Weak-localization theory for quasiparticle dynamical conductivities of disordered *d*-wave superconductors

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By the diagrammatic technique, we calculate the quantum interference (QI) contributions to the quasiparticle dynamical conductivities of two-dimensional *d*-wave superconductors with dilute nonmagnetic impurities either near the Born or near the unitary limit. For generic situations, only the 0-mode cooperon has QI contributions to the charge and spin conductivities. It is found that with decreasing frequency, the spin conductivity increases logarithmically while the charge one decreases logarithmically. Such a qualitative difference originates from a novel QI process related to the intrinsic particle-hole symmetry. In the combined limit of unitarity and nested Fermi surface, there exist additional diffusive π modes in addition to the usual diffusive 0 modes. The QI corrections in this limit are shown to vanish due to the cancellation of the contributions from 0-mode and π-mode cooperons, so that both the charge and spin conductivities approach their universal values.

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I. INTRODUCTION

Over the past decade, there have been considerable activities on the study of disorder effects on quasiparticle transport properties of *d*-wave superconductors. This is in part due to the convincing evidences that the superconducting state of the hole-doped cuprate materials is characterized by spin-singlet *d*-wave paring,[1] and partly due to the fact that significant disorder—perhaps originating with charge-donor impurities—is present in nearly all cuprate superconductors. The low-lying quasiparticle excitations in the *d*-wave superconducting state have a Dirac-type spectrum because of the existence of gap nodes. In stark contrast with the conventional superconductors, the low-energy states and transport properties of the *d*-wave superconductor are strongly affected by the disorder.[2] Experimental measurements of the transport properties such as microwave,[3–5] optical,[6–9] and thermal[10–14] conductivities of the cuprates have provided substantial useful information on the quasiparticle features in the disordered *d*-wave superconductor. Theoretically, the self-consistent *T*-matrix approximation (SCTMA) for impurity scattering has been widely used, with some success, to calculate the charge,[15–22] spin,[22] and thermal[22,23] conductivities.

The simplest theories based on the SCTMA ignore the quantum interference (QI) effects. It has been shown that the QI effects have important influences on the quasiparticle states, as well as on transport properties in disordered *d*-wave superconductors.[24–29] More recently, it was argued[30] that the maximum of optical conductivity at a disorder-dependent frequency observed in disordered cuprates[8,9] is related to the QI effects of quasiparticles, and cannot be interpreted by the simple SCTMA analysis.

It is well known that the QI effects in disordered metals result from the existence of the diffusive modes (cooperon and diffuson) (for reviews, see Refs. 31–33). Due to the intrinsic particle-hole symmetry, the diffusive modes of a superconductor exist both in retarded-advanced (RA) (or AR) and in retarded-retarded (RR) (or AA) channels,[34] quite different from those of a normal conductor. For the *d*-wave superconductor, each of the usual diffusive 0 modes has a π-mode counterpart near the combined limit of the unitarity and nested Fermi surface [the unitary and nesting(UN) limit].[26,27] The QI effects on quasiparticle dc conductivities of disordered *d*-wave superconductors have been studied by several groups with the nonlinear-sigma-model approach.[24,25,28] By carrying out a weak-localization calculation with the diagrammatic technique, Yashenkin *et al.*[27] pointed out that the existence of diffusive π modes can be used to account for the previous contradictory predictions for the density of states (DOS) of disordered *d*-wave superconductors. Recently, in the spirit of the method of Ref. 27, Yang *et al.*[29] recalculated the QI corrections to the quasiparticle dc conductivities. They found an additional conductivity diagram, which was not considered previously. This new diagram describes a novel QI process related to the intrinsic particle-hole symmetry. While this QI process has a vanishing contribution to the charge conductivity, it gives rise to an antilocalization correction to the spin conductivity.

Recent numerical solutions of the Bogoliubov–de Gennes equations suggested that the QI effects may also play an important role in the quasiparticle *dynamical* transport of cuprate superconductors.[30] Up to now, to our knowledge, dynamical transport properties of the cuprates have not been yet investigated within the weak-localization theory. This is the principal motivation for the current work. On the other hand, the weak-localization theory for the random Dirac fermions in *d*-wave superconductors is far from well established, and thus needs further development.

In this work, the method of Ref. 29 is extended to investigate the QI contributions to the quasiparticle dynamical conductivities of a two-dimensional (2D) $d_{x^2-y^2}$-wave superconductor with dilute nonmagnetic impurities. We restrict ourselves near the Born or unitary limit, as these two limiting cases are considered to be relevant to the disorder effects of the cuprates. Both the frequency-dependent charge and spin conductivities are calculated for generic Fermi surfaces, as well as for the nesting case. In the singlet $d_{x^2-y^2}$-wave superconductor, the spin is conserved while the charge is not. As a result, the charge and spin conductivities are found to have *qualitatively* different frequency dependences due to the existence of the novel QI process. In generic situations (far from the UN limit), only the 0-mode cooperon contributes to the QI effects. We show that the charge conductivity is subject to a logarithmic suppression with decreasing frequency, which is qualitatively in agreement with the result at low frequency of the numerical study,[30] and is also supported by the experimental observations of the cuprates.[8,9] In contrast, the spin conductivity is found to increase logarithmically with decreasing frequency. Near the UN limit, both the 0-mode and π-mode cooperons contribute to the QI effects. Their contributions to the charge or spin dynamical conductivity are shown to cancel out exactly, and thus both conductivities approach their universal values in the UN limit. We also show that both the 0-mode and π-mode diffusons have vanishing contributions to the QI effects in all situations considered.

This paper is structured as follows. In Sec. II, we outline the SCTMA, which is a starting point of the present weak-localization theory. In Sec. III, we derive in detail the expressions of diffusive modes for a d-wave superconductor. The QI contributions to the dynamical charge and spin conductivities in generic situations are calculated in Sec. IV. Section V is devoted to study the QI effects near the UN limit. A summary of the results is given in Sec. VI. Some useful mathematical formulas are presented in Appendix A. In Appendixes B and C, we show, respectively, that the 0-mode and π-mode diffusons have vanishing contributions to the QI effects. For completeness, we present in Appendix D the demonstration that the lowest-order conductivity diagrams with nonsingular ladders do not contribute to the QI corrections.

II. SCTMA FOR THE $D_{x^2-y^2}$-WAVE SUPERCONDUCTOR

We begin with the most extensively studied model for a 2D d-wave superconductor, in which the normal-state dispersion of a square lattice is given by $\xi_{\boldsymbol{k}}=-t(\cos k_x a+\cos k_y a)-\mu$, where a is the lattice constant, t is the nearest-neighbor hopping integral, and μ is the chemical potential. The nested Fermi surface corresponds to a half-filled band ($\mu=0$). The order parameter of the $d_{x^2-y^2}$-wave pairing state is expressed by $\Delta_{\boldsymbol{k}}=\Delta_0(\cos k_x a-\cos k_y a)$. The four gap nodes are given by $\boldsymbol{k}_n=\pm(k_0,\pm k_0)$ $(n=1,2,3,4)$ with $k_0=(1/a)\arccos(-\mu/2t)$. The quasiparticle spectrum near the gap nodes can be linearized as $\epsilon_{\boldsymbol{k}}=\sqrt{\xi_{\boldsymbol{k}}^2+\Delta_{\boldsymbol{k}}^2}\approx\sqrt{(\boldsymbol{v}_f\cdot\tilde{\boldsymbol{k}})^2+(\boldsymbol{v}_g\cdot\tilde{\boldsymbol{k}})^2}$, where $v_f=v_g t/\Delta_0=\sqrt{2}ta\sqrt{1-(\mu/2t)^2}$, and $\tilde{\boldsymbol{k}}$ is the momentum measured from the node $\boldsymbol{k}_n$. The directions of the Fermi velocity $\boldsymbol{v}_f$ and "gap velocity" $\boldsymbol{v}_g$ are depicted in Fig. 1.

The disorder model considered in the present work is exactly the same as that of Ref. 27, i.e., the random distribution

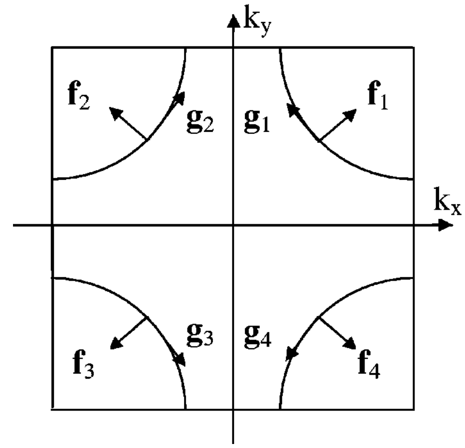

FIG. 1. Directions of the Fermi velocity $\boldsymbol{v}_f$ and "gap velocity" $\boldsymbol{v}_g$. $\boldsymbol{f}_n$ and $\boldsymbol{g}_n$ represent, respectively, the unity vectors parallel to $\boldsymbol{v}_f$ and $\boldsymbol{v}_g$ at the nth gap node ($n=1,2,3,4$).

of pointlike nonmagnetic impurities with potential V and low concentration n_i. Then the time-reversal and spin-rotational symmetries remain preserved, meaning that the system belongs to symmetry class CI in the classification of Ref. 34. In the SCTMA, the quasiparticle self-energy can be expressed in the Nambu spinor representation as[27]

$$\Sigma^{R(A)}(\epsilon)=n_i T^{R(A)}(\epsilon)=(\lambda\epsilon\mp i\gamma)\tau_0+\eta\gamma\tau_3, \quad (2.1)$$

for $|\epsilon|\ll\gamma$. Here γ is the impurity-induced relaxation rate, λ and η are dimensionless parameters, $\eta\gamma$ represents the decrease of chemical potential induced by the impurity scattering, τ_0 and τ_i ($i=1,2,3$) stand for the 2×2 unity and Pauli matrices, respectively. A use of Dyson's equation immediately yields the impurity-averaged one-particle Green's functions as

$$G_{\boldsymbol{k}}^{R(A)}(\epsilon)=\frac{[(1-\lambda)\epsilon\pm i\gamma]\tau_0+\Delta_{\boldsymbol{k}}\tau_1+\xi_{\boldsymbol{k}}\tau_3}{[(1-\lambda)\epsilon\pm i\gamma]^2-\epsilon_{\boldsymbol{k}}^2}, \quad (2.2)$$

where the shift of chemical potential is absorbed in μ. The impurity-induced density of states at zero energy is calculated as $\rho_0=-(1/\pi)\mathrm{Im}\Sigma_{\boldsymbol{k}}\mathrm{Tr}G_{\boldsymbol{k}}^R(0)=4l\gamma/\pi^2 v_f v_g$, where $l=\ln(\Gamma/\gamma)>1$ with $\Gamma\sim\sqrt{v_f v_g}/a$.

The parameters γ, λ, and η can be evaluated consistently via the T-matrix equation

$$T^{R(A)}(\epsilon)=V\tau_3+V\tau_3 g^{R(A)}(\epsilon)T^{R(A)}(\epsilon), \quad (2.3)$$

with $g^{R(A)}(\epsilon)=\Sigma_{\boldsymbol{k}}G_{\boldsymbol{k}}^{R(A)}(\epsilon)$. Using Eq. (2.2), we can show that

$$g^{R(A)}(\epsilon)=\frac{\pi\rho_0}{2\gamma}[(\lambda-1)(1-l^{-1})\epsilon\mp i\gamma]\tau_0+(V^{-1}-U^{-1})\tau_3, \quad (2.4)$$

for $|\epsilon|\ll\gamma$, where U is the effective impurity potential given by $U^{-1}=V^{-1}+\Sigma_{\boldsymbol{k}}\xi_{\boldsymbol{k}}/(\epsilon_{\boldsymbol{k}}^2+\gamma^2)$. A substitution of Eqs. (2.1) and (2.4) into Eq. (2.3) leads to $\gamma=2n_i/\pi\rho_0(1+\eta^2)$, $\lambda=(1-\eta^2)(l-1)/(\eta^2+2l-1)$, and $\eta=2/\pi\rho_0 U$.

The Born limit corresponds to $\eta^2\gg 2l$, yielding

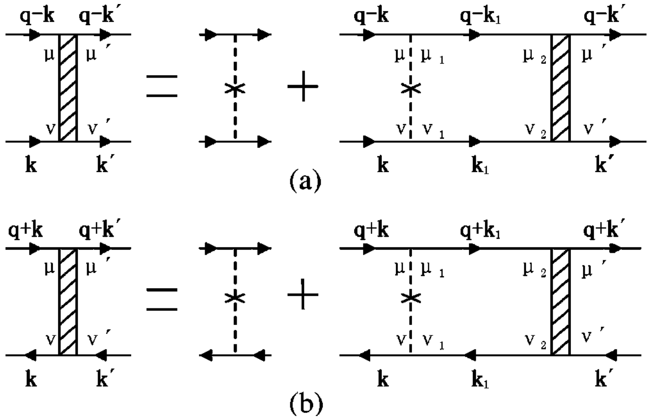

FIG. 2. Ladder diagrams for the 0-mode cooperon (a) and 0-mode diffuson (b). The dashed lines denote the T matrices instead of the impurity potential. The corresponding diagrams for the π-mode cooperon and π-mode diffuson can be generated by the replacement of $\boldsymbol{q}\to\boldsymbol{Q}+\boldsymbol{q}$ in these 0-mode diagrams.

$$\gamma=\frac{\pi}{2}n_i\rho_0U^2,\quad \lambda=1-l; \tag{2.5}$$

and the unitary limit corresponds to $\eta\to 0$, meaning that

$$\gamma=\frac{2n_i}{\pi\rho_0},\quad \lambda=\frac{l-1}{2l-1}. \tag{2.6}$$

The above results of SCTMA will be used in the evaluations of the quasiparticle diffusive modes and transport coefficients.

III. THE DIFFUSIVE MODES

Since the QI effects are related to the diffusive modes, we first derive their expressions for a disordered d-wave superconductor.

A. 0-mode cooperon and diffuson

The 0-mode cooperon and diffuson exist both in RA and in RR channels due to the local particle-hole symmetry $\tau_2G_{\boldsymbol{k}}^R(\epsilon)\tau_2=-G_{\boldsymbol{k}}^A(-\epsilon)$. The ladder diagrams for the cooperon are shown in Fig. 2(a). According to the Feynman rules in the diagrammatic technique, the equation for 0-mode cooperon can be expressed as

$$C(\boldsymbol{q};\epsilon,\epsilon')_{\mu\nu,\mu'\nu'}=W(\epsilon,\epsilon')_{\mu\nu,\mu'\nu'}+W(\epsilon,\epsilon')_{\mu\nu,\mu_1\nu_1}\times H(\boldsymbol{q};\epsilon,\epsilon')_{\mu_1\nu_1,\mu_2\nu_2}C(\boldsymbol{q};\epsilon,\epsilon')_{\mu_2\nu_2,\mu'\nu'}, \tag{3.1}$$

where the repeated Nambu indices mean summations, and the two-particle irreducible vertex and the integral kernel in the RR (or RA) channel are defined, respectively, as

$$W(\epsilon,\epsilon')^{RR(A)}_{\mu\nu,\mu'\nu'}=n_iT^R(\epsilon)_{\mu\mu'}T^{R(A)}(\epsilon')_{\nu\nu'} \tag{3.2}$$

and

$$H(\boldsymbol{q};\epsilon,\epsilon')^{RR(A)}_{\mu_1\nu_1,\mu_2\nu_2}=\sum_{\boldsymbol{k}} G^R_{\boldsymbol{q}-\boldsymbol{k}}(\epsilon)_{\mu_1\mu_2}G^{R(A)}_{\boldsymbol{k}}(\epsilon')_{\nu_1\nu_2}. \tag{3.3}$$

We introduce tensor product $\mathcal{Z}=X\otimes Y$ (where X and Y denote 2×2 matrices) with the definition of $Z_{\mu\mu',\nu\nu'}=X_{\mu\nu}Y_{\mu'\nu'}$. Then Eqs. (3.1)–(3.3) can be reexpressed as

$$\mathcal{C}(\boldsymbol{q};\epsilon,\epsilon')=\mathcal{W}(\epsilon,\epsilon')+\mathcal{W}(\epsilon,\epsilon')\mathcal{H}(\boldsymbol{q};\epsilon,\epsilon')\mathcal{C}(\boldsymbol{q};\epsilon,\epsilon'), \tag{3.4}$$

with

$$\mathcal{W}(\epsilon,\epsilon')^{RR(A)}=n_iT^R(\epsilon)\otimes T^{R(A)}(\epsilon') \tag{3.5}$$

and

$$\mathcal{H}(\boldsymbol{q};\epsilon,\epsilon')^{RR(A)}=\sum_{\boldsymbol{k}} G^R_{\boldsymbol{q}-\boldsymbol{k}}(\epsilon)\otimes G^{R(A)}_{\boldsymbol{k}}(\epsilon'). \tag{3.6}$$

One can make a decomposition of $\mathcal{X}=\Sigma_{ij}X_{ij}\tau_i\otimes\tau_j$ for $\mathcal{X}=\mathcal{W},\mathcal{H},$ and $\mathcal{C}$. A substitution of Eq. (2.1) into Eq. (3.5) immediately yields

$$W(\epsilon,\epsilon')^{RR(A)}_{00}=\mp\frac{2\gamma}{\pi\rho_0(1+\eta^2)}\left[1+i\frac{\lambda}{\gamma}(\epsilon\pm\epsilon')\right], \tag{3.7}$$

$$W(\epsilon,\epsilon')^{RR(A)}_{33}=\frac{2\gamma\eta^2}{\pi\rho_0(1+\eta^2)}, \tag{3.8}$$

$$W(\epsilon,\epsilon')^{RR(A)}_{03}=-i\frac{2\gamma\eta}{\pi\rho_0(1+\eta^2)}\left(1+i\frac{\lambda}{\gamma}\epsilon\right), \tag{3.9}$$

and

$$W(\epsilon,\epsilon')^{RR(A)}_{30}=\mp i\frac{2\gamma\eta}{\pi\rho_0(1+\eta^2)}\left(1\pm i\frac{\lambda}{\gamma}\epsilon'\right), \tag{3.10}$$

where the terms bilinear in ϵ or ϵ' are omitted, consistent with an approximation made throughout this paper. The above equations indicate that the dominant component of $\mathcal{W}(\epsilon,\epsilon')$ near the Born limit ($\eta^2\gg 2l$) is $W(\epsilon,\epsilon')_{33}$, while that near the unitary limit ($\eta^2\ll 1$) is $W(\epsilon,\epsilon')_{00}$.

Now let us calculate $\mathcal{H}(\boldsymbol{q};\epsilon,\epsilon')$. For small values of $\boldsymbol{q}$, ϵ, and ϵ', we have

$$G^R_{\boldsymbol{q}-\boldsymbol{k}}(\epsilon)\approx G^R_{\boldsymbol{k}}+\epsilon\frac{\partial}{\partial\epsilon'}G^R_{\boldsymbol{k}}(\epsilon')\big|_{\epsilon'=0}-\boldsymbol{q}\cdot\boldsymbol{\nabla}G^R_{\boldsymbol{k}}+\frac{1}{2}\boldsymbol{q}\boldsymbol{q}:\boldsymbol{\nabla}\boldsymbol{\nabla}G^R_{\boldsymbol{k}} \tag{3.11}$$

and

$$G^{R(A)}_{\boldsymbol{k}}(\epsilon')\approx G^{R(A)}_{\boldsymbol{k}}+\epsilon'\frac{\partial}{\partial\epsilon}G^{R(A)}_{\boldsymbol{k}}(\epsilon)\big|_{\epsilon=0}, \tag{3.12}$$

with $G^{R(A)}_{\boldsymbol{k}}=G^{R(A)}_{\boldsymbol{k}}(0)$. Substituting Eqs. (3.11) and (3.12) into Eq. (3.6), and using Eqs. (A1)–(A8) in Appendix A, we can show that all the nonvanishing components of $\mathcal{H}(\boldsymbol{q};\epsilon,\epsilon')$ are given by

$$H(\boldsymbol{q};\epsilon,\epsilon')_{00}^{RR(A)}=\mp\frac{1}{\pi v_f v_g}\left(1-\frac{v_f^2+v_g^2}{12\gamma^2}q^2\right), \quad (3.13)$$

$$H(\boldsymbol{q};\epsilon,\epsilon')_{11}^{RR(A)}=\frac{1}{2\pi v_f v_g}\left[(2l-1)+i\frac{1-\lambda}{\gamma}(\epsilon\pm\epsilon') - \frac{v_f^2+3v_g^2}{12\gamma^2}q^2\right], \quad (3.14)$$

and

$$H(\boldsymbol{q};\epsilon,\epsilon')_{33}^{RR(A)}=\frac{1}{2\pi v_f v_g}\left[(2l-1)+i\frac{1-\lambda}{\gamma}(\epsilon\pm\epsilon') - \frac{v_g^2+3v_f^2}{12\gamma^2}q^2\right]. \quad (3.15)$$

In order to evaluate the expression of cooperon, we reexpress Eq. (3.4) as

$$\mathcal{A}(\boldsymbol{q};\epsilon,\epsilon')\mathcal{C}(\boldsymbol{q};\epsilon,\epsilon')=\mathcal{W}(\epsilon,\epsilon'), \quad (3.16)$$

where

$$\mathcal{A}(\boldsymbol{q};\epsilon,\epsilon')=\mathcal{I}-\mathcal{W}(\epsilon,\epsilon')\mathcal{H}(\boldsymbol{q};\epsilon,\epsilon'), \quad (3.17)$$

with $\mathcal{I}=\tau_0\otimes\tau_0$. Substituting Eqs. (3.7), (3.8), and (3.13)–(3.15) into Eq. (3.17), we obtain

$$A(\boldsymbol{q};\epsilon,\epsilon')_{00}^{RR(A)}=-\frac{1}{4l(1+\eta^2)}\left[-2(2l-1)-\eta^2(2l+1) + i\frac{2\lambda+\eta^2(1-\lambda)}{\gamma}(\epsilon\pm\epsilon') - \frac{2(v_f^2+v_g^2)+\eta^2(3v_f^2+v_g^2)}{12\gamma^2}q^2\right], \quad (3.18)$$

$$A(\boldsymbol{q};\epsilon,\epsilon')_{11}^{RR(A)}=\pm\frac{1}{4l(1+\eta^2)}\left[(2l-1)+i\frac{1+2\lambda(l-1)}{\gamma} \times(\epsilon\pm\epsilon') - \frac{v_f^2+3v_g^2}{12\gamma^2}q^2\right], \quad (3.19)$$

$$A(\boldsymbol{q};\epsilon,\epsilon')_{22}^{RR(A)}=\frac{\eta^2}{4l(1+\eta^2)}\left[(2l-1)+i\frac{1-\lambda}{\gamma}(\epsilon\pm\epsilon') - \frac{v_f^2+3v_g^2}{12\gamma^2}q^2\right], \quad (3.20)$$

and

$$A(\boldsymbol{q};\epsilon,\epsilon')_{33}^{RR(A)}=\pm\frac{1}{4l(1+\eta^2)}\left[(2l-1)+2\eta^2 + i\frac{1+2\lambda(l-1)}{\gamma}(\epsilon\pm\epsilon') - \frac{v_g^2+3v_f^2+2\eta^2(v_f^2+v_g^2)}{12\gamma^2}q^2\right]. \quad (3.21)$$

In the formulism of components, Eq. (3.16) reads

$$A_{\mu\nu,\mu_1\nu_1}C_{\mu_1\nu_1,\mu'\nu'}=W_{\mu\nu,\mu'\nu'}.$$

Using the decomposition of $X_{\mu\mu',\nu\nu'}=\Sigma_{ij}X_{ij}(\tau_i)_{\mu\nu}(\tau_j)_{\mu'\nu'}$ for $X=A,C,W$, and completing the matrix products of Pauli matrices, one obtains

$$\sum_{ijkl}A_{ij}C_{kl}(\tau_i\tau_k)_{\mu\mu'}(\tau_j\tau_l)_{\nu\nu'}=\sum_{il}W_{il}(\tau_i)_{\mu\mu'}(\tau_l)_{\nu\nu'}.$$

There is no simple elegant way to evaluate the fourfold sum on the left-hand side of the above equation. However, near the Born or unitary limit, the nondiagonal components of $\mathcal{W}(\epsilon,\epsilon')$ are negligible, and thus only the diagonal components of $\mathcal{A}(\boldsymbol{q};\epsilon,\epsilon')$ and $\mathcal{C}(\epsilon,\epsilon')$ need to be taken into account. Therefore Eq. (3.16) is equivalent to the following group of equations:

$$A_{00}C_{00}+A_{11}C_{11}+A_{22}C_{22}+A_{33}C_{33}=W_{00}, \quad (3.22)$$

$$A_{00}C_{11}+A_{11}C_{00}-A_{22}C_{33}-A_{33}C_{22}=0, \quad (3.23)$$

$$A_{00}C_{22}-A_{11}C_{33}+A_{22}C_{00}-A_{33}C_{11}=0, \quad (3.24)$$

$$A_{00}C_{33}-A_{11}C_{22}-A_{22}C_{11}+A_{33}C_{00}=W_{33}, \quad (3.25)$$

where the arguments ($\boldsymbol{q}$, ϵ, and ϵ') of A_{ii}, C_{ii}, and W_{ii} have been omitted.

Since $C(\boldsymbol{q};\epsilon,\epsilon')_{ii}$ is singular for small values of $\boldsymbol{q}$, ϵ, and ϵ', the singular terms in the left-hand side of every one of Eqs. (3.22)–(3.25) cancel out, meaning that

$$\bar{A}_{00}C_{00}+\bar{A}_{11}C_{11}+\bar{A}_{22}C_{22}+\bar{A}_{33}C_{33}=0, \quad (3.26)$$

$$\bar{A}_{00}C_{11}+\bar{A}_{11}C_{00}-\bar{A}_{22}C_{33}-\bar{A}_{33}C_{22}=0, \quad (3.27)$$

$$\bar{A}_{00}C_{22}-\bar{A}_{11}C_{33}+\bar{A}_{22}C_{00}-\bar{A}_{33}C_{11}=0, \quad (3.28)$$

$$\bar{A}_{00}C_{33}-\bar{A}_{11}C_{22}-\bar{A}_{22}C_{11}+\bar{A}_{33}C_{00}=0, \quad (3.29)$$

where $\bar{A}_{ii}=A(0;0,0)_{ii}$. In the Born or unitary limit, it is easy to show that the group of Eqs. (3.26)–(3.29) has a sole solution as

$$C_{00}^{RR(A)}=\mp C_{11}^{RR(A)}=-C_{22}^{RR(A)}=\mp C_{33}^{RR(A)}. \quad (3.30)$$

For the Born limit, W_{00} can be neglected. Substituting Eqs. (3.8), (3.18)–(3.21), and (3.30) into Eq. (3.25), and using Eq. (2.5), we obtain

$$C(\boldsymbol{q};\epsilon,\epsilon')_{00}^{RR(A)}=\mp\frac{4\gamma^2}{\pi\rho_0}\frac{1}{Dq^2-i(\epsilon\pm\epsilon')},\qquad(3.31)$$

where $D=(v_f^2+v_g^2)/4l\gamma$ is the quasiparticle diffusion coefficient. One can show that the above expression is also valid for small but finite η^{-2}. Combining Eq. (3.30) with Eq. (3.31), and noting that the 0-mode diffuson has the same expression as that of 0-mode cooperon due to the time-reversal symmetry, we obtain

$$\mathcal{C}(\boldsymbol{q};\epsilon,\epsilon')^{RR(A)}=\mathcal{D}(\boldsymbol{q};\epsilon,\epsilon')^{RR(A)}=\frac{4\gamma^2}{\pi\rho_0}\frac{1}{Dq^2-i(\epsilon\pm\epsilon')}\times(\mp\tau_0\otimes\tau_0+\tau_1\otimes\tau_1\pm\tau_2\otimes\tau_2+\tau_3\otimes\tau_3),\qquad(3.32)$$

which is in agreement with that of Ref. 26 (apart from a disputed prefactor 2). Similarly, we can show that Eq. (3.32) is also valid near the unitary limit. The above evaluations indicate that any small deviations from either limit do not make the Goldstone 0 modes gapped.

B. π-mode cooperon and diffuson

In the UN limit, there exist the additional π-mode cooperon and diffuson due to the global particle-hole symmetry[27]

$$\tau_2G_{\boldsymbol{k}}^{R(A)}(\epsilon)\tau_2=G_{\boldsymbol{Q}+\boldsymbol{k}}^{R(A)}(\epsilon),\qquad(3.33)$$

with $\boldsymbol{Q}=\pm(\pi/a,\pm\pi/a)$ the nesting vector. Any small deviations from this combined limit can be shown to make the Goldstone π modes gapped. The ladder diagrams for π-mode cooperon can be obtained from those of 0-mode cooperon by replacing $\boldsymbol{q}$ by $\boldsymbol{Q}+\boldsymbol{q}$. The equation for π-mode cooperon is given by

$$\mathcal{A}_\pi(\boldsymbol{q};\epsilon,\epsilon')\mathcal{C}_\pi(\boldsymbol{q};\epsilon,\epsilon')=\mathcal{W}(\epsilon,\epsilon'),\qquad(3.34)$$

where

$$\mathcal{A}_\pi(\boldsymbol{q};\epsilon,\epsilon')=\mathcal{I}-\mathcal{W}(\epsilon,\epsilon')\mathcal{H}_\pi(\boldsymbol{q};\epsilon,\epsilon'),\qquad(3.35)$$

with

$$\mathcal{H}_\pi(\boldsymbol{q};\epsilon,\epsilon')^{RR(A)}=\sum_{\boldsymbol{k}}G_{\boldsymbol{Q}+\boldsymbol{q}-\boldsymbol{k}}^{R}(\epsilon)\otimes G_{\boldsymbol{k}}^{R(A)}(\epsilon').\qquad(3.36)$$

In order to calculate $\mathcal{H}_\pi(\boldsymbol{q};\epsilon,\epsilon')$ and $\mathcal{A}_\pi(\boldsymbol{q};\epsilon,\epsilon')$, one needs to exploit the relations $\Delta_{\boldsymbol{Q}+\boldsymbol{k}}=-\Delta_{\boldsymbol{k}}$ and $\xi_{\boldsymbol{Q}+\boldsymbol{k}}=-\xi_{\boldsymbol{k}}-2\mu$. For a nearly nested Fermi surface ($|\mu|\ll\gamma$), we have

$$G_{\boldsymbol{Q}+\boldsymbol{q}-\boldsymbol{k}}^{R}(\epsilon)\approx\tau_2G_{\boldsymbol{q}-\boldsymbol{k}}^{R}(\epsilon)\tau_2+2\mu\frac{2\xi_{\boldsymbol{k}}(i\gamma\tau_0-\Delta_{\boldsymbol{k}}\tau_1)+(\gamma^2+\Delta_{\boldsymbol{k}}^2-\xi_{\boldsymbol{k}}^2)\tau_3}{(\gamma^2+\epsilon_{\boldsymbol{k}}^2)^2}+(2\mu)^2\frac{(\gamma^2+\Delta_{\boldsymbol{k}}^2-3\xi_{\boldsymbol{k}}^2)(i\gamma\tau_0-\Delta_{\boldsymbol{k}}\tau_1)-\xi_{\boldsymbol{k}}(3\gamma^2+3\Delta_{\boldsymbol{k}}^2-\xi_{\boldsymbol{k}}^2)\tau_3}{(\gamma^2+\epsilon_{\boldsymbol{k}}^2)^3}.\qquad(3.37)$$

Substituting Eqs. (3.12) and (3.37) into Eq. (3.36), and using Eqs. (A1)–(A8), we can show that all the nonvanishing components of $\mathcal{H}_\pi(\boldsymbol{q};\epsilon,\epsilon')$ are given by

$$H_\pi(\boldsymbol{q};\epsilon,\epsilon')_{00}^{RR(A)}=H(\boldsymbol{q};\epsilon,\epsilon')_{00}^{RR(A)}\pm\frac{2\mu^2}{3\pi v_fv_g\gamma^2},\qquad(3.38)$$

$$H_\pi(\boldsymbol{q};\epsilon,\epsilon')_{11}^{RR(A)}=-H(\boldsymbol{q};\epsilon,\epsilon')_{11}^{RR(A)}+\frac{\mu^2}{3\pi v_fv_g\gamma^2},\qquad(3.39)$$

$$H_\pi(\boldsymbol{q};\epsilon,\epsilon')_{33}^{RR(A)}=-H(\boldsymbol{q};\epsilon,\epsilon')_{33}^{RR(A)}+\frac{\mu^2}{3\pi v_fv_g\gamma^2},\qquad(3.40)$$

and

$$H_\pi(\boldsymbol{q};\epsilon,\epsilon')_{03}^{RR(A)}=\pm H_\pi(\boldsymbol{q};\epsilon,\epsilon')_{30}^{RR(A)}=-i\frac{\mu}{\pi v_fv_g\gamma}.\qquad(3.41)$$

A substitution of Eqs. (3.13)–(3.15) and (3.38)–(3.41) into Eq. (3.35) yields the diagonal components of $\mathcal{A}_\pi(\boldsymbol{q};\epsilon,\epsilon')$ as

$$A_\pi(\boldsymbol{q};\epsilon,\epsilon')_{00}^{RR(A)}=-\frac{1}{4l(1+\eta^2)}\bigg[-2(2l-1)-\eta^2(6l-1)+i\frac{2\lambda-\eta^2(1-\lambda)}{\gamma}(\epsilon\pm\epsilon')-\frac{2(v_f^2+v_g^2)-\eta^2(3v_f^2+v_g^2)}{12\gamma^2}q^2-\frac{(4-6\eta^2)\mu^2}{3\gamma^2}-\frac{4\eta\mu}{\gamma}\bigg],\qquad(3.42)$$

$$A_{\pi}(\boldsymbol{q};\epsilon,\epsilon')_{11}^{RR(A)}=\mp\frac{1}{4l(1+\eta^2)}\Bigg[(2l-1)+i\,\frac{1+2\lambda(l-1)}{\gamma}\times(\epsilon\pm\epsilon')-\frac{v_f^2+3v_g^2}{12\gamma^2}q^2-\frac{2\mu^2}{3\gamma^2}\Bigg], \quad (3.43)$$

$$A_{\pi}(\boldsymbol{q};\epsilon,\epsilon')_{22}^{RR(A)}=-\frac{\eta^2}{4l(1+\eta^2)}\Bigg[(2l-1)+i\,\frac{1-\lambda}{\gamma}(\epsilon\pm\epsilon')-\frac{v_f^2+3v_g^2}{12\gamma^2}q^2-\frac{2\mu^2}{3\gamma^2}\Bigg], \quad (3.44)$$

and

$$A_{\pi}(\boldsymbol{q};\epsilon,\epsilon')_{33}^{RR(A)}=\mp\frac{1}{4l(1+\eta^2)}\Bigg[(2l-1)-2\eta^2+i\,\frac{1+2\lambda(l-1)}{\gamma}(\epsilon\pm\epsilon')-\frac{v_g^2+3v_f^2-2\eta^2(v_f^2+v_g^2)}{12\gamma^2}q^2-\frac{(6-4\eta^2)\mu^2}{3\gamma^2}-\frac{4\eta\mu}{\gamma}\Bigg]. \quad (3.45)$$

Equations (3.22)–(3.29) are also suitable for the π-mode cooperon. Therefore, using Eqs. (3.42)–(3.45), one can similarly show that (near the unitary limit)

$$\mathcal{C}_{\pi}(\boldsymbol{q};\epsilon,\epsilon')^{RR(A)}=\mathcal{D}_{\pi}(\boldsymbol{q};\epsilon,\epsilon')^{RR(A)}=-\frac{4\gamma^2}{\pi\rho_0}\frac{1}{Dq^2-i(\epsilon\pm\epsilon')+2\delta}\times(\pm\tau_0\otimes\tau_0+\tau_1\otimes\tau_1\mp\tau_2\otimes\tau_2+\tau_3\otimes\tau_3), \quad (3.46)$$

with $\delta=2\eta^2\gamma+2\eta\mu/\gamma+\mu^2/l\gamma\ll\gamma$. Near the Born limit, one gets $C_{\pi ii}^{RR(A)}=0$ $(i=0,1,2,3)$, indicating that the diffusive π modes exist only near the UN limit. Contrary to the diffusive 0 modes, the Goldstone π modes are gapped by any small deviations from the UN limit measured by δ. For the situations far from the UN limit, the contributions of diffusive π modes to the QI effects are completely suppressed due to the large gap. Equations (3.32) and (3.46) will be used to calculate the QI corrections to the charge and spin conductivities.

IV. DYNAMICAL CONDUCTIVITIES IN GENERIC SITUATIONS

We use the Kubo formula to calculate the quasiparticle conductivities. At zero temperature the real part of charge ($\chi=e$) or spin ($\chi=s$) dynamical conductivity can be evaluated via[35]

$$\sigma^{\chi}(\omega)=\frac{1}{\omega}\int_0^{\omega}\frac{d\epsilon}{2\pi}\mathrm{Re}\Pi^{\chi}(\epsilon,\epsilon-\omega), \quad (4.1)$$

with $\Pi^{\chi}(\epsilon,\epsilon')=\Pi^{\chi}(\epsilon,\epsilon')^{RA}-\Pi^{\chi}(\epsilon,\epsilon')^{RR}$ where $\Pi^{\chi}(\epsilon,\epsilon')^{RA}$ and $\Pi^{\chi}(\epsilon,\epsilon')^{RR}$ stand for the corresponding current-current correlation functions in RA and RR channels, respectively. Throughout this paper we only consider the low-frequency region of $\omega\ll\gamma$. In the static case, Eq. (4.1) reduces to be

$$\lim_{\omega\to 0}\sigma^{\chi}(\omega)=\frac{1}{2\pi}\mathrm{Re}\Pi^{\chi}(0,0). \quad (4.2)$$

The Boltzmann dc conductivity corresponds to the contribution of "bare bubble" diagram, and the relevant correlation function in RA channel is given by

$$\Pi^{\chi}(0,0)_0^{RA}=\frac{1}{2}\sum_{\boldsymbol{k}}\mathrm{Tr}(\Lambda_{\boldsymbol{k}}^{\chi}G_{\boldsymbol{k}}^{R}\cdot\Lambda_{\boldsymbol{k}}^{\chi}G_{\boldsymbol{k}}^{A}), \quad (4.3)$$

where $\Lambda_{\boldsymbol{k}}^{e}=-e\boldsymbol{v}_f(\boldsymbol{k})\tau_0$ and $\boldsymbol{\Lambda}_{\boldsymbol{k}}^{s}=(1/2)[\boldsymbol{v}_g(\boldsymbol{k})\tau_1+\boldsymbol{v}_f(\boldsymbol{k})\tau_3]$ denote, respectively, the charge and spin current vectors.[22] The corresponding expression in RR channel can be easily obtained by a replacement of $A\to R$ in Eq. (4.3). The same replacement is also suitable for the correlation functions responsible for the QI effects. Hereafter we only provide the expressions for correlation functions in RA channel, but present the calculated results in RR channel where necessary. Substituting Eq. (2.2) into Eq. (4.3) and its counterpart in RR channel, and using Eqs. (A1)–(A4), one can easily show that $\Pi^{e}(0,0)_0^{RA}=-\Pi^{e}(0,0)_0^{RR}=e^2v_f/\pi v_g$, and $\Pi^{s}(0,0)_0^{RA}=-\Pi^{s}(0,0)_0^{RR}=(v_f^2+v_g^2)/4\pi v_f v_g$. Thus we obtain the universal conductivities as

$$\sigma_0^{e}=\frac{e^2v_f}{\pi^2v_g},\quad \sigma_0^{s}=\frac{v_f^2+v_g^2}{4\pi^2v_fv_g}. \quad (4.4)$$

Evidently, the spin conductivity satisfies the Einstein relation $\sigma_0^{s}=\rho_0D/4$, for the quasiparticle spin is a good quantum number. Equation (4.4) is exactly in agreement with those of Refs. 15 and 22, and was widely used by other authors. Here we wish to point out that the correlation functions in RR channel have nonvanishing contributions to the quasiparticle conductivities,[15,22] contrary to the situation of normal state. This is because the quasiparticle state is a mixture of an electron and a hole, while the carries of charge and spin in a normal conductor are either electrons or holes.

We now turn to the QI contributions to the dynamical conductivities. In generic situations, only the diffusive 0 modes contribute to the QI effects. All the lowest-order conductivity diagrams with cooperon and those with diffuson are depicted in Figs. 3 and 4, respectively. These diagrams can be generated from the self-energy diagrams, Figs. 1(c) and 1(d) in Ref. 27, as in the study of disordered interacting electron systems.[32,36] As shown in Appendix B, none of the diagrams in Fig. 4 contributes to the conductivities. Those conductivity diagrams containing the nonsingular ladders are shown to have also vanishing contributions (see Appendix

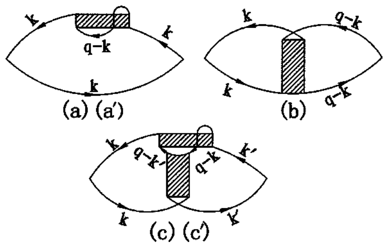

FIG. 3. Lowest-order conductivity diagrams with 0-mode cooperon (shaded blocks). Figures 3(a′) and 3(c′) denote, respectively, the symmetrical conjugates of Figs. 3(a) and 3(c), with the upper and lower quasiparticle lines interchanged.

D). Therefore the QI corrections to the conductivities come only from the cooperon contribution, as shown by diagrams in Fig. 3.

Figure 3(b) denotes a sum of the well-known maximally crossed diagrams.[31–33] Figure 3(a) was first proposed by Altland and Zirnbauer in the random-matrix theory of mesoscopic normal/superconducting systems,[34] its physical effects have also been studied by the nonlinear-sigma-model approach in disordered d-wave superconductors,[24] as well as in the mixed superconducting state.[25] Figure 3(c) represents a novel QI process; its physical picture has been described in Ref. 29 at the semiclassical level. As will be shown below, it is the existence of Fig. 3(c) that leads to a qualitative difference between the frequency dependences of the charge and spin conductivities.

A. Correlation functions due to 0-mode cooperon

The contributions of Figs. 3(a), 3(b), and 3(c) to the current-current correlation function in RA channel are expressed, respectively, by

$$\Pi^{\chi}(\epsilon,\epsilon')^{RA}_{3a}=\frac{1}{2}\sum_{\boldsymbol{kq}}\sum_{i} C(\boldsymbol{q};\epsilon,\epsilon)^{RR}_{ii} \times\mathrm{Tr}(\boldsymbol{\Lambda}^{\chi}_{\boldsymbol{k}}G^{R}_{\boldsymbol{k}}\tau_i G^{R}_{-\boldsymbol{k}}\tau_i G^{R}_{\boldsymbol{k}}\cdot\boldsymbol{\Lambda}^{\chi}_{\boldsymbol{k}}G^{A}_{\boldsymbol{k}}), \quad (4.5)$$

$$\Pi^{\chi}(\epsilon,\epsilon')^{RA}_{3b}=\frac{1}{2}\sum_{\boldsymbol{kq}}\sum_{i} C(\boldsymbol{q};\epsilon,\epsilon')^{RA}_{ii} \times\mathrm{Tr}(\boldsymbol{\Lambda}^{\chi}_{\boldsymbol{k}}G^{R}_{\boldsymbol{k}}\tau_i G^{R}_{-\boldsymbol{k}}\cdot\boldsymbol{\Lambda}^{\chi}_{-\boldsymbol{k}}G^{A}_{-\boldsymbol{k}}\tau_i G^{A}_{\boldsymbol{k}}), \quad (4.6)$$

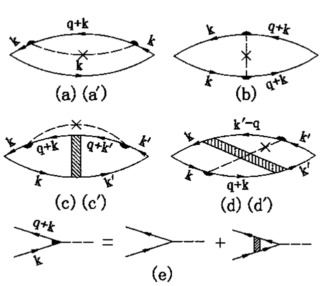

FIG. 4. Lowest-order conductivity diagrams with 0-mode diffuson (shaded blocks) 4(a)–4(d′), and the diagrams for the vertex function 4(e).

and

$$\Pi^{\chi}(\epsilon,\epsilon')^{RA}_{3c}=\frac{1}{2}\sum_{\boldsymbol{kk'q}}\sum_{ij} C(\boldsymbol{q};\epsilon,\epsilon)^{RR}_{ii}C(\boldsymbol{q};\epsilon,\epsilon')^{RA}_{jj} \times\mathrm{Tr}(\boldsymbol{\Lambda}^{\chi}_{\boldsymbol{k}}G^{R}_{\boldsymbol{k}}\tau_i G^{R}_{\boldsymbol{q}-\boldsymbol{k}'}\tau_j G^{R}_{\boldsymbol{q}-\boldsymbol{k}}\tau_i G^{R}_{\boldsymbol{k}'}\cdot\boldsymbol{\Lambda}^{\chi}_{\boldsymbol{k}'}G^{A}_{\boldsymbol{k}'}\tau_j G^{A}_{\boldsymbol{k}}). \quad (4.7)$$

Using Eqs. (A9) and (A10), and noting that $\Pi^{\chi}(\epsilon,\epsilon')_{3a}=\Pi^{\chi}(\epsilon,\epsilon')^{RA}_{3a}-\Pi^{\chi}(\epsilon,\epsilon')^{RR}_{3a}$, we can rewrite Eqs. (4.5)–(4.7) as

$$\Pi^{\chi}(\epsilon,\epsilon')_{3a}=\sum_{\boldsymbol{q}}\sum_{i}[\mathcal{C}(\boldsymbol{q};\epsilon,\epsilon)^{RR}(\mathcal{M}^{\chi})_{3a}]_{ii}, \quad (4.8)$$

$$\Pi^{\chi}(\epsilon,\epsilon')^{RA}_{3b}=\sum_{\boldsymbol{q}}\sum_{i}[\mathcal{C}(\boldsymbol{q};\epsilon,\epsilon')^{RA}(\mathcal{M}^{\chi})^{RA}_{3b}]_{ii}, \quad (4.9)$$

and

$$\Pi^{\chi}(\epsilon,\epsilon')^{RA}_{3c}=\sum_{\boldsymbol{q}}\sum_{i}[\mathcal{C}(\boldsymbol{q};\epsilon,\epsilon)^{RR}\boldsymbol{\mathcal{M}}^{\chi}(\boldsymbol{q})^{RA}_{3c}\cdot\mathcal{C}(\boldsymbol{q};\epsilon,\epsilon')^{RA} \times\tilde{\boldsymbol{\mathcal{M}}}^{\chi}(\boldsymbol{q})^{RA}_{3c}]_{ii}, \quad (4.10)$$

where

$$(\mathcal{M}^{\chi})_{3a}=\sum_{\boldsymbol{k}} G^{R}_{-\boldsymbol{k}}\otimes[G^{R}_{\boldsymbol{k}}\boldsymbol{\Lambda}^{\chi}_{\boldsymbol{k}}(G^{A}_{\boldsymbol{k}}-G^{R}_{\boldsymbol{k}})\cdot\boldsymbol{\Lambda}^{\chi}_{\boldsymbol{k}}G^{R}_{\boldsymbol{k}}], \quad (4.11)$$

$$(\mathcal{M}^{\chi})^{RA}_{3b}=\sum_{\boldsymbol{k}}(G^{R}_{-\boldsymbol{k}}\boldsymbol{\Lambda}^{\chi}_{-\boldsymbol{k}}G^{A}_{-\boldsymbol{k}})\otimes\cdot(G^{A}_{\boldsymbol{k}}\boldsymbol{\Lambda}^{\chi}_{\boldsymbol{k}}G^{R}_{\boldsymbol{k}}), \quad (4.12)$$

$$\boldsymbol{\mathcal{M}}^{\chi}(\boldsymbol{q})^{RA}_{3c}=\sum_{\boldsymbol{k}} G^{R}_{\boldsymbol{q}-\boldsymbol{k}}\otimes(G^{R}_{\boldsymbol{k}}\boldsymbol{\Lambda}^{\chi}_{\boldsymbol{k}}G^{A}_{\boldsymbol{k}}), \quad (4.13)$$

and

$$\tilde{\boldsymbol{\mathcal{M}}}^{\chi}(\boldsymbol{q})^{RA}_{3c}=\sum_{\boldsymbol{k}} G^{R}_{\boldsymbol{q}-\boldsymbol{k}}\otimes(G^{A}_{\boldsymbol{k}}\boldsymbol{\Lambda}^{\chi}_{\boldsymbol{k}}G^{R}_{\boldsymbol{k}}). \quad (4.14)$$

For the asymmetrical diagrams ξ and ξ' with $\xi=3a,3c,4a,4c,4d$, one can readily show that $\mathrm{Re}\Pi^{\chi}(\epsilon,\epsilon')_{\xi'}=\mathrm{Re}\Pi^{\chi}(\epsilon',\epsilon)_{\xi}$, so that

$$\sigma^{\chi}(\omega)_{\xi'}=\sigma^{\chi}(-\omega)_{\xi}. \quad (4.15)$$

Equations (4.5)–(4.15) are general expressions, and will be used to calculate both the charge and spin conductivities.

B. QI correction to the charge conductivity

Let us first calculate the charge conductivity. Substituting the electrical-current vector $\boldsymbol{\Lambda}^{e}_{\boldsymbol{k}}=-e\boldsymbol{v}_f(\boldsymbol{k})\tau_0$ into Eqs. (4.11)–(4.14), and noting that the vector $\boldsymbol{k}$ in these equations is restricted in the vicinity of the four gap nodes, we obtain

$$(\mathcal{M}^{e})_{3a}=e^2v_f^2\sum_{\boldsymbol{k}} G^{R}_{\boldsymbol{k}}\otimes[G^{R}_{\boldsymbol{k}}(G^{A}_{\boldsymbol{k}}-G^{R}_{\boldsymbol{k}})G^{R}_{\boldsymbol{k}}], \quad (4.16)$$

$$(\mathcal{M}^e)_{3b}^{RA} = -e^2 v_f^2 \sum_{\boldsymbol{k}} (G_{\boldsymbol{k}}^R G_{\boldsymbol{k}}^A) \otimes (G_{\boldsymbol{k}}^A G_{\boldsymbol{k}}^R), \tag{4.17}$$

$$\boldsymbol{\mathcal{M}}^e(\boldsymbol{q})_{3c}^{RA} = e \sum_{\boldsymbol{k}} \boldsymbol{v}_f(\boldsymbol{k}) \boldsymbol{q} \cdot \nabla G_{\boldsymbol{k}}^R \otimes (G_{\boldsymbol{k}}^R G_{\boldsymbol{k}}^A), \tag{4.18}$$

and

$$\tilde{\boldsymbol{\mathcal{M}}}^e(\boldsymbol{q})_{3c}^{RA} = e \sum_{\boldsymbol{k}} \boldsymbol{v}_f(\boldsymbol{k}) \boldsymbol{q} \cdot \nabla G_{\boldsymbol{k}}^R \otimes (G_{\boldsymbol{k}}^A G_{\boldsymbol{k}}^R). \tag{4.19}$$

Substituting Eq. (2.2) into Eqs. (4.16)–(4.19), and using Eqs. (A1)–(A8), we find that

$$(\mathcal{M}^e)_{3b}^{RA} = -\frac{e^2 v_f}{\pi \gamma^2 v_g} \tau_0 \otimes \tau_0, \tag{4.20}$$

$$(\mathcal{M}^e)_{3b}^{RR} = (\mathcal{M}^e)_{3a} = -\frac{e^2 v_f}{3\pi \gamma^2 v_g} (\tau_0 \otimes \tau_0 - \tau_1 \otimes \tau_1 - \tau_3 \otimes \tau_3), \tag{4.21}$$

$$\boldsymbol{\mathcal{M}}^e(\boldsymbol{q})_{3c}^{RA} = \tilde{\boldsymbol{\mathcal{M}}}^e(\boldsymbol{q})_{3c}^{RA} = -\frac{e}{4\pi \gamma^2 v_g} (\hat{\boldsymbol{q}} v_g \tau_1 \otimes \tau_0 + \boldsymbol{q} v_f \tau_3 \otimes \tau_0), \tag{4.22}$$

and

$$\boldsymbol{\mathcal{M}}^e(\boldsymbol{q})_{3c}^{RR} = \tilde{\boldsymbol{\mathcal{M}}}^e(\boldsymbol{q})_{3c}^{RR} = \frac{e}{12\pi \gamma^2 v_g} [\hat{\boldsymbol{q}} v_g (\tau_1 \otimes \tau_0 - 2\tau_0 \otimes \tau_1) + \boldsymbol{q} v_f (\tau_3 \otimes \tau_0 - 2\tau_0 \otimes \tau_3)], \tag{4.23}$$

with $\hat{\boldsymbol{q}} = (\boldsymbol{q} \cdot \boldsymbol{f}_1)\boldsymbol{g}_1 + (\boldsymbol{q} \cdot \boldsymbol{g}_1)\boldsymbol{f}_1$. Substituting Eqs. (3.32) and (4.20)–(4.23) into Eqs. (4.8)–(4.10), as well as into the counterparts of Eqs. (4.9) and (4.10) in RR channel, we obtain the electrical current-current correlation functions as

$$\Pi^e(\epsilon, \epsilon')_{3a} = -\frac{4e^2 v_f}{\alpha v_g} \sum_{\boldsymbol{q}} \frac{D}{Dq^2 - i2\epsilon}, \tag{4.24}$$

$$\Pi^e(\epsilon, \epsilon')_{3b} = -\frac{4e^2 v_f}{\alpha v_g} \sum_{\boldsymbol{q}} \left[\frac{D}{Dq^2 - i(\epsilon - \epsilon')} - \frac{D}{Dq^2 - i(\epsilon + \epsilon')} \right], \tag{4.25}$$

and

$$\Pi^e(\epsilon, \epsilon')_{3c} = 0, \tag{4.26}$$

with $\alpha = (v_f^2 + v_g^2)/2v_f v_g$. The first and second terms in the right-hand side of Eq. (4.25) correspond to the contributions in RA and RR channels, respectively. Equation (4.26) indicates that Fig. 3(c) does not contribute to the charge conductivity.

The upper cutoff of $|\boldsymbol{q}|$ in the above equations is set to be $1/l_e$, with $l_e = \sqrt{D/2\gamma}$ the elastic mean free path. Substituting Eqs. (4.24)–(4.26) into Eq. (4.1), and completing the integrals over $\boldsymbol{q}$ and ϵ, we obtain the contributions of diagrams in Fig. 3 to the charge conductivity as

$$\sigma^e(\omega)_{3a} = \sigma^e(\omega)_{3a'} = -\frac{e^2 v_f}{2\pi^2 \alpha v_g} \ln \frac{\gamma}{|\omega|} \tag{4.27}$$

and

$$\sigma^e(\omega)_{3b} = \sigma^e(\omega)_{3c} = \sigma^e(\omega)_{3c'} = 0. \tag{4.28}$$

Here we have neglected the nonsingular terms of the order σ_0^e. The above evaluations show that Fig. 3(b) has also a vanishing contribution to the charge conductivity, due to the cancellation of contributions from RA and RR channels. This feature is considerably different from that of a disordered normal metal, in which the QI effect results just from the maximally crossed diagrams.[31–33] The total QI correction is thus the sum of $\sigma^e(\omega)_{3a}$ and $\sigma^e(\omega)_{3a'}$, given by

$$\frac{\Delta \sigma^e(\omega)}{\sigma_0^e} = -\frac{1}{\alpha} \ln \frac{\gamma}{|\omega|}. \tag{4.29}$$

The weak-localization correction shown by Eq. (4.29) implies that the charge conductivity is suppressed with decreasing frequency. This result is qualitatively in agreement with the numerical study,[30] as well as supported by the experimental observations of the cuprates.[8,9]

C. QI correction to the spin conductivity

The QI contribution to the dynamical spin conductivity can be similarly evaluated by the approach for the charge conductivity. The explicit expressions of Eqs. (4.11)–(4.14) for the spin conductivity are given by

$$(\mathcal{M}^s)_{3a} = \frac{1}{4} v_g^2 \sum_{\boldsymbol{k}} G_{\boldsymbol{k}}^R \otimes [G_{\boldsymbol{k}}^R \tau_1 (G_{\boldsymbol{k}}^A - G_{\boldsymbol{k}}^R) \tau_1 G_{\boldsymbol{k}}^R] + \frac{1}{4} v_f^2 \sum_{\boldsymbol{k}} G_{\boldsymbol{k}}^R \otimes [G_{\boldsymbol{k}}^R \tau_3 (G_{\boldsymbol{k}}^A - G_{\boldsymbol{k}}^R) \tau_3 G_{\boldsymbol{k}}^R], \tag{4.30}$$

$$(\mathcal{M}^s)_{3b}^{RA} = -\frac{1}{4} v_g^2 \sum_{\boldsymbol{k}} (G_{\boldsymbol{k}}^R \tau_1 G_{\boldsymbol{k}}^A) \otimes (G_{\boldsymbol{k}}^A \tau_1 G_{\boldsymbol{k}}^R) - \frac{1}{4} v_f^2 \sum_{\boldsymbol{k}} (G_{\boldsymbol{k}}^R \tau_3 G_{\boldsymbol{k}}^A) \otimes (G_{\boldsymbol{k}}^A \tau_3 G_{\boldsymbol{k}}^R), \tag{4.31}$$

$$\boldsymbol{\mathcal{M}}^s(\boldsymbol{q})_{3c}^{RA} = -\frac{1}{2} \sum_{\mathbf{k}} \boldsymbol{v}_g(\boldsymbol{k}) \boldsymbol{q} \cdot \nabla G_{\boldsymbol{k}}^R \otimes (G_{\boldsymbol{k}}^R \tau_1 G_{\boldsymbol{k}}^A) - \frac{1}{2} \sum_{\mathbf{k}} \boldsymbol{v}_f(\boldsymbol{k}) \boldsymbol{q} \cdot \nabla G_{\boldsymbol{k}}^R \otimes (G_{\boldsymbol{k}}^R \tau_3 G_{\boldsymbol{k}}^A), \tag{4.32}$$

and

$$\tilde{\boldsymbol{\mathcal{M}}}^s(\boldsymbol{q})^{RA}_{3c} = -\frac{1}{2}\sum_{\boldsymbol{k}} \boldsymbol{v}_g(\boldsymbol{k})\boldsymbol{q}\cdot\nabla G^R_{\boldsymbol{k}}\otimes(G^A_{\boldsymbol{k}}\tau_1 G^R_{\boldsymbol{k}}) - \frac{1}{2}\sum_{\boldsymbol{k}} \boldsymbol{v}_f(\boldsymbol{k})\boldsymbol{q}\cdot\nabla G^R_{\boldsymbol{k}}\otimes(G^A_{\boldsymbol{k}}\tau_3 G^R_{\boldsymbol{k}}). \tag{4.33}$$

By completing the summations over $\boldsymbol{k}$ in the above equations, one obtains

$$(\mathcal{M}^s)_{3a} = -\frac{\alpha}{6\pi\gamma^2}(\tau_0\otimes\tau_0 - \tau_1\otimes\tau_1 - \tau_3\otimes\tau_3), \tag{4.34}$$

$$(\mathcal{M}^s)^{RA}_{3b} = -\frac{1}{12\pi\gamma^2}[(2\alpha-\beta)\tau_1\otimes\tau_1 - 2\alpha\tau_2\otimes\tau_2 + (2\alpha+\beta)\tau_3\otimes\tau_3], \tag{4.35}$$

$$(\mathcal{M}^s)^{RR}_{3b} = -\frac{1}{12\pi\gamma^2}[(2\alpha-\beta)\tau_1\otimes\tau_1 - 2\alpha\tau_0\otimes\tau_0 + (2\alpha+\beta)\tau_3\otimes\tau_3], \tag{4.36}$$

$$\boldsymbol{\mathcal{M}}^s(\boldsymbol{q})^{RA}_{3c} = \tilde{\boldsymbol{\mathcal{M}}}^s(\boldsymbol{q})^{RA}_{3c} = -\frac{1}{12\pi\gamma^2}[\boldsymbol{q}\beta(\tau_1\otimes\tau_1 - \tau_3\otimes\tau_3) - \hat{\boldsymbol{q}}(\tau_1\otimes\tau_3 + \tau_3\otimes\tau_1)], \tag{4.37}$$

and

$$\boldsymbol{\mathcal{M}}^s(\boldsymbol{q})^{RR}_{3c} = \tilde{\boldsymbol{\mathcal{M}}}^s(\boldsymbol{q})^{RR}_{3c} = \frac{1}{12\pi\gamma^2}\{\boldsymbol{q}[2\alpha\tau_0\otimes\tau_0 - (2\alpha-\beta)\tau_1\otimes\tau_1 - (2\alpha+\beta)\tau_3\otimes\tau_3] - \hat{\boldsymbol{q}}(\tau_1\otimes\tau_3 + \tau_3\otimes\tau_1)\}, \tag{4.38}$$

with $\beta = (v_f^2 - v_g^2)/2v_f v_g$. A substitution of Eqs. (3.32) and (4.34)–(4.38) into Eqs. (4.8)–(4.10) leads to the spin current-current correlation functions as

$$\Pi^s(\epsilon,\epsilon')_{3a} = -\sum_{\boldsymbol{q}} \frac{2D}{Dq^2 - i2\epsilon}, \tag{4.39}$$

$$\Pi^s(\epsilon,\epsilon')_{3b} = -\sum_{\boldsymbol{q}} \left[\frac{2D}{Dq^2 - i(\epsilon-\epsilon')} + \frac{2D}{Dq^2 - i(\epsilon+\epsilon')}\right], \tag{4.40}$$

and

$$\Pi^s(\epsilon,\epsilon')_{3c} = \sum_{\boldsymbol{q}} \frac{8D^2q^2}{(Dq^2 - i2\epsilon)[Dq^2 - i(\epsilon+\epsilon')]}. \tag{4.41}$$

Substituting Eqs. (4.39)–(4.41) into Eq. (4.1), and completing the integrals over $\boldsymbol{q}$ and ϵ, we obtain the contributions of diagrams in Fig. 3 to the spin conductivity as

$$4\sigma^s(\omega)_{3a} = 4\sigma^s(\omega)_{3a'} = 2\sigma^s(\omega)_{3b} = -\sigma^s(\omega)_{3c} = -\sigma^s(\omega)_{3c'} = -\frac{1}{\pi^2}\ln\frac{\gamma}{|\omega|}. \tag{4.42}$$

Quite different from the case of charge conductivity, both Figs. 3(b) and 3(c) have nontrivial contributions to the spin conductivity. While Figs. 3(a) and 3(b) give rise to negative corrections to the spin conductivity, Fig. 3(c) yields a *positive* one. By summing up the contributions of all the diagrams in Fig. 3, we obtain the total QI correction to the spin conductivity as

$$\frac{\Delta\sigma^s(\omega)}{\sigma_0^s} = \frac{2}{\alpha}\ln\frac{\gamma}{|\omega|}. \tag{4.43}$$

Contrary to the charge conductivity, the spin conductivity is subject to a *weak-antilocalization* correction, due to the fact that the positive contribution from Fig. 3(c) exceeds the sum of the negative ones from Figs. 3(a) and 3(b). Clearly, the qualitative difference in the frequency dependence between these two conductivities stems from the existence of the QI process described by Fig. 3(c).

V. DYNAMICAL CONDUCTIVITIES NEAR THE UN LIMIT

Since the d-wave superconductor is fundamentally sensitive to the details of disorder, as well as to certain symmetries of the normal-state Hamiltonian,[2] it is worthy to investigate the QI effects near the UN limit. In addition to the 0-mode cooperon, the diffusive π modes may also contribute to the QI effects. All the lowest-order conductivity diagrams with the diffusive π modes can be readily obtained by replacing all $\boldsymbol{q}$ by $\boldsymbol{Q}+\boldsymbol{q}$ in Figs. 3 and 4. In Appendix C we show that the contributions of those diagrams with π-mode diffuson are summed to vanish. As a result, the total QI correction to the charge or spin conductivity near the UN limit is a sum of the contributions from 0-mode and π-mode cooperons, i.e.,

$$\Delta\sigma^\chi_{\mathrm{UN}}(\omega) = \Delta\sigma^\chi(\omega) + \Delta\sigma^\chi_\pi(\omega). \tag{5.1}$$

Here $\Delta\sigma^\chi_\pi(\omega)$ denotes the contribution of π-mode cooperon to the conductivity, which can be similarly evaluated by the approach used in the case of 0-mode cooperon.

A. Correlation functions due to π-mode cooperon

Equations (4.8)–(4.10) are also suitable for the case of π-mode cooperon, provided that one makes the replacements of $\mathcal{C}(\boldsymbol{q};\epsilon,\epsilon')^{RR(A)} \to \mathcal{C}_\pi(\boldsymbol{q};\epsilon,\epsilon')^{RR(A)}$, $(\mathcal{M})^\chi_{3a} \to (\mathcal{M}_\pi)^\chi_{3a}$, etc., with

$$(\mathcal{M}^{\chi}_{\pi})_{3a}=\sum_{\boldsymbol{k}} G^R_{\boldsymbol{Q}-\boldsymbol{k}}\otimes[G^R_{\boldsymbol{k}}\boldsymbol{\Lambda}^{\chi}_{\boldsymbol{k}}(G^A_{\boldsymbol{k}}-G^R_{\boldsymbol{k}})\cdot\boldsymbol{\Lambda}^{\chi}_{\boldsymbol{k}}G^R_{\boldsymbol{k}}], \tag{5.2}$$

$$(\mathcal{M}^{\chi}_{\pi})^{RA}_{3b}=\sum_{\boldsymbol{k}} (G^R_{\boldsymbol{Q}-\boldsymbol{k}}\boldsymbol{\Lambda}^{\chi}_{\boldsymbol{Q}-\boldsymbol{k}}G^A_{\boldsymbol{Q}-\boldsymbol{k}})\otimes\cdot(G^A_{\boldsymbol{k}}\boldsymbol{\Lambda}^{\chi}_{\boldsymbol{k}}G^R_{\boldsymbol{k}}), \tag{5.3}$$

$$\boldsymbol{\mathcal{M}}^{\chi}_{\pi}(\boldsymbol{q})^{RA}_{3c}=\sum_{\boldsymbol{k}} G^R_{\boldsymbol{q}+\boldsymbol{Q}-\boldsymbol{k}}\otimes(G^R_{\boldsymbol{k}}\boldsymbol{\Lambda}^{\chi}_{\boldsymbol{k}}G^A_{\boldsymbol{k}}), \tag{5.4}$$

and

$$\tilde{\boldsymbol{\mathcal{M}}}^{\chi}_{\pi}(\boldsymbol{q})^{RA}_{3c}=\sum_{\boldsymbol{k}} G^R_{\boldsymbol{q}+\boldsymbol{Q}-\boldsymbol{k}}\otimes(G^A_{\boldsymbol{k}}\boldsymbol{\Lambda}^{\chi}_{\boldsymbol{k}}G^R_{\boldsymbol{k}}). \tag{5.5}$$

By comparing Eqs. (4.11)–(4.14) with Eqs. (5.2)–(5.5), we see that the results of the summations over $\boldsymbol{k}$ in the above equations can be obtained by a transformation from the results in the case of 0-mode cooperon, due to the the global particle-hole symmetry.

B. QI correction to the charge conductivity

For the charge conductivity, we have $\boldsymbol{\Lambda}^e_{\boldsymbol{Q}+\boldsymbol{k}}=-\boldsymbol{\Lambda}^e_{\boldsymbol{k}}=-\tau_2\boldsymbol{\Lambda}^e_{\boldsymbol{k}}\tau_2$ in the nesting case. Substituting this relation and Eq. (3.33) into Eqs. (5.2)–(5.5), and taking into account Eqs. (4.20)–(4.23), one can easily show that

$$(\mathcal{M}^{e}_{\pi})^{RA}_{3b}=\frac{e^2v_f}{\pi\gamma^2v_g}\tau_0\otimes\tau_0, \tag{5.6}$$

$$(\mathcal{M}^{e}_{\pi})^{RR}_{3b}=-(\mathcal{M}^{e}_{\pi})_{3a}=\frac{e^2v_f}{3\pi\gamma^2v_g}(\tau_0\otimes\tau_0+\tau_1\otimes\tau_1+\tau_3\otimes\tau_3), \tag{5.7}$$

$$\boldsymbol{\mathcal{M}}^{e}_{\pi}(\boldsymbol{q})^{RA}_{3c}=\tilde{\boldsymbol{\mathcal{M}}}^{e}_{\pi}(\boldsymbol{q})^{RA}_{3c}=\frac{e}{4\pi\gamma^2v_g}(\hat{\boldsymbol{q}}v_g\tau_1\otimes\tau_0+\boldsymbol{q}v_f\tau_3\otimes\tau_0), \tag{5.8}$$

and

$$\boldsymbol{\mathcal{M}}^{e}_{\pi}(\boldsymbol{q})^{RR}_{3c}=\tilde{\boldsymbol{\mathcal{M}}}^{e}_{\pi}(\boldsymbol{q})^{RR}_{3c}=-\frac{e}{12\pi\gamma^2v_g}[\hat{\boldsymbol{q}}v_g(\tau_1\otimes\tau_0+2\tau_0\otimes\tau_1)+\boldsymbol{q}v_f(\tau_3\otimes\tau_0+2\tau_0\otimes\tau_3)]. \tag{5.9}$$

A substitution of Eqs. (3.46) and (5.6)–(5.9) into Eqs. (4.8)–(4.10) yields the electrical current-current correlation functions due to π-mode cooperon as

$$\Pi^e_{\pi}(\epsilon,\epsilon')_{3a}=\frac{4e^2v_f}{\alpha v_g}\sum_{\boldsymbol{q}}\frac{D}{Dq^2-i2\epsilon+2\delta}, \tag{5.10}$$

$$\Pi^e_{\pi}(\epsilon,\epsilon')_{3b}=-\frac{4e^2v_f}{\alpha v_g}\sum_{\boldsymbol{q}}\left[\frac{D}{Dq^2-i(\epsilon-\epsilon')+2\delta}-\frac{D}{Dq^2-i(\epsilon+\epsilon')+2\delta}\right], \tag{5.11}$$

and

$$\Pi^e_{\pi}(\epsilon,\epsilon')_{3c}=0. \tag{5.12}$$

Substituting Eqs. (5.10)–(5.12) into Eq. (4.1), we get the contributions of the diagrams with π-mode cooperon to the charge conductivity as

$$\sigma^e_{\pi}(\omega)_{3a}=\sigma^e_{\pi}(\omega)_{3a'}=\frac{e^2v_f}{2\pi^2\alpha v_g}\ln\frac{\gamma}{\sqrt{\omega^2+\delta^2}} \tag{5.13}$$

and

$$\sigma^e_{\pi}(\omega)_{3b}=\sigma^e_{\pi}(\omega)_{3c}=\sigma^e_{\pi}(\omega)_{3c'}=0. \tag{5.14}$$

Same as in the case of 0-mode cooperon, Figs. 3(b) and 3(c) for the π-mode cooperon have also vanishing contributions to the charge conductivity. Thus, the contribution of π-mode cooperon is the sum of $\sigma^e_{\pi}(\omega)_{3a}$ and $\sigma^e_{\pi}(\omega)_{3a'}$, yielding

$$\frac{\Delta\sigma^e_{\pi}(\omega)}{\sigma^e_0}=\frac{1}{\alpha}\ln\frac{\gamma}{\sqrt{\omega^2+\delta^2}}. \tag{5.15}$$

Contrary to the 0-mode cooperon, the π-mode cooperon yields a *positive* correction to the charge conductivity. This is because the phase differences of coherent paths corresponding to 0-mode cooperon differ by π from those to π-mode cooperon. In addition, while $\Delta\sigma^e(\omega)$ has a logarithmic singularity at zero frequency [Eq. (4.29)], the same singularity for $\Delta\sigma^e_{\pi}(\omega)$ is cut off by the gap δ, as indicated by Eq. (5.15). If the deviation from the UN limit is large enough so that $\delta\sim\gamma$, the diffusion poles of the π modes are sufficiently killed out, and the contribution of π-mode cooperon is significantly suppressed. By summing up Eqs. (4.29) and (5.15), we obtain the total QI correction to the charge conductivity near the UN limit as

$$\frac{\Delta\sigma^e_{\mathrm{UN}}(\omega)}{\sigma^e_0}=-\frac{1}{2\alpha}\ln\left(1+\frac{\delta^2}{\omega^2}\right). \tag{5.16}$$

Equation (5.16) indicates that the weak-localization correction contributed by 0-mode cooperon to the charge conductivity is suppressed due to the existence of π-mode cooperon. At the UN limit ($\delta=0$), the contributions of 0-mode and π-mode cooperons just cancel out, $\Delta\sigma^e_{\mathrm{UN}}(\omega)=0$, and thus the charge conductivity remains its universal value σ^e_0.

C. QI correction to the spin conductivity

Similarly, substituting Eq. (3.33) and the relation $\boldsymbol{\Lambda}^s_{\boldsymbol{Q}+\boldsymbol{k}}=-\boldsymbol{\Lambda}^s_{\boldsymbol{k}}=\tau_2\boldsymbol{\Lambda}^s_{\boldsymbol{k}}\tau_2$ into Eqs. (5.2)–(5.5), and using Eqs. (4.34)–(4.38), we immediately obtain

$$(\mathcal{M}^{s}_{\pi})_{3a}=-\frac{\alpha}{6\pi\gamma^2}(\tau_0\otimes\tau_0+\tau_1\otimes\tau_1+\tau_3\otimes\tau_3), \tag{5.17}$$

$$(\mathcal{M}^{s}_{\pi})^{RA}_{3b}=\frac{1}{12\pi\gamma^2}[(2\alpha-\beta)\tau_1\otimes\tau_1+2\alpha\tau_2\otimes\tau_2+(2\alpha+\beta)\tau_3\otimes\tau_3], \tag{5.18}$$

$$(\mathcal{M}^{s}_{\pi})^{RR}_{3b}=\frac{1}{12\pi\gamma^2}[(2\alpha-\beta)\tau_1\otimes\tau_1+2\alpha\tau_0\otimes\tau_0+(2\alpha+\beta)\tau_3\otimes\tau_3], \tag{5.19}$$

$$\boldsymbol{\mathcal{M}}^{s}_{\pi}(\boldsymbol{q})^{RA}_{3c}=\tilde{\boldsymbol{\mathcal{M}}}^{s}_{\pi}(\boldsymbol{q})^{RA}_{3c}=\frac{1}{12\pi\gamma^2}[\boldsymbol{q}\beta(\tau_1\otimes\tau_1-\tau_3\otimes\tau_3)-\hat{\boldsymbol{q}}(\tau_1\otimes\tau_3+\tau_3\otimes\tau_1)], \tag{5.20}$$

and

$$\boldsymbol{\mathcal{M}}^{s}_{\pi}(\boldsymbol{q})^{RR}_{3c}=\tilde{\boldsymbol{\mathcal{M}}}^{s}_{\pi}(\boldsymbol{q})^{RR}_{3c}=\frac{1}{12\pi\gamma^2}\{\boldsymbol{q}[2\alpha\tau_0\otimes\tau_0+(2\alpha-\beta)\tau_1\otimes\tau_1+(2\alpha+\beta)\tau_3\otimes\tau_3]+\hat{\boldsymbol{q}}(\tau_1\otimes\tau_3+\tau_3\otimes\tau_1)\}. \tag{5.21}$$

A substitution of Eqs. (3.46) and (5.17)–(5.21) into Eqs. (4.8)–(4.10) leads to the spin current-current correlation functions for π-mode cooperon as

$$\Pi^{s}_{\pi}(\epsilon,\epsilon')_{3a}=\sum_{\boldsymbol{q}}\frac{2D}{Dq^2-i2\epsilon+2\delta}, \tag{5.22}$$

$$\Pi^{s}_{\pi}(\epsilon,\epsilon')_{3b}=\sum_{\boldsymbol{q}}\left[\frac{2D}{Dq^2-i(\epsilon-\epsilon')+2\delta}+\frac{2D}{Dq^2-i(\epsilon+\epsilon')+2\delta}\right], \tag{5.23}$$

and

$$\Pi^{s}_{\pi}(\epsilon,\epsilon')_{3c}=-\sum_{\boldsymbol{q}}\frac{8D^2q^2}{(Dq^2-i2\epsilon+2\delta)[Dq^2-i(\epsilon+\epsilon')+2\delta]}. \tag{5.24}$$

Substituting Eqs. (5.22)–(5.24) into Eq. (4.1), and completing the integrals over $\boldsymbol{q}$ and ϵ, we obtain the contributions of the diagrams with π-mode cooperon to the spin conductivity as

$$4\sigma^{s}_{\pi}(\omega)_{3a}=4\sigma^{s}_{\pi}(\omega)_{3a'}=2\sigma^{s}_{\pi}(\omega)_{3b}=-\sigma^{s}_{\pi}(\omega)_{3c}=-\sigma^{s}_{\pi}(\omega)_{3c'}=\frac{1}{\pi^2}\ln\frac{\gamma}{\sqrt{\omega^2+\delta^2}}. \tag{5.25}$$

Comparing Eq. (4.42) with Eq. (5.25), one finds that the contributions of the 0-mode and π-mode cooperons have opposite signs for each diagram in Fig. 3. The contribution of π-mode cooperon to the spin conductivity is the sum of all contributions of diagrams in Fig. 3, given by

$$\frac{\Delta\sigma^{s}_{\pi}(\omega)}{\sigma^{s}_{0}}=-\frac{2}{\alpha}\ln\frac{\gamma}{\sqrt{\omega^2+\delta^2}}. \tag{5.26}$$

By summing up Eqs. (4.43) and (5.26), we obtain the total QI correction to the spin conductivity near the UN limit as

$$\frac{\Delta\sigma^{s}_{\mathrm{UN}}(\omega)}{\sigma^{s}_{0}}=\frac{1}{\alpha}\ln\left(1+\frac{\delta^2}{\omega^2}\right). \tag{5.27}$$

In the UN limit ($\delta\to 0$), we get $\Delta\sigma^{s}_{\mathrm{UN}}=0$, meaning that the spin conductivity also approaches its universal value σ^{s}_{0}. This result is in agreement with the numerical result in the weak-disorder limit.[37]

VI. SUMMARY

Within the weak-localization theory, we have calculated the QI contributions to the quasiparticle dynamical conductivities in a weakly disordered *d*-wave superconductor near the Born or unitary limit. By neglecting the quasiparticle interactions, we regard the random Dirac fermions in cuprate superconductors as a disordered nodal gas. The intrinsic particle-hole symmetry is generally preserved in the superconducting state, while the additional global particle-hole symmetry appears only for the nested Fermi surface. In the singlet *d*-wave superconductor under consideration, the spin is conserved but the charge is not. All these characteristic features make the QI effects of such a nodal gas quite different from those of a disordered normal conductor.

In generic situations, the QI effects result only from the contributions of 0-mode cooperon. The charge conductivity is shown to be subject to a logarithmic suppression at low frequency, which is qualitatively in agreement with the numerical study,[30] and supported by the experimental observations of the disordered cuprate superconductors.[8,9] Here the weak-localization effect on charge arises only from the QI process described by Fig. 3(a), and thus remains persistent in the presence of a weak magnetic field.[24,25] Contrarily, the usual weak-localization effect in a normal conductor is substantially suppressed by a magnetic field through the orbital coupling, as it stems from the contribution of the maximally crossed diagrams, Fig. 3(b).[31–33]

Opposite to the charge conductivity, the spin conductivity is found to increase with decreasing frequency. As shown above, all diagrams in Fig. 3 have nontrivial contributions to the QI correction of the spin conductivity. Figures 3(a) and 3(b) correspond, respectively, to a suppression of forward scattering and to an enhancement of backscattering of the quasiparticles, hence both of them lead to negative corrections to the spin conductivity. The coherent result of Fig. 3(c) includes an enhancement of forward scattering and a suppression of backscattering, yielding a positive correction.[29] This positive contribution is shown to prevail over the sum of the negative ones. Therefore, it is the existence of the additional QI process, Fig. 3(c), that leads to the *qualitative* difference in the frequency dependence between the charge and spin conductivities. Since the quasiparticle spin is a good quantum number, such an antilocalization correction to the spin conductivity immediately signals the existence of extended low-lying quasiparticle states. It is worthy to point out that, like the usual weak-localization theory, the present evaluations are valid only for the weak-disorder case, as it is based on the SCTMA. Therefore we do not rule out the possibility of localized quasiparticle states at higher impurity concentration.

In the UN (unitary and nesting) limit, neither 0-mode nor π-mode diffuson has contributions to the conductivities. At the same time, the QI corrections from the 0-mode cooperon are just canceled out by the additional contributions of π-mode cooperon, both for the charge and for the spin conductivities. As a result, these two conductivities approach their universal values in this limit.[38] The same cancellation law has been found in the QI effect on the quasiparticle DOS.[27,30] It seems that the UN-limit cancellation of the contributions from the 0-mode and π-mode cooperons is a general feature for the disordered *d*-wave superconductor. However, such a rule is not valid for the diffusons in all senses, even though in the present case either the 0-mode or π-mode diffuson does not contribute to the conductivities. For example, the 0-mode diffuson has no contribution to the QI correction of the DOS, but the π mode was shown to produce an enhanced zero-energy quasiparticle DOS.[27] In addition, the diffusons have been shown to play important roles in quasiparticle interaction effects in disordered *d*-wave superconductors.[26]

The problem whether or not the low-energy quasiparticle states are localized in a disordered *d*-wave superconductor still remains controversial. Balatsky *et al.* have shown that a single strong impurity produces a virtual-bound state at zero energy, and the long-range overlaps between these impurity states can yield an extended quasiparticle band.[39] The possibilities of critical states[40] and localization-delocalization transitions[41] in random Dirac fermions have been also discussed in the literature. We note that the measurements on the thermal conductivities in optimally-doped $YBa_2Cu_3O_{6.9}$ (Ref. 10) and $Bi_2Sr_2CaCu_2O_8$ (Ref. 12) do not show any localization-induced suppression down to 0.1 K. On the other hand, the nonlinear-sigma-model approaches[24,28] predicted a localization correction to the spin conductivity. However, the disorder models used in Refs. 24 and 28 are different from the present binary alloy model. While the uncorrelated zero-mean local Gaussian fields were used in Ref. 24, the authors of Ref. 28 treated the hopping matrix elements between nearest-neighbor sites as independent random variables. It turns out that different disorder models may lead to various theoretical predictions for the quasiparticle transport coefficients due to the anisotropy of the order parameter in *d*-wave superconductors. A similar situation appears in the studies of the quasiparticle DOS.[2] The numerical study in Ref. 42 has shown that the binary alloy and random site energy disorder models yield qualitatively different predictions for the low-energy DOS in the *d*-wave superconductor.

Finally, we wish to point out that the QI process, described by Fig. 3(c), also exists in superconductors that belong to symmetry classes *C* and *D* in the classification of Ref. 34, for the cooperon in RR channel is not influenced by the time-reversal breaking.[24,25,34] The QI effects contributed by this process on the quasiparticle transport in the mixed superconducting state deserve further investigation. How the spin-orbit coupling or magnetic impurities affects this QI process is also an interesting and open problem.

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APPENDIX A: SOME USEFUL MATHEMATICAL FORMULAS

In this appendix we present some useful mathematical formulas.

(i) Making use of the Dirac-type quasiparticle spectrum, we can show that

$$\sum_{\mathbf{k}} \frac{\xi_{\mathbf{k}}^2 \Delta_{\mathbf{k}}^2}{(\gamma^2+\epsilon_{\mathbf{k}}^2)^4} = \frac{1}{24\pi v_f v_g \gamma^2}, \tag{A1}$$

$$\sum_{\mathbf{k}} \frac{\xi_{\mathbf{k}}^4}{(\gamma^2+\epsilon_{\mathbf{k}}^2)^4} = \sum_{\mathbf{k}} \frac{\Delta_{\mathbf{k}}^4}{(\gamma^2+\epsilon_{\mathbf{k}}^2)^4} = \frac{1}{8\pi v_f v_g \gamma^2}, \tag{A2}$$

$$\sum_{\mathbf{k}} \frac{1}{(\gamma^2+\epsilon_{\mathbf{k}}^2)^n} = \frac{1}{(n-1)\pi v_f v_g \gamma^{2(n-1)}} \quad \text{for } n \geqslant 2, \tag{A3}$$

and

$$\sum_{\mathbf{k}} \frac{\xi_{\mathbf{k}}^2}{(\gamma^2+\epsilon_{\mathbf{k}}^2)^n} = \sum_{\mathbf{k}} \frac{\Delta_{\mathbf{k}}^2}{(\gamma^2+\epsilon_{\mathbf{k}}^2)^n} = \frac{1}{(n-1)(n-2)2\pi v_f v_g \gamma^{2(n-2)}} \quad \text{for } n \geqslant 3. \tag{A4}$$

As an example, we shall prove Eq. (A1). Noting that there exist four gap nodes, we have

$$\sum_{\mathbf{k}} \frac{\xi_{\mathbf{k}}^2 \Delta_{\mathbf{k}}^2}{(\gamma^2+\epsilon_{\mathbf{k}}^2)^4} = 4\int\int \frac{d\tilde{k}_f d\tilde{k}_g}{(2\pi)^2} \frac{v_f^2 v_g^2 \tilde{k}_f^2 \tilde{k}_g^2}{(\gamma^2+v_f^2\tilde{k}_f^2+v_g^2\tilde{k}_g^2)^4}.$$

By means of the transformations of $p_f=\sqrt{v_f/v_g}\tilde{k}_f$ and $p_g=\sqrt{v_g/v_f}\tilde{k}_g$, the above equation can be changed as

$$\sum_{\mathbf{k}} \frac{\xi_{\boldsymbol{k}}^2\Delta_{\boldsymbol{k}}^2}{(\gamma^2+\epsilon_{\boldsymbol{k}}^2)^4}=4\int\int \frac{dp_f dp_g}{(2\pi)^2}\frac{v_f^2v_g^2p_f^2p_g^2}{[\gamma^2+v_fv_g(p_f^2+p_g^2)]^4}$$
$$=\frac{1}{2\pi^2v_fv_g\gamma^2}\int_0^{x_0}dx\frac{x^2}{(1+x)^4}$$
$$\times\int_0^{2\pi}d\theta\cos^2\theta\sin^2\theta,$$

where $x_0=v_fv_gp_0^2/\gamma^2$ with $p_0\sim 1/a$. For the weak-disorder case (γ is small enough) considered, we can set $x_0=\infty$. Thus, a completion of the integrals over x and θ in the above equation immediately yields Eq. (A1).

(ii) If $\varphi(\xi_{\boldsymbol{k}},\Delta_{\boldsymbol{k}})$ stands for an arbitrary function of $\xi_{\boldsymbol{k}}$ and $\Delta_{\boldsymbol{k}}$, we have

$$\sum_{\mathbf{k}} \boldsymbol{q}\cdot\boldsymbol{v}_f(\boldsymbol{k})\boldsymbol{v}_f(\boldsymbol{k})\varphi(\xi_{\boldsymbol{k}},\Delta_{\boldsymbol{k}})=\frac{1}{2}v_f^2\boldsymbol{q}\sum_{\mathbf{k}}\varphi(\xi_{\boldsymbol{k}},\Delta_{\boldsymbol{k}}), \tag{A5}$$

$$\sum_{\mathbf{k}} \boldsymbol{q}\cdot\boldsymbol{v}_g(\boldsymbol{k})\boldsymbol{v}_g(\boldsymbol{k})\varphi(\xi_{\boldsymbol{k}},\Delta_{\boldsymbol{k}})=\frac{1}{2}v_g^2\boldsymbol{q}\sum_{\mathbf{k}}\varphi(\xi_{\boldsymbol{k}},\Delta_{\boldsymbol{k}}), \tag{A6}$$

$$\sum_{\boldsymbol{k}} \boldsymbol{q}\cdot\boldsymbol{v}_f(\boldsymbol{k})\boldsymbol{v}_g(\boldsymbol{k})\varphi(\xi_{\boldsymbol{k}},\Delta_{\boldsymbol{k}})=\frac{1}{2}v_fv_g\hat{\boldsymbol{q}}\sum_{\boldsymbol{k}}\varphi(\xi_{\boldsymbol{k}},\Delta_{\boldsymbol{k}}), \tag{A7}$$

and

$$\sum_{\mathbf{k}} \boldsymbol{q}\cdot\boldsymbol{v}_g(\boldsymbol{k})\boldsymbol{v}_f(\boldsymbol{k})\varphi(\xi_{\boldsymbol{k}},\Delta_{\boldsymbol{k}})=\frac{1}{2}v_fv_g\hat{\boldsymbol{q}}\sum_{\mathbf{k}}\varphi(\xi_{\boldsymbol{k}},\Delta_{\boldsymbol{k}}), \tag{A8}$$

with $\hat{\boldsymbol{q}}=(\boldsymbol{q}\cdot\boldsymbol{f}_1)\boldsymbol{g}_1+(\boldsymbol{q}\cdot\boldsymbol{g}_1)\boldsymbol{f}_1$. Here $\boldsymbol{f}_n$ and $\boldsymbol{g}_n$ represent, respectively, the unity vectors parallel to $\boldsymbol{v}_f$ and $\boldsymbol{v}_g$ at the nth node ($n=1,2,3,4$), as shown in Fig. 1.

Here we only prove Eq. (A5). Due to the existence of four gap nodes, one can write

$$\sum_{\boldsymbol{k}} \boldsymbol{q}\cdot\boldsymbol{v}_f(\boldsymbol{k})\boldsymbol{v}_f(\boldsymbol{k})\varphi(\xi_{\boldsymbol{k}},\Delta_{\boldsymbol{k}})$$
$$=v_f^2\sum_{n=1}^{4}(\boldsymbol{q}\cdot\boldsymbol{f}_n)\boldsymbol{f}_n\sum_{\boldsymbol{k}}^{(n)}\varphi(\xi_{\boldsymbol{k}},\Delta_{\boldsymbol{k}}),$$

where $\Sigma_{\boldsymbol{k}}^{(n)}$ represents the summation over $\boldsymbol{k}$ only in the vicinity of the nth node. The above equation is easily shown to be equivalent to Eq. (A5) by noting that

$$\sum_{\boldsymbol{k}}^{(n)}\varphi(\xi_{\boldsymbol{k}},\Delta_{\boldsymbol{k}})=\frac{1}{4}\sum_{\boldsymbol{k}}\varphi(\xi_{\boldsymbol{k}},\Delta_{\boldsymbol{k}})$$

and

$$\sum_{n=1}^{4}(\boldsymbol{q}\cdot\boldsymbol{f}_n)\boldsymbol{f}_n=2\boldsymbol{q}.$$

(iii) If A, A', B, and B' are arbitrary linear superimpositions of τ_i ($i=0, 1, 2, 3$), we have

$$\frac{1}{2}\sum_i C_{ii}\mathrm{Tr}(\tau_iA\tau_iB)=\sum_i(\mathcal{C}M)_{ii} \tag{A9}$$

and

$$\frac{1}{2}\sum_{ij} C_{ii}C'_{jj}\mathrm{Tr}(\tau_iA\tau_jA'\tau_iB\tau_jB')=\sum_i(\mathcal{C}M\mathcal{C}'M')_{ii}, \tag{A10}$$

where $\mathcal{C}=\Sigma_iC_{ii}\tau_i\otimes\tau_i$, $\mathcal{C}'=\Sigma_iC'_{ii}\tau_i\otimes\tau_i$, $\mathcal{M}=A\otimes B$, and $\mathcal{M}'=A'\otimes B'$.

As an example, we shall prove Eq. (A10). Assuming that $\tau_iA\tau_jA'=\Sigma_kx_{ijk}\tau_k$ and $\tau_iB\tau_jB'=\Sigma_ly_{ijl}\tau_l$, we have

$$\frac{1}{2}\sum_{ij} C_{ii}C'_{jj}\mathrm{Tr}(\tau_iA\tau_jA'\tau_iB\tau_jB')$$
$$=\frac{1}{2}\sum_{ijkl} C_{ii}C'_{jj}x_{ijk}y_{ijl}\mathrm{Tr}(\tau_k\tau_l)$$
$$=\sum_{ijkl} C_{ii}C'_{jj}x_{ijk}y_{ijl}\delta_{kl}=\sum_{ijk} C_{ii}C'_{jj}x_{ijk}y_{ijk} \tag{A11}$$

and

$$\mathcal{C}\mathcal{M}\mathcal{C}'\mathcal{M}'=\sum_{ij} C_{ii}C'_{jj}(\tau_iA\tau_jA')\otimes(\tau_iB\tau_jB')$$
$$=\sum_{ijkl} C_{ii}C'_{jj}x_{ijk}y_{ijl}\tau_k\otimes\tau_l. \tag{A12}$$

Equation (A12) yields

$$\sum_k(\mathcal{C}M\mathcal{C}'\mathcal{M}')_{kk}=\sum_{ijk} C_{ii}C'_{jj}x_{ijk}y_{ijk}. \tag{A13}$$

A combination of Eq. (A11) with Eq. (A13) immediately leads to Eq. (A10).

APPENDIX B: VANISHING CONTRIBUTION OF 0-MODE DIFFUSON TO THE QI EFFECTS

In this appendix, we shall show that 0-mode diffuson does not contribute to the QI effects. As shown by Fig. 4(e), the impurity scattering from $\boldsymbol{k}$ state to $\boldsymbol{q}+\boldsymbol{k}$ state ($Dq^2\ll\gamma$) is subject to a vertex correction by the diffuson. The vertex-corrected retarded T matrix can be expressed by

$$\overline{T}^R(\boldsymbol{q},\boldsymbol{\epsilon})_{\mu\mu'}=\sum_{\nu\nu'}\mathcal{J}(\boldsymbol{q},\boldsymbol{\epsilon})^{RR}_{\mu\mu',\nu\nu'}T^R(\boldsymbol{\epsilon})_{\nu\nu'}, \tag{B1}$$

where the vertex function $\mathcal{J}(\boldsymbol{q},\boldsymbol{\epsilon})^{RR}$ is given by

$$\mathcal{J}(\boldsymbol{q},\boldsymbol{\epsilon})^{RR}=\mathcal{I}+\mathcal{D}(\boldsymbol{q};\boldsymbol{\epsilon},\boldsymbol{\epsilon})^{RR}\mathcal{H}(\boldsymbol{q};\boldsymbol{\epsilon},\boldsymbol{\epsilon})^{RR}. \tag{B2}$$

In order to calculate the vertex function, we exploit the equation for 0-mode diffuson in the RR channel,

$$\mathcal{D}(\boldsymbol{q};\epsilon,\epsilon)^{RR}=\mathcal{W}(\epsilon,\epsilon)^{RR}+\mathcal{W}(\epsilon,\epsilon)^{RR}\mathcal{H}(\boldsymbol{q};\epsilon,\epsilon)^{RR}\mathcal{D}(\boldsymbol{q};\epsilon,\epsilon)^{RR}, \tag{B3}$$

yielding

$$\mathcal{H}(\boldsymbol{q};\epsilon,\epsilon)^{RR}=\mathcal{W}^{-1}(\epsilon,\epsilon)^{RR}-\mathcal{D}^{-1}(\boldsymbol{q};\epsilon,\epsilon)^{RR}. \tag{B4}$$

A substitution of Eq. (B4) into Eq. (B2) leads to

$$\mathcal{J}(\boldsymbol{q},\epsilon)^{RR}=\mathcal{D}(\boldsymbol{q};\epsilon,\epsilon)^{RR}\mathcal{W}^{-1}(\epsilon,\epsilon)^{RR}. \tag{B5}$$

Making use of Eqs. (3.7) and (3.8), we get

$$\mathcal{W}^{-1}(\epsilon,\epsilon)^{RR}\approx\frac{\pi\rho_0}{2\gamma}\times\begin{cases}\tau_3\otimes\tau_3 & \text{for the Born limit}\\ -\tau_0\otimes\tau_0 & \text{for the unitary limit.}\end{cases} \tag{B6}$$

Substituting Eqs. (3.32) and (B6) into Eq. (B5), we obtain the expression of the vertex function as (for $Dq^2\ll\gamma$ and $|\epsilon|\ll\gamma$)

$$\mathcal{J}(\boldsymbol{q},\epsilon)^{RR}=\frac{2\gamma}{Dq^2-i2\epsilon}(\tau_0\otimes\tau_0-\tau_1\otimes\tau_1-\tau_2\otimes\tau_2-\tau_3\otimes\tau_3), \tag{B7}$$

which is suitable both near the Born and near the unitary limits. Substituting Eqs. (2.1) and (B7) into Eq. (B1), we can easily show that

$$\overline{T}^R(\boldsymbol{q},\epsilon)=\sum_i J(\boldsymbol{q},\epsilon)_{ii}^{RR}\tau_i T^R(\epsilon)\tau_i^*=0. \tag{B8}$$

Since every conductivity diagram in Fig. 4 contains the vertex correction by the diffuson, Eq. (B8) indicates that none of them contributes to the conductivities.

APPENDIX C: VANISHING CONTRIBUTION OF π-MODE DIFFUSON TO THE QI EFFECTS

In this appendix, we shall show that the contributions of all the lowest-order conductivity diagrams with π-mode diffuson to the QI effects are summed to vanish. These diagrams are generated by replacing all $\boldsymbol{q}$ by $\boldsymbol{Q}+\boldsymbol{q}$ in Fig. 4. In the case of π-mode diffuson, the vertex-dressed T matrices are shown to be

$$\overline{T}_\pi^{R(A)}(\boldsymbol{q},\epsilon)=\phi^{R(A)}(\boldsymbol{q},\epsilon)\tau_0, \tag{C1}$$

for $Dq^2\ll\gamma$ and $|\epsilon|\ll\gamma$, where

$$\phi^{R(A)}(\boldsymbol{q},\epsilon)=\mp i\frac{16\gamma}{\pi\rho_0}\frac{1}{Dq^2\mp i2\epsilon+2\delta}. \tag{C2}$$

The contributions of the diagrams with π-mode diffuson to the current-current correlation function in RA channel are expressed as

$$\Pi_\pi^\chi(\epsilon,\epsilon')_{4a}^{RA}=\frac{n_i}{2}\sum_{\mathbf{k}\boldsymbol{q}}\mathrm{Tr}[\boldsymbol{\Lambda}_{\boldsymbol{k}}^\chi G_{\boldsymbol{k}}^R\overline{T}_\pi^R(\boldsymbol{q},\epsilon)\times G_{\boldsymbol{Q}+\boldsymbol{k}}^R\overline{T}_\pi^R(\boldsymbol{q},\epsilon)G_{\boldsymbol{k}}^R\cdot\boldsymbol{\Lambda}_{\boldsymbol{k}}^\chi G_{\boldsymbol{k}}^A], \tag{C3}$$

$$\Pi_\pi^\chi(\epsilon,\epsilon')_{4b}^{RA}=\frac{n_i}{2}\sum_{\mathbf{k}\boldsymbol{q}}\mathrm{Tr}[\boldsymbol{\Lambda}_{\boldsymbol{k}}^\chi G_{\boldsymbol{k}}^R\overline{T}_\pi^R(\boldsymbol{q},\epsilon)\times G_{\boldsymbol{Q}+\boldsymbol{k}}^R\cdot\boldsymbol{\Lambda}_{\boldsymbol{Q}+\boldsymbol{k}}^\chi G_{\boldsymbol{Q}+\boldsymbol{k}}^A\overline{T}_\pi^A(\boldsymbol{q},\epsilon')G_{\boldsymbol{k}}^A], \tag{C4}$$

$$\Pi_\pi^\chi(\epsilon,\epsilon')_{4c}^{RA}=\frac{n_i}{2}\sum_{\mathbf{q}}\sum_{\mathbf{k}\boldsymbol{k}'}\sum_i D_\pi(\boldsymbol{q};\epsilon,\epsilon')_{ii}^{RA}\times\mathrm{Tr}[\boldsymbol{\Lambda}_{\boldsymbol{k}}^\chi G_{\boldsymbol{k}}^R\overline{T}_\pi^R(\boldsymbol{q},\epsilon)G_{\boldsymbol{Q}+\boldsymbol{q}+\boldsymbol{k}}^R\tau_i G_{\boldsymbol{Q}+\boldsymbol{q}+\boldsymbol{k}'}^R\times\overline{T}_\pi^R(\boldsymbol{q},\epsilon)G_{\boldsymbol{k}'}^R\cdot\boldsymbol{\Lambda}_{\boldsymbol{k}'}^\chi G_{\boldsymbol{k}'}^A\tau_i^*G_{\boldsymbol{k}}^A], \tag{C5}$$

and

$$\Pi_\pi^\chi(\epsilon,\epsilon')_{4d}^{RA}=\frac{n_i}{2}\sum_{\mathbf{q}}\sum_{\mathbf{k}\boldsymbol{k}'}\sum_i D_\pi(\boldsymbol{q};\epsilon,\epsilon')_{ii}^{RA}\times\mathrm{Tr}[\boldsymbol{\Lambda}_{\boldsymbol{k}}^\chi G_{\boldsymbol{k}}^R\tau_i G_{\boldsymbol{k}'-\boldsymbol{q}-\boldsymbol{Q}}^R\overline{T}_\pi^R(\boldsymbol{q},\epsilon)\times G_{\boldsymbol{k}'}^R\cdot\boldsymbol{\Lambda}_{\boldsymbol{k}'}^\chi G_{\boldsymbol{k}'}^A\tau_i^*G_{\boldsymbol{Q}+\boldsymbol{q}+\boldsymbol{k}}^A\overline{T}_\pi^A(\boldsymbol{q},\epsilon')G_{\boldsymbol{k}}^A]. \tag{C6}$$

By means of Eqs. (A9) and (C1), one can reexpress Eqs. (C3)–(C6) as

$$\Pi_\pi^\chi(\epsilon,\epsilon')_{4a}^{RA}=\frac{n_i}{2}\sum_{\mathbf{k}\boldsymbol{q}}\phi^R(\boldsymbol{q},\epsilon)^2\mathrm{Tr}(\boldsymbol{\Lambda}_{\boldsymbol{k}}^\chi G_{\boldsymbol{k}}^R G_{\boldsymbol{Q}+\boldsymbol{k}}^R G_{\boldsymbol{k}}^R\cdot\boldsymbol{\Lambda}_{\boldsymbol{k}}^\chi G_{\boldsymbol{k}}^A), \tag{C7}$$

$$\Pi_\pi^\chi(\epsilon,\epsilon')_{4b}^{RA}=\frac{n_i}{2}\sum_{\mathbf{k}\boldsymbol{q}}\phi^R(\boldsymbol{q},\epsilon)\phi^A(\boldsymbol{q},\epsilon')\times\mathrm{Tr}(\boldsymbol{\Lambda}_{\boldsymbol{k}}^\chi G_{\boldsymbol{k}}^R G_{\boldsymbol{Q}+\boldsymbol{k}}^R\cdot\boldsymbol{\Lambda}_{\boldsymbol{Q}+\boldsymbol{k}}^\chi G_{\boldsymbol{Q}+\boldsymbol{k}}^A G_{\boldsymbol{k}}^A), \tag{C8}$$

$$\Pi_\pi^\chi(\epsilon,\epsilon')_{4c}^{RA}=n_i\sum_{\mathbf{q}}\sum_i\phi^R(\boldsymbol{q},\epsilon)^2\{\mathcal{D}_\pi'(\boldsymbol{q};\epsilon,\epsilon')^{RA}\times[\boldsymbol{M}_\pi^\chi(\boldsymbol{q})_{4c}^{RA}\otimes\cdot\tilde{\boldsymbol{M}}_\pi^\chi(\boldsymbol{q})_{4c}^{RA}]\}_{ii}, \tag{C9}$$

and

$$\Pi_\pi^\chi(\epsilon,\epsilon')_{4d}^{RA}=n_i\sum_{\mathbf{q}}\sum_i\phi^R(\boldsymbol{q},\epsilon)\phi^A(\boldsymbol{q},\epsilon')\times\{\mathcal{D}_\pi'(\boldsymbol{q};\epsilon,\epsilon')^{RA}[\boldsymbol{M}_\pi^\chi(\boldsymbol{q})_{4d}^{RA}\otimes\cdot\tilde{\boldsymbol{M}}_\pi^\chi(\boldsymbol{q})_{4d}^{RA}]\}_{ii}, \tag{C10}$$

where

$$\boldsymbol{M}_\pi^\chi(\boldsymbol{q})_{4c}^{RA}=\sum_{\mathbf{k}}G_{\boldsymbol{q}+\boldsymbol{Q}+\boldsymbol{k}}^R G_{\boldsymbol{k}}^R\boldsymbol{\Lambda}_{\boldsymbol{k}}^\chi G_{\boldsymbol{k}}^A, \tag{C11}$$

$$\tilde{\boldsymbol{M}}_{\pi}^{\chi}(\boldsymbol{q})_{4c}^{RA} = \sum_{\mathbf{k}} G_{\boldsymbol{k}}^{A} \boldsymbol{\Lambda}_{\boldsymbol{k}}^{\chi} G_{\boldsymbol{k}}^{R} G_{\boldsymbol{q}+\boldsymbol{Q}+\boldsymbol{k}}^{R}, \tag{C12}$$

$$\boldsymbol{M}_{\pi}^{\chi}(\boldsymbol{q})_{4d}^{RA} = \sum_{\mathbf{k}} G_{\boldsymbol{q}+\boldsymbol{Q}-\boldsymbol{k}}^{R} G_{\boldsymbol{k}}^{R} \boldsymbol{\Lambda}_{\boldsymbol{k}}^{\chi} G_{\boldsymbol{k}}^{A}, \tag{C13}$$

$$\tilde{\boldsymbol{M}}_{\pi}^{\chi}(\boldsymbol{q})_{4d}^{RA} = \sum_{\mathbf{k}} G_{\boldsymbol{q}+\boldsymbol{Q}+\boldsymbol{k}}^{A} G_{\boldsymbol{k}}^{A} \boldsymbol{\Lambda}_{\boldsymbol{k}}^{\chi} G_{\boldsymbol{k}}^{R}, \tag{C14}$$

and

$$\mathcal{D}_{\pi}'(\boldsymbol{q};\epsilon,\epsilon')^{RR(A)} = \sum_{i} D_{\pi}(\boldsymbol{q};\epsilon,\epsilon')_{ii}^{RR(A)} \tau_i \otimes \tau_i^{*}. \tag{C15}$$

The impurity concentration n_i is related to γ through Eq. (2.6). The above correlation functions can be calculated by the same method as used in the previous sections.

(i) For the charge conductivity, a completion of the summations over $\boldsymbol{k}$ in Eqs. (C7), (C8), and their counterparts in RR channel yields

$$\Pi_{\pi}^{e}(\epsilon,\epsilon')_{4a} = \frac{64 e^2 v_f}{\alpha v_g} \sum_{\boldsymbol{q}} \frac{D\gamma}{(Dq^2 - i2\epsilon + 2\delta)^2} \tag{C16}$$

and

$$\Pi_{\pi}^{e}(\epsilon,\epsilon')_{4b} = -\frac{64 e^2 v_f}{\alpha v_g} \sum_{\boldsymbol{q}} \frac{D}{Dq^2 - i2\epsilon + 2\delta} \times \left(\frac{\gamma}{Dq^2 + i2\epsilon' + 2\delta} + \frac{\gamma}{Dq^2 - i2\epsilon' + 2\delta} \right). \tag{C17}$$

Substituting Eqs. (C16) and (C17) into Eq. (4.1), we obtain

$$\frac{\sigma_{\pi}^{e}(\omega)_{4a}}{\sigma_0^{e}} = \frac{\sigma_{\pi}^{e}(\omega)_{4a'}}{\sigma_0^{e}} = \frac{4\gamma}{\alpha\delta} F_1\left(\frac{\omega}{\delta}\right) \tag{C18}$$

and

$$\frac{\sigma_{\pi}^{e}(\omega)_{4b}}{\sigma_0^{e}} = -\frac{8\gamma}{\alpha\delta} F_2\left(\frac{\omega}{\delta}\right), \tag{C19}$$

where

$$F_1(x) = \frac{1}{x}\arctan x = \begin{cases} 1 & \text{if } x \ll 1 \\ \pi/2x & \text{if } x \gg 1, \end{cases} \tag{C20}$$

and

$$F_2(x) = \frac{1}{x}\int_0^1 dy \left[\frac{1+y}{2y}\arctan\frac{x(1+y)}{2} - \frac{1-y}{2y}\arctan\frac{x(1-y)}{2} \right] = \begin{cases} 1 & \text{if } x \ll 1 \\ \pi/2x & \text{if } x \gg 1. \end{cases} \tag{C21}$$

From Eqs. (C18)–(C21), it follows that

$$\sigma_{\pi}^{e}(\omega)_{4a} + \sigma_{\pi}^{e}(\omega)_{4a'} + \sigma_{\pi}^{e}(\omega)_{4b} = 0, \tag{C22}$$

for $\omega \ll \delta$ or $\omega \gg \delta$. Therefore, the sum of the contributions from Figs. 4(a) and 4(b) has no singular correction to the dynamical charge conductivity.

A completion of the summations over $\boldsymbol{k}$ in Eqs. (C11)–(C14) yields

$$\begin{aligned} \boldsymbol{M}_{\pi}^{e}(\boldsymbol{q})_{4c}^{RA} &= \tilde{\boldsymbol{M}}_{\pi}^{e}(\boldsymbol{q})_{4c}^{RA} = -\boldsymbol{M}_{\pi}^{e}(\boldsymbol{q})_{4c}^{RR} = -\tilde{\boldsymbol{M}}_{\pi}^{e}(\boldsymbol{q})_{4c}^{RR} \\ &= -\boldsymbol{M}_{\pi}^{e}(\boldsymbol{q})_{4d}^{RA} = -\tilde{\boldsymbol{M}}_{\pi}^{e}(\boldsymbol{q})_{4d}^{RA} = \boldsymbol{M}_{\pi}^{e}(\boldsymbol{q})_{4d}^{RR} \\ &= \tilde{\boldsymbol{M}}_{\pi}^{e}(\boldsymbol{q})_{4d}^{RR} = -\frac{e}{4\pi v_g \gamma^2}(\hat{\boldsymbol{q}} v_g \tau_1 + \boldsymbol{q} v_f \tau_3). \end{aligned} \tag{C23}$$

Substituting Eqs. (C15) and (C23) into Eqs. (C9) and (C10), and completing the summations over i, we get

$$\Pi_{\pi}^{e}(\epsilon,\epsilon')_{4c}^{RA} = \Pi_{\pi}^{e}(\epsilon,\epsilon')_{4c}^{RR} = \Pi_{\pi}^{e}(\epsilon,\epsilon')_{4d}^{RA} = \Pi_{\pi}^{e}(\epsilon,\epsilon')_{4d}^{RR} = 0,$$

leading to

$$\sigma_{\pi}^{e}(\omega)_{4c} = \sigma_{\pi}^{e}(\omega)_{4d} = 0. \tag{C24}$$

Equations (C22) and (C24) indicate that the π-mode diffuson has a vanishing contribution to the QI correction of the charge conductivity.

(ii) For the spin conductivity, a completion of the summations over $\boldsymbol{k}$ in Eqs. (C7) and (C8) yields

$$\Pi_{\pi}^{s}(\epsilon,\epsilon')_{4a} = \sum_{\boldsymbol{q}} \frac{32 D\gamma}{(Dq^2 - i2\epsilon + 2\delta)^2} \tag{C25}$$

and

$$\Pi_{\pi}^{s}(\epsilon,\epsilon')_{4b} = -\sum_{\boldsymbol{q}} \frac{32D}{Dq^2 - i2\epsilon + 2\delta}\left(\frac{\gamma}{Dq^2 + i2\epsilon' + 2\delta} + \frac{\gamma}{Dq^2 - i2\epsilon' + 2\delta} \right). \tag{C26}$$

Substituting Eqs. (C25) and (C26) into Eq. (4.1), we obtain

$$\frac{\sigma_{\pi}^{s}(\omega)_{4a}}{\sigma_0^{s}} = \frac{\sigma_{\pi}^{s}(\omega)_{4a'}}{\sigma_0^{s}} = \frac{4\gamma}{\alpha\delta} F_1\left(\frac{\omega}{\delta}\right) \tag{C27}$$

and

$$\frac{\sigma_{\pi}^{s}(\omega)_{4b}}{\sigma_0^{s}} = -\frac{8\gamma}{\alpha\delta} F_2\left(\frac{\omega}{\delta}\right). \tag{C28}$$

A combination of Eqs. (C20), (C21), (C27), and (C28) leads to

$$\sigma_{\pi}^{s}(\omega)_{4a} + \sigma_{\pi}^{s}(\omega)_{4a'} + \sigma_{\pi}^{s}(\omega)_{4b} = 0, \tag{C29}$$

for $\omega \ll \delta$ or $\omega \gg \delta$, implying that the sum of the contributions from Figs. 4(a) and 4(b) to the dynamical spin conductivity is nonsingular.

A completion of the summations over $\boldsymbol{k}$ in Eqs. (C11)–(C14) for the spin case yields

$$\boldsymbol{M}^s_\pi(\boldsymbol{q})^{RA}_{4c} = \tilde{\boldsymbol{M}}^s_\pi(\boldsymbol{q})^{RA}_{4c} = \boldsymbol{M}^s_\pi(\boldsymbol{q})^{RA}_{4d} = \tilde{\boldsymbol{M}}^s_\pi(\boldsymbol{q})^{RA}_{4d} = 0 \tag{C30}$$

and

$$\boldsymbol{M}^s_\pi(\boldsymbol{q})^{RR}_{4c} = \tilde{\boldsymbol{M}}^s_\pi(\boldsymbol{q})^{RR}_{4c} = -\boldsymbol{M}^s_\pi(\boldsymbol{q})^{RR}_{4d} = -\tilde{\boldsymbol{M}}^s_\pi(\boldsymbol{q})^{RR}_{4d} = -\frac{\alpha \boldsymbol{q}}{2\pi\gamma^2}\tau_0. \tag{C31}$$

Substituting Eqs. (C15), (C30), and (C31) into Eqs. (C9) and (C10), and completing the summations over i, we get

$$\Pi^s_\pi(\epsilon,\epsilon')_{4c} = -\sum_{\boldsymbol{q}} \frac{128 D^2 q^2 \gamma}{(Dq^2 - i2\epsilon + 2\delta)^2[Dq^2 - i(\epsilon+\epsilon') + 2\delta]}, \tag{C32}$$

and

$$\Pi^s_\pi(\epsilon,\epsilon')_{4d} = \sum_{\boldsymbol{q}} \frac{128 D^2 q^2 \gamma}{(Dq^2 - i2\epsilon + 2\delta)(Dq^2 - i2\epsilon' + 2\delta)[Dq^2 - i(\epsilon+\epsilon') + 2\delta]}. \tag{C33}$$

A substitution of Eqs. (C32) and (C33) into Eq. (4.1) leads to

$$\frac{\sigma^s_\pi(\omega)_{4c}}{\sigma^s_0} = \frac{\sigma^s_\pi(\omega)_{4c'}}{\sigma^s_0} = -\frac{8\gamma}{\alpha\delta} F_3\left(\frac{\omega}{\delta}\right) \tag{C34}$$

and

$$\frac{\sigma^s_\pi(\omega)_{4d}}{\sigma^s_0} = \frac{\sigma^s_\pi(\omega)_{4d'}}{\sigma^s_0} = \frac{8\gamma}{\alpha\delta} F_4\left(\frac{\omega}{\delta}\right), \tag{C35}$$

where

$$F_3(x) = \frac{2}{x^2}\ln(1+x^2) - \frac{4}{x^2}\ln\left(1+\frac{x^2}{4}\right) + \frac{2}{x^3}\arctan x + 2\left(\frac{1}{x} - \frac{4}{x^3}\right)\arctan\frac{x}{2} = \begin{cases} 1 & \text{if } x \ll 1 \\ \pi/x & \text{if } x \gg 1, \end{cases} \tag{C36}$$

and

$$F_4(x) = -\frac{4}{x^2}\ln(1+x^2) + \frac{4}{x^2}\ln\left(1+\frac{x^2}{4}\right) + 4\left(\frac{1}{x} - \frac{1}{x^3}\right) \times \arctan x - 2\left(\frac{1}{x} - \frac{4}{x^3}\right)\arctan\frac{x}{2} = \begin{cases} 1 & \text{if } x \ll 1 \\ \pi/x & \text{if } x \gg 1. \end{cases} \tag{C37}$$

Therefore, we obtain

$$\sigma^s_\pi(\omega)_{4c} + \sigma^s_\pi(\omega)_{4c'} + \sigma^s_\pi(\omega)_{4d} + \sigma^s_\pi(\omega)_{4d'} = 0, \tag{C38}$$

for $\omega \ll \delta$ or $\omega \gg \delta$. Equations (C29) and (C38) indicate that the π-mode diffuson has also a vanishing contribution to the QI correction of the spin conductivity.

APPENDIX D: VANISHING CONTRIBUTIONS OF NONSINGULAR LADDERS TO THE QI EFFECTS

In this appendix, we show that the lowest-order conductivity diagrams with nonsingular ladders, shown in Fig. 5, do not contribute to the QI effects. The contribution of Fig. 5(a) to the electrical current-current correlation function is given by

$$\Pi^e(\epsilon,\epsilon')_{5a} = \frac{1}{2} e^2 v_f^2 \sum_{\boldsymbol{q}} \sum_{\boldsymbol{kk'}} \sum_{ijk} L(\epsilon,\epsilon)^{RR}_{ij} C(\boldsymbol{q};\epsilon,\epsilon)^{RR}_{kk} \times \mathrm{Tr}[\tau_i G^R_{\boldsymbol{k'}} \tau_k G^R_{\boldsymbol{q}-\boldsymbol{k'}} \tau_k G^R_{\boldsymbol{k'}} \tau_j^* G^R_{\boldsymbol{k}} (G^A_{\boldsymbol{k}} - G^R_{\boldsymbol{k}}) G^R_{\boldsymbol{k}}], \tag{D1}$$

with $\mathcal{L}(\epsilon,\epsilon')^{RR(A)} = \Sigma_{ij} L(\epsilon,\epsilon')^{RR(A)}_{ij} \tau_i \otimes \tau_j$, standing for the expression of the nonsingular ladders. By means of Eqs. (2.2) and (A1)–(A4), one can readily show that

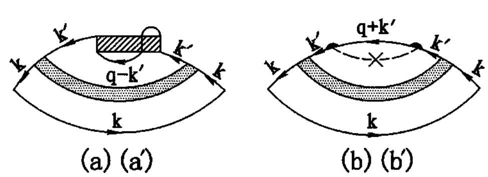

FIG. 5. Lowest-order conductivity diagrams with nonsingular ladders (gray blocks).

$$\sum_{\boldsymbol{k}} G^R_{\boldsymbol{k}}(G^A_{\boldsymbol{k}}-G^R_{\boldsymbol{k}})G^R_{\boldsymbol{k}}=0. \tag{D2}$$

A substitution of Eq. (D2) into Eq. (D1) immediately yields $\Pi^e(\epsilon,\epsilon')_{5a}=0$, indicating that Fig. 5(a) has a vanishing contribution to the charge conductivity. Obviously, the same conclusion is valid for the spin conductivity, as well as for the case of π-mode cooperon. Similarly, it is easy to show that Fig. 5(b) has also a vanishing contribution to the QI effects due to Eq. (D2).

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