Theory of the quantum Hall effect at $\nu=\frac{1}{2}$ in a wide quantum well

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In a monolayer system a composite (c) fermion with two flux quanta (fluxons) is known to be generated at the Landau level occupation number (filling factor) $\nu=\frac{1}{2}$. In a bilayer system two c fermions are bound via the phonon exchange to form a pair (p) c boson with parallel electron spins and the system of condensed pc bosons generates a quantum Hall effect (QHE) state at $\nu=\frac{1}{2}$. Similar pc bosons are formed more efficiently at $\nu=1$, and the QHE state developed is stronger. These QHE states become weaker at a tilted magnetic field since the pc boson has a p-wave-type charge distribution with the preferred axis along [001].

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I. INTRODUCTION

Suen *et al.*[1] (Eisenstein *et al.*[2]) discovered the quantum Hall effect (QHE) state at the Landau Level (LL) occupation ratio (filling factor) $\nu=\frac{1}{2}$ in a single (double) well Al-GaAs/GaAs/AlGaAs. This is quite remarkable since in a heterojunction GaAs/AlGaAs the Fermi liquid state is observed at $\nu=\frac{1}{2}$, while the QHE is observed at $\nu=P/Q$, odd Q. When the magnetic field **B** is tilted, the deep resistivity minimum at $\nu=\frac{1}{2}$ disappears for tilt angle $\theta>10°$, see Fig. 1, indicating a spin-sensitive QHE state. Ezawa surveyed in his book[3] a number of the experimental and theoretical papers, and characterized the $\nu=\frac{1}{2}$ state as the bilayer-"locked" state. In the present work we shall show that the $\frac{1}{2}$ state arises from the Bose-Einstein condensation (BEC) of the electron-pairs with parallel spins containing four quantum fluxes (fluxons) bound by the phonon exchange, thus clarifying the microscopic origin of the "locking."

Laughlin introduced the ground-state wave function for the correlated electrons in the description of the single-layer fractional QHE.[4] Laughlin[4] and Haldane[5] argued that the quasiparticle (elementary excitation) over the ground state has a fractional charge $e^*=e/Q$ for the QHE state at $\nu=P/Q$, odd Q. Their prediction was experimentally confirmed by Clark *et al.* and others.[6] The composite (c) boson (fermion), each containing an electron and an odd (even) number of fluxons, were introduced by Zhang *et al.* and others[7] (Jain[8]) for the discussion of the QHE at $\nu=P/Q$, odd Q (the Fermi liquid state at $\nu=\frac{1}{2}$). The prevalent theories[3,9] based on the Laughlin ground-state wave function deal with the QHE at 0 K and immediately above. The resistivity data under fields up to 50 T by Leadley *et al.*,[10] indicate that the QHE is temperature dependent. We need a finite temperature theory. The same data also indicate that the QHE state is more stable at $\nu=\frac{1}{3}$ than at $\nu=1$, suggesting some kind of bonding between the electron and the fluxon. The theories based on the Hamiltonian with the repulsive Coulomb interaction is unlikely to describe this behavior. We need a new bonding Hamiltonian. Laughlin pointed out a remarkable similarity between the QHE and the high-temperature superconductivity (HTSC), both occurring in two-dimensional (2D) systems.[11] The major superconducting properties observed in the HTSC are (a) zero resistance, (b) a sharp phase change at the critical temperature T_c, (c) the energy gap below T_c, (d) the flux quantization, (e) Meissner effect, and (f) Josephson effects. The Josephson effects can be observed in the bilayer QHE systems.[3] All others have been observed in single-layer GaAs/AlGaAs. Following Bardeen, Cooper, and Schrieffer (BCS),[12] we regard the phonon exchange as the causes of both effects. Starting with a reasonable Hamiltonian, we calculate everything using the standard statistical mechanical methods. Since our theory is quite different from the prevalent theories, we briefly summarize our theory of the monolayer QHE in Secs. II and III, and treat the bilayer systems in Sec. IV.

II. THE HAMILTONIAN

Let us take a dilute system of electrons moving in the plane subject to no fields. Applying a magnetic field **B** perpendicular to the plane, each electron with the effective mass m^* will be in the Landau state with the energy

$$E=(N_L+1/2)\hbar\omega_0, \quad N_L=0,1,2,..., \tag{1}$$

where $\omega_0\equiv|eB/m^*|$ is the cyclotron frequency. In this state the electron can be viewed as circulating around the guiding center. The radius of circulation $l\equiv(\hbar/eB)^{1/2}$ is about 81 Å at a typical field 10 T. We now apply a weak electric field **E**

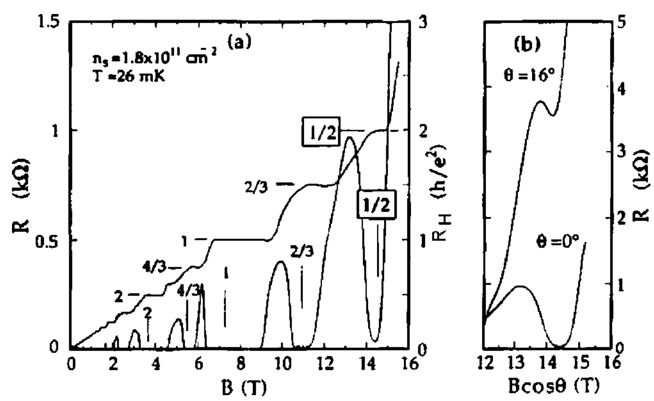

FIG. 1. (a) The magnetotransport data at $T=26$ mK for $n_s=1.8\times10^{11}$ cm^{-2} after Suen *et al.* (Ref. 1). A QHE state at $\nu=\frac{1}{2}$ is observed with a deep minimum in ρ and a plateau in ρ_H quantized at $2h/e^2$ to within 0.3%. This $\frac{1}{2}$ state becomes weaker in a tilted magnetic field as shown in (b).

in the x direction. With the scatterers (impurities, phonons) the guiding centers can jump from place to place preferrentially in the x direction, generating a current. The theory can simply be extended to the c particle. We regard the fluxon as a half-spin fermion with zero mass and zero charge. Our view is supported by the fact that the magnetic (electric) flux line cannot (can) terminate at a sink, and hence the associated fluxon (photon) is fermionic (bosonic). No half-spin fermion can disappear by itself because of angular momentum conservation. Fujita and Morabito[13] showed that the center-of-mass (c.m.) of *any* composite moves as a fermion (boson), that is, the c.m. momentum occupation number is limited to 0 or 1 (unlimited), if it contains an odd (even) number of elementary fermions. Hence the composite containing an electron and Q fluxons moves as a boson (fermions) if Q is odd (even). The c particle (boson, fermion) in the prevalent theories[3,9] is defined as the complex comprising an electron and Chern-Simons statistical field objects.[15] These objects are neither bosons nor fermions. In our theory the fluxons are fermions and hence the quantum statistics of the c particles is well founded. The countability and statistics of the fluxons are fundamental quantum particle properties. Hence they cannot be derived from any Hamiltonian and must be postulated.

Assume the interface in AlGaAs/GaAs being in (001). The planner arrays of $Ga^{3+}(A)$ and $As^{3-}(B)$ are located alternately along $\langle 100\rangle$ as $ABA'B'AB\cdots$. The longitudinal phonon, acoustic, or optical, proceeding in $\langle 100\rangle$ can generate a density wave, which affects the fluxon (electron) motion by the lattice-ionic charge current (displacement), establishing the fluxon (electron)-phonon interaction. The same condition also holds for the phonon proceeding in $\langle 010\rangle$. The lattice wave proceeding in the (001) plane can be regarded as a superposition of the waves proceeding in $\langle 100\rangle$ and $\langle 010\rangle$, and hence the corresponding phonon can also generate the electron (fluxon)-phonon interaction. The electron (fluxon)-phonon interaction can be represented by a vertex bilinear in the electron (fluxon) creation and the annihilation operator and linear in the phonon creation (or annihilation) operator having the interaction strength $V_q(V_q')$, where q is the phonon momentum transfer. The phonon exchange between an electron and a fluxon generates a transition in the electron states with the effective interaction

$$V_{\rm ef}\equiv|V_qV_q'|\,\frac{\hbar\omega_q}{(\epsilon_{|\mathbf{k}+\mathbf{q}|s}-\epsilon_{ks})^2-\hbar^2\omega_q^2},\tag{2}$$

where the Landau quantum number N_L is omitted; the bold $\mathbf{k}$ denotes the two dimensional (2D) guiding center momentum and the italic k the magnitude. The interaction is attractive when the electron states before and after the exchange have the same energy as in the degenerate LL so that $V_{\rm ef}=-|V_qV_q'|(\hbar\omega_q)^{-1}$, see more detail in the work by Fujita, Tamura, and Suzuki.[14]

Following Bardeen, Cooper, and Schrieffer (BCS),[12] we start with a Hamiltonian H with the phonon variables eliminated:

$$\begin{aligned}H={}&\sum_{\mathbf{k}}{}'\sum_s \epsilon_k^{(1)}n_{\mathbf{k}s}^{(1)}+\sum_{\mathbf{k}}{}'\sum_s \epsilon_k^{(2)}n_{\mathbf{k}s}^{(2)}+\sum_{\mathbf{k}}{}'\sum_s \epsilon_k^{(3)}n_{\mathbf{k}s}^{(3)}\\&-v_1\sum_{\mathbf{q}}{}'\sum_{\mathbf{k}}{}'\sum_{\mathbf{k}'}{}'\sum_s[B_{\mathbf{k}'\mathbf{q}s}^{(1)\dagger}B_{\mathbf{k}\mathbf{q}s}^{(1)}+B_{\mathbf{k}'\mathbf{q}s}^{(1)\dagger}B_{\mathbf{k}\mathbf{q}s}^{(2)\dagger}\\&+B_{\mathbf{k}'\mathbf{q}s}^{(2)}B_{\mathbf{k}\mathbf{q}s}^{(1)}+B_{\mathbf{k}'\mathbf{q}s}^{(2)}B_{\mathbf{k}\mathbf{q}s}^{(2)\dagger}],\end{aligned}\tag{3}$$

where $n_{\mathbf{k}s}^{(j)}$ is the number operator for the "electron" (1) ["hole" (2), fluxon (3)] at momentum $\mathbf{k}$ and spin s with the energy $\epsilon_{ks}^{(j)}$. We represent the "electron" ("hole") number $n_{\mathbf{k}s}^{(j)}$ by $c_{\mathbf{k}s}^{(j)\dagger}c_{\mathbf{k}s}^{(j)}$, where c ($c^\dagger$) are annihilation (creation) operators satisfying the Fermi anticommutation rules

$$\{c_{\mathbf{k}s}^{(i)},c_{\mathbf{k}'s'}^{(j)\dagger}\}\equiv c_{\mathbf{k}s}^{(i)}c_{\mathbf{k}'s'}^{(j)\dagger}+c_{\mathbf{k}'s'}^{(j)\dagger}c_{\mathbf{k}s}^{(i)}=\delta_{\mathbf{k},\mathbf{k}'}\delta_{s,s'}\delta_{i,j},$$
$$\{c_{\mathbf{k}s}^{(i)},c_{\mathbf{k}'s'}^{(j)}\}=0.\tag{4}$$

We represent the fluxon number $n_{\mathbf{k}s}^{(3)}$ by $a_{\mathbf{k}s}^\dagger a_{\mathbf{k}s}$, with $a(a^\dagger)$, satisfying the anticommutation rules. $B_{\mathbf{k}\mathbf{q}s}^{(1)\dagger}\equiv c_{\mathbf{k}+\mathbf{q}/2s}^{(1)\dagger}a_{-\mathbf{k}+\mathbf{q}/2-s}^{\dagger}$, $B_{\mathbf{k}\mathbf{q}s}^{(2)}\equiv c_{\mathbf{k}+\mathbf{q}/2s}^{(2)}a_{-\mathbf{k}+\mathbf{q}/2-s}$. The prime on the summation means the restriction: $0<\epsilon_{ks}^{(j)}<\hbar\omega_D$, ω_D = Debye frequency. If the fluxons are replaced by the conduction electrons ("electrons," "holes") our Hamiltonian H is reduced to the original BCS Hamiltonian, Eq. (24) of Ref. 12. The "electron" and "hole" are generated, depending on the energy contour curvature sign.[16] For example only "electrons" ("holes") are generated for a circular Fermi surface with the negative (positive) curvature whose inside (outside) is filled with electrons. Since the phonon has no charge, the phonon exchange cannot change the net charge. The pairing interaction terms in Eq. (3) conserve the charge. The term $-v_1B_{\mathbf{k}'\mathbf{q}s}^{(1)\dagger}B_{\mathbf{k}\mathbf{q}s}^{(1)}$, where $v_1\equiv|V_qV_q'|(\hbar\omega_0A)^{-1}$, A = sample area, is the pairing strength, generates the transition in the "electron" states. Similarly, the exchange of a phonon generates a transition in the "hole" states, represented by $-v_1B_{\mathbf{k}'\mathbf{q}s}^{(2)}B_{\mathbf{k}\mathbf{q}s}^{(2)\dagger}$. The phonon exchange can also pair-create and pair-annihilate "electron" ("hole")-fluxon composites, represented by $-v_1B_{\mathbf{k}'\mathbf{q}s}^{(1)\dagger}B_{\mathbf{k}\mathbf{q}s}^{(2)\dagger}$, $-v_1B_{\mathbf{k}'\mathbf{q}s}^{(2)}B_{\mathbf{k}\mathbf{q}s}^{(1)}$. At 0 K the system can have equal numbers of $-(+)c$ bosons, "electrons" ("holes") composites, generated by $-v_1B_{\mathbf{k}'\mathbf{q}s}^{(1)\dagger}B_{\mathbf{k}\mathbf{q}s}^{(2)\dagger}$.

III. THE HALL RESISTIVITY PLATEAU

The c bosons, each with one fluxon, will be called the fundamental (f) c bosons. The *fc* boson with one fluxon can be viewed as the electron circulating around one magnetic flux. Their energies $w_q^{(j)}$ are obtained from[14]

$$w_q^{(j)}\Psi(\mathbf{k},\mathbf{q})=\epsilon_{|\mathbf{k}+\mathbf{q}|}^{(j)}\Psi(\mathbf{k},\mathbf{q})-(2\pi\hbar)^{-2}v_0\int{}'d^2k'\,\Psi(\mathbf{k}',\mathbf{q}),\tag{5}$$

where $\Psi(\mathbf{k},\mathbf{q})$ is the reduced wave function for the *fc* boson; we neglected the fluxon energy. The energy $w_q^{(j)}$ is negative, which is obtained after an indefinite number of phonon exchanges between the electron and the fluxon, each generated by $B^{(j)\dagger}_{\mathbf{k}'\mathbf{q}s}B^{(j)}_{\mathbf{kq}s}$, called the ladder-binding process. The v_0 represents the attraction strength after the ladder diagram binding. For small q, we obtain

$$w_q^{(j)}=w_0+(2/\pi)v_F^{(j)}q, \quad w_0=\frac{-\hbar\omega_D}{\exp(v_0D_0)^{-1}-1}, \tag{6}$$

where $v_F^{(j)}\equiv(2\epsilon_F/m_j)^{1/2}$ is the Fermi velocity, and $D_0\equiv D(\epsilon_F)$ the density of states per spin. Note that the energy $w_q^{(j)}$ depends *linearly* on the momentum q.

The system of free *fc* bosons undergoes a BEC in 2D at the critical temperature[14]

$$k_BT_c=1.24\ \ \hbar v_F n_0^{1/2}, \tag{7}$$

where n_0 is the boson density. The interboson distance $R_0\equiv n_0^{-1/2}$ calculated from this expression is $1.24\hbar v_F(k_BT_c)^{-1}$. The boson size r_0 calculated from Eq. (6), using the uncertainty relation ($q_{\max}r_0\sim\hbar$) and $|w_0|\sim k_BT_c$, is $r_0=(2/\pi)\hbar v_F(k_BT_c)^{-1}$, which is a few times smaller than R_0. Hence, the bosons do not overlap in space, and the model of free bosons is justified. Let us take GaAs, $m^*=0.067m_c$, m_c= electron mass. For the 2D electron density 10^{11} cm^{-2}, we have $v_F=1.36\times10^6$ cm s^{-1}. Note all electrons are bound with fluxons since the simultaneous generations of $\pm fc$ bosons is required. The minority carrier (''hole'') density controls the *fc*-boson density. For $n_0=10^{10}$ cm^{-2}, $T_c=1.29$ K, which is reasonable.

In the presence of Bose condensate below T_c the unfluxed electron carries the energy $E_k^{(j)}=\sqrt{\epsilon_k^{(j)2}+\Delta^2}$, where the quasielectron energy gap Δ is the solution of

$$1=v_0D_0\int_0^{\hbar\omega_D}d\epsilon\frac{1}{(\epsilon^2+\Delta^2)^{1/2}}\{1+\exp[-\beta(\epsilon^2+\Delta^2)^{1/2}]\}^{-1},$$

$\beta\equiv(k_BT)^{-1}$.

Note that the gap Δ depends on T. At T_c, there is no condensate and hence Δ vanishes.

Now the moving *fc* boson below T_c has the energy $\tilde{w}_q$ obtained from

$$\tilde{w}_q^{(j)}\Psi(\mathbf{k},\mathbf{q})=E^{(j)}_{|\mathbf{k}+\mathbf{q}|}\Psi(\mathbf{k},\mathbf{q})-(2\pi\hbar)^{-2}v_0\int' d^2k'\,\Psi(\mathbf{k}',\mathbf{q}), \tag{8}$$

where $E^{(j)}$ replaced $\epsilon^{(j)}$ in Eq. (5). We obtain from Eq. (8)

$$\tilde{w}_q^{(j)}=\tilde{w}_0+(2/\pi)v_F^{(j)}q\equiv w_0+\epsilon_g+(2/\pi)v_F^{(j)}q, \tag{9}$$

where $\tilde{w}_0(T)$ is determined from $1=D_0v_0\int_0^{\hbar\omega_D}d\epsilon[|\tilde{w}_0|+(\epsilon^2+\Delta^2)^{1/2}]^{-1}$. The energy difference $\tilde{w}_0(T)-w_0\equiv\epsilon_g(T)$ represents the T-dependent *energy gap* between the moving and the stationary *fc* bosons. The energy $\tilde{w}_q$ is negative. Otherwise, the *fc* boson should break up. This limits $\epsilon_g(T)$ to be $|w_0|$ at 0 K. The ϵ_g declines to zero as the temperature approaches T_c from below.

The *fc* boson, having the linear dispersion (9), can move in all directions in the plane with the constant speed $(2/\pi)v_F^{(j)}$. The supercurrent is generated by the $\pm fc$ bosons condensed monochromatically at the momentum directed along the sample length. The supercurrent density (magnitude) J, calculated by the rule (charge e^*) $\times$(carrier density n_0)$\times$(drift velocity v_d) is

$$J\equiv e^*n_0v_d=e^*n_0(2/\pi)|v_F^{(1)}-v_F^{(2)}|. \tag{10}$$

The induced Hall field (magnitude) E_H equals v_dB. The magnetic flux is quantized $B=n_\phi(h/e)$, n_ϕ= fluxon density. Hence we obtain

$$\rho_H\equiv\frac{E_H}{J}=\frac{v_dB}{e^*n_0v_d}=\frac{1}{e^*n_0}n_\phi\left(\frac{h}{e}\right). \tag{11}$$

If $e^*=e$, $n_\phi=n_0$, we obtain $\rho_H=h/e^2$, explaining the plateau value observed.

The same model can be extended to the integer QHE at $\nu=P$ $(Q=1)$. The field magnitude is less. The LL degeneracy (eBA/h) is linear in B, and hence the lowest P LL's must be considered. The *fc* boson density n_0 per LL is

$$n_0=n_e/P, \tag{12}$$

where n_e is the electron density and the fluxon density n_ϕ is

$$n_\phi=n_0/P. \tag{13}$$

At $\nu=\frac{1}{2}$ there are c fermions, each with two fluxons. The c fermions moving in the crystal have a Fermi energy. The $\pm c$ fermions have effective masses. The Hall resistivity ρ_H has a B-linear behavior while the resistivity ρ is finite.

Let us now take a general case $\nu=P/Q$, odd Q. Assume that there are P sets of c fermions with $Q-1$ fluxons, which occupy the lowest P LL's. The c fermions subject to the available B-field form c bosons with Q fluxons. In this configuration the c-boson density n_0 per LL is given by Eq. (12), and the fluxon density n_ϕ is given by Eq. (13). Using Eqs. (11) and (13) and assuming the factional charge[4,5]

$$e^*=e/Q, \tag{14}$$

we obtain

$$\rho_H\equiv\frac{E_H}{J}=\frac{v_d}{e^*n_0v_d}n_\phi\left(\frac{h}{e}\right)=\frac{Q}{P}\,\frac{h}{e^2}, \tag{15}$$

as observed. We note that the integer Q indicates the number of fluxons in the c boson and the integer P the number of the LL's occupied by the parental c fermions, each with $Q-1$ fluxons.

We note that our Hamiltonian in Eq. (3) can generate and stabilize the c particles with an arbitrary number of fluxons. For example a c fermion with two fluxons is generated by two sets of the ladder diagram bindings, each between the electron and the fluxon. The ladder diagram binding arises as follows. Consider a hydrogen atom problem. The Hamiltonian contains kinetic energies of the electron and the proton and the attractive Coulomb interaction. If we regard the Coulomb interaction as a perturbation and use a perturbation theory, we can represent the interaction process by an infinite set of ladder diagrams, each ladder step connecting the electron and the proton. The energy eigenvalues of this system is

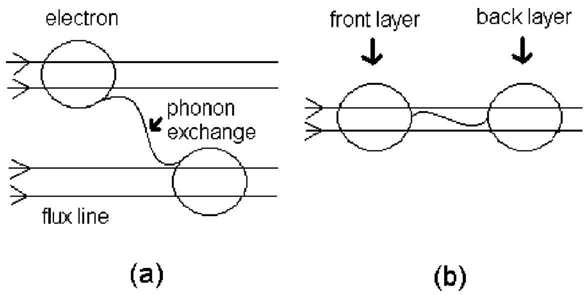

FIG. 2. (a) Two composite fermions, each one electron (circle) with two fluxons (lines), are bound by the phonon exchange (wavy line). The pair composite boson formed at $\nu=\frac{1}{2}$ carries two electrons (circles) and four fluxons. (b) The *pc* boson is more efficiently formed at $\nu=1$.

not obtained by using the perturbation theory but they are obtained by solving the Schrödinger equation directly. This example indicates that the binding energy (the negative of the ground-state energy) is calculated by a nonperturbative method, see Sec. IV for the case of the *fc* boson. The binding energy of the *c* boson increases with the number of fluxons in it, which is experimentally supported. Leadley *et al.*[10] found that the strength, measured by the width, of the QH state at $\nu=\frac{1}{3}$ is greater than at $\nu=1$.

IV. PAIR COMPOSITE BOSON

Suppose that a pair of *c* fermions with two fluxons, one each in the two layers, are bound by the phonon exchange as shown in Fig. 2. The composite moves as a boson since it contains six elementary fermions (two electrons, four fluxons). It carries two electronic charge $2e$ divided by the number of fluxons 4, that is, the net charge $(1/2)e$. To describe this composite it is necessary to introduce a pairing Hamiltonian.

The phonon exchange between two electrons generates a transition in the electron states with the effective interaction[12]

$$V_{\rm ee}\equiv|V_q|^2\frac{\hbar\omega_q}{(\epsilon_{|\mathbf{k}+\mathbf{q}|s}-\epsilon_{ks})^2-\hbar^2\omega_q^2}$$

$$=-|V_q|^2(\hbar\omega_q)^{-1}, \tag{16}$$

where the last line was obtained since the states before and after the phonon exchange have the same energy. Eliminating the phonon variables, we obtain the pairing Hamiltonian

$$H_p=-v_2{\sum_{\mathbf{q}}}'{\sum_{\mathbf{k}}}'{\sum_{\mathbf{k}'}}'\sum_s[D^{(1)\dagger}_{\mathbf{k}'\mathbf{q}s}D^{(1)}_{\mathbf{kq}s}+D^{(1)\dagger}_{\mathbf{k}'\mathbf{q}s}D^{(2)\dagger}_{\mathbf{kq}s}$$

$$+D^{(2)}_{\mathbf{k}'\mathbf{q}s}D^{(1)}_{\mathbf{kq}s}+D^{(2)}_{\mathbf{k}'\mathbf{q}s}D^{(2)\dagger}_{\mathbf{kq}s}],$$

$$D^{(1)\dagger}_{\mathbf{kq}s}\equiv c^{(1)\dagger}_{\mathbf{k}+\mathbf{q}/2s}c^{(1)\dagger}_{-\mathbf{k}+\mathbf{q}/2s}, \quad D^{(2)}_{\mathbf{kq}s}\equiv c^{(2)}_{\mathbf{k}+\mathbf{q}/2s}c^{(2)}_{-\mathbf{k}+\mathbf{q}/2s},$$

$$v_2\equiv -V_{\rm ee}/A. \tag{17}$$

This Hamiltonian H_p is now added to the Hamiltonian in Eq. (3). The energy $W_q^{(j)}$ of the pair (p) c boson can be calculated by solving the Cooper-like equation

$$W_q a(\mathbf{k},\mathbf{q})=[\epsilon(|\mathbf{k}+\mathbf{q}/2|)+\epsilon(|-\mathbf{k}+\mathbf{q}/2|)]a(\mathbf{k},\mathbf{q})$$

$$-\frac{1}{(2\pi\hbar)^2}v_p\int' d^2k' a(\mathbf{k}',\mathbf{q}), \tag{18}$$

where the prime on the k' integral means the restriction $0<\epsilon(|\mathbf{k}+\mathbf{q}/2|)+\epsilon(|-\mathbf{k}+\mathbf{q}/2|)<\hbar\omega_D$ and the v_p represents the binding strength after the ladder diagram binding between two *c* fermions. We note that the net momentum $\mathbf{q}$ is a constant of motion, which arises from the fact that the phonon exchange is an internal process and hence cannot change the net momentum. The pair wave functions $a(\mathbf{k},\mathbf{q})$ are coupled with respect to the other variable $\mathbf{k}$, meaning that the exact (or energy-eigenstate) wave functions are superpositions of the pair wave functions $a(\mathbf{k},\mathbf{q})$. We solve Eq. (18) and obtain to the linear in q

$$W_q^{(j)}=W_0+(2/\pi)v_F^{(j)}q, \tag{19}$$

$$W_0=\frac{-2\hbar\omega_D}{\exp(2/v_pD_0)-1}<0. \tag{20}$$

The brief derivation of Eqs. (18)–(20) is given in the Appendix. As expected, the zero-momentum *pc* boson has the lowest energy. The excitation energy is continuous with no energy gap. The energy W_q increases *linearly* with momentum q for small q rather than quadratically. This fact arises since the density of states is strongly reduced with increasing momentum q, and this behavior dominates the q^2 increase of the kinetic energy. *pc* bosons move similar to massless particles with a common speed $2v_F/\pi$.

Since the phonon has no charge, the phonon exchange generates in pair the $\pm$ *pc* bosons having four fluxons and having different charges $\pm e^*$ via $D^{(1)\dagger}_{\mathbf{k}'\mathbf{q}s}D^{(2)\dagger}_{\mathbf{kq}s}$. The "electron" *pc* bosons, having the greater speed, dominates the BEC, and the critical temperature T_c is given by

$$k_BT_c=1.24\hbar v_F^{(1)}n_p^{1/2}, \tag{21}$$

where n_p is the *pc*-boson density.

The *pc* boson containing two electrons and four fluxons is similar to the Cooper pair in HTSC's. But it is different in that it has a net magnetic moment due to the *parallel* electron spin-pairs. This makes the *c* boson the magnetic field-direction sensitive, see below.

We are now ready to analyze the experimental data. Suen *et al.*[1] observed that the resistivity ρ has a deep drop and the Hall resistivity ρ_H has a near-plateau quantized at $2h/e^2$ within 0.3% for the QHE state at $\nu=\frac{1}{2}$, $T=26$ mK, and $B\sim 15$ T. Since ρ is finite (nonsuperconducting), we must assume that the critical temperature T_c is less than 26 mK. The deep drop in ρ can be interpreted as follows.

The *c* bosons with the linear dispersion relation (19) can move in all directions in the plane, and they are scattered by impurities and phonons. The equation of motion for the *c*-boson subject to the electric field in the x direction is

$$\frac{p}{c}\frac{dv_x}{dt}=e^*E, \quad \epsilon=(2/\pi)v_F p\equiv cp, \tag{22}$$

where we renamed $W\rightarrow\epsilon$, $q\rightarrow p$, see Eq. (19). Hence the drift velocity v_d is

$$v_d=c^2e^*E\langle\epsilon^{-1}\rangle\tau, \tag{23}$$

where τ is the time between scatterings and the angular brackets mean the thermal average. If the Bose-Einstein distribution for the pc bosons above T_c is approximated by the Boltzmann distribution, we have

$$\langle\epsilon^{-1}\rangle=(k_BT)^{-1}. \tag{24}$$

Using Ohm's law $j=e^*n_pv_d=\sigma E$, we obtain the following expression for the conductivity σ:

$$\sigma=(e^*)^2n_pc^2(k_bT)^{-1}\tau. \tag{25}$$

Hence the resistivity $\rho\equiv 1/\sigma$ decreases at least inverse-linearly with the temperature T. The actual decrease is steeper since the time between the scatterings, τ, becomes greater with fewer phonons and since the BEC takes place at a finite temperature. We predict that the dip minimum reaches zero (superconductivity) at a finite critical temperature. This is significant since the resistivity ρ at $\nu=\frac{1}{2}$ for a monolayer system remains finite.

Let us now turn to the Hall resistivity ρ_H. Below T_c the current density J is given in the form (10) with $n_0=n_p$, with the assumption that the c bosons condenses in the momentum along the sample length. Here the drift velocity v_d is given by the *unaveraged* velocity difference and hence the exact cancellation of the v_d occurs in the calculation of ρ_H in Eq. (11), giving rise to the extreme accuracy (10^{-8}) for the plateau value. The plateau is formed due to the Meissner effect. Consider the case of zero temperature near $\nu=1$. Only the energy E matters. The fc bosons are condensed with the ground-state energy w_0, and hence the system energy E at $\nu=1$ is $2N_0w_0$, where N_0 is the number of $-fc$ bosons (or $+fc$ bosons). The factor 2 arises since there are $\pm fc$ bosons. Away from $\nu=1$ we must add the magnetic field energy $(2\mu_0)^{-1}A(B')^2$, so that

$$E=2N_0w_0+(2\mu_0)^{-1}A(B')^2, \tag{26}$$

where

$$B'\equiv B-B_1 \tag{27}$$

is the effective field relative to the field B_1 at $\nu=1$. If B' is small, the fc-boson number N_0 should be unchanged and the superconducting state be maintained. When the field is reduced, the system tries to keep the same number N_0 by sucking in the flux lines. Thus the magnetic field becomes inhomogeneous outside the sample, generating the magnetic field energy $(2\mu_0)^{-1}A(B')^2$. If the field is raised, the system tries to keep the same number N_0 by expeling out the flux lines. The inhomogeneous fields outside raise the field energy as well. There is a critical field $B'=(4\mu_0|w_0|)^{1/2}$. Beyond this value, the superconducting state is destroyed. This is significant. Some authors stated that the Meissner effect is absent in the QH state.[2,17] We note that the effective field B' is defined in the same form as that for the c-fermion model by Jain.[8] Only the reference state is bosonic (fermionic) in our (Jain's) case. Our theory based on Eq. (26) predicts the left-(low field) right (high field) symmetry relative to the center position $(\nu=1)$ in the resistivity dip in agreement with the experiments. The plateau stability with respect to the applied electric field can be explained in terms of the gap ϵ_g in the c-boson excitation spectrum, (9). The Bose distribution near absolute zero is reduced to the Boltzmann distribution f_c so that

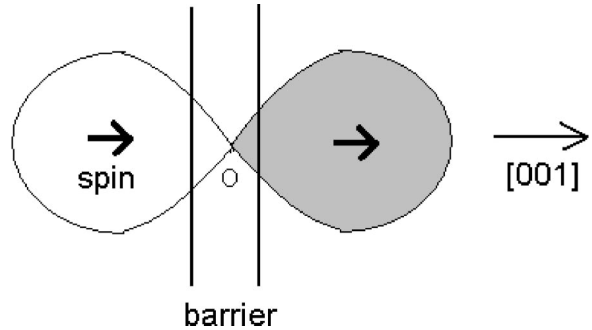

FIG. 3. The pc boson with parallel electron spins have a p-wave-type wave function, preferrably aligned in [001], positive on one side and negative on the other.

$$f_c(E)=\alpha e^{-\beta(\epsilon_g+cp)}=\alpha e^{-\beta\epsilon_g}e^{-\beta cp}$$

$$\alpha=\text{const}, \tag{28}$$

which contains the activation-energy type Boltzmann factor $\exp[-\epsilon_g/(k_BT)]$. In the prevalent theories the so-called "energy gap" is regarded as the energy required to create a quasielectron and a quasihole. The connection between this theoretical gap and the experimental gap is not clear. Our interpretation is more logical. Above T_c the c bosons move in all directions and hence the drift velocity v_d is the quantity averaged over angles. In addition there are $\pm$ c bosons. Hence the cancellation of v_d in the calculation of ρ_H is an approximation. The observed accuracy of 0.3% reported by Suen *et al.*[1] is reasonable. It is highly desirable to lower the temperature to observe the superconducting state and the stable plateau at $\nu=\frac{1}{2}$ since this proves the existence of the c boson at 0 K unquestionably.

The theory dealing with the temperature behavior of ρ and ρ_H applies to all QHE states. In particular the temperature dependence of the QHE state at $\nu=\frac{1}{3}$ in a monolayer system observed by Leadley *et al.*[10] follows the same trend.

Suen *et al.*[1] observed the $\frac{1}{2}$ state becomes weaker in a tilted magnetic field $(\theta=16°)$. The pc boson with parallel spins at $\theta=0°$ has a p-wave-type charge distribution, shown in Fig. 3, which is symmetric around the preferred direction [001]. For θ in addition 0° the symmetry is broken. It can align its spins along the field; but its cyclotronic motion only occurs in the plane, making the diamagnetic (energy-lowering) effect weaker and therefore the binding energy smaller. This explains qualitatively the observed behavior. In contrast the c boson with three fluxons generating the QHE state at $\nu=\frac{2}{3}$ has a s-wave-type charge distribution, and hence the state is stable against the tilting.

The QHE state at $\nu=1$ is closely connected with that at $\nu=\frac{1}{2}$. The *pc* boson as depicted in Fig. 2(b) may be formed at $\nu=1$. Since this is formed more symmetrically and more efficiently, the binding energy is greater at $\nu=1$ than at $\nu=\frac{1}{2}$. In fact the superconducting state with a stable plateau at $\nu=1$ is visible at 26 mK for the sample of the nominal electron density $n_s=2.8\times10^{11}$ cm^{-2}, see Ref. 1, Fig. 2. The weakening of this state observed in a tilt magnetic field ($\theta=31°$) supports our model.

Eisenstein *et al.*[2] observed the $\frac{1}{2}$ QHE state in a double quantum wells. The strength of the state decreases with increasing interwell center-to-center distance d. Otherwise the near-plateau in ρ_H and the deep drop in ρ are similar to those observed by Suen *et al.*[1] in a single wide well. This is a little surprising since a barrier (AlAs) exists between the two layers. The puzzle may be solved as follows.

The two electrons in the *pc* boson have parallel spins unlike the Cooper pair[12] since their spins are aligned in the external magnetic field. Then the electron exchange requires that the space part of the wave function be antisymmetric as shown in Fig. 3, making the node of the wave function reside at the barrier center. In this antisymmetric configuration the thin barrier is a small hindrance for a quantum state extending over the two layers. If the width of the barrier increases, the quantum state becomes weaker and eventually disappears, which is supported by the experiments.

In the formation of a *pc* boson we chose a two-step scenario: first, c fermions are formed and then two c fermions are bound to form a *pc* boson. This appears to suggest that the phonon exchange interaction between electron and fluxon ($|V_q|$) is stronger than that between two electrons ($|V'_q|$) contrary to the common belief. This puzzle may be solved as follows. The c fermions have spins aligned due to the applied field and hence the *pc* boson, if formed by the phonon exchange, must have antisymmetric wave function. This restriction makes the c-fermion binding very weak although the phonon exchange interaction between the two electrons is the stronger one ($|V_q|>|V'_q|$). Here the fermionic nature of the c fermions played an important role for the binding.

This is somewhat similar to the atomic binding. Consider a neutral atom containing Z electrons and a nucleus having the charge Ze. The cause of the binding is the Coulomb attraction between the electrons and the nucleus and the Coulomb repulsion among the electrons. In the self-consistent field approximation each electron moves in the medium of the total charge e concentrated at the nucleus just as for the electron in the hydrogen atom. Hence the hydrogenlike orbital description is justified. Applying the Fermi-Dirac statistics to the electrons and considering the spin degeneracy, we can explain the periodic nature of the atomic binding (ionization) energy and the reason why there are eight columns in the Mendeleev's periodic table. There is however a difference. The c particle (boson, fermion) is bonded only between the electron and a number of fluxons with no interaction between the fluxons. This condition makes the binding greater in proportion to the number of fluxons. This is borne out by the experiments.[10] The strength of the QH state measured by the plateau width at $\nu=\frac{1}{3}$ is greater than that at $\nu=1$.

These examples indicate that the binding energy of a composite depends not only on the principal attraction but also on the quantum statistics of the constituting particles and the interaction among the particles. Prior to the experimental discovery Yoshioka, MacDonald, and Girvin[18] and He *et al.*[19] discussed the QHE state at $\nu=\frac{1}{2}$ in a bilayer system, starting with the Coulomb interaction Hamiltonian and using numerical methods for few-particle systems. The predicted optimum layer separation d is approximately $2l_B\equiv2(\hbar/eB)^{1/2}$ while the experimental one is found a few times greater. Our theory is significantly different. We assume that the phonon exchange between the electron and the fluxon and among the electrons generates attractive interactions, creating c particles (bosons, fermions) and *pc* bosons. The c bosons condense below some critical temperature T_c and generate the superconducting QHE state with an energy gap ϵ_g for each $\nu=P/Q$. For a bilayer system, our theory predicts that p-wave-type *pc* bosons with parallel electron spins are formed at $\nu=\frac{1}{2}$, generating a deep resistivity drop as observed in the experiments.[1,2] The *pc* boson has a net spin and anisotropic charge distribution and hence it is unstable against the tilted magnetic field.

Before closing we briefly discuss a connection between our theory and the Laughlin wave function. The ground-state wave function for *any* quantum particle can be represented by a positive near-constant everywhere except at the sample boundary. The state in which all c bosons with (odd) Q fluxons occupy the same state is the many-boson ground state at $\nu=1/Q$. If this state is viewed in terms of the N electrons in the system, the Laughlin wave function for the $1/Q$ state is represented by

$$\Psi_Q(\mathbf{r}_1,\mathbf{r}_2,\ldots,\mathbf{r_N})=\Pi_{i<j}\ (z_i-z_j)^Q\exp(-\Sigma_i|z_i|^2),$$

$$z\equiv(x-iy)/l_B. \tag{29}$$

The highly correlated electron state can be developed by the phonon exchange and/or the longitudinal photon exchange (Coulomb) interaction. As in the HTSC the phonon exchange is more relevant here since this can generate an attractive interaction needed to form the c bosons and *pc* bosons. The ground-state wave function can carry no current. The wave functions

$$C\exp(ip_nx/\hbar),\quad p_n\equiv\hbar(2\pi n/L),\quad C=\text{const} \tag{30}$$

can carry currents, where L=sample length and $n=\pm1$, ±2,..., and a periodic boundary condition is assumed. Since L is macroscopic, the momentum p_n is small and so is the associated energy $\epsilon=cp$. If all $\pm$ c bosons occupy a single p_n, the supercurrent density J is given by Eq. (10). Any other p's will have an energy gap and hence the supercurrent is stable.

APPENDIX A: DERIVATION OF EQS. (18)–(20)

If we drop the "hole" contribution and the "electron" indices from our Hamiltonian $H+H_p$, see Eqs. (3) and (17), we obtain the Cooper Hamiltonian

$$H_c=\sum_{\mathbf{k}}{}' \sum_{\mathbf{q}}{}' (\epsilon_{|\mathbf{k}+\mathbf{q}/2|}+\epsilon_{|-\mathbf{k}+\mathbf{q}/2|})D^\dagger_{\mathbf{kq}}D_{\mathbf{kq}} - \sum_q{}' \sum_k{}' \sum_{k'}{}' v_p D^\dagger_{\mathbf{k}'\mathbf{q}}D_{\mathbf{kq}}. \tag{A1}$$

Using Eqs. (A1) and (4), we obtain

$$[H_c,D^\dagger_{\mathbf{kq}}]=[\epsilon(|\mathbf{k}+\mathbf{q}/2|)+\epsilon(|-\mathbf{k}+\mathbf{q}/2|)]D^\dagger_{\mathbf{kq}} - v_p\sum_{\mathbf{k}'} D^\dagger_{\mathbf{k}'\mathbf{q}}(1-n_{\mathbf{k}+\mathbf{q}/2}-n_{-\mathbf{k}+\mathbf{q}/2}). \tag{A2}$$

The net momentum $\mathbf{q}$ is a constant of motion, which arises from the momentum conservation in the phonon exchange process. The Hamiltonian H_c is bilinear in $(D,D^\dagger)$, and can therefore be diagonalized:

$$H_c=\sum_{\mathbf{q}} W_q\phi^\dagger_{\mathbf{q}}\phi_{\mathbf{q}}, \tag{A3}$$

where W_q is the boson energy and $\phi_{\mathbf{q}}$ the annihilation operator at the simultaneous eigenstate of c.m. momentum $\mathbf{q}$ and energy. We multiply Eq. (A2) by $\phi_{\mathbf{q}}\rho_{gc}$ from the right and take a grand ensemble trace and obtain

$$W_q a_{\mathbf{kq}}=[\epsilon(|\mathbf{k}+\mathbf{q}/2|)+\epsilon(|-\mathbf{k}+\mathbf{q}/2|)]a_{\mathbf{kq}} - v_p\sum_{\mathbf{k}'} \langle D^\dagger_{\mathbf{k}'\mathbf{q}}(1-n_{\mathbf{k}+\mathbf{q}/2}-n_{-\mathbf{k}+\mathbf{q}/2})\phi_{\mathbf{q}}\rangle, \tag{A4}$$

where $a_{\mathbf{kq}}\equiv\langle D^\dagger_{\mathbf{kq}}\phi_{\mathbf{q}}\rangle$; the angular brackets means the grand canonical ensemble average: $\langle A\rangle\equiv\mathrm{Tr}\{A\rho_{gc}\}\equiv\mathrm{Tr}\{A\exp(\alpha N-\beta H)\}/\mathrm{Tr}\{\exp(\alpha N-\beta H)\}$. Denoting the wave function in the bulk limit by $a(\mathbf{k},\mathbf{q})$ and using a factorization approximation, we obtain from Eq. (A4)

$$W_q a(\mathbf{k},\mathbf{q})=[\epsilon(|\mathbf{k}+\mathbf{q}/2|)+\epsilon(|-\mathbf{k}+\mathbf{q}/2|)]a(\mathbf{k},\mathbf{q})-(2\pi\hbar)^{-2}v_p \times\int{}' d^2k'\,a(\mathbf{k}',\mathbf{q})\{1-f_F[\epsilon(|\mathbf{k}+\mathbf{q}/2|)]+f_F[\epsilon(|1-\mathbf{k}+\mathbf{q}/2|)]\}, \tag{A5}$$

where $\langle n_p\rangle=1/\{\exp(\beta\epsilon_p)+1\}\equiv f_F(\epsilon_p)$ is the Fermi distribution function with the energy ϵ_p being measured relative to the Fermi energy. c bosons and electrons are quite different particles, and hence their motion is correlated weakly, and the factorization is justified. In the low-temperature limit $(T\to0,\beta\to\infty)$ $f_F(\epsilon_p)\to0$ $(\epsilon_p>0)$. Equation (A5) then becomes Eq. (18). Assuming that the *pc* boson is bound so that $W_q<0$, we then obtain from Eq. (A5)

$$1=\frac{v_p}{(2\pi\hbar)^2}\int{}' d^2k'[\epsilon(|\mathbf{k}+\mathbf{q}/2)+\epsilon(|-\mathbf{k}+\mathbf{q}/2|)+|W_q|]^{-1}. \tag{A6}$$

We now assume a free fermion model, whose Fermi surface is a circle of radius (momentum) $k_F\equiv(2m_1\epsilon_F)^{1/2}$, where m_1 represents the effective mass.

The prime on the k integral means the restriction $0<\epsilon(|\mathbf{k}+\mathbf{q}/2|),\ \epsilon(|-\mathbf{k}+\mathbf{q}/2|)<\hbar\omega_D$. We may choose the x axis along $\mathbf{q}$. The k integral can then be expressed by

$$\frac{(2\pi\hbar)^2}{v_2}=4\int_0^{\pi/2}d\theta\int_{k_F+(1/2)q\cos\theta}^{k_F+k_D-(1/2)q\cos\theta}\frac{kdk}{|W_q|+(k^2-k_F^2)/m_1} = 2m_1\int_0^{\pi/2}d\theta\ln\left|\frac{|W_q|+2\hbar\omega_D-v_Fq\cos\theta}{|W_q|+v_Fq\cos\theta}\right|, \tag{A7}$$

where $k_D\equiv m_1\omega_D\hbar k_F^{-1}$, and we retained the linear term in (k_D/k_F) only. We assume a small q and keep terms up to the first order in q. After performing the θ integration, we obtain Eqs. (19) and (20).

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