Transport theory of interacting mesoscopic systems: A memory-function approach to charge-counting statistics

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We address the subtle problem of formulating mesoscopic transport phenomena in the language of many-body physics. We propose a microscopic description in which the whole system (sample, leads, and detectors) is given fully-quantum-mechanical treatments. The dynamics of the system is obtained from the projection, or memory-function, formalism of nonequilibrium statistical mechanics combined with recent prescriptions for measurements that are extended in the time domain. The associated irreversible quantum dynamics contains an intrinsic doubling of the degrees of freedom identical to the real-time path-integral representation of the Keldysh formalism. We derive a simple formula relating the generating function of the charge-counting statistics to the single-particle matrix Green's function of the interacting system, thereby generalizing the Levitov-Lesovik functional determinant formula. We report an interesting sample-leads duality in our description, which has a simple interpretation in the regime of noninteracting particles, thus establishing the equivalence between the Green's-function technique and the random scattering-matrix approach. We discuss the physical conditions, within the present scheme, for the validity of the Landauer-Büttiker description of mesoscopic transport. We conclude by providing an exact solution for the problem of charge transfer into a noninteracting ballistic cavity in the presence of a periodic time-dependent voltage, using the supersymmetry method.

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I. INTRODUCTION

The physics of bulk many-body systems has reached a staggering level of maturity both at the phenomenological level of response functions and relaxation processes and that of rigorous microscopic calculations of correlation functions using quantum-field-theoretical methods. Achievements include an essentially complete understanding of equilibrium and nonequilibrium phenomena in systems as varied as normal quantum fluids, magnetic materials, liquid crystals, superfluids, and superconductors, both at stable phases and in the vicinity of critical points. To be sure, much of our current understanding of irreversible transport processes in these systems stems from unifying concepts derived from linear-response theory and time-dependent correlation functions, such as Onsager's reciprocity relations, dynamic stability conditions, fluctuation-dissipation theorems, causality relations, Kubo formulas, sum rules, Goldstone modes, and hydrodynamic collective fluctuations.

In the 1980s, however, technological breakthroughs in nanolithographic processes made available electron devices in which novel nonequilibrium conditions became experimentally accessible.[1] It was soon realized that for these so-called mesoscopic systems new theoretical tools were needed to account for challenging experimental data in new transport regimes. An important step forward in this direction was the early recognition of the inadequacy of the conventional Boltzmann picture in which one neglects the microscopic details of the impurity potential by replacing it by an average smooth function with a constant gradient along the sample, i.e., a homogeneous transport field that accelerates the carriers throughout the whole device. Such an approach is entirely justified for bulk systems, in which the many-body description is contracted by projecting the dynamics of the initially well-controlled density matrix onto a subspace of states perturbed only on scales much larger than atomic distances. These semiclassical concepts cannot, however, account for the coherent processes inside the sample that lies at the heart of mesoscopic phenomena.

The correct procedure came with the rediscovery of Landauer's pioneering insight on the conduction problem,[2,3] which consisted of inverting the traditional viewpoint of regarding the transport field as a cause and the current flow as a response. In Landauer's approach, self-consistent inhomogeneous transport fields appear as a many-body response (Coulomb screening) to carrier injection in the contacts implying that transport coefficients could be calculated by considering a sequence of highly localized voltage drops across impurity barriers. For a one-dimensional disordered conductor, this prescription leads, at zero temperature, to a direct connection between the conductance G and the transmission coefficient T of the sample, $G=G_0T/(1-T)$, where $G_0=2e^2/h$ is the conductance quantum. The singlemost appealing feature of this approach is the ease with which quantum-mechanical coherence effects can be accounted for. If one assumes that the phase-breaking length is larger than the sample itself, one may neglect many-body effects and calculate the conductance from the transmission coefficient of the corresponding single-particle scattering problem. Such procedure turned out to be highly successful.

After this initial breakthrough, it was soon realized that the usefulness of the Landauer scattering approach in describing mesoscopic phenomena hinged upon the possibility of generalizing it to conductors of finite cross sections, i.e., with a finite number of open transport channels. This enterprize, however, turned out to be much harder and controversial than expected.[3] The correct answer was finally put forward by Büttiker,[4] who explained in the process the puzzling manifestation of Onsager-Casimir reciprocity relations in resistance coefficient measurements. Büttiker's solution demonstrated the crucial role played by the measurement process in the description of mesoscopic dynamics. By assuming the measurement time, i.e. the average time it would take for the signal in the detector to be distinguished from the background noise, to be equal to the dephasing time (in modern language this is equivalent to assuming an ideal detector), Büttiker established a direct link between the measurement process and Landauer's scattering formalism. Furthermore, the spatial separation of phase-breaking events allowed for, as pointed out earlier by Landauer, a specification of boundary conditions in terms of certain fictitious reservoirs, which were supposed to incorporate all the complexities of the underlying many-body problem. In particular, the existence of well-defined electrochemical potentials in these reservoirs, which after all are integral parts of a connected many-body system, could, at least, in principle, be justified by appealing to hidden irreversible phase-breaking scattering processes, similar to the way local equilibrium conditions appear in the hydrodynamic regime of quantum fluids. The remarkable simplicity of such formalism, which absorbs many-body effects as boundary conditions and allows transport observables such as conductance and noise power spectrum to be directly related to the scattering characteristics of a noninteracting system, is only matched by its surprising experimental accuracy. In the trail of this success a massive amount of work both theoretical and experimental followed, exploiting essentially all aspects of the single-particle description and bringing the area of mesoscopic transport to the level of a mature research field.

Notwithstanding the fully experimental corroboration of the quantitative predictions of the Landauer-Büttiker scattering approach and its close relatives, such as the impurity average single-particle Green's-function technique with reservoir boundary conditions,[1] its true microscopic foundation in terms of a controlled sequence of conserving approximations starting from a full-fledged many-body description is largely unknown. This fundamental problem might be considered, from a practical point of view, somewhat academic, was it not for the fact that recent experimental observations may well be stretching the current theory beyond its breaking point. These include nonlinear-response measurements such as charge pumping into quantum dots[5] and the integer quantum Hall effect at large currents, time-dependent effects in photon-assisted tunneling through single[6] and double[7] quantum dots, interaction driven phenomena such as the mesoscopic Coulomb drag effect,[8] phase relaxation in open ballistic cavities,[9] Fermi-edge singularities in impurity assisted tunneling through tunnel junctions,[10] transport properties in hybrid normal-superconductor systems,[11] tunneling magnetoresistance in quantum dots coupled to ferromagnetic leads,[12] and electron transmission through atomic-sized conductors,[13] to mention just a few. Although much effort has been made to account for these effects by generating the appropriate extensions to the theory via suitable, phenomenology guided, generalizations of the boundary conditions and scattering characteristics with reasonable success, the general expectation is that eventually irreducible many-body effects will become dominant and the use of sophisticated nonperturbative tools, such as field-theoretical renormalization-group procedures, shall be imperative. For that, one would need a more general theory in which the dynamical aspects of the many-body problem appear in its natural form. In this way, one would, in principle, be able to establish the precise physical conditions, in terms of ratios of relevant time scales, for the validity of the local equilibrium conditions underlying the Landauer-Büttiker approach.

Interesting efforts in this direction can already be seen in the recent literature, where extensions of the Landauer-Büttiker formalism were introduced by admitting arbitrary interactions in the mesoscopic region, but maintaining the leads as noninteracting and by using reservoir boundary conditions with different electrochemical potentials. Within the Kubo formalism, for instance, Oguri[14] developed a finite-temperature linear-response approach and expressed the transmission probability in terms of a three-point correlation function. Using the more general Keldysh formalism, Meir and Wingreen[15] managed to go beyond the linear-response regime and expressed the finite-temperature differential conductance in terms of nonequilibrium Green's functions. Their approach was later extended to address time-dependent phenomena as well.[16] Much effort has also been devoted to developing efficient calculation schemes, such as numerical procedures and renormalization-group techniques. In Ref. 17, the authors expressed the zero-temperature linear-response conductance in terms of the persistent current of an auxiliary noninteracting system. This method turned out to be well suited for numerical studies. In Ref. 18, a numerical recipe was put forward for an approximate evaluation of the Meir-Wingreen conductance formula. In Ref. 19 a powerful framework was presented to study transport of interacting electrons through nearly closed ballistic chaotic quantum dots. By separating the regimes in which interactions can be described by Landau Fermi-liquid parameters, these authors were able to recover the universal Hamiltonian description[20] in a weak-coupling phase and predicted a surprising strong-coupling phase with a spontaneous Fermi-surface distortion. For open systems, time scales related to electron decay widths enter the picture leading to much complex transport regimes. An important progress in this problem has recently been reported by Kindermann and Nazarov,[21] who developed a renormalization-group approach to address the weak interacting regime. They showed the existence of two universality classes for the power-law behavior in the low-voltage and low-temperature differential conductance. A third universality class, related to a quantum transition at intermediate coupling and driven by resonance trapping, has recently been reported.[22]

In this work, we address the problem in its full generality and propose a formulation of mesoscopic transport in the language of many-body physics. In Sec. II, we describe the physical system together with its measurement setup. Using the language of modern quantum measurement theory,[25] we relate the internal dynamics of the system to the reduced density matrix of the detector. The resulting doubling of degrees of freedom is conveniently accommodated in the

Keldysh time-contour formalism.[26] In Sec. III, we apply the memory-function technique[27] to deduce the relevant Dyson's equation for the coupled leads-sample-detector system. A generalization of Levitov-Lesovik determinant formula[28] for the generating function of charge-counting statistics is derived in Sec. IV. This formula includes interaction effects and uncovers a striking duality in the problem that allows the development of two equivalent approaches: one that eliminates the degrees of freedom in the leads (inside problem) and another that eliminates the degrees of freedom in the sample (outside problem). In Sec. V, we adopt the approach of eliminating the degrees of freedom in the leads and propose an approximation, for its influence on the dynamics of the remaining degrees of freedom, that exploits a wide separation of certain relevant time scales. The Landauer-Büttiker regime is obtained as a particular limit. We analyze, in Sec. VI, an exactly solvable model of a noninteracting ballistic chaotic cavity and compute the average transmitted charge, during a finite observation time, in the presence of a periodic time-dependent voltage. A discussion and a summary are presented in Sec. VII.

II. THE MODEL SYSTEM AND THE MEASUREMENT PROCEDURE

One of the fundamental problems of a quantum description of a given physical system is to establish the precise connection between the dynamics of the information acquisition process of its observable features and the quantum dynamics of the coupling to the associated measurement probes. In general, both the quantum system and the probes are complex many-body systems and the resulting irreversible dynamics of the measurement process must, in principle, be dealt with using techniques from nonequilibrium statistical mechanics. Although a universal scheme for solving such a problem is not yet available, significant progress have been made recently by considering certain simple processes, such as indirect quantum measurements.[25] In this case, the measurement is described as a two-step process consisting of (i) an interaction with a previously prepared quantum probe, described by a unitary evolution that produces correlations between quantum states of both the system and the probe and (ii) a direct measurement of a chosen observable in the probe leading to an entropy production associated with information extraction and thus causing, via back-action effects, an irreversible change in the system's initial quantum state.

The great advantage of the indirect quantum measurement formalism in comparison with the traditional ensemble approach is the possibility to address processes that are extended in the time domain and therefore one can access subtle time-dependent correlations in the system. Following the time evolution of the system plus detector one realizes the existence of three distinct time scales:[29] the dephasing time τ_φ over which the system loses phase coherence; the measurement time τ_{meas} after which information about the system's state is extracted, and finally there is the detector's back-action on the system destroying the information about the initial state on a mixing time scale τ_{mix}. The question of the efficiency of a quantum measurement, i.e., finding the conditions for minimal detector-induced dephasing, has received much attention of the recent literature, particularly in connection with quantum state engineering[29] and with the quest to construct quantum-limited devices.[30]

In the context of mesoscopic physics, Büttiker was the first to realize the importance of the measurement process in describing transport observables. He pointed out[4] that the Landauer conductance formula encompasses a scattering-matrix description of the system-detector interaction and that the very notion of particle reservoirs in the presence of transport currents can only make physical sense in such a context. More recently, Levitov and Lesovik,[28] building on the similarity between the theory of photodetection in quantum optics and current measurement in mesoscopic devices, proposed the concept of full counting statistics of particle transfer through the system during a given time interval T_0. Using a current detection scheme based on the dynamics of a spin-1/2 galvanometer electromagnetically coupled at the system's interface, they derived a microscopic representation of the counting statistics generating function, defined as

$$\chi_\lambda(T_0)=\sum_n e^{in\lambda}p_n(T_0), \tag{1}$$

where $p_n(T_0)$ is the probability that a discrete number n of particles traverse the interface during the observation time T_0. Adopting the Landauer-Büttiker scattering formalism, they were able to derive a very useful determinant formula for $\chi_\lambda(T_0)$, which could be generalized to time-dependent problems and was successfully applied to ac transport[31] and quantum pumps.[32]

In this section we shall formulate the above current detection scheme as a nonequilibrium mesoscopic many-body problem. Our model system is a many-body generalization of that of Ref. 33 and consists of a phase-coherent sample connected via M macroscopic conducting leads to the terminals of $M/2$ generators supplying time-dependent electromotive forces. The system has thus the topology of a multiply connected network in which, as shown in Ref. 34, Maxwell's equations, though locally conservative, admit globally dissipative solutions. As pointed out in Ref. 23, the importance of the latter result resides in the fact that it provides the physical grounds for using quantum kinetic methods with locally conservative Hamiltonians (such as the Keldysh technique) to describe the globally dissipative dynamics in the system.

Following the standard prescription for applying the Keldysh formalism,[35] we assume that at times prior to t_0 the macroscopic leads and the sample are uncoupled and the system is taken to be in contact with a thermal bath at temperature T and with a *unique* particle reservoir of chemical potential μ. The Hamiltonian of the uncoupled system reads $H^0_{sys}=H_l+H_s$, where $H_l=H^0_{leads}+H^{int}_{leads}$ and $H_s=H^0_{sample}+H^{int}_{sample}$ are the Hamiltonians for the electrons in the leads and sample, respectively. More specifically, introducing the field operators $\varphi_\alpha(\mathbf{r})$ and $\psi_s(\mathbf{r})$ for electrons in lead α and in the sample, respectively, we may write

$$H^0_{leads}=\sum_{\alpha=1}^{M}\int_{\Omega_\alpha}d\mathbf{r}\varphi^\dagger_\alpha(\mathbf{r})\left[\frac{1}{2m}\left(\mathbf{p}+\frac{e}{c}\mathbf{A}_\alpha(\mathbf{r},t)\right)^2+U_\alpha(\mathbf{r})\right]\varphi_\alpha(\mathbf{r}) \tag{2}$$

and

$$H^{int}_{leads}=\sum_{\alpha=1}^{M}\int_{\Omega_\alpha}d\mathbf{r}d\mathbf{r}'u_\alpha(\mathbf{r},\mathbf{r}')\varphi^\dagger_\alpha(\mathbf{r})\varphi^\dagger_\alpha(\mathbf{r}')\varphi_\alpha(\mathbf{r}')\varphi_\alpha(\mathbf{r}) \tag{3}$$

for the leads, and

$$H^0_{sample}=\int_{\Omega_s}d\mathbf{r}\psi^\dagger_s(\mathbf{r})\left[\frac{p^2}{2m}+V_s(r)\right]\psi_s(\mathbf{r}) \tag{4}$$

and

$$H^{int}_{sample}=\int_{\Omega_s}d\mathbf{r}d\mathbf{r}'v_s(\mathbf{r},\mathbf{r}')\psi^\dagger_s(\mathbf{r})\psi^\dagger_s(\mathbf{r}')\psi_s(\mathbf{r}')\psi_s(\mathbf{r}) \tag{5}$$

for the mesoscopic sample. The total configuration space is given by $\Omega=\Omega_s\cup\Omega_l$, where $\Omega_l=\cup_{\alpha=1}^{M}\Omega_\alpha$ and $\partial\Omega_s\cap\partial\Omega_l=0$. The electron spin can be accounted for by introducing two identical copies of Ω one for each spin projection, so that $\Omega=\Omega(\uparrow)\cup\Omega(\downarrow)$. The vector potential $\mathbf{A}_\alpha(\mathbf{r},t)$ describes the effect, in lead α, of switching on the emf, while the scalar functions $U_\alpha(\mathbf{r})$ and $V_s(\mathbf{r})$ represent internal potential profiles in lead α and in the sample, respectively. We remark that the quantum dynamics of this uncoupled system can, in principle, be described by conventional many-body techniques.

Let us now turn to the measurement problem associated to the emergence of current-carrying states in the coupled leads plus sample system. Following Levitov and Lesovik,[28] we consider a spin-1/2 galvanometer, represented by the following fictitious vector potential in the sample region:

$$\mathbf{A}_s(\mathbf{r},t)=\frac{\Phi_0}{4\pi}\sigma_z\sum_{\alpha=1}^{M}\lambda_\alpha(t)\boldsymbol{\nabla}\theta(f_\alpha(\mathbf{r})), \tag{6}$$

in which $\Phi_0=hc/e$ is the flux quantum, σ_z is a Pauli spin matrix, and the equation $f_\alpha(\mathbf{r})=0$ specifies the leads-sample interface $C_\alpha=\partial\Omega_s\cap\partial\Omega_\alpha$. The counting field $\lambda_\alpha(t)$ has a constant value λ_α in the measurement time interval $0\leqslant t<T_0$ and goes smoothly to zero for $t_0\leqslant t<0$ and $t>T_0$. We assume that at time t_0 both the system-detector and the leads-sample couplings are switched on and the total time-dependent Hamiltonian is given by

$$H_{tot}=H_{sys}+H_{sd}+H_d, \tag{7}$$

where $H_{sys}=H^0_{sys}+H_{ls}=H_l+H_s+H_{ls}$ is the Hamiltonian of the coupled leads plus sample system and H_{sd} represents the system-detector interaction. We neglect, in the following, the internal dynamics of the spin system by setting $H_d=0$. The total density matrix at $t=t_0$ is prepared so that it reads

$$\rho^0_{tot}=\rho^0_{sys}\otimes\rho^0_d, \tag{8}$$

where ρ^0_{sys} is diagonal in charge representation.[36] For $t>t_0$ we may describe the first step of the indirect measurement prescription by using von Neumann's equation

$$\rho_{tot}(t)=U(t,t_0)\rho^0_{tot}U(t_0,t), \tag{9}$$

in which the evolution operator $U(t,t_0)$ satisfies

$$i\hbar\frac{\partial}{\partial t}U(t,t_0)=H_{tot}U(t,t_0). \tag{10}$$

In the second step, a direct measurement is performed on the detector to extract information about the system. Accordingly, we may thus project the total density matrix onto the subspace of the detector's degrees of freedom by performing the partial trace

$$\rho_d(t)=\mathrm{Tr}_{sys}(U(t,t_0)\rho^0_{tot}U(t_0,t)). \tag{11}$$

Expressing the evolution operator in the interaction picture with respect to H_{sys}, we get

$$U(t,t_0)=U^0(t,t_0)\vec{T}\exp\left[-\frac{i}{\hbar}\int_{t_0}^{t}dt'H^0_{sd}(t')\right]U^0(t_0,t) \tag{12}$$

and

$$U(t_0,t)=U^0(t_0,t)\overleftarrow{T}\exp\left[\frac{i}{\hbar}\int_{t_0}^{t}dt'H^0_{sd}(t')\right]U^0(t,t_0), \tag{13}$$

where $\vec{T}$ ($\overleftarrow{T}$) is the (anti)time-ordering operator. Using the above expressions in Eq. (11) we get

$$\langle\sigma|\rho_d(t)|\sigma'\rangle=\langle\sigma|\rho^0_d|\sigma'\rangle\chi^{\sigma\sigma'}_\lambda(t), \tag{14}$$

where $\chi^{\uparrow\uparrow}_\lambda(t)=1=\chi^{\downarrow\downarrow}_\lambda(t)$ and $\chi^{\uparrow\downarrow}_\lambda(t)=\chi^{\downarrow\uparrow}_{-\lambda}(t)$ with

$$\chi^{\uparrow\downarrow}_\lambda(t)=\left\langle T_\gamma\exp\left(-\frac{i}{\hbar}\int_\gamma dt'H^0_{sd}(t')\right)\right\rangle_{sys}, \tag{15}$$

where $\gamma=[t_0\to t\to t_0]$ is the Keldysh time contour, and

$$\langle\cdots\rangle_{sys}\equiv\mathrm{Tr}(\rho^0_{sys}(t)\ \ldots), \tag{16}$$

in which

$$\rho^0_{sys}(t)=U^0(t_0,t)\rho^0_{sys}U^0(t,t_0). \tag{17}$$

The counting field has been redefined so that it changes sign when going from the lower to the upper part of the Keldysh contour.

Equation (15) is, as expected, in agreement with the result of Ref. 37. Note that the doubling of degrees of freedom, characteristic of the Keldysh approach, is generated directly from the dynamics and, as pointed out by Rammer,[38] this is a generic feature of the density-matrix description to quantum dynamics. Interestingly, the authors of Ref. 39, analyzing the microscopic foundations of this formula, concluded that it is independent of the specific charge detection scheme, provided one neglects the internal dynamics of the detector.

We remark that the measurement problem in mesoscopic physics has become an area of its own, with several interesting recent developments such as optimal quantum detectors and controlled entanglement effects, topics of great interest in quantum-information theory.[40]

III. THE MEMORY-FUNCTION FORMALISM

The measurement prescription described in Sec. II specifies rigorously the observational level. The information extracted in the detection process must be obtained from the full many-body dynamics of the coupled system via a suitably defined projection. The memory-function formalism[27] is a systematic way of implementing such a program. This technique, also known as the projection method, was developed in the 1960s by Nakajima, Zwanzig, Mori, and Robertson[41] and is one of the cornerstones of nonequilibrium statistical mechanics. Its main features can be summarized as follows: (i) it allows for a detailed analysis of the time scales in the problem, (ii) it provides a well-controlled procedure to exploit possible separations of time scales; and (iii) it permits systematic inclusion of finite memory effects in the dynamics. This method has been given a powerful geometric interpretation in Ref. 42 which allows for many useful generalizations. In the present problem, it will prove convenient to introduce a geometrical extension of the formalism by using noncommutative Grassmann coordinates.

In the projection technique the dynamics of the system is projected onto a level of description, defined by a small, selected set of relevant operators. We choose the following set:

$$\mathcal{B}=\{\varphi_1(\mathbf{r}),\varphi_1^\dagger(\mathbf{r}),\ldots,\varphi_M(\mathbf{r}),\varphi_M^\dagger(\mathbf{r}),\psi_s(\mathbf{r}),\psi_s^\dagger(\mathbf{r})\}. \tag{18}$$

The expectation values of the relevant operators are fundamental constraints on the projected dynamics and thus, in order to get a well-posed problem with nonvanishing average values, we must introduce the following fictitious time-dependent perturbation in the system:[43]

$$\delta H_t=\sum_{\alpha=1}^{M}\int_{\Omega_\alpha} d\mathbf{r}[\eta_\alpha(\mathbf{r},t)\varphi_\alpha(\mathbf{r})+\eta_\alpha^*(\mathbf{r},t)\varphi_\alpha^\dagger(\mathbf{r})] + \int_{\Omega_s} d\mathbf{r}[\xi_s(\mathbf{r},t)\psi_s(\mathbf{r})+\xi_s^*(\mathbf{r},t)\psi_s^\dagger(\mathbf{r})], \tag{19}$$

in which Grassmann fields have been introduced as Lagrange multipliers. The average values of the electron fields are given by $\chi_\alpha(\mathbf{r},t)=\langle\varphi_\alpha(\mathbf{r},t)\rangle_{\mathbf{J},\mathbf{J}^*}$, $\chi_\alpha^*(\mathbf{r},t)=\langle\varphi_\alpha^\dagger(\mathbf{r},t)\rangle_{\mathbf{J},\mathbf{J}^*}$, $\zeta_s(\mathbf{r},t)=\langle\psi_s(\mathbf{r},t)\rangle_{\mathbf{J},\mathbf{J}^*}$, and $\zeta_s^*(\mathbf{r},t)=\langle\psi_s^\dagger(\mathbf{r},t)\rangle_{\mathbf{J},\mathbf{J}^*}$, in which we defined the following vector field $\mathbf{J}=(\eta_1,\ldots,\eta_M,\xi_s)$. The expectation value $\langle\cdots\rangle_{\mathbf{J},\mathbf{J}^*}$ is calculated with the following canonical density matrix:[43]

$$\rho(\mathbf{J},\mathbf{J}^*)=\frac{Z(0)}{Z(\mathbf{J},\mathbf{J}^*)}\rho(0)T_\gamma\exp\left(-\frac{i}{\hbar}\int_\gamma dt'\,\delta H_{t'}(t')\right). \tag{20}$$

The canonical partition function $Z(\mathbf{J},\mathbf{J}^*)$ is determined from the identity $\mathrm{Tr}\rho(\mathbf{J},\mathbf{J}^*)=1$.

Following up the conventional implementation of the projection formalism the next step would be to project out the evolution equation of the vector field of average values, $\mathbf{K}=(\chi_1,\ldots,\chi_M,\zeta_s)$, and thus derive the associated memory function. Here, we shall instead take an alternative route that has the advantage of allowing a direct link with Keldysh Green's-function technique. We start by defining the connected contour-ordered Green's function[44]

$$G_{\sigma\sigma'}(\mathbf{r},t|\mathbf{r}',t')=i\left.\frac{\delta^2\ln Z(\mathbf{J},\mathbf{J}^*)}{\delta J_{\sigma'}^*(\mathbf{r}',t')\,\delta J_\sigma(\mathbf{r},t)}\right|_{\mathbf{J},\mathbf{J}^*=0}, \tag{21}$$

where $\sigma,\sigma'=1,2,\ldots,M+1$. Since the mathematical structure of the projection formalism is quite similar to that of equilibrium statistical mechanics, we may define different ''thermodynamic potentials'' via generalized Legendre transformations. Introducing the compact notation, $x=(\mathbf{r},t/\hbar)$, we consider the following potential:

$$\mathcal{F}(\mathbf{K},\mathbf{K}^*)=-\ln Z(\mathbf{J},\mathbf{J}^*)+i\int dx[\mathbf{K}(x)\cdot\mathbf{J}(x) + \mathbf{K}^*(x)\cdot\mathbf{J}^*(x)]. \tag{22}$$

The Legendre transformation is completed by using the relations $\mathbf{J}(x)=-i\,\delta\mathcal{F}(\mathbf{K},\mathbf{K}^*)/\delta\mathbf{K}(x)$ and $\mathbf{J}^*(x)=-i\,\delta\mathcal{F}(\mathbf{K},\mathbf{K}^*)/\delta\mathbf{K}^*(x)$. Taking $\mathcal{F}(\mathbf{K},\mathbf{K}^*)$ as a generating functional we may define the contour-ordered vertex function[44]

$$\Gamma_{\sigma\sigma'}(\mathbf{r},t|\mathbf{r}',t')=-i\left.\frac{\delta^2\mathcal{F}(\mathbf{K},\mathbf{K}^*)}{\delta K_{\sigma'}(\mathbf{r}',t')\,\delta K_\sigma^*(\mathbf{r},t)}\right|_{\mathbf{K},\mathbf{K}^*=0}. \tag{23}$$

Combining Eq. (21) with Eq. (23) we obtain the important relation

$$\sum_\nu\int dy\,G_{\sigma\nu}(x|y)\Gamma_{\nu\sigma'}(y|x')=\delta_{\sigma\sigma'}\delta(x-x'). \tag{24}$$

We remark that a similar equation applies for the Green's function $g_{\sigma\sigma'}(x|x')$ and the vertex function $\gamma_{\sigma\sigma'}(x|x')$ of the uncoupled leads-sample system at times prior to t_0. We proceed by introducing the contour-ordered memory function

$$\Sigma_{\sigma\sigma'}(x|x')\equiv\gamma_{\sigma\sigma'}(x|x')-\Gamma_{\sigma\sigma'}(x|x'), \tag{25}$$

which, in turn, implies the validity of Dyson's equation

$$\mathbf{G}=\mathbf{g}+\mathbf{g}*\mathbf{\Sigma}*\mathbf{G}=\mathbf{g}+\mathbf{G}*\mathbf{\Sigma}*\mathbf{g}, \tag{26}$$

expressed here in a self-evident compact notation. The memory function is a central concept in the projection technique, since it contains detailed information about the relaxation processes and can thus be used to exploit separations in time scales and to describe retardation effects.

We are now in position to make contact with the Keldysh Green's-function formalism. As can be shown, by using Lengreth theorem,[45] adopting the Keldysh contour in the above derivations leads to matrix equations in a fictitious 2×2 space. We shall take the standard definition[35]

$$\overline{A}=\begin{pmatrix} A^r & A^K \\ 0 & A^a \end{pmatrix}, \tag{27}$$

where $A^{r(a)}$ is the retarded (advanced) component and $A^K=A^>+A^<$ is the Keldysh component. The less-than (more-than) functions $A^{<(>)}$ satisfy the following relation $A^r-A^a=A^>-A^<$. We may thus write Dyson's equation (26) using Keldysh matrix functions as follows:

$$\overline{\mathbf{G}}=\overline{\mathbf{g}}+\overline{\mathbf{g}}*\overline{\boldsymbol{\Sigma}}*\overline{\mathbf{G}}=\overline{\mathbf{g}}+\overline{\mathbf{G}}*\overline{\boldsymbol{\Sigma}}*\overline{\mathbf{g}}. \tag{28}$$

Note that the self-energy matrix $\overline{\boldsymbol{\Sigma}}$ of the Keldysh-Green's function formalism plays a role similar to that of the memory function in the projection technique. We stress that the Keldysh method, although less general, has the advantage of being perfectly adapted to powerful field-theoretical methods such as path-integral representations, saddle-point analysis, and Feynman diagrams. This is particularly relevant in applications that require implementations of sophisticated techniques such as the dynamical renormalization-group approach.[46]

IV. A DIFFERENT FORMULA FOR THE GENERATING FUNCTION OF CHARGE-COUNTING STATISTICS

In this section we shall derive an explicit connection between Eq. (15) and the single-particle Green's function of the Keldysh formalism, thereby providing a different useful formula for charge-counting statistics. We begin by writing all matrices in Dysons's equation (28) in a leads-sample block structure, so that we get

$$\overline{\mathbf{G}}=\begin{pmatrix} \overline{G}_l & \overline{G}_{ls} \\ \overline{G}_{sl} & \overline{G}_s \end{pmatrix} \tag{29}$$

for the coupled Green's function,

$$\overline{\mathbf{G}}=\begin{pmatrix} \overline{g}_l & 0 \\ 0 & \overline{g}_s \end{pmatrix} \tag{30}$$

for the uncoupled Green's function, and

$$\overline{\boldsymbol{\Sigma}}=\begin{pmatrix} 0 & \overline{\Sigma}_{ls} \\ \overline{\Sigma}_{sl} & 0 \end{pmatrix} \tag{31}$$

for the self-energy describing the coupling. Inserting Eqs. (29)–(31) into Eq. (28) and gauging out the influence of the detector we obtain the following matrix equations:

$$\overline{G}_l=\overline{g}_l+\overline{g}_l\overline{\Sigma}_{ls}\overline{G}_{sl}=\overline{g}_l+\overline{G}_{ls}\overline{\Sigma}_{sl}\overline{g}_l, \tag{32}$$

$$\overline{G}_{ls}=\overline{g}_l\overline{\Sigma}_{ls}\overline{G}_s=\overline{G}_l\overline{\Sigma}_{ls}\overline{g}_s^{(-\lambda)}, \tag{33}$$

$$\overline{G}_{sl}=\overline{g}_s^{(-\lambda)}\overline{\Sigma}_{sl}\overline{G}_l=\overline{G}_s\overline{\Sigma}_{sl}\overline{g}_l, \tag{34}$$

and

$$\overline{G}_s=\overline{g}_s^{(-\lambda)}+\overline{g}_s^{(-\lambda)}\overline{\Sigma}_{sl}\overline{G}_{ls}=\overline{g}_s^{(-\lambda)}+\overline{G}_{sl}\overline{\Sigma}_{ls}\overline{g}_s^{(-\lambda)}, \tag{35}$$

where the gauge transformation is defined by

$$\overline{g}_s^{\lambda}(x,x')=e^{i\overline{a}_\lambda(x)}\overline{g}_s(x,x')e^{-i\overline{a}_\lambda(x)}, \tag{36}$$

with

$$\overline{a}_\lambda(x)=\frac{1}{2}\overline{\tau}\sum_{\alpha=1}^{M}\lambda_\alpha(t)\,\theta(f_\alpha(\mathbf{r})), \tag{37}$$

where

$$\overline{\tau}=\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \tag{38}$$

Combining Eqs. (32) and (35) with Eqs. (33) and (34) we get the renormalized Dyson equations

$$\overline{G}_l=\overline{g}_l+\overline{g}_l\overline{\Sigma}_l\overline{G}_l=\overline{g}_l+\overline{G}_l\overline{\Sigma}_l\overline{g}_l \tag{39}$$

and

$$\overline{G}_s=\overline{g}_s^{(-\lambda)}+\overline{g}_s^{(-\lambda)}\overline{\Sigma}_s\overline{G}_s=\overline{g}_s^{(-\lambda)}+\overline{G}_s\overline{\Sigma}_s\overline{g}_s^{(-\lambda)}, \tag{40}$$

where the renormalized self-energy functions are given by

$$\overline{\Sigma}_l=\overline{\Sigma}_{ls}\overline{g}_s^{(-\lambda)}\overline{\Sigma}_{sl} \tag{41}$$

and

$$\overline{\Sigma}_s=\overline{\Sigma}_{sl}\overline{g}_l\overline{\Sigma}_{ls}. \tag{42}$$

In the Keldysh formalism it is usual to define a matrix current that satisfies extended conservation laws thus generalizing standard charge conservation. For the present system the total matrix current inside the sample at time t in the presence of the counting field $\lambda_\alpha(t)$ reads simply[1]

$$\overline{I}_s^{\lambda}(t)=\frac{e}{\hbar}\overline{\tau}\int_{\Omega_s}d\mathbf{r}[\overline{\Sigma}_s^{\lambda}\overline{G}_s^{\lambda}-\overline{G}_s^{\lambda}\overline{\Sigma}_s^{\lambda}](\mathbf{r},t|\mathbf{r},t), \tag{43}$$

in which the gauge transformed functions are defined as in Eq. (36). The total charge transferred to the system during the measurement is given by $Q_s(T_0)=\lim_{\lambda\to 0}Q_s^{\lambda}(T_0)$, where

$$Q_s^{\lambda}(T_0)=\frac{1}{2}\int_0^{T_0}dt\,\mathrm{Tr}_{\mathrm{K}}(\overline{I}_s^{\lambda}(t)), \tag{44}$$

in which Tr_K is the trace over Keldysh 2×2 matrix space. Inserting Eq. (43) into Eq. (44) and using Eq. (36) we get

$$Q_s^{\lambda}(T_0)=-ie\sum_{\alpha=1}^{M}\mathrm{Tr}\left(\overline{G}_s^{\lambda}\frac{\partial\overline{\Sigma}_s^{\lambda}}{\partial\lambda_\alpha}\right), \tag{45}$$

where $\mathrm{Tr} \equiv \mathrm{Tr}_K \int dx$. This expression can be further simplified by using the following relation obtained from Dyson's equation (40):

$$\frac{\partial \overline{G}_s^{\lambda}}{\partial \lambda_\alpha} = \overline{G}_s^{\lambda} \frac{\partial \overline{\Sigma}_s^{\lambda}}{\partial \lambda_\alpha} \overline{G}_s^{\lambda}, \tag{46}$$

thus we get

$$Q_s^{\lambda}(T_0) = -ie \sum_{\alpha=1}^{M} \frac{\partial}{\partial \lambda_\alpha} \mathrm{Tr} \ln(\overline{G}_s^{\lambda}). \tag{47}$$

Defining $Q_{s\alpha}(T_0) = \lim_{\lambda \to 0} Q_{s\alpha}^{\lambda}(T_0)$ as the charge transferred into the sample through interface α during measurement time T_0, we may write $Q_s^{\lambda}(T_0) = \Sigma_{\alpha=1}^{M} Q_{s\alpha}^{\lambda}(T_0)$ and therefore

$$Q_{s\alpha}^{\lambda}(T_0) = -ie \frac{\partial}{\partial \lambda_\alpha} \mathrm{Tr} \ln(\overline{G}_s^{\lambda}). \tag{48}$$

From the measurement prescription, i.e., Eq. (15), we get

$$Q_{s\alpha}^{\lambda}(T_0) = ie \frac{\partial}{\partial \lambda_\alpha} \ln \chi_{\lambda}^{\uparrow\downarrow}(T_0). \tag{49}$$

Combining Eq. (48) with Eq. (49) and using the normalization condition $\chi_0^{\uparrow\downarrow}(T_0) = 1$, we obtain

$$\chi_{\lambda}^{\uparrow\downarrow}(T_0) = \det\left(\frac{\overline{G}_s^{0}}{\overline{G}_s^{\lambda}} \right). \tag{50}$$

From Dyson's equation one can show that

$$\overline{G}_s^{\lambda} = \overline{G}_s^{0} + \overline{G}_s^{\lambda} (\overline{\Sigma}_s^{\lambda} - \overline{\Sigma}_s^{0}) \overline{G}_s^{0}, \tag{51}$$

where $\overline{\Sigma}_s^{0} = \overline{\Sigma}_s = \overline{\Sigma}_{sl} \overline{g}_l \overline{\Sigma}_{ls}$. Inserting Eq. (51) into Eq. (50) we get the central result of this section

$$\chi_{\lambda}^{\uparrow\downarrow}(T_0) = \det[1 + \overline{G}_s^{\lambda} (\overline{\Sigma}_s^{\lambda} - \overline{\Sigma}_s^{0})]^{-1}. \tag{52}$$

It represents a generalization to many-body physics of the Levitov-Lesovik functional determinant formula,[28,37] which has been successfully used to describe charge-counting statistics in noninteracting mesoscopic systems in both dc and ac transport regimes.[47] We remark that Eq. (52) can be used to establish a link between physical observables (transferred charge, conductance, shot noise, etc.) and many-body correlation functions.

There is a remarkable duality in this problem that can be demonstrated by using Eq. (50) and Dyson's equations (32)–(35) as follows:

$$\begin{aligned}
\chi_{\lambda}^{\uparrow\downarrow}(T_0) &= \det\left(\frac{\overline{G}_s^{0}}{\overline{G}_s^{\lambda}} \right) = \det\left(\frac{\overline{G}_s^{0}}{\overline{G}_s} \right) = \frac{\det(1 + \overline{\Sigma}_s^{0} \overline{G}_s^{0})}{\det(1 + \overline{\Sigma}_s \overline{G}_s)} \\
&= \frac{\det(1 + \overline{\Sigma}_{sl} \overline{g}_l \overline{\Sigma}_{ls} \overline{G}_s^{0})}{\det(1 + \overline{\Sigma}_{sl} \overline{g}_l \overline{\Sigma}_{ls} \overline{G}_s)} = \frac{\det(1 + \overline{g}_l \overline{\Sigma}_{ls} \overline{G}_s^{0} \overline{\Sigma}_{sl})}{\det(1 + \overline{g}_l \overline{\Sigma}_{ls} \overline{G}_s \overline{\Sigma}_{sl})} \\
&= \det\left(\frac{\overline{G}_l^{0}}{\overline{G}_l} \right) = \det\left(\frac{\overline{G}_l^{0}}{\overline{G}_l^{\lambda}} \right),
\end{aligned} \tag{53}$$

therefore

$$\chi_{\lambda}^{\uparrow\downarrow}(T_0) = \det[1 + \overline{G}_l^{\lambda} (\overline{\Sigma}_l^{\lambda} - \overline{\Sigma}_l^{0})]^{-1}, \tag{54}$$

where $\overline{\Sigma}_l^{0} = \overline{\Sigma}_{ls} \overline{g}_s \overline{\Sigma}_{sl}$.

The existence of such duality in the description of mesoscopic transport underlines the equivalence of two well-developed approaches to describe coherent transport of noninteracting particles: the scattering-matrix method[48] and the impurity average Green's-function technique.[1] In the former free asymptotic states are defined in the leads and the microscopic details of the dynamics inside the sample are eliminated by introducing scattering matrices with certain stochastic properties, which can be justified by means of maximum-entropy arguments. The impurity average Green's function, on the other hand, satisfies a microscopic equation of motion with open boundary conditions, derived from an appropriate elimination of degrees of freedom in the leads. These methods are considered complementary and have been used to produce a number of reliable results both perturbative, such as conductance fluctuations and weak localization effects,[1,48] and nonperturbative, such as the full scaling function for the ballistic-diffusive crossover in quantum wires[49] and nonanalytic scaling of conductance cumulants in dirty unconventional superconductors.[50]

V. TRANSFERRED CHARGE DRIVEN BY A PERIODIC TIME-DEPENDENT VOLTAGE

In this section we carry out our approach in a concrete example. Needless to say that the exactness of the equations cannot be maintained indefinitely and approximations will eventually be necessary to perform actual calculations of observable quantities. An important merit of the present formalism is the physical transparency (conservation laws, sum rules, etc.) with which such approximations can be introduced, thus allowing unrestricted use of the traditional literature on nonequilibrium many-body physics.

Assuming absence of direct processes between different leads, we may decompose the self-energy in terms of functions localized at the interfaces as follows $\overline{\Sigma}_s^{\lambda} = \overline{\Sigma}_{s1}^{\lambda} + \cdots + \overline{\Sigma}_{sM}^{\lambda}$. The charge transferred into the sample through interface α during measurement time T_0 is thus given by

$$Q_{s\alpha}(T_0) = \frac{1}{2} \lim_{\lambda \to 0} \int_0^{T_0} dt \, \mathrm{Tr}_K (\overline{I}_{s\alpha}^{\lambda}(t)), \tag{55}$$

where

$$\overline{I^{\lambda}_{s\alpha}}(t)=\frac{e}{\hbar}\overline{\tau}\int_{\Omega_s}d\mathbf{r}[\overline{\Sigma^{\lambda}_{s\alpha}}\,\overline{G^{\lambda}_{s}}-\overline{G^{\lambda}_{s}}\,\overline{\Sigma^{\lambda}_{s\alpha}}](\mathbf{r},t|\mathbf{r},t). \tag{56}$$

Let us now introduce our central approximation. Taking inspiration from the general theory of quantum dissipative systems,[51] we can emulate the influence of the leads in the sample by means of an effective electromagnetic environment, which acts as the source and sink of the current flowing through the system. From a physical point of view, one expects that the many-body dynamics (quasiparticles and collective modes) in the leads should imply a wide separation of time scales associated with (i) tunneling into the sample, (ii) occupation and pinning of single-particle states at the interfaces, and (iii) quantum coherence of charge propagation. In principle, one could set up a projection formalism to rigorously derive the reduced dynamics of the measurement process and exploit the resulting time-scale separation by performing expansions about the Markovian limit. Here, we shall instead take advantage of the convenient structure of the Keldysh formalism and propose a symmetry guided conserving approximation for the memory function that implements the separation in time scales described above, and then use it to derive the appropriate boundary condition for charge transport through the open sample. It goes without saying that a rigorous microscopic many-body justification for this approximation together with its regime of validity, which will probably involve ratios of the above time scales, is an important issue for future research. We remark that this approximation is physically equivalent to the wide-band limit, introduced by Jauho, Wingreen, and Meir[16] within a nonequilibrium Green's-function formalism for time-dependent transport.

The components of the self-energy matrix function, $\overline{\Sigma}_{s\alpha}(t,t')\equiv\overline{\Sigma}_{s\alpha}(\mathbf{r},t|\mathbf{r}',t')$, describing the coupling of the system to the leads during the measurement time T_0, are approximated as follows:

$$\Sigma^{K}_{s\alpha}(t,t')=i\Gamma_{s\alpha}[2f_{\alpha}(t-t')e^{-i\Delta_{\alpha}(t,t')}-\hbar\,\delta(t-t')] \tag{57}$$

and

$$\Sigma^{r,a}_{s\alpha}(t,t')=\mp\frac{i\hbar}{2}\Gamma_{s\alpha}\delta(t-t'), \tag{58}$$

in which $\Gamma_{s\alpha}\equiv\Gamma_{s\alpha}(\mathbf{r},\mathbf{r}')$ is the linewidth function that controls the tunneling rates between lead α and the sample. Note that only the Keldysh component $\Sigma^{K}_{s\alpha}(t,t')$ contains finite memory effects. The single-particle occupation function at lead α, $f_{\alpha}(t)$, is defined as the Fourier transform of the reduced distribution function $n_{\alpha}(E)$, which is considered constant during the observation time T_0 and includes interaction effects,

$$f_{\alpha}(t)=\int_{-\infty}^{\infty}\frac{dE}{2\pi}n_{\alpha}(E)e^{-iEt/\hbar}. \tag{59}$$

The function $\Delta_{\alpha}(t,t')$ accounts for the phase accumulated by the carrier as it transverses the lead-sample interface in the presence of an effective self-consistent inhomogeneous time-dependent mean-field $V_{\alpha}(t)$, which, in principle, could be related to the local electrostatic potential associated with the formation of Landauer dipoles (a many-body effect), so that

$$\Delta_{\alpha}(t,t')=\varphi_{\alpha}(t)-\varphi_{\alpha}(t'), \tag{60}$$

where

$$\varphi_{\alpha}(t)=-\frac{e}{\hbar}\int_{-\infty}^{t}dt'V_{\alpha}(t'). \tag{61}$$

In the linear regime $V_{\alpha}(t)$ is proportional to the external ac voltage.

Let $\Phi_{\alpha}(t,t')=z_{\alpha}(t)f_{\alpha}(t-t')z^{*}_{\alpha}(t')$, where $z_{\alpha}(t)=\exp[-i\varphi_{\alpha}(t)]$ and define

$$\Phi_{\alpha}(E,E')=\int_{-\infty}^{\infty}\frac{dt}{\hbar}\int_{-\infty}^{\infty}\frac{dt'}{\hbar}e^{i(Et-E't')/\hbar}\Phi_{\alpha}(t,t'), \tag{62}$$

then, in the above-described approximation, Eq. (55) can be written as

$$Q_{s\alpha}=e\int_{0}^{T_0}dt\int_{-\infty}^{\infty}\frac{dE}{2\pi}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}e^{-i\omega t}R(E+\hbar\omega,E), \tag{63}$$

where

$$R(E,E')=\sum_{\beta=1}^{M}\Phi_{\beta}(E,E')\mathrm{Tr}\{\Gamma_{s\alpha}G^{0r}_{s}(E)\times[\Gamma_{s\beta}+\delta_{\alpha\beta}(i(E-E')-\Gamma_s)]G^{0a}_{s}(E')\}. \tag{64}$$

This expression can be further simplified by assuming $V_{\alpha}(t)$ to be a periodic function of period τ_0 and by taking the measurement time to cover several cycles, i.e., $T_0=n\tau_0$ with n an arbitrary integer. We obtain

$$Q_{s\alpha}=\frac{e}{h}\sum_{\beta=1}^{M}\int_{-\infty}^{\infty}dE\,C_{\alpha\beta}(E)[K_{\alpha}(E)-K_{\beta}(E)], \tag{65}$$

where

$$C_{\alpha\beta}(E)=\mathrm{Tr}[\Gamma_{s\alpha}G^{0r}_{s}(E)\Gamma_{s\beta}G^{0a}_{s}(E)] \tag{66}$$

and

$$K_{\alpha}(E)=\int_{0}^{T_0}dt\,z_{\alpha}(t)\exp\left(i\hbar\frac{\partial^2}{\partial E\,\partial t}\right)n_{\alpha}(E)z^{*}_{\alpha}(t). \tag{67}$$

Applying the Hubbard-Stratonovitch transformation

$$e^{i\hbar\partial^2/\partial E\partial t}=\int\frac{d\zeta d\zeta^*}{2\pi}\exp\left(-|\zeta|^2-\frac{\hbar}{\eta}\zeta^*\frac{\partial}{\partial E}-i\,\eta\zeta\frac{\partial}{\partial t}\right), \tag{68}$$

with $\eta\to0^{+}$, we get

$$K_{\alpha}(E)=\int_{0}^{T_0}dt\,n_{\alpha}[E+eV_{\alpha}(t)], \tag{69}$$

which together with Eqs. (65) and (66) is our final result.

From a formal point of view it coincides exactly with a multiprobe generalization of the time-average conductance formula derived in Ref. 16 in the wide-band limit and assuming barrier heights that are time independent. As such, all numerical methods available in the literature, such as, for instance, that of Ref. 18, to evaluate such expressions can be used here as well. One should be mindful, however, of the different physical conditions under which this result was derived here. First, in our derivation we found no necessity to introduce different local chemical potentials in the system. This crucial point has been the center of much debate in the recent literature.[23,24] Second, the projection technique offers a systematic method to reintroduce the memory effects neglected in the wide-band limit, if they turn out to be relevant for comparison with experimental data, via expansions in ratios of widely separated time scales. The approximation, therefore, can, at least, in principle, be well controlled. Third, when combined with the many-body extension of Levitov-Lesovik functional determinant formula, Eq. (52), our formalism allows for a unified description of the full counting statistics of the charge measurement, not just the conductance. Finally, the projection formalism provides a natural framework for implementing powerful renormalization-group (RG) schemes, such as those of Refs. 19 and 21, by interpreting each RG step as a projection in Liouville space.

We conclude this section by providing a short derivation of the Landauer-Büttiker formula for conductance coefficients of noninteracting systems. Taking the dc limit $[V_\alpha(t)=V_\alpha]$ in Eq. (69) and assuming identical leads $[n_\alpha(E)=n(E)]$ we obtain

$$K_\alpha(E)=T_0 n(E+eV_\alpha). \tag{70}$$

Inserting Eq. (70) into Eq. (65) using the definition of current $I_\alpha=Q_{s\alpha}/T_0$ and taking the linear regime limit we get

$$I_\alpha=\sum_{\beta=1}^{M} G_{\alpha\beta}\Delta V_{\beta\alpha}, \tag{71}$$

where $\Delta V_{\beta\alpha}\equiv V_\beta - V_\alpha$ and the conductance coefficients $G_{\alpha\beta}$ are given by

$$G_{\alpha\beta}=\frac{G_0}{2}\int_{-\infty}^{\infty} dE\, C_{\alpha\beta}(E)\left(-\frac{dn}{dE}\right), \quad \alpha\neq\beta. \tag{72}$$

Assuming further that the electrons in the sample are noninteracting, the self-energy contribution to the Green's function in $C_{\alpha\beta}(E)$ contains only terms related to tunneling processes, which can be described as a scattering problem. Using the Weidenmüller-Mahaux relation[52] between the on-shell scattering matrix $S_{nm}^{\alpha\beta}$ of the leads-sample system and the corresponding Green's function and taking the zero-temperature limit, we recover the Landauer-Büttiker formula

$$G_{\alpha\beta}=G_0\sum_{n=1}^{N_\alpha}\sum_{m=1}^{N_\beta}|S_{nm}^{\alpha\beta}|^2, \tag{73}$$

in which the sums run over the propagating channels in the leads.

VI. AN EXACT SOLUTION: BALLISTIC CHAOTIC CAVITY

A central topic in mesoscopic physics is the description of universal (model independent) features of coherent transport in the presence of complex chaotic dynamics. In this scenario, two categories of physical systems stand out as particularly relevant from an experimental point of view: disordered conductors with elastic impurity scattering and ballistic cavities with specular boundary scattering. A fundamental problem in the field is to account for the interplay between interaction effects and fluctuating phenomena arising from phase-coherent chaotic scattering in such systems. Although considerable progress has been achieved recently in the perturbative regime with the development of a field-theory approach[53] to the Keldysh formalism, the nonperturbative sector of the theory appears beyond the reach of current techniques.[54] This is rather unfortunate, since it is precisely in the nonperturbative regime that beautiful universal quantum transport effects are observed.

In this section, we shall illustrate such universal features by calculating exactly the average total charge transferred, during an observation time T_0, to a noninteracting ballistic chaotic cavity coupled to an arbitrary number of terminals and in the presence of a time-dependent periodic field. As a motivation one could regard the result of this problem as a necessary first step towards the development of a nonperturbative renormalization-group treatment of the interacting system. Furthermore, it may also help by setting the appropriate language and concepts, on which to build the full theory. Note that the absence of nontrivial interaction effects, i.e., beyond setting up the relevant quasiparticles and stabilizing the boundary conditions, in the dynamics of the carriers inside the sample, makes it possible to use the powerful nonperturbative supersymmetry method.[55]

We start by making the standard simplifying assumption that chaotic scattering is confined to the sample region, so that $C_{\alpha\beta}(E)$ is a strongly fluctuating function, while $K_\alpha(E)$ is sharp. Our objective is then to calculate the average value $\langle C_{\alpha\beta}\rangle$. From Eq. (66) one can easily verify that

$$\langle C_{\alpha\beta}\rangle=\frac{\partial^2}{\partial h_{\alpha 1+}\partial h_{\beta 2+}}\langle \mathcal{Z}(h)\rangle\bigg|_{h=0}, \tag{74}$$

where $\mathcal{Z}(h)$ is a generating function, represented by the following superdeterminant (see Ref. 55 for the definitions of the basic operations and functions in supermathematics):

$$\mathcal{Z}(h)=\mathrm{Sdet}^{-1/2}[1+J_s(h)G_s^0], \tag{75}$$

in which we have defined a block-diagonal matrix of retarded and advanced Green's functions

$$G_s^0=\mathrm{diag}(1_4\otimes G_s^{0r},1_4\otimes G_s^{0a}), \tag{76}$$

where 1_4 is the 4×4 unit matrix. The source field is represented by

$$J_s(h)=\sum_{\alpha=1}^{M}\sum_{l=1}^{2}\sum_{\sigma=\pm} h_{\alpha l\sigma}F_{l\sigma}\otimes\Gamma_{s\alpha}, \tag{77}$$

where

$$F_{1\sigma}=\begin{pmatrix}0 & K_{2\sigma}\\ K_{1\sigma} & 0\end{pmatrix},\quad F_{2\sigma}=\begin{pmatrix}0 & K_{1\sigma}\\ K_{2\sigma} & 0\end{pmatrix},\tag{78}$$

and the submatrices $K_{l\sigma}$ are defined as

$$K_{l+}=\begin{pmatrix}k_l & 0\\ 0 & 0\end{pmatrix},\quad K_{l-}=\begin{pmatrix}0 & 0\\ 0 & k_l\end{pmatrix},\tag{79}$$

with $k_1=\mathrm{diag}(1,0)$ and $k_2=\mathrm{diag}(0,1)$.

The average generating function $\langle\mathcal{Z}(h)\rangle$ can be calculated using the standard supersymmetry technique. It consists of performing a map onto a nonlinear σ model defined in a coset manifold that describes the target space of massless Goldstone modes associated with a spontaneously broken hidden global symmetry. There are ten universality classes of such σ models, as has recently been pointed out by Zirnbauer.[56] Here we shall consider the orthogonal Wigner-Dyson class, appropriate to systems with time-reversal symmetry and spin-rotation invariance. We end up with a representation of $\langle\mathcal{Z}(h)\rangle$ as an integral over a coset space, whose points are parametrized by supermatrices Q satisfying the constraint $Q^2=1$,

$$\langle\mathcal{Z}(h)\rangle=\int dQ\prod_{\alpha=1}^{M}\prod_{n=1}^{N_\alpha}Z_{\alpha n}(Q,P_\alpha),\tag{80}$$

where

$$Z_{\alpha n}(Q,Q')=\mathrm{Sdet}^{-1/2}(1+e^{-\kappa_{\alpha n}}QQ'),\tag{81}$$

with $\kappa_{\alpha n}$ being parameters related to the tunneling probabilities as follows $T_{\alpha n}=\mathrm{sech}^2(\kappa_{\alpha n}/2)$. Furthermore

$$P_\alpha=\Lambda-2i\sum_{l=1}^{2}\sum_{\sigma=\pm}h_{\alpha l\sigma}F_{l\sigma},\tag{82}$$

where $\Lambda=\mathrm{diag}(1_4,-1_4)$. Inserting Eq. (80) into Eq. (74) yields

$$\langle C_{\alpha\beta}\rangle=-\sum_{n=1}^{N_\alpha}\sum_{m=1}^{N_\beta}\langle\mathrm{Str}(\Theta_{\alpha n}^{1+})\mathrm{Str}(\Theta_{\beta m}^{2+})\rangle_Q-2\delta_{\alpha\beta}\sum_{n=1}^{N_\alpha}\langle\mathrm{Str}(\Theta_{\alpha n}^{1+}\Theta_{\alpha n}^{2+})\rangle_Q,\tag{83}$$

where

$$\Theta_{\alpha n}^{l\sigma}=(1+e^{\kappa_{\alpha n}}\Lambda Q)^{-1}\Lambda F_{l\sigma}\tag{84}$$

and the average $\langle\cdots\rangle_Q$ is defined as

$$\langle A\rangle_Q=\int dQA(Q)\prod_{\alpha=1}^{M}\prod_{n=1}^{N_\alpha}Z_{\alpha n}(Q,\Lambda).\tag{85}$$

The matrix condition $Q^2=1$ can be solved explicitly using Efetov's parametrization

$$Q=U^{-1}\begin{pmatrix}\cosh\hat\theta & i\sinh\hat\theta\\ i\sinh\hat\theta & -\cosh\hat\theta\end{pmatrix}U,\tag{86}$$

where U is an 8×8 supermatrix containing all Grassmann and phase variables, integration over which can be done explicitly in our problem. The matrix $\hat\theta$ contains the relevant variables and is given by

$$\hat\theta=\begin{pmatrix}\theta_B & 0\\ 0 & \theta_F\end{pmatrix},\quad \theta_B=\begin{pmatrix}\theta_1 & \theta_2\\ \theta_2 & \theta_1\end{pmatrix},\qquad \theta_F=i\theta_0 1_2,\tag{87}$$

where $\theta_1,\theta_2>0$ and $0<\theta_0<\pi$. The integration measure in these coordinates reads

$$dQ=\frac{(1-\lambda_0^2)d\lambda_0 d\lambda_1 d\lambda_2 dU}{2^5(\lambda_0^2+\lambda_1^2+\lambda_2^2-2\lambda_0\lambda_1\lambda_2-1)^2},\tag{88}$$

where $\lambda_0=\cos\theta_0$, $\lambda_1=\cosh\theta_1$, and $\lambda_2=\cosh\theta_2$. It turns out to be convenient to perform a further change of variables through

$$\lambda_0=1-2\mu_0,$$
$$\lambda_1^2=1+\mu_1+\mu_2+2\mu_1\mu_2+2\sqrt{\mu_1(1+\mu_1)\mu_2(1+\mu_2)},$$
$$\lambda_2^2=1+\mu_1+\mu_2+2\mu_1\mu_2-2\sqrt{\mu_1(1+\mu_1)\mu_2(1+\mu_2)},\tag{89}$$

where $\mu_1,\mu_2>0$ and $0<\mu_0<1$. Evaluating Eq. (83) using the coordinates above, we obtain

$$\langle C_{\alpha\beta}\rangle=\sum_{n=1}^{N_\alpha}\sum_{m=1}^{N_\beta}A_{nm}^{\alpha\beta}+\delta_{\alpha\beta}\sum_{n=1}^{N_\alpha}(A_{nn}^{\alpha\alpha}+B_n^\alpha+e^{-\kappa_{\alpha n}}T_{\alpha n}),\tag{90}$$

in which

$$A_{nm}^{\alpha\beta}=\int_{\{\mu\}}\mathcal{P}(\{T\},\{\mu\})\frac{f(T_{\alpha n}^{-1},\{\mu\})-f(T_{\beta m}^{-1},\{\mu\})}{T_{\beta m}^{-1}-T_{\alpha n}^{-1}}\tag{91}$$

and

$$B_n^\alpha=\int_{\{\mu\}}\mathcal{P}(\{T\},\{\mu\})\frac{f(T_{\alpha n}^{-1},\{\mu\})^2}{T_{\alpha n}^{-1}(T_{\alpha n}^{-1}-1)}.\tag{92}$$

We defined

$$\int_{\{\mu\}}=\int_0^1 d\mu_0\int_0^\infty d\mu_1\int_0^\infty d\mu_2\mathcal{J}(\{\mu\}),\tag{93}$$

where

$$\mathcal{J}(\{\mu\})=\frac{\mu_0(1-\mu_0)|\mu_1-\mu_2|}{2^3\prod_{s=1}^{2}(\mu_0+\mu_s)^2\sqrt{\mu_s(1+\mu_s)}}.\tag{94}$$

Furthermore

$$\mathcal{P}(\{T\},\{\mu\})=\prod_{\alpha=1}^{M}\prod_{n=1}^{N_\alpha}\frac{1-T_{\alpha n}\mu_0}{\sqrt{(1+T_{\alpha n}\mu_1)(1+T_{\alpha n}\mu_2)}},\tag{95}$$

and finally

$$f(x,\{\mu\})=x(1-x)\left(\frac{2}{x-\mu_0}-\sum_{s=1}^{2}\frac{1}{x+\mu_s}\right). \tag{96}$$

Equation (90) is the central result of this section. Combined with Eq. (65) it represents the exact result for the problem of charge transfer into a noninteracting chaotic ballistic cavity in the presence of a periodic time-dependent driving field. As such, it may serve as a useful starting point to address the full interacting problem by means of renormalization-group concepts. Interestingly, in the Coulomb blockade regime, in which the system is weakly coupled to the leads, $T_\alpha \equiv \Sigma_n T_{\alpha n} \ll 1$, so that the electron decay widths are much smaller than the charging energy, a working framework for treating the interplay between interaction and phase coherence in the presence of chaotic dynamics is already available, with remarkable predictions.[19] In the other extreme of weakly interacting systems, strongly coupled to the measurement leads, charging effects, such as the dynamical Coulomb blockade, are irrelevant and the RG approach of Ref. 21 can be used to study the low-frequency behavior of charge transport. For systems with arbitrary coupling to the leads and interaction strength, however, competitions of several relevant time scales will lead to a very rich diagram of physical regimes, which appears to be beyond the reach of current frameworks. The projection technique, implicit in the present approach, has the potential to handle such difficulties and the development of the associated RG scheme is an exciting subject for future research.

We conclude this section by presenting an important relation between the average coefficients $\langle C_{\alpha\beta}\rangle$ that follows from a Ward identity. It reads

$$\sum_{\beta=1}^{M}\langle C_{\alpha\beta}\rangle=\sum_{\beta=1}^{M}\langle C_{\beta\alpha}\rangle=\sum_{n=1}^{N_\alpha}\frac{4}{e^{\kappa_{\alpha n}}+1}. \tag{97}$$

To appreciate the usefulness of Eq. (97), consider the simple limit of ideal contacts, $T_{\alpha n}=1$, in which case Eq. (90) reads

$$\langle C_{\alpha\beta}\rangle=2N_\alpha N_\beta A_N+\delta_{\alpha\beta}N_\alpha(1+2A_N), \tag{98}$$

where A_N is a constant and $N=N_1+\cdots+N_M$. From Eq. (97) we get

$$\sum_{\beta=1}^{M}\langle C_{\alpha\beta}\rangle=2N_\alpha, \tag{99}$$

therefore $A_N=1/[2(N+1)]$ and from Eq. (98) we obtain

$$\langle C_{\alpha\beta}\rangle=\frac{N_\alpha N_\beta}{N+1}+\delta_{\alpha\beta}\frac{N_\alpha(N+2)}{N+1}. \tag{100}$$

In the Landauer-Büttiker regime, the resulting average conductance coefficients $G_{\alpha\beta}/G_0=N_\alpha N_\beta/(N+1)$ agree with independent calculations using a maximum-entropy approach.[57] For systems with broken time-reversal symmetry, a detailed analysis of the multiterminal average conductance coefficients in several regimes of interest has been carried out in Ref. 58 and most conclusions apply here mutatis mutandi.

VII. DISCUSSION AND SUMMARY

Mesoscopic transport is a very challenging and fast growing field of research. From an experimental point of view it provides an almost unique and versatile ground to address fundamental issues related to quantum kinetics and has led to a substantial enhancement in our understanding of subtle topics such as measurement theory and decoherence phenomena. From a theoretical viewpoint, it gives unprecedented opportunities to test in laboratories the consequences of highly abstract mathematical constructions such as Riemannian supermanifolds and Liouville operator space. In this context, the Landauer-Büttiker approach (LBA) stands out as a great intellectual achievement. Its phenomenological success helped shaping up the field during the last decade and has, as a by-product, deeply influenced an entire generation of researchers in mesoscopic physics. However, pressing new challenges brought about by recent experiments and technological demands may be stretching the theory well beyond its regime of validity. Generalizing LBA, while keeping as much as possible its conceptual simplicity, has proved to be a very hard problem indeed. The crux of the matter is the fact that LBA is deeply founded upon single-particle physics and on a phenomenological description of the measurement process. As such, there appears to be no simple derivation of its results following from standard many-body physics through a well-defined logical sequence of conserving approximations. This unfortunate state of affairs has led to much controversy and criticisms in recent literature.[23]

In this work, we proposed a formulation of mesoscopic transport using the language of nonequilibrium many-body physics combined with recent developments in measurement theory. To be concrete, we focused on the problem of measuring the counting statistics of particle transfer through a phase-coherent sample, during a finite observation time, in the presence of a time-dependent inhomogeneous driving field. Following pioneering ideas by Levitov and Lesovik,[28] we derived the fundamental connection between an observable quantity, the off-diagonal components of the detector's reduced density matrix, and the generating function of charge-counting statistics. The final expression is largely independent from the specific charge detection scheme and provides the microscopic foundation for the phenomenological measurement description that is implicit in LBA. Furthermore, it brings out the singlemost important feature of our approach, which is the systematic exploitation of the time-scale separations in the underlying many-body problem leading ultimately to simplified effective descriptions, with LBA being just the simplest case, i.e., single particles in contact with local reservoirs.

The formalism is set up using a conjunction of the memory-function approach with Keldysh Green's-function technique, thereby circumventing the need to specify the details of the system-detector and system-leads couplings. This permitted us to derive a different formula for charge-counting statistics that generalize, by including many-body effects, the determinant formula by Levitov and Lesovik.[28] Our fully quantum-mechanical description of both the sample and the leads helped us in uncovering a fundamental duality in the problem, thereby providing a justification for the equivalence between two complementary descriptions of single-particle mesoscopic transport: (i) the Green's-function technique, which contains an implicit elimination of the leads by adopting a specific set of boundary conditions in the diffusionlike equations for diffusons and cooperons, and (ii) the scattering-matrix approach, which contains hypothesis of stochastic nature that implies an elimination of the degrees of freedom inside the sample.

We illustrated the use of the approach by studying in detail the average charge transferred to a sample, during an observation time T_0, in the presence of a time-dependent voltage. We adopted the point of view of eliminating the leads and approximated its influence on the sample by proposing a mathematical expression for the memory function (or self-energy function) that accommodates a wide separation in time scales associated with tunneling, occupation of single-particle states at the boundaries, and phase memory. Interestingly, our hypothesis bears some similarity with that of Ref. 39, in which the influence of the leads was simulated by a fluctuating electromagnetic environment. Our central result, Eq. (65), is physically equivalent to a multiprobe generalization of the time-average conductance formula derived in Ref. 16 and reduces to the Landauer-Büttiker expression in the noninteracting linear time-independent limit. We completed the analysis by calculating an exactly solvable system: a noninteracting ballistic cavity with chaotic dynamics. The results, obtained using the supersymmetry method, may be used to set the standards of the kind of universal description we are looking for in a yet to be constructed nonperturbative approach to interacting systems with chaotic single-particle dynamics.

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