Theory of superconductivity in ferromagnetic superconductors with triplet pairing

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We point out that ferromagnetic superconductors with triplet pairing and strong spin-orbit coupling are even in the simplest case at least two-band superconductors. The Gor'kov type formalism for such superconductors is developed and the Ginzburg-Landau equations are derived. The dependence of the critical temperature on the concentration of ordinary pointlike impurities is found. Its nonuniversality could serve as a qualitative measure of the two-band character of ferromagnetic superconductors. The problem of upper critical field determination is also discussed.

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I. INTRODUCTION

The extension of the Bardeen-Cooper-Schrieffer (BCS) theory of superconductivity to the case with two bands of itinerant electrons was developed soon after the appearance of the BCS theory.[1] This theory has been the subject of renewed interest following the recent discovery of MgB_2—the first superconducting material where the existence of two energy gaps has been unambiguously demonstrated by thermodynamic and spectroscopic measurements.[2] Certainly, there are many other superconducting compounds where multiband effects are less pronounced and experimentally invisible because the Cooper pairing occurs mostly in one band of the itinerant electrons or holes. On the other hand, there is a whole class of superconductors where two-band (or more generally multiple band) superconductivity is an inherent property: the so called ferromagnetic superconductors where the different bands with "spin-up" and "spin-down" electrons are always present. UGe_2,[3,4] $ZrZn_2$,[5] URhGe (Ref. 6) are recent examples of such materials where the superconducting states are expected to be spin triplet in order to avoid the large depairing influence of the exchange field due to ferromagnetism.

The symmetry classification of the superconducting states for itinerant ferromagnetic spin-triplet superconductors has been proposed recently by several authors.[7–9] At the same time a general Gor'kov-type mathematical description of multiband superconductivity in a ferromagnetic metal with triplet pairing has not been developed. The principal goal of this paper is to present such a description for two-band ferromagnetic metal with anisotropic spectrum of quasiparticles and general form of pairing interaction. Being useful for a concrete calculation with particular form of spectrum and pairing interaction this approach allows to solve in general terms several typical for superconductivity theory problems such as critical temperature determination, derivation of Ginzburg-Landau equations, suppression of superconductivity by impurities, upper critical field calculation.

We begin with the general form of the order parameter and the pairing interaction in a two-band itinerant ferromagnet. Then the Gor'kov equations will be written that permits to calculate the spectrum of quasiparticles and the critical temperature and to derive the system of coupled equations for the order parameters from two bands. Then a law of suppression of critical temperature by pointlike nonmagnetic impurities is found. Its characteristic nonuniversal behavior can serve as a qualitative measure of the two-band character of ferromagnetic superconductors. The problem of upper critical field determination is finally discussed.

II. FERROMAGNETIC SUPERCONDUCTORS WITH TRIPLET PAIRING

A. Two-band superconductivity

For a triplet superconductivity the order parameter is written as[10]

$$\Delta_{\alpha\beta}(\mathbf{R},\mathbf{k})=\begin{pmatrix}\Delta_\uparrow & \Delta_0\\ \Delta_0 & \Delta_\downarrow\end{pmatrix}=(\mathbf{d}^\Gamma(\mathbf{R},\mathbf{k})\boldsymbol{\sigma})i\sigma_y$$
$$=\begin{pmatrix}-d_x(\mathbf{R},\mathbf{k})+id_y(\mathbf{R},\mathbf{k}) & d_z(\mathbf{R},\mathbf{k})\\ d_z(\mathbf{R},\mathbf{k}) & d_x(\mathbf{R},\mathbf{k})+id_y(\mathbf{R},\mathbf{k})\end{pmatrix}, \quad (1)$$

where $\boldsymbol{\sigma}=(\sigma_x,\sigma_y,\sigma_z)$ are the Pauli matrices. Superconducting states $\mathbf{d}^\Gamma(\mathbf{R},\mathbf{k})$ with different critical temperatures in the ferromagnetic crystals are classified in accordance with irreducible co-representations Γ of the magnetic group M of crystal.[7–9] All the co-representations in ferromagnets with orthorombic and cubic symmetries are one dimensional. However, they obey of multicomponent order parameters determined through the coordinate dependent pairing amplitudes: one per each band populated by electrons with spins up or down and one per each pair of the bands with the opposite spins (zero-spin projection states). Owing to the big difference in the Fermi momenta the pairing of electrons from the different bands is negligibly small. Hence we shall neglect by pairing amplitude with zero spin projection, another words $\Delta_0=d_z(\mathbf{R},\mathbf{k})=0$ will be taken throughout the paper. Also we limit ourself by the consideration of two-band ferromagnetic superconductor with strong spin-orbital coupling where two-component order parameter has a form

$$\mathbf{d}^\Gamma(\mathbf{R},\mathbf{k})=\tfrac{1}{2}[-(\hat{x}+i\hat{y})\Delta_\uparrow(\mathbf{R},\mathbf{k})+(\hat{x}-i\hat{y})\Delta_\downarrow(\mathbf{R},\mathbf{k})] \quad (2)$$

as it was first pointed out in Ref. 11. Here $\hat{x},\hat{y}$ are the unit vectors of the spin (or, more exactly, pseudospin[10]) coordinate system pinned to the crystal axes.

$$\Delta_{\uparrow}(\mathbf{R},\mathbf{k}) = -\eta_1(\mathbf{R}) f_-(\mathbf{k}), \quad \Delta_{\downarrow}(\mathbf{R},\mathbf{k}) = \eta_2(\mathbf{R}) f_+(\mathbf{k}). \tag{3}$$

Functions $f_{\pm}(\mathbf{k}) = f_x(\mathbf{k}) \pm i f_y(\mathbf{k})$ and the projections $f_i(\mathbf{k})$, $i = x,y$ are odd functions of momentum directions of pairing particles on the Fermi surface. The general forms of these functions for the different corepresentations in ferromagnetic superconductors with orthorhombic and cubic symmetries are listed in the paper.[7] For instance in the case of A_1 representation in orthorombic crystal they are

$$f_x(\mathbf{k}) = k_x u_1^{A_1} + i k_y u_2^{A_1}, \quad f_y(\mathbf{k}) = k_y u_3^{A_1} + i k_x u_4^{A_1}, \tag{4}$$

where $u_1^{A_1}, \ldots$ are real functions of k_x^2, k_y^2, k_z^2. The simple consequence of this is that the only symmetry dictated nodes in quasiparticle spectrum of superconducting A states in orthorombic ferromagnets are the nodes lying on the northern and southern poles of the Fermi surface $k_x = k_y = 0$. On the contrary for the B states they are on the line of equator $k_z = 0$.

The coordinate dependent complex order parameter amplitudes $\eta_1(\mathbf{R})$ and $\eta_2(\mathbf{R})$ have been discussed in the paper[7] as equal that is not in general truth. However, even in general case they are not completely independent:

$$\eta_1(\mathbf{R}) = |\eta_1(\mathbf{R})| e^{i\varphi(\mathbf{R})}, \quad \eta_2(\mathbf{R}) = \pm |\eta_2(\mathbf{R})| e^{i\varphi(\mathbf{R})}. \tag{5}$$

Thus, being different by their modulos they have the same phase with an accuracy $\pm\pi$. The latter property guarantees the consistency of transformation of both parts of the order parameter under the time reversal.

The BCS Hamiltonian in two-band ferromagnet with triplet pairing is

$$H = \sum_{\mathbf{k},\mathbf{k}',\alpha} \langle \mathbf{k} | \hat{h}_\alpha | \mathbf{k}' \rangle a^\dagger_{\mathbf{k}\alpha} a_{\mathbf{k}'\alpha} + \frac{1}{2} \sum_{\mathbf{k},\mathbf{k}',\mathbf{q},\alpha,\beta} V_{\alpha\beta}(\mathbf{k},\mathbf{k}') a^\dagger_{-\mathbf{k}+\mathbf{q}/2,\alpha} a^\dagger_{\mathbf{k}+\mathbf{q}/2,\alpha} a_{\mathbf{k}'+\mathbf{q}/2,\beta} \times a_{-\mathbf{k}'+\mathbf{q}/2,\beta}, \tag{6}$$

where the band indices α and β are $(\uparrow,\downarrow)$ or (1,2),

$$\hat{h}_\alpha = \hat{\varepsilon}_\alpha - \mu_B \hat{\mathbf{g}}_\alpha \mathbf{H}_{ext}/2 + U(\mathbf{r}) - \varepsilon_F, \tag{7}$$

are one particle band energy operators, the functions $\hat{\varepsilon}_\alpha$ (including the exchange splitting) and $\hat{\mathbf{g}}_\alpha$—factor depend of gauge invariant operator $-i\boldsymbol{\nabla} + (e/c)\mathbf{A}(\mathbf{r})$ and crystallographic directions. In the simplest case of isotropic bands without a spin-orbital coupling $\mathbf{g}_{1,2} = \pm 2\mathbf{H}_{ext}/H_{ext}$. $U(\mathbf{r})$ is an impurity potential, $\mathbf{H}_{\rm ext}$ is an external magnetic field,

$$\nabla \times \mathbf{A} = \mathbf{B} = \mathbf{H}_{\rm ext} + 4\pi\mathbf{M}, \tag{8}$$

and $\mathbf{M}$ is the magnetic moment of the ferromagnet.

The pairing potential interaction is expanded over

$$V_{\alpha\beta}(\mathbf{k},\mathbf{k}') = -V_{\alpha\beta}\varphi_\alpha(\mathbf{k})\varphi^*_\beta(\mathbf{k}'), \tag{9}$$

where

$$\varphi_{\uparrow}(\mathbf{k}) = -f_-(\mathbf{k}), \quad \varphi_{\downarrow}(\mathbf{k}) = f_+(\mathbf{k}). \tag{10}$$

It contains four different interaction terms corresponding to (i) a pairing between electrons with the same spin polarization (intraband interaction) and (ii) the interband scattering terms with $V_{\uparrow\downarrow} = V_{\downarrow\uparrow}$ describing the transitions of the pair electron from one sheet of the Fermi surface to the other sheet by reversing the pair spin orientation with the help of the spin-orbit coupling.

When the interband scattering is negligible $V_{\uparrow\downarrow} = V_{\downarrow\uparrow} = 0$, the pairing of the electrons occurs first only in one of the sheets of the Fermi surface as in the A_1 phase of ^{3}He. In general the superconductivity in each band is not independent.

B. Gor'kov equations

We want to determine the Green's functions of ferromagnetic superconductors in the absence of external perturbations and impurity scattering. Even under these simple conditions, the system is not spatially uniform due to the inherent presence of $4\pi M$. If we neglect $4\pi M$, the system is spatially uniform. Then, we can write the Gor'kov equations in the form

$$(i\omega_n - \xi_{\mathbf{k}\alpha}) G_\alpha(\mathbf{k},\omega_n) + \Delta_\alpha(\mathbf{k}) F^\dagger_\alpha(\mathbf{k},\omega_n) = 1$$

$$(i\omega_n + \xi_{\mathbf{k}\alpha}) F^\dagger_\alpha(\mathbf{k},\omega_n) + \Delta^\dagger_\alpha(\mathbf{k}) G_\alpha(\mathbf{k},\omega_n) = 0, \tag{11}$$

where $\xi_{\mathbf{k}\alpha} = \varepsilon_{\mathbf{k}\alpha} - \varepsilon_F$ and $\omega_n = \pi T(2n+1)$ are Matsubara frequencies. The equations for each band are only coupled through the order parameter given by the self-consistency condition

$$\Delta_\alpha(\mathbf{k}) = -T\sum_n \sum_{\mathbf{k}'} \sum_{\beta=\uparrow,\downarrow} V_{\alpha,\beta}(\mathbf{k},\mathbf{k}') F_\beta(\mathbf{k}',\omega_n). \tag{12}$$

The superconductor Green's functions are

$$G_\alpha(\mathbf{k},\omega_n) = -\frac{i\omega_n + \xi_{\mathbf{k}\alpha}}{\omega_n^2 + E^2_{\mathbf{k},\alpha}} \tag{13}$$

$$F_\alpha(\mathbf{k},\omega_n) = \frac{\Delta_\alpha(\mathbf{k})}{\omega_n^2 + E^2_{\mathbf{k},\alpha}}, \tag{14}$$

where $E_{\mathbf{k},\alpha} = \sqrt{\xi^2_{\mathbf{k}\alpha} + |\Delta_\alpha(\mathbf{k})|^2}$. Obviously, the superconductivity in ferromagnetic superconductors is non-unitary.

The Gor'kov equations taking into consideration the magnetic moment $4\pi M$, an external field and nonmagnetic pointlike impurities can be easily written according to the general procedure described in Ref. 10. We shall not overload the article by this and just write the self-consistency equations near the superconducting transition.

C. The order parameter equations near the superconducting transition

This system consists of two equations for the order parameter components with spin polarizations up and down,

$$\Delta_\alpha(\mathbf{R},\mathbf{r}) = -T\sum_{n,\beta}\int d\mathbf{r}' V_{\alpha,\beta}(\mathbf{r},\mathbf{r}')G_\beta(\mathbf{r}',\tilde{\omega}_n^\beta) \times G_\beta(\mathbf{r}',-\tilde{\omega}_n^\beta)\exp(i\mathbf{r}'\mathbf{D}(\mathbf{R})) \times\{\Delta_\beta(\mathbf{R},\mathbf{r}')+\Sigma_\beta(\tilde{\omega}_n^\beta,\mathbf{R})\}, \quad (15)$$

and two equations for the impurity self-energy components

$$\Sigma_\alpha(\tilde{\omega}_n^\alpha,\mathbf{R}) = n_i u_\alpha^2\int d\mathbf{r} G_\alpha(\mathbf{r},\tilde{\omega}_n^\alpha)G_\alpha(\mathbf{r},-\tilde{\omega}_n^\alpha) \times\exp(i\mathbf{r}\mathbf{D}(\mathbf{R}))\{\Delta_\alpha(\mathbf{R},\mathbf{r})+\Sigma_\alpha(\tilde{\omega}_n^\alpha,\mathbf{R})\}, \quad (16)$$

where $\tilde{\omega}_n^\alpha = \omega_n + \text{sign}\ \omega_n/2\tau_\alpha$, and τ_α is the quasiparticle mean-free time in the different bands. These mean-free times are related in the Born approximation to the impurity concentration n_i through

$$\frac{1}{2\tau_\alpha} = \pi n_i N_{0\alpha} u_\alpha^2, \quad (17)$$

with u_α—the amplitude of the impurity scattering and $N_{0\alpha}$—the density of electronic states in each band.

The operator of covariant differentiation is

$$\mathbf{D}(\mathbf{R}) = -i\frac{\partial}{\partial\mathbf{R}} + \frac{2e}{c}\mathbf{A}(\mathbf{R}).$$

The normal metal electron Green functions are

$$G_\alpha(\mathbf{r},\tilde{\omega}_n^\alpha) = \int \frac{d\mathbf{p}}{(2\pi)^3} e^{i\mathbf{p}\cdot\mathbf{r}}(i\tilde{\omega}_n^\alpha - \xi_{\mathbf{p},\alpha} + \mu_B \mathbf{g}_{\mathbf{p},\alpha}\mathbf{H}_{\text{ext}}/2)^{-1}. \quad (18)$$

The order parameter components in different bands are determined in accordance with Eq. (3):

$$\Delta_\uparrow(\mathbf{R},\mathbf{r}) = -\eta_1(\mathbf{R})f_-(\mathbf{r}), \quad \Delta_\downarrow(\mathbf{R},\mathbf{r}) = \eta_2(\mathbf{R})f_+(\mathbf{r}). \quad (19)$$

D. The critical temperature T_{c0}

In the absence of impurities and an external field let us first find the critical temperature T_{c0} in the formally spatially uniform situation of negligible ferromagnetic moment $\mathbf{M}=0$. This case the anomalous impurity self-energy part $\Sigma_\alpha(\tilde{\omega}_n^\alpha,\mathbf{R})=0$ and from Eq. (15) we obtain the system of equations

$$\eta_1 = (g_1\eta_1 + g_{12}\eta_2)\lambda(T_{c0}),$$
$$\eta_2 = (g_{21}\eta_1 + g_2\eta_2)\lambda(T_{c0}), \quad (20)$$

where $g_1 = V_{\uparrow\uparrow}\langle|f_-(\mathbf{k})|^2 N_{0\uparrow}(\hat{\mathbf{k}})\rangle$, the angular brackets mean the averaging over the Fermi surface, $N_{0\uparrow}(\hat{\mathbf{k}})$ is the angular dependent density of electronic states at the Fermi surface of the band $\uparrow$. Correspondingly $g_{12} = V_{\uparrow\downarrow}\langle|f_+(\mathbf{k})|^2 N_{0\downarrow}(\hat{\mathbf{k}})\rangle$, $g_{21} = V_{\downarrow\uparrow}\langle|f_-(\mathbf{k})|^2 N_{0\uparrow}(\hat{\mathbf{k}})\rangle$, $g_2 = V_{\downarrow\downarrow}\langle|f_+(\mathbf{k})|^2 N_{0\downarrow}(\hat{\mathbf{k}})\rangle$. The function $\lambda(T)$ is

$$\lambda(T) = 2\pi T\sum_{n\geq 0}\frac{1}{\omega_n} = \ln\frac{2\gamma\epsilon}{\pi T}, \quad (21)$$

$\ln\gamma = 0{,}577\ldots$ is the Euler constant, ϵ is an energy cutoff.

Thus, similar to Ref. 1 the critical temperature is given by

$$T_{c0} = (2\gamma\epsilon/\pi)\exp(-1/g), \quad (22)$$

where g is defined by the maximum of zeros of determinant of the system, Eq. (20),

$$g = (g_1+g_2)/2 + \sqrt{(g_1-g_2)^2/4 + g_{12}g_{21}}. \quad (23)$$

In particular at $g_{12}, g_{21} \ll g_1, g_2$ the critical temperature is determined by

$$g = \max(g_1, g_2). \quad (24)$$

E. The critical temperature dependence on impurities concentration

Triplet superconductivity is suppressed by nonmagnetic impurities.[12] Moreover, the law of suppression of superconductivity is described by the universal Abrikosov-Gor'kov (AG) dependence[13]

$$-\ln t = \Psi\left(\frac{1}{2}+\frac{x}{4\gamma t}\right) - \Psi\left(\frac{1}{2}\right) \quad (25)$$

valid for any unconventional superconducting state and applicable in particular to a concrete unconventional superconductor independently of the pressure.[10] Here Ψ is the digamma function. The variable $t = T_c/T_{c0}$ is the ratio of the critical temperature of the superconductor with a given concentration of impurities n_i to the critical temperature of the clean superconductor, and $x = n_i/n_{ic} = \tau_c/\tau$ is the ratio of the impurity concentration in the superconductor to the critical impurity concentration destroying superconductivity, or the inverse ratio of the corresponding mean-free particle lifetimes. The critical mean-free time is given by $\tau_c = \gamma/\pi T_{c0}$. This dependence has been demonstrated (although with some dispersion of the experimental points) for the triplet superconductor Sr_2RuO_4.[14]

Deviations from the universality of the AG law can be caused by the anisotropy of the scattering which takes place in the presence of extended imperfections in the crystal. Such a modification of the theory applied to UPt_3 has been considered previously.[15] However, a complete experimental investigation of the suppression of superconductivity by impurities in this unconventional superconductor, in particular the study of the universality of the behavior, has not been performed.

The nonuniversality of the suppression of superconductivity can also be caused by any inelastic scattering mechanism by impurities with internal degrees of freedom of magnetic or nonmagnetic origin. For the simplest discussion of this, see Ref. 16.

Finally, universality is certainly not expected in multiband superconductors. Theories for this case have been developed with regard to the unconventional superconductivity in

Sr_2RuO_4 (p wave, two-band two-dimensional model[17]) and conventional superconductivity in MgB_2 (anisotropic scattering two-band model[18]).

A simple modification of the universal AG law for the suppression of the superconductivity by impurities in a two-band ferromagnetic superconductor is derived here. Our consideration is limited to the simplest case of scattering by ordinary pointlike impurities. Then, due to spin conservation, one can neglect interband quasiparticle scattering and take into account only the intraband quasiparticle scattering on impurities. At finite impurity concentration the similar to Eq. (20) system of equations is

$$\eta_1 = g_1\Lambda_1(T)\eta_1 + g_{12}\Lambda_2(T)\eta_2,$$

$$\eta_2 = g_{21}\Lambda_1(T)\eta_1 + g_2\Lambda_2(T)\eta_2, \tag{26}$$

where

$$\Lambda_{1,2}(T) = \Psi\left(\frac{1}{2}\right) - \Psi\left(\frac{1}{2} + \frac{1}{4\pi\tau_{1,2}T}\right) + \ln\frac{T_{c0}}{T} + \lambda(T_{c0}). \tag{27}$$

Hence the critical temperature is determined from the equation

$$[g_1\Lambda_1(T) - 1][g_2\Lambda_2(T) - 1] - g_{12}g_{21}\Lambda_1(T)\Lambda_2(T) = 0. \tag{28}$$

In particular at $g_{12}, g_{21} \ll g_1, g_2$ the critical temperature is determined by the max(T_{c1}, T_{c2}) of the solutions of equations

$$\ln\frac{T_{c0}}{T_{c1}} = \Psi\left(\frac{1}{2} + \frac{1}{4\pi\tau_1 T_{c1}}\right) - \Psi\left(\frac{1}{2}\right) + \frac{1}{g_1} - \lambda(T_{c0}), \tag{29}$$

$$\ln\frac{T_{c0}}{T_{c2}} = \Psi\left(\frac{1}{2} + \frac{1}{4\pi\tau_2 T_{c2}}\right) - \Psi\left(\frac{1}{2}\right) + \frac{1}{g_2} - \lambda(T_{c0}). \tag{30}$$

Let us accept for determiness that $g_1 > g_2$ hence the maximal critical temperature in absence of impurities is defined by $1/g_1 = \lambda(T_{c0})$. Then at small impurity concentrations the solutions of Eqs. (29) and (30) are the linear functions of impurities concentration:

$$T_{c1} = T_{c0} - \frac{\pi}{8\tau_1}, \tag{31}$$

$$T_{c2} = T_{c0} - \frac{1}{g_2} + \frac{1}{g_1} - \frac{\pi}{8\tau_2}. \tag{32}$$

These lines can in principle intersect each other, as result an upturn on the critical temperature dependence of impurity concentration $T_c(n_i)$ is appeared. Such the type of deviations of the $T_c(n_i)$ dependence from the AG law present the direct manifestation of the two-band character of the superconductivity. On the other hand, an absence of strong deviations from the universal one-band curve if it would found experimentally in a ferromagnetic superconductor means that the superconductivity is developed in one-band with only electrons with up spins paired and the down spin electrons leave normal (or vice versa).

Another specific feature of the ferromagnetic superconductors is that even in the absence of an external magnetic field the exchange field $H_{ex} \sim E_{ex}/\mu_B$ acting on the electron spins in a ferromagnet produces in addition an electromagnetic field $4\pi M \sim 4\pi\mu_B k_F^3$ acting via the electronic charges on the orbital motion of electrons, and suppressing the superconductivity.[19] Hence, the actual critical temperature in ferromagnetic superconductors is always smaller by the value $\sim 4\pi M/H_{c2}(T=0)$ relative to the (imaginary) ferromagnetic superconductor without $4\pi M$. The upper critical field H_{c2} is also purity dependent. That is why the impurity concentration dependence of the actual T_c in a ferromagnetic superconductor might be determined not only directly by the suppression of superconducting correlations by the impurity scattering as in any nonconventional superconductor but also indirectly through the suppression of H_{c2}. In fact the second indirect mechanism has a smaller influence because the ratio $4\pi M/H_{c2}(T=0)$ is less than 1/10 for superconductors with an upper critical field of the order of several Tesla.

Thus the problem of determination of the critical temperature in superconducting ferromagnet is at bottom the problem of determination of the upper critical field in single domain ferromagnet.

F. The upper critical field

The equations for determination of upper critical field at least near T_c is easily derived from the systems (15), (16). Keeping only the lowest order gradient terms we have

$$\Delta_\alpha(\mathbf{R},\mathbf{r}) = -T\sum_{n,\beta}\int d\mathbf{r}' V_{\alpha,\beta}(\mathbf{r},\mathbf{r}')G_\beta(\mathbf{r}',\tilde{\omega}_n^\beta) \times G_\beta(\mathbf{r}',-\tilde{\omega}_n^\beta)([1-(\mathbf{r}'\mathbf{D}(\mathbf{R})]^2/2)\Delta_\beta(\mathbf{R},\mathbf{r}') + (i\mathbf{r}'\mathbf{D}(\mathbf{R}))\Sigma_\beta(\tilde{\omega}_n^\beta,\mathbf{R})), \tag{33}$$

and

$$\Sigma_\alpha(\tilde{\omega}_n^\alpha,\mathbf{R}) = n_i u_\alpha^2\int d\mathbf{r}\, G_\alpha(\mathbf{r},\tilde{\omega}_n^\alpha)G_\alpha(\mathbf{r},-\tilde{\omega}_n^\alpha) \times\{(i\mathbf{r}\mathbf{D}(\mathbf{R}))\Delta_\alpha(\mathbf{R},\mathbf{r}) + \Sigma_\alpha(\tilde{\omega}_n^\alpha,\mathbf{R})\}. \tag{34}$$

Finding $\Sigma_\alpha(\tilde{\omega}_n^\alpha,\mathbf{R})$ from the last equation and substituting to Eq. (33) we obtain after all the necessary integrations the pair of the Ginzburg-Landau equations for two components of the order parameter

$$\eta_1 = V_{\uparrow\uparrow}\hat{\alpha}_1\eta_1 + V_{\uparrow\downarrow}\hat{\alpha}_2\eta_2,$$

$$\eta_2 = V_{\uparrow\downarrow}\hat{\alpha}_1\eta_1 + V_{\downarrow\downarrow}\hat{\alpha}_2\eta_2, \tag{35}$$

where operator $\hat{\alpha}_1$ consists of previously determined homogeneous part and second order gradient terms

$$\hat{\alpha}_1=\langle|f_-(\mathbf{k})|^2N_{0\uparrow}(\hat{\mathbf{k}})\rangle\Lambda_1(T)-K_{\uparrow ij}D_iD_j. \tag{36}$$

The gradient terms coefficients are

$$K_{\uparrow ij}=\langle|f_-(\mathbf{k})|^2N_{0\uparrow}(\hat{\mathbf{k}})v_{F\uparrow i}(\hat{\mathbf{k}})v_{F\uparrow j}(\hat{\mathbf{k}})\rangle\frac{\pi T}{2}\sum_{n\geqslant 0}\frac{1}{|\tilde{\omega}_n^{\uparrow}|^3}$$
$$+\langle f_-(\mathbf{k})N_{0\uparrow}(\hat{\mathbf{k}})v_{F\uparrow i}(\hat{\mathbf{k}})\rangle$$
$$\times\langle f_-^*(\mathbf{k})N_{0\uparrow}(\hat{\mathbf{k}})v_{F\uparrow j}(\hat{\mathbf{k}})\rangle\frac{\pi^2 Tn_iu_\uparrow{}^2}{2}\sum_{n\geqslant 0}\frac{1}{\omega_n^2\tilde{\omega}_n^{\uparrow 2}}. \tag{37}$$

Operator $\hat{\alpha}_2$ is obtained from here by the natural substitutions $1\rightarrow 2$, $\uparrow\rightarrow\downarrow$, $+\rightarrow -$.

Now the problem of the upper critical field finding is just the problem of solution of the two coupled equations (35). There are a lot of different situations depending of crystal symmetry, direction of spontaneous magnetization and the external field orientation. The simplest case is when the external magnetic field is parallel or antiparallel to the easy magnetization axis. If the latter coincides with fourth order symmetry axis in the cubic crystal such as it is in $ZrZn_2$ then the gradient terms in the perpendicular plane are isotropic and described by two constants $K_{\uparrow ij}=K_\uparrow\delta_{ij}$, $K_{\downarrow ij}=K_\downarrow\delta_{ij}$. This case formally corresponds to the problem of determination of upper critical field parallel to the c direction in two-band hexagonal superconductor MgB_2 solved in Ref. 20. Then the linearized Ginzburg-Landau equations describe a system of two coupled oscillators and have the solution in the form $\eta_1=c_1f_0(x)$ and $\eta_2=c_2f_0(x)$, where $f_0(x)=\exp(-hx^2/2)$ and h is related to the upper critical field by means

$$|H_{c2}\pm 4\pi M|=\frac{h(\tau)\Phi_0}{2\pi}, \tag{38}$$

where Φ_0 is the flux quantum.

Let us for the simplicity limit ourself by the impurity less case. Then $\tau=1-T/T_{c0}$ and the equation for the determination of upper critical field is

$$\{g_1[\tau+\lambda(T_{c0})]+V_{\uparrow\uparrow}K_\uparrow h-1\}\{g_2[\tau+\lambda(T_{c0})]$$
$$+V_{\downarrow\downarrow}K_\downarrow h-1\}-\{g_{12}[\tau+\lambda(T_{c0})]+V_{\uparrow\downarrow}K_\uparrow h-1\}$$
$$\times\{g_{21}[\tau+\lambda(T_{c0})]+V_{\uparrow\downarrow}K_\downarrow h-1\}=0. \tag{39}$$

This is a simple square equation and as before if we consider the case $g_{12},g_{21}\ll g_1,g_2$ and $g_1>g_2$ then we obtain the following two roots

$$h_1(\tau)=\frac{g_1\tau}{V_{\uparrow\uparrow}K_\uparrow}, \tag{40}$$

$$h_2(\tau)=\frac{g_2}{V_{\downarrow\downarrow}K_\downarrow}\left(\tau+\frac{1}{g_1}-\frac{1}{g_2}\right). \tag{41}$$

This two lines can in principle intersect each other, then an upturn on the temperature dependence of the upper critical field given by $\max(h_1,h_2)$ is appeared.

In the more anisotropic situation such as in orthorombic crystals UGe_2 and URhGe even for the external field direction parallel or antiparallel to the easy magnetization axis all the coefficients $K_{\uparrow xx}$, $K_{\uparrow yy}$, $K_{\downarrow xx}$, $K_{\downarrow yy}$ are different. Then our system of equations can be solved following a variational approach developed in Ref. 20. Again an upturn in $h(\tau)$ dependence can be possible.

The comparison with experiment shall be always not easy masked by the presence of many ferromagnetic domains. The monodomain measurements are possible in high enough fields. To work in this region one can easily obtain the fourth order gradient terms to the Ginzburg-Landau equations. However the problem of theoretical determination of the upper critical field at arbitrary temperature has the same principal difficulties as in any conventional anisotropic superconductor.[21]

III. CONCLUSION

Ferromagnetic superconductors are in general multiband metals. The two-band description of ferromagnetic superconductors with triplet pairing developed in this paper presents the simplest model applicable to this type of material. One-band superconductivity in these superconductors arises only at a negligibly small spin-orbit coupling, as is the case for the A_1 phase of ^{3}He. We have studied the dependence of the critical temperature T_c on the concentration of ordinary pointlike impurities in the framework of a two-band weak coupling BCS theory. It was demonstrated that the nonuniversal $T_c(x)$ dependence can serve as a qualitative measure of the two-band character of the superconductivity in ferromagnetic superconductors. Also the general equations for the determination of the upper critical field at arbitrary temperature and impurity concentration were derived. The solution of these equations near the critical temperature was found in the simplest case of cubic crystalline symmetry for the field orientation parallel or antiparallel to the fourth order symmetry axis.

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