Simulation of vortex noise in superconductors in weak applied magnetic fields

Anders Vestergren and Mats Wallin
Condensed Matter Theory, Department of Physics, KTH, SE-106 91 Stockholm, Sweden

(Received 8 January 2004; revised manuscript received 19 February 2004; published 28 April 2004)

The magnetic-field dependence of flux noise spectra in superconductors is studied by Monte Carlo simulation of vortex dynamics in two- and three-dimensional vortex models. Without an applied magnetic field the flux noise power spectra typically vary with frequency approximately like $1/f^{3/2}$. In this paper we show that in the presence of a weak magnetic field, the spectra instead typically vary approximately like $1/f$ in a range of temperatures and frequencies. Both types of behavior have been observed in different experiments.

DOI: 10.1103/PhysRevB.69.144522 PACS number(s): 74.50.+r, 75.40.Mg

I. INTRODUCTION

Fluctuations and noise are always present and affect most properties of superconductors, and it is thus highly desirable to reach better understanding of these phenomena, in particular in the presence of a magnetic field where most important applications take place. Flux noise in Josephson-junction arrays and high-temperature superconductors has been studied extensively, both experimentally and theoretically.[1–13] However, most theoretical calculations of the vortex noise spectrum has considered only the case of zero applied magnetic field. In this paper we study the effects of a weak applied magnetic field on the vortex noise power spectra in both two- and three-dimensional models for vortex fluctuations.

Experiments on two-dimensional (2D) Josephson-junction array (JJA) systems by Shaw *et al.*[6] determine the flux noise through a superconducting quantum interference device (SQUID) loop placed above a JJA. The measured flux noise power spectrum typically has two different frequency regimes. Below a characteristic frequency f_ξ, a white-noise power spectrum $S(f)=\text{const}$ is obtained, and above f_ξ, $S(f)\sim 1/f$ power spectrum is obtained, with no strong temperature dependence of the power spectrum visible in the $1/f$ regime. In 3D thin films of $YBa_2Cu_3O_{7-x}$ (YBCO) similar $S\sim 1/f$ results are obtained, but with a quite different temperature dependence with a distinct peak around $T=T_c$ in S at a fixed frequency.[4,5] Somewhat different results are obtained in the flux noise experiments of Rogers *et al.*[8] on $Bi_2Sr_2CaCu_2O_{8+x}$ films of single unit-cell thickness. The power spectrum is nearly frequency independent below a characteristic frequency, and varies like $S\sim 1/f^{3/2}$ above the characteristic frequency. Festin *et al.* obtained similar results in experiments on thin YBCO films.[1] In zero applied magnetic field, the 2D flux noise problem has been extensively studied theoretically and by simulations of both resistively shunted junction models and time-dependent Ginzburg-Landau models.[9–14]

In this paper we study the dependence of the flux noise power spectrum on a weak applied magnetic field, by Monte Carlo simulations of both two- and three-dimensional vortex models. The main results of this paper is to demonstrate that by tuning the strength of the applied field, different types power spectra are obtained, which can be directly compared with corresponding experimental results. In particular, we obtain approximate $S\sim 1/f$ power spectra for a range of temperatures, magnetic fields, and frequencies.

II. MODELS, MONTE CARLO DETAILS, AND QUANTITIES STUDIED

As a model for the electromagnetic properties of a 2D superconductor, we use a Coulomb gas model for vortex-antivortex pair fluctuations. The partition function is $Z=\mathrm{Tr}e^{-H/T}$, where Tr denotes the sum over all configurations, T is the temperature, and the Hamiltonian is

$$H=\frac{1}{2}\sum_{i,j} q_i q_j G_{ij}. \tag{1}$$

Here q_i is an integer denoting the vorticity on site i of a square lattice with $N=L\times L$ sites with periodic boundary conditions, and G is a lattice Green's function defined by

$$G_{ij}=\frac{2\pi}{L^2}\sum_{\mathbf{k}}\frac{e^{i\mathbf{k}\cdot(\mathbf{r}_i-\mathbf{r}_j)}}{4-2\cos k_x-2\cos k_y}. \tag{2}$$

In zero applied magnetic field the total vorticity is $\Sigma_i q_i=0$, i.e., there are equally many vortices and antivortices present in the system. In addition to the vortex-antivortex pair fluctuations, an applied perpendicular magnetic field contributes with a total vorticity of N_Φ, such that the flux of B through the system is $BN=N_\Phi\Phi_0$, where Φ_0 is the flux quantum.

In our Monte Carlo (MC) simulations the trial moves are attempts to insert vortex-antivortex pairs on randomly selected nearest-neighbor (NN) sites. The moves are accepted with probability $1/[1+\exp(\beta\Delta H)]$. One MC sweep consists of on average one attempt to update each NN pair of sites. We typically use up to 10^6 MC sweeps to compute averages and discard some 30% initial data to approach equilibrium. The model has a Kosterlitz-Thouless (KT) transition with a critical temperature $T_c\approx 0.211$, which can be computed by finite-size scaling analysis of MC data for the helicity modulus.[15] The MC calculation of dynamic quantities, such as the vortex noise spectrum, involves an assumption about the dynamics. The natural dynamics for the discrete vortex variables is purely dissipative, such as the dynamics generated by the Monte Carlo simulation where MC time is equated to real time. This choice of dynamics is expected to work close to T_c, where vortex motion is slow and overdamped.[16]

Flux noise experiments typically measure the time-dependent magnetic flux through a SQUID loop placed at a distance above the superconductor sample.[4,5] Following Ref. 11, we model this quantity as the time-dependent total vorticity in a fixed pickup area in the lattice. We usually take a half plane as pickup area. The shape and size of the pickup area has no qualitative effect on the frequency dependence of the power spectrum in our simulations, but the magnitude of the power spectrum is proportional to the linear dimension l of the pickup area, and we eliminate this dependence by dividing the power spectra by l. We assume that vortex noise closely corresponds to flux noise measured in the SQUID loop.[17] Let $N_a(t_i)$, $(t_i=i\Delta t;i=0,\ldots,n-1)$, denote the MC time series generated in the simulation of total vorticity in the pickup area. With zero applied field, we obtain nonzero vorticity in the pickup area due to vortex-antivortex pair fluctuations, where one member of the pair is inside the pickup area and the other member outside. In an applied magnetic field, we obtain contributions both from motion of field-induced vortices and from neutral pair fluctuations, and in this case we subtract the average vorticity in the pickup area from the time series. The power spectrum S is obtained from

$$S(f_k)=\frac{1}{n\Delta t}\left|\sum_{i=0}^{n-1}\Delta t N_a(t_i)e^{i2\pi f_k t_i}\right|^2, \qquad (3)$$

where $f_k=k/n\Delta t, k=0,\ldots,n-1$. We evaluate power spectra using standard fast Fourier transform and window techniques.[18] We also consider the correlation time τ, which is computed from $\tau=(1/2)S(0)/\langle\Phi^2\rangle$. The power spectra in the figures below are averages over 50 to 100 simulations, and the statistical errors are of the same magnitude as the fluctuations in the data curves.

III. MONTE CARLO RESULTS IN 2D

Monte Carlo results for the vortex noise in zero magnetic field are shown in Fig. 1. At low frequency we obtain a white-noise regime with $S(f)=\text{const}$, i.e., uncorrelated fluctuations. At high frequency we have a regime with $S(f)\sim 1/f^{3/2}$, indicating vortex diffusion across the boundary of the pickup area. Similar results have been obtained in previous simulations.[11,12] The spectra are similar at all temperatures and system sizes (up to 200×200) we tried, and we observe no $1/f$ regime in zero field.

Figures 2–4 show simulation results for different values of the applied magnetic field. Figure 2 shows the effect of applying a magnetic field that corresponds to one single vortex to a system of size 100×100. The addition of the single vortex significantly alters the frequency dependence in the white-noise region obtained for zero field in Fig. 1. The subsequent figures show that this frequency dependence approaches an approximate $1/f$ dependence upon increasing the number of field-induced vortices, in the limit of large system size. There is also an upper limit for the range of applied

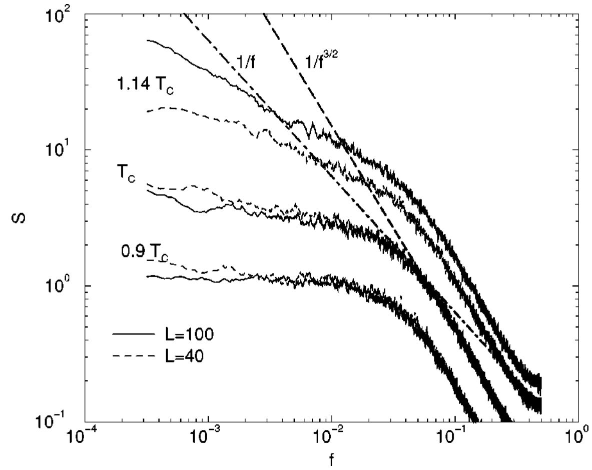

FIG. 1. MC results for the power spectrum of the 2D Coulomb gas model for system size 40^2 and zero applied field. The power spectrum looks similar for all temperatures close to T_c, with no observable $1/f$ regime.

fields where we obtain $1/f$ power spectra in our MC data, corresponding to a vortex density of about 4%. We note that similar results with power spectrum exponents 1 and 2/3 are obtained in Ref. 19, for problems with a boundary with interacting particles injected at the system boundaries.

Figure 3 shows MC data for S vs f for various system sizes $L\times L$, at $T=T_c$ and at $T=1.14T_c$, where T_c denotes the zero field transition temperature. For system size $L=50$ the data curves for $T=T_c$ and for $T=1.14T_c$ nearly coincide, but for $L=200$ they are separated by nearly an order of magnitude in the white noise regime. This indicates that for systems with $L\sim1000$ the corresponding curves would be separated by several orders of magnitude. Hence the number of decades of $1/f$ behavior grows with system size. All our data curves also show a high frequency crossover to $S\sim1/f^{\alpha}$ with $\alpha>1$. Such a crossover must in principle always exist, since

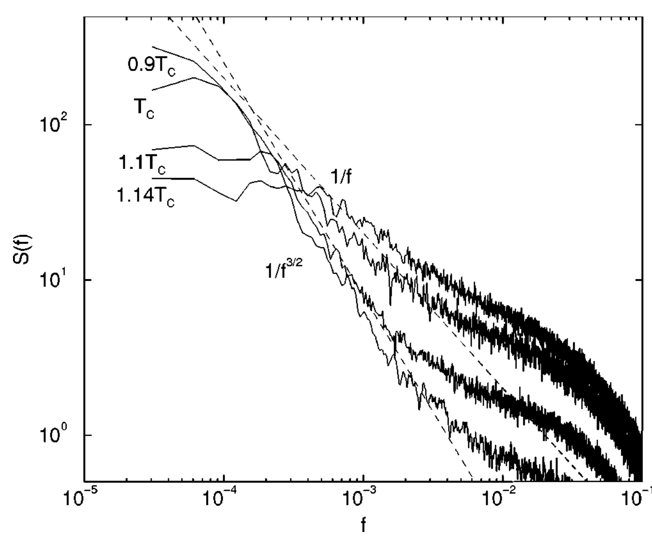

FIG. 2. MC results for the power spectrum of the 2D Coulomb gas model for system size 100^2. The applied field corresponds to $N_\Phi=1$ vortex.

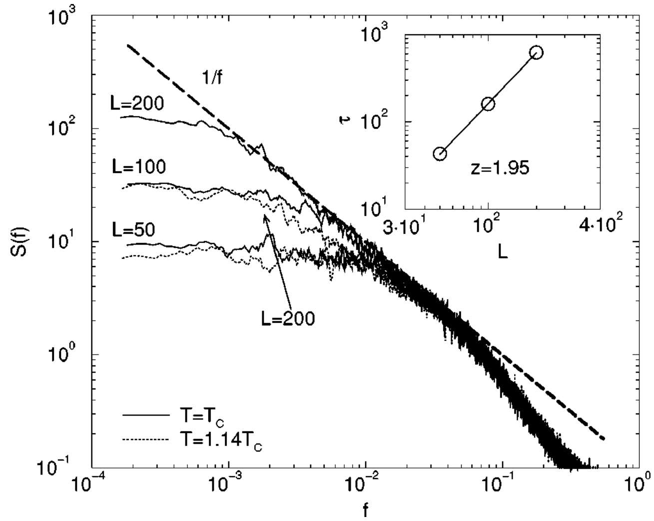

FIG. 3. System size dependence of MC data for the vortex noise of a 2D Coulomb gas model in an applied magnetic field corresponding to a vortex density of 0.2%. Inset: Correlation time τ vs system size L at T_c. We find $\tau \sim L^z$ with $z \approx 2 \pm 0.1$.

the frequency integral of the power spectrum must be finite. This crossover corresponds to some short time scale associated with fast motion of vortices on short length scales, where we do not expect our model to accurately correspond to experiments. Away from T_c we obtain a slight temperature dependence in the power spectrum exponent, and we obtain $\alpha = 1$ with high accuracy only very close to T_c.

To analyze the L dependence quantitatively, we assume standard critical slowing down of the form $\tau \sim \xi^z$, where τ is the correlation time, ξ the correlation length, and z the dynamic critical exponent. Since the power spectrum has the

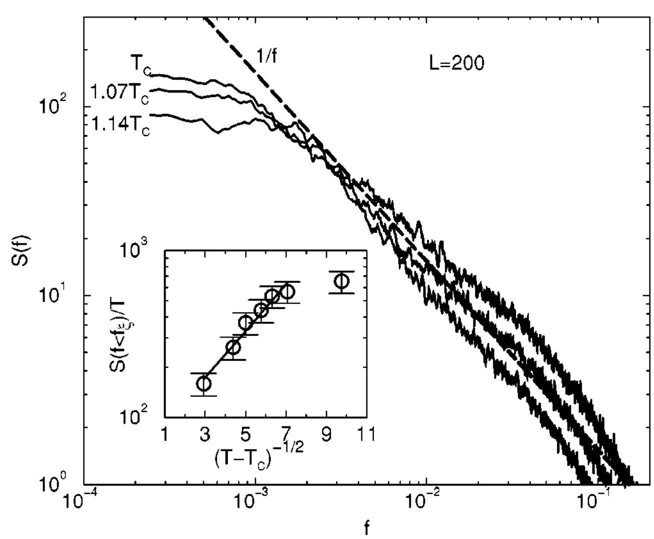

FIG. 4. Temperature dependence of MC data for the vortex noise of a 2D Coulomb gas model in an applied magnetic field corresponding to a vortex density of 0.2%, i.e., $N_\Phi = 80$ vortices at system size 200^2. Inset: Temperature scaling of the white-noise power spectra at $T > T_c$ and $f < f_\xi$. The deviation of the rightmost data point is a finite-size effect visible close to T_c.

dimension of time, or inverse frequency, we make the finite size scaling ansatz

$$S(f) = \frac{1}{f} g(f\xi^z, L/\xi) = L^z \tilde{g}(fL^z, L/\xi), \tag{4}$$

where g and $\tilde{g}$ are universal scaling functions. For times $t > \tau$, i.e., frequencies $f < 1/\tau$, the vorticity inside the pickup area is uncorrelated, and the power spectrum is white noise. Hence the finite size crossover, at the zero field $T = T_c$, between $S \sim 1/f$ and white noise scales as $f_\xi \sim L^{-z}$. From the inset in Fig. 3 we obtain $z \approx 2$, i.e. the usual value for the dynamic critical exponent at the KT transition. Similar dynamic scaling arguments have been used for power spectra of the order parameter in Ising systems.[20]

Figure 4 shows results for the flux noise spectrum for a 2D system in an applied magnetic field. In the figure we observe a crossover from $S \sim 1/f$ to $S \sim$ const at a frequency f_ξ. This crossover is related to the temperature dependence of the correlation length at the KT transition. From the usual KT form of the correlation length, $\xi \sim \exp(a/\sqrt{T-T_c})$, $fS(f)$ should be a function of f/f_ξ, with $f_\xi \sim \exp(az/\sqrt{T-T_c})$. The inset in Fig. 4 shows data for the white-noise spectrum, $S(f) =$ const for $f < f_\xi$ vs $\sqrt{T-T_c}$, which displays a reasonable agreement with the KT functional form. The deviation from the straight line in the plot of the data point closest to T_c is a finite-size effect.

IV. MONTE CARLO RESULTS IN 3D

We have also simulated a 3D vortex loop model, using similar methods as in 2D. In the 3D case the Hamiltonian is $H = (1/2)\Sigma_j q_j^2$, where q_j the integer vorticity on link j of a simple cubic lattice with $L \times L \times L$ sites and periodic boundary conditions (PBC) in all directions. This model assumes an onsite interaction, which applies in the limit of strong screening of the interaction between vortices. The use of PBC means that we ignore surface effects that may actually be important in experiments. The MC trial moves are attempts to insert closed vortex loops around the elementary plaquettes of the lattice.[16] This loop model has $T_c \approx 0.333$.[16] In zero field, we find a $1/f$ power spectrum only at $T = T_c$ (data not shown). MC results for power spectra with a small applied field are shown in Fig. 5. With a finite number of net vortices, we observe $1/f$ power spectra for a range of temperatures. The range of temperatures where we observe $1/f$ is actually broader than in 2D. We also studied random-T_c disorder,[16] but this does not qualitatively modify the power spectra. Finally, in Fig. 6 we study the power spectrum at a fixed frequency, as a function of the strength of the applied magnetic field. We find that the power spectrum is roughly proportional to the field, which is in agreement with the experimental result for YBCO in Ref. 5.

V. CONCLUSION

From simulations of vortex dynamics in two- and three-dimensional vortex models, we study the magnetic-field dependence of the power spectrum, and compare the results to

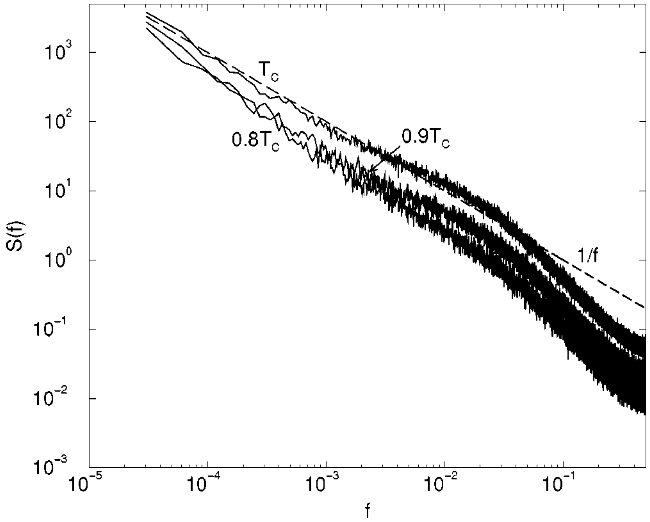

FIG. 5. Monte Carlo results for power spectra of a 3D vortex loop model for system size 40^3 and an applied field corresponding to $N_\Phi = 16$ vortex lines.

experiments on flux noise in superconductors. In zero applied magnetic field, we obtain a crossover from white noise at low frequency to $1/f^{3/2}$ at high frequency, which is similar to the experiments by Rogers *et al.*[8] and Festin *et al.*[1] With a small net field we instead find approximate $1/f$ behavior of the power spectra, which is similar to the experimental results of Shaw *et al.*[6] We also present simulations for a 3D model, where we obtain a magnetic-field dependence of the power spectrum that is similar to experiments by Ferrari *et al.*[5] It would be of interest with further, systematic experimental results on the effects of applied magnetic fields on flux noise power spectra.

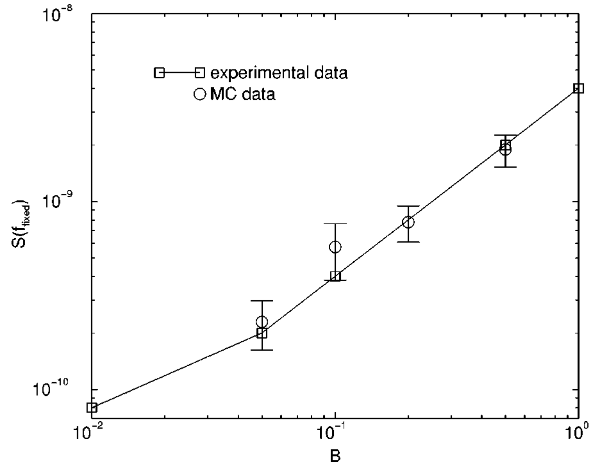

FIG. 6. Field dependence of MC data in Fig. 5 for $T = 0.8T_c$ together with experimental data from Ref. 5. The MC data have been multiplied by a constant to simplify the comparison with experimental curve.

ACKNOWLEDGMENTS

We acknowledge illuminating discussions with Ö. Festin, S. M. Girvin, and P. Svedlindh. This work was supported by the Swedish Research Council, the Göran Gustafsson Foundation, PDC, and NCS.

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