] In this approximation the rapidly oscillating terms, such as terms with frequency $\pm[\omega_L+(\varepsilon_\pm-\varepsilon_1)/\hbar]$, are dropped from the Hamiltonian, yielding

$$\begin{aligned}\hat{H} = \frac{\hbar}{2}[&\Omega(t)e^{-i\Delta t}|1\rangle\langle 3| + \Omega(t)e^{-i\Delta t}|2\rangle \\ &\times\langle 3| + \Omega(t)e^{-i(\Delta+2U)t}|1\rangle\langle 4| - \Omega(t)e^{-i(\Delta+2U)t}|2\rangle \\ &\times\langle 4| + \text{H.c.}],\end{aligned} \tag{5}$$

where $\Delta=(\varepsilon_+-\varepsilon_1)/\hbar-\omega_L$ is the field detuning from resonance with the transition $|1\rangle(|2\rangle)\leftrightarrow|3\rangle$, and $\Omega(t)=\Omega'(t)/\sqrt{2}$.

The Hamiltonian (5), which describes the interaction of the driving pulse with the quantum dots, can be simplified if we assume that the detuning Δ is much smaller than the coupling frequency U, $|\Delta|\ll U$. This approximation is known as the resonant approximation. In this case the transitions $|1\rangle(|2\rangle)\leftrightarrow|4\rangle$ can be ignored and the Hamiltonian (5) can be approximated in the rotating-wave picture[34] as

$$\hat{H}' = \hbar\Delta|3\rangle\langle 3| + \frac{\hbar}{2}[\Omega(t)|1\rangle\langle 3| + \Omega(t)|2\rangle\langle 3| + \text{H.c.}]. \tag{6}$$

This Hamiltonian will be our starting point in the following section, where we describe a scheme for the controlled rotation of the quantum state of the double quantum dot structure.

III. IMPLEMENTING THE ROTATION

We are going to present a scheme, with which the probability amplitudes on the two lower states $|1\rangle$ and $|2\rangle$ can be controlled. It proves to be useful to introduce new basis states, which are composed of these two states, hence we can define a coupled state $|C\rangle$:

$$|C\rangle = \frac{1}{\sqrt{2}}(|1\rangle + |2\rangle), \tag{7}$$

and an uncoupled state

$$|NC\rangle = \frac{1}{\sqrt{2}}(-|1\rangle + |2\rangle), \tag{8}$$

with respect to the driving pulse.[31] State (8) is uncoupled as $\hat{H}'|NC\rangle=0$. In this basis the Hamiltonian (6) reads

$$\hat{H}' = \hbar\Delta|\tilde{3}\rangle\langle\tilde{3}| + \frac{\hbar}{\sqrt{2}}\bar{\Omega}(t)(|C\rangle\langle\tilde{3}| + |\tilde{3}\rangle\langle C|), \tag{9}$$

where $\Omega(t)=\bar{\Omega}(t)e^{i\phi}$, $\bar{\Omega}(t)$ is real, with ϕ being a constant phase and $|\tilde{3}\rangle=|3\rangle e^{i\phi}$. The Hamiltonian (9) is the one describing the interaction of a two-level system with a pulsed field under the rotating-wave approximation.[34] The dynamics is governed by the Schrödinger equation

$$i\hbar\frac{d}{dt}|\psi\rangle = \hat{H}'|\psi\rangle. \tag{10}$$

The state vector $|\psi\rangle$ is written in a superposition of the two lower localized states $|1\rangle$, $|2\rangle$ and the symmetric excited state $|\tilde{3}\rangle$,

$$|\psi\rangle = c_1(t)|1\rangle + c_2(t)|2\rangle + \tilde{c}_3(t)|\tilde{3}\rangle, \tag{11}$$

or in a superposition of $|C\rangle$, $|NC\rangle$, $|\tilde{3}\rangle$,

$$|\psi\rangle = b_{NC}(t)|NC\rangle + b_C(t)|C\rangle + \tilde{b}_3(t)|\tilde{3}\rangle. \tag{12}$$

The component vectors $\mathbf{c}(t)=[c_1(t),c_2(t),\tilde{c}_3(t)]^T$ and $\mathbf{b}(t)=[b_{NC}(t),b_C(t),\tilde{b}_3(t)]^T$ are coupled through the relation

$$\mathbf{b}(t) = \mathcal{W}\mathbf{c}(t), \tag{13}$$

with $\mathcal{W}$ being a constant matrix

$$\mathcal{W}=\begin{bmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{14}$$

Note that $\mathcal{W}^{-1}=\mathcal{W}$. In the following we assume that at the initial time t_i the system occupies only the two lower states, hence $\tilde{c}_3(t_i)=\tilde{b}_3(t_i)=0$.

There are several analytically solvable models for two-level systems interacting with pulsed fields that can be applied here.[34,35] A general solution can be obtained via the transition matrix method. The columns of the transition matrix $\tilde{\mathcal{U}}(t_f,t_i)$ correspond to the state vector of the system at time t_f if it was initially, at time t_i, in states $|NC\rangle$,$|C\rangle$, and $|\tilde{3}\rangle$, respectively. The transition matrix for our system in general can be parametrized as

$$\tilde{\mathcal{U}}(t_f,t_i)=\begin{bmatrix} 1 & 0 & 0 \\ 0 & \alpha & -\beta^* \\ 0 & \beta & \alpha^* \end{bmatrix}, \tag{15}$$

in the basis $\{|NC\rangle,|C\rangle,|\tilde{3}\rangle\}$. Note that in the original basis $\{|1\rangle,|2\rangle,|\tilde{3}\rangle\}$ the transition matrix is given by $\mathcal{U}(t_f,t_i)=\mathcal{W}\tilde{\mathcal{U}}(t_f,t_i)\mathcal{W}$. The unity on the top-left corner means that the uncoupled state $|NC\rangle$ is not altered in this process. The 2×2 block describes the time evolution in the subspace of the coupled state $|C\rangle$ and the state $|\tilde{3}\rangle$. Here, we need to apply a model such that after the pulse has passed the coupled state $|C\rangle$ acquires a phase shift $-\delta$ and the state $|\tilde{3}\rangle$ is not populated. Hence the parameters of the transition matrix $\tilde{\mathcal{U}}(t_f,t_i)$ are chosen to be

$$\alpha=e^{-i\delta}, \quad \beta=0. \tag{16}$$

Inserting the above parameters into the transition matrix, Eq. (15), we obtain the final state of the system in the form

$$|\psi_f\rangle=\langle NC|\psi_i\rangle|NC\rangle+e^{-i\delta}\langle C|\psi_i\rangle|C\rangle. \tag{17}$$

By inserting the explicit forms of the scalar products $\langle NC|\psi_i\rangle$ and $\langle C|\psi_i\rangle$ using Eq. (11), and Eqs. (7), (8) into Eq. (17) we obtain

$$|\psi_f\rangle=e^{-i\delta/2}\hat{R}_{\mathbf{n}}(\delta)|\psi_i\rangle, \tag{18}$$

where $\hat{R}_{\mathbf{n}}(\delta)$ is an element of the SU(2) group, with $\mathbf{n}=[1,0,0]^T$ and δ being the axis and angle of rotation, respectively. Hence, the scheme described above results in a rotation about the x axis in the Hilbert space of a two-state system. In this scheme we cannot control the axis of rotation $\mathbf{n}$, because the coupling strengths between the two localized lower states $|1\rangle$, $|2\rangle$ and the spread out excited state $|3\rangle$ are the same, since both couplings are provided by the same electromagnetic field. For a more general rotation, one would need selective addressing of the lower levels.[33] In that case the rotation axis $\mathbf{n}$ could be also varied.

Let us take the initial state to be $|\psi_i\rangle=|1\rangle$, i.e., $c_1(t_i)=1$ and $c_2(t_i)=\tilde{c}_3(t_i)=0$ in Eq. (11), then

$$|\psi_f\rangle=e^{-i(\delta/2)}\left[\cos\left(\frac{\delta}{2}\right)|1\rangle-i\sin\left(\frac{\delta}{2}\right)|2\rangle\right], \tag{19}$$

a coherent superposition state of the two lower states of the two quantum dots is obtained. The angle δ determines the probability amplitudes, which should be controlled in the experimental realization.

We will now briefly discuss possible interaction schemes that can be used to realize the dynamics described above. First we consider the Rabi model with rectangular pulse shape, $\bar{\Omega}(t)=\text{const}$, if $0<t<T$, and zero elsewhere. In the Rabi model the elements of the transition matrix (15) are given by

$$\alpha=\left[\cos\left(\frac{1}{2}\tilde{\Omega}T\right)+i\frac{\Delta}{\tilde{\Omega}}\sin\left(\frac{1}{2}\tilde{\Omega}T\right)\right]e^{-i\Delta T/2}, \tag{20a}$$

$$\beta=-i\frac{\sqrt{2}\bar{\Omega}}{\tilde{\Omega}}\sin\left(\frac{1}{2}\tilde{\Omega}T\right)e^{-i\Delta T/2}, \tag{20b}$$

where $\tilde{\Omega}=\sqrt{2\bar{\Omega}^2+\Delta^2}$. For effective pulse area

$$\tilde{\Omega}T=2\pi m \quad \text{with} \quad m=1,2,\ldots, \tag{21}$$

the transition amplitudes become

$$\alpha=\exp\left[-i\left(\frac{\Delta}{\tilde{\Omega}}+1\right)m\pi\right], \tag{22a}$$

$$\beta=0. \tag{22b}$$

Thus, the conditions of Eq. (16) are fulfilled. Hence, the angle of rotation δ reads

$$\delta=\left(\frac{\Delta}{\tilde{\Omega}}+1\right)m\pi. \tag{23}$$

Another model is the one obtained under pulsed excitation in the case of exact resonance, i.e., $\Delta=0$. This is the case studied by Openov[20] for rectangular pulse shape excitation. However, for arbitrary pulse shape the parameters of the transition matrix read

$$\alpha=\cos\left[\frac{1}{\sqrt{2}}\int_{t_i}^{t_f}\bar{\Omega}(t)dt\right],$$

$$\beta=-i\sin\left[\frac{1}{\sqrt{2}}\int_{t_i}^{t_f}\bar{\Omega}(t)dt\right]. \tag{24}$$

For pulse area

$$\int_{t_i}^{t_f}\bar{\Omega}(t)dt=\sqrt{2}\pi m \quad \text{with} \quad m=0,1,2,\ldots, \tag{25}$$

the phase δ becomes

$$\delta = m\pi. \tag{26}$$

Another, rather general model, is the one obtained in the case of large detuning $|\Delta| \gg |\overline{\Omega}|/\sqrt{2}$, i.e., the case of Raman coupling between states $|1\rangle$ and $|2\rangle$. Then, the upper state $|3\rangle$ can be adiabatically eliminated from the dynamics[34,36] and the parameters of the transition matrix are given by

$$\alpha \approx \exp\left[\frac{i}{2\Delta}\int_{t_i}^{t_f} \overline{\Omega}(t)^2 dt\right], \tag{27a}$$

$$\beta \approx 0. \tag{27b}$$

Therefore, the angle δ reads

$$\delta = -\frac{1}{2\Delta}\int_{t_i}^{t_f} \overline{\Omega}(t)^2 dt. \tag{28}$$

Finally, the last model that we will discuss here is the Rosen-Zener model[35,37] with hyperbolic-secant pulse shape, where we have $\overline{\Omega}(t) = \Omega_0 \,\mathrm{sech}[(t-\tau_0)/\tau]$. The initial t_i and final t_f times of the interaction are set to $\mp\infty$, respectively, in the Schrödinger equation (10). In practice this means that $t_i \ll \tau_0 - \tau$ and $t_f \gg \tau_0 + \tau$. Then,

$$\alpha = \frac{\left[\Gamma\left(\frac{1}{2} + i\overline{\Delta}\right)\right]^2}{\Gamma\left(\frac{1}{2} + \overline{\Omega}_0 + i\overline{\Delta}\right)\Gamma\left(\frac{1}{2} - \overline{\Omega}_0 + i\overline{\Delta}\right)}, \tag{29}$$

where $\Gamma(\cdot)$ is the Gamma function, $\overline{\Delta} = \Delta\tau/2$ and $\overline{\Omega}_0 = \Omega_0\tau/\sqrt{2}$. A simple expression for the probability amplitude can be found when

$$\overline{\Omega}_0 = n \quad \text{with} \quad n = 1,2,\ldots. \tag{30}$$

Then, using the recurrence relation $\Gamma(z+1) = z\Gamma(z)$ we obtain[35]

$$\alpha = (-1)^n \prod_{j=0}^{n-1} \frac{j + \frac{1}{2} - i\overline{\Delta}}{j + \frac{1}{2} + i\overline{\Delta}}, \tag{31a}$$

$$\beta = 0. \tag{31b}$$

Here, the phase δ is not given by a closed-form relation as in the previous cases. However, requiring $\alpha = \exp(-i\delta)$, for specific δ, we obtain an algebraic equation for $\overline{\Delta}$ which may have n real solutions. These solutions will complete the parameters for the wanted superposition.

As an example of controlled response of the system we choose to study the case of coherent population transfer from state $|1\rangle$ to state $|2\rangle$ or vice versa. This is an application of the rotation scheme we described in the paper. This sort of rotation can be quite useful in the area of quantum computation as it is the basis for the NOT quantum gate.[29] We use the full Hamiltonian of Eq. (1) and study the unitary evolution of the system. This way both the rotating-wave and the

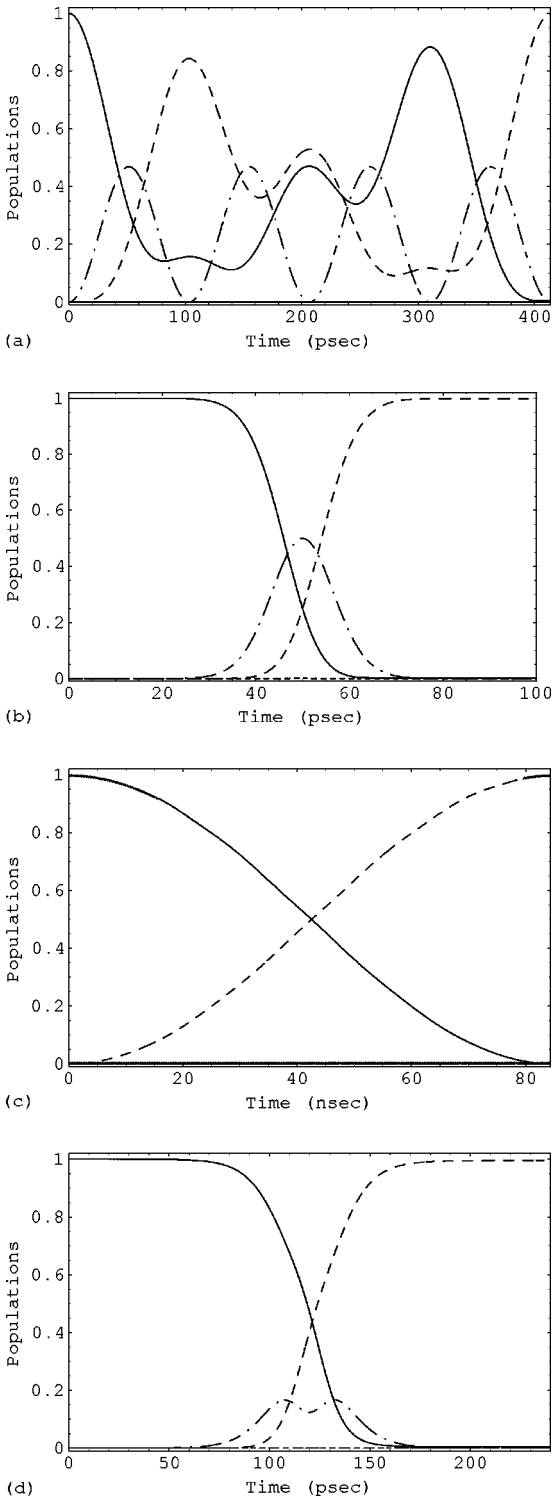

FIG. 2. Time evolution of the populations in states $|1\rangle$ (solid curve), $|2\rangle$ (long dashed curve), $|3\rangle$ (dot-dashed curve), and $|4\rangle$ (short dashed curve) using the Hamiltonian of Eq. (1). In all figures $\varepsilon_1 = 0.1$ eV, $\varepsilon_2 = 1$ eV, $\hbar U = 10^{-3}$ eV. (a) is for the Rabi model with $\hbar\Delta = 10^{-5}$ eV and $\delta = 5\pi$. (b) is for the resonant model with Gaussian pulse shape $\overline{\Omega}(t) = \Omega_0 \exp[-(t-\tau_0)^2/2\tau^2]$ with $\hbar\Omega_0 = 10^{-4}$ eV and $\delta = \pi$. (c) is for the off-resonant Raman model for rectangular pulse shape with $\hbar\overline{\Omega} = 7\times10^{-7}$ eV, $\hbar\Delta = 10^{-5}$ eV, and $\delta = -\pi$. (d) is for the Rosen-Zener model with $\hbar\Omega_0 = 10^{-4}$ eV, $n = 2$, and $\delta = \pi$.

resonant approximations, which are the basis of the effective three-level model of Eq. (6) that describes our system, are assessed. We choose typical parameters for the material system, see Sec. II of Ref. 20. The results of our numerical calculations are shown in Fig. 2. In all the cases the required population transfer is succeeded. We note that the off-resonant Raman coupling gives quite longer response times for the system than the other cases, see Fig. 2(c). This shows that this last interaction model may not be useful in the area of quantum computation due to the short decoherence times of these materials.[20]

IV. SUMMARY

In summary, we have considered the coherent manipulation of the quantum state of an electron bounded in a double quantum dot structure. The speciality of the system studied is that each quantum dot has only two bound states. We have shown that, by applying a coherent and appropriately detuned electromagnetic field, the controlled manipulation of the electronic state can be realized. Mathematically, we implement a rotation in a two-dimensional Hilbert subspace, representing the state of the electron. In physical terms, we realize a coherent population transfer between the two lower states of the two quantum dots. We have shown that apart from a global phase $-\delta/2$ a superposition state of the two lower levels can be created in a controlled manner with our method. If the system is isolated, then the global phase $-\delta/2$ is unimportant. If the system is part of a larger system, such that there are several couples of quantum dots which form a quantum computer, then the global phase is clearly relevant, however, it may be incorporated into the algorithm being implemented on the quantum computer. We have considered several models for two-level systems interacting with pulsed fields, which may serve as a basis for a physical realization. Finally, we have performed numerical simulations to justify the validity of our analytic considerations. The simulations verified the validity of the rotating-wave and resonant approximations that lead to the Hamiltonian of Eq. (6) and tested the excitation schemes discussed in Sec. III. We have found very good agreement in all the cases we studied.

ACKNOWLEDGMENTS

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