Thermally assisted magnetization reversal in the presence of a spin-transfer torque

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We propose a generalized stochastic Landau-Lifshitz equation and its corresponding Fokker-Planck equation for the magnetization dynamics in the presence of spin-transfer torques. Since the spin-transfer torque can pump a magnetic energy into the magnetic system, the equilibrium temperature of the magnetic system is ill defined. We introduce an effective temperature based on a stationary solution of the Fokker-Planck equation. In the limit of high-energy barriers, the law of thermal agitation is derived. We find that the Néel-Brown relaxation formula remains valid as long as we replace the temperature by an effective one that is linearly dependent on the spin torque. We carry out the numerical integration of the stochastic Landau-Lifshitz equation to support our theory. Our results agree with existing experimental data.

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I. INTRODUCTION

Thermally assisted magnetization reversal has been the subject of intensive theoretical and experimental study for many decades. Aside from the relevance of the subject to emerging magnetic technology such as heat-assisted magnetic recording[1] and thermal stability of magnetic random access memory,[2] fundamental physics of magnetization reversal process driven by white noise is very rich. Classical transition-rate theory of Kramer[3] has supplied a framework in understanding thermal activation of a single domain magnetic element.[4] Namely, the thermal switching probability $P(t)$ can be described by the Néel-Brown (NB) relaxation-time formula $P(t)=1-\exp(-t/\tau)$, where the relaxation time is $\tau=f_0^{-1}\exp(E_b/k_BT)$, f_0 is an attempt frequency, E_b is the energy barrier, and T is the temperature. For a multidomain structure, the energy surface becomes extremely complicated and identifying energy barriers are numerically nontrivial. Nevertheless, with recent development of micromagnetic modeling, one can understand thermal reversal reasonably well for a not-too-complicated structure.[5]

An implicit and yet critical assumption in the NB theory is that magnetization dynamics is governed by a torque from an effective magnetic field $\mathbf{H}_{eff}=-\boldsymbol{\nabla}_{\mathbf{M}}E(\mathbf{M})$, where $E(\mathbf{M})$ is the total magnetic energy, i.e., the effective field is derivable from the derivative of an energy function with respect to the magnetization vector. Therefore, an energy barrier is well defined in the NB relaxation formula. If the torque is not derivable from an energy function, one would expect breakdown of the NB formula. Recently, a new class of torques, called spin-transfer torque (STT), has been proposed[6,7] and verified experimentally.[8,9] STT is derived from a spin-polarized current in magnetic multilayers. For a spin valve structure, STT is written as[7]

$$\boldsymbol{\Gamma}_s=\frac{\gamma a_J}{M_s}\mathbf{M}\times(\mathbf{M}\times\hat{\mathbf{M}}_p), \tag{1}$$

where a_J represents the strength of STT; it is proportional to the current density. γ is gyromagnetic ratio, $\hat{\mathbf{M}}_p$ is a *unit* vector representing the direction of the magnetization of the pinned layer, $\mathbf{M}$ is the magnetization vector of the free layer, and $M_s=|\mathbf{M}|$ is the saturation magnetization. If we define an effective field $\mathbf{H}_J'\equiv(a_J/M_s)\mathbf{M}\times\hat{\mathbf{M}}_p$ from STT, it is evident that $\mathbf{H}_J'$ cannot be written as a total derivative of a function with respect to the magnetization vector, i.e., there is no well-defined energy associated with the field $\mathbf{H}_J'$.

Recent experiments[10–13] on the thermal effect of the spin torque had also indicated that the thermally assisted magnetization reversal cannot be simply described by the NB formula. Urazhdin *et al.*[10] found that the activation energy strongly depends on the magnitude as well as the direction of the current. To capture the gloss features of the observed experiments, they have to introduce an effective temperature unrelated to the true temperature in the NB formula. Their proposed effective temperature was then interpreted via a possible magnetic heating and magnetic excitations from the spin-transfer torque. The current directional dependence of the effective temperature indicated that the heating is not of the ordinary current-induced Joule heating. However, no attempt has been made to mathematically link the effective temperature with the spin-transfer torque of Eq. (1).

The problem of thermally assisted escape process driven by a nongradient driven force, not derivable from a potential, is an unresolved outstanding problem in statistical physics. While there are already some efforts to formulate the escape time in this case, the general conclusion is that the law of escape time lacks universality and a variety of scaling relations exist.[14] The standard treatment of the thermal escape problem in the presence of a nongradient field would start from the Fokker-Planck equation and one numerically solves for the probability distribution.[14] This procedure involves proper averaging over the possible escape trajectories. Let us consider the total work done by the conservative torque and the nonconservative STT along an arbitrary trajectory,

$$\delta W=-\int(\mathbf{H}_{eff}+\mathbf{H}_J')\cdot d\mathbf{M}=E_b-\frac{a_J}{M_s}\int_{\mathbf{M}_0}^{\mathbf{M}_f}(\mathbf{M}\times\hat{\mathbf{M}}_p)\cdot d\mathbf{M}, \tag{2}$$

where we have assumed that the magnetization vector starts at an initial equilibrium point $\mathbf{M}_0=M_s\mathbf{e}_x$ and reaches to an energy saddle point $\mathbf{M}_f$, and we have defined the energy barrier from the conservative torque $E_b=E(\mathbf{M}_f)-E(\mathbf{M}_0)$.

One immediately realizes that δW defined above depends on the escape trajectory. In the absence of STT, one relies on the assumption that the most probable path of the escape is through a minimum energy barrier, i.e., $\mathbf{M}_f$ would be an energy saddle point. In the presence of STT, such assumption breaks down in general. In the present paper, we do not intend to address the general problem of the thermal escape in an open system, rather we want to answer a very focused question: to what extent one can formulate the thermal agitation in terms of a simplified Néel-Brown activation process and to what accuracy one can analyze the relevant experimental data through a simple effective formula we will develop in the later sections? The paper is organized as follows. In Sec. II, we propose the stochastic Landau-Lifshitz equation and its corresponding Fokker-Planck equation in the presence of the current. In Sec. III, we introduce a stationary solution for the probability density of magnetization by identifying an effective barrier or an effective temperature associated with spin torques. In Sec. IV, we present a numerical calculation to demonstrate the validity of our theory in several realistic cases. Finally, we compare our theory with existing experiments and summarize our theory.

II. STOCHASTIC LANDAU-LIFSHITZ EQUATION IN THE PRESENCE OF CURRENTS

Let us first explicitly propose the following generalized stochastic Landau-Lifshitz (LL) equation that describes dynamics of the magnetization vector subject to a STT at finite temperatures

$$\frac{d\mathbf{M}}{dt}=-\gamma\mathbf{M}\times(\mathbf{H}_{eff}+\mathbf{h}_r)-\gamma\frac{\alpha}{M_s}\mathbf{M}\times[\mathbf{M}\times(\mathbf{H}_{eff}+\mathbf{h}_r)]+\mathbf{\Gamma}_s, \tag{3}$$

where α is the damping constant, $\mathbf{H}_{eff}$ is the effective magnetic field including the external field, the anisotropy field, the exchange field, and the demagnetization field, and $\mathbf{h}_r$ is a fluctuating field with a Gaussian stochastic process whose statistical properties are defined as

$$\langle h_r^i(t)\rangle=0,\quad \langle h_r^i(t)h_r^j(t')\rangle=2D\delta_{ij}\delta(t-t'), \tag{4}$$

where i and j are Cartesian indices, D represents the strength of the thermal fluctuations whose value will be determined later. $\langle\,\rangle$ denotes an average taken over all realization of the fluctuating field. In the absence of the spin torque, the above equation is the standard stochastic LL equation. Note that we have conveniently dropped the customary renormalized gyromagnetic ratio $\gamma/(1+\alpha^2)$ when compared with the standard Landau-Lifshitz-Gilbert (LLG) equation. The critical assumption of our proposed stochastic LL equation, Eq. (3), is that *the spin torque does not contain a fluctuating field* $\mathbf{h}_r$. The justification for this choice is that the spin torque comes from the conduction electrons whose transport properties are less affected by thermal fluctuations since the Fermi level is much higher than the thermal energy. Therefore, the thermal fluctuation would not appear to affect a_J which represents the strength of the spin torque. We believe that our proposed stochastic LL equation captures the main random processes induced by thermal fluctuation. Nevertheless, one could, in principle, have introduced a second random field or torques to take into account the fluctuation of the spin torque. In our proposed LL equation, the thermal effect on the spin torque is only encoded in the dependence of the magnetization vector.

To establish the thermal properties from the above stochastic equation, one must take a proper thermal average. Fortunately, much of theoretical work on the stochastic LL in the absence of the spin torque had been carried out.[4,15] Here we will follow and generalize the procedure pioneered by Brown.[4] We define $P(\mathbf{M},t)$ as a nonequilibrium probability density for magnetic orientation vectors associated with the stochastic equation (3). The rate equation for $P(\mathbf{M},t)$ is

$$\frac{\partial P}{\partial t}+\nabla\cdot\mathbf{J}-\lambda\nabla^2P=0, \tag{5}$$

where the probability current density $\mathbf{J}=Pd\mathbf{M}/dt$, ∇ is a short notation for the gradient operator on the magnetization vector $\nabla_{\mathbf{M}}$, and λ is the diffusion constant whose value is determined by fluctuation-dissipation theorem. In the present case, λ is related to D by $\lambda=D\gamma^2(1+\alpha^2)$. The above rate equation, Eq. (5), is a simple statement for the angular momentum conservation: the change of probability density in an enclosed small volume (first term) has to be balanced by the net probability in-flowing flux (second term) plus the probability density loss via spin diffusion (third term). By inserting Eq. (3) into Eq. (5), after a straightforward but rather tedious algebra manipulation,[16] the resulting equation is the Fokker-Planck equation,

$$\frac{\partial P}{\partial t}=-\nabla\cdot\left\{\left[-\gamma\mathbf{M}\times\mathbf{H}_{eff}+\mathbf{\Gamma}_s-\frac{\gamma\alpha}{M_s}\mathbf{M}\times(\mathbf{M}\times\mathbf{H}_{eff})+\gamma^2(1+\alpha^2)D\mathbf{M}\times(\mathbf{M}\times\nabla)\right]P\right\}. \tag{6}$$

In the absence of the spin torque, i.e., $\mathbf{\Gamma}_s=0$, the thermal equilibrium distribution density P demands to take the form of the Boltzmann distribution function, i.e., $P(a_J=0,T)\propto\exp(-E/k_BT)$ where T is the temperature and E is the energy defined by $H_{eff}=-\nabla E$. Inserting this equilibrium $P(a_J=0,T)$ into Eq. (6), one finds that

$$D=\frac{\alpha}{1+\alpha^2}\frac{k_BT}{\gamma M_s}; \tag{7}$$

this is the well-known dissipation-fluctuation relation. We now postulate that *the fluctuating field is independent of the spin torque*. This hypothesis is consistent with our notion that the spin torque is a deterministic action so that the spin torque does not alter the randomness induced by the thermal fluctuation. With this identification, the stochastic LL equation, Eq. (3), and its corresponding Fokker-Planck equation, Eq. (6), completely determine the dynamics of the magnetization vector at finite temperature T.

III. STATIONARY SOLUTION AND EFFECTIVE TEMPERATURES

Before we numerically solve the above stochastic LL equation for a number of interesting cases, we should first look for a stationary solution in Eq. (6), i.e., $P=P_0(\mathbf{M})$ is independent of time. Without the spin torque, this solution is known as the equilibrium Boltzmann distribution function mentioned earlier. With the spin torque, the system is no more in an equilibrium state because the system is subject to the spin torque and thus it is not a closed system. For an open system, the law of thermal dynamics does not require the minimum free energy and the concept of thermal equilibrium breaks down. Nevertheless, it is still meaningful to obtain a stationary solution P_s where Fokker-Planck probability density is time independent $\partial P_s/\partial t=0$. Thus,

$$\nabla\cdot\left\{\left[-\gamma\mathbf{M}\times\mathbf{H}_{eff}+\Gamma_s-\frac{\gamma\alpha}{M_s}\mathbf{M}\times(\mathbf{M}\times\mathbf{H}_{eff})\right.\right.$$
$$\left.\left.+\gamma^2(1+\alpha^2)D\mathbf{M}\times(\mathbf{M}\times\nabla)\right]P_s\right\}=0. \tag{8}$$

Unfortunately, the above eigenstate problem for an arbitrary field $\mathbf{H}_{eff}$ is generally difficult to solve. One can immediately verify that the Boltzmann probability density $P_0 \propto \exp(-E/k_BT)$ is no more a solution of the above equation. To make progress, we need to consider a special case below. First, we recall that the Néel-Brown formula for the thermal agitation is in fact most useful where the energy barrier constructed by $\mathbf{H}_{eff}$ is much higher than the thermal energy k_BT. In this limit, the probability density will be very small if the direction of the magnetization vector is away from the energy minimum. Here we should also consider this case. We now tentatively seek a solution of P_s in the form of $P_s \propto \exp(-E/k_BT^*)$ where we have introduced an effective temperature T^* that will be determined next. By placing this P_s into Eq. (8) and by noticing

$$\nabla P_s=-\frac{\nabla E}{k_BT^*}P_s\equiv\frac{\mathbf{H}_{eff}}{k_BT^*}P_s, \tag{9}$$

the first term of Eq. (8) is

$$-\gamma\nabla\cdot[(\mathbf{M}\times\mathbf{H}_{eff})P_s]=-\gamma k_BT^*\nabla\cdot(\mathbf{M}\times\nabla P_s). \tag{10}$$

By realizing the vector algebra relation $[\nabla_{\mathbf{r}}\cdot(\mathbf{r}\times\nabla_{\mathbf{r}}f)=0$ for any function $f(\mathbf{r})]$, one immediately sees that Eq. (10) is identically zero. Next, we combine the last two terms of Eq. (8) by using Eq. (9) again, and we have

$$\nabla\cdot\left\{\left[a_J\mathbf{M}\times(\mathbf{M}\times\hat{\mathbf{M}}_p)+\alpha\left(\frac{T}{T^*}-1\right)\mathbf{M}\times(\mathbf{M}\times\mathbf{H}_{eff})\right]P_s\right\}$$
$$=0. \tag{11}$$

Clearly, the above equation does not necessarily have a solution for an arbitrary effective field. However, as we point out earlier, we have limited ourselves to a high barrier case such that the magnetization vector at the stationary condition is nearly at the direction of $\mathbf{M}_p$. For concreteness, let us consider a most experimentally relevant geometry where $\mathbf{M}_p=\mathbf{e}_x$ and

$$\mathbf{H}_{eff}=\left(H_{ext}+\frac{H_K}{M_s}M_x\right)\mathbf{e}_x-4\pi M_z\mathbf{e}_z, \tag{12}$$

where H_{ext} is the external field which is applied at x direction, H_K is the anisotropy field, and $-4\pi M_z\mathbf{e}_z$ is the demagnetization field perpendicular to the plane of the film. In this case, the energy minimum are at $M_x=\pm M_s$, $M_y=M_z=0$. We simply keep the first order in M_y and M_z, and set $M_x=\sqrt{M_s^2-M_y^2-M_z^2}=M_s$ up to the first order; we find

$$\mathbf{M}\times(\mathbf{M}\times\mathbf{H}_{eff})\approx M_y(H_{ext}+H_K)\mathbf{e}_y$$
$$+M_z(H_{ext}+H_K+4\pi M_s)\mathbf{e}_z, \tag{13}$$

and $\mathbf{M}\times(\mathbf{M}\times\mathbf{M}_p)\approx M_y\mathbf{e}_y+M_z\mathbf{e}_z$. By placing these expressions into Eq. (11) and carrying out the divergence ∇ for M_y and M_z components we find [one noticed that the divergence operator on P_s produces higher orders since Eq. (13) is already the first order in M_y and M_z, thus it is consistent with our approximation by neglecting the terms of divergence on P_s],

$$\alpha(2\pi M_s+H_{ext}+H_K)\left(\frac{T}{T^*}-1\right)+a_J=0, \tag{14}$$

or

$$T^*=T\left(1-\frac{a_J}{a_c}\right)^{-1}, \tag{15}$$

where we defined a critical spin torque $a_c=\alpha(H_{ext}+H_K+2\pi M_s)$. Coincidentally, this critical spin torque is precisely the minimum spin torque required to switch the magnetization at zero temperature.[17]

We should point out that the concept of the effective temperature introduced here should be understood in terms of the stationary solution of the probability density. The thermally averaged dynamical variable, for example, the magnetization vector $\langle\mathbf{M}\rangle=\int P_s\mathbf{M}\sin\theta d\theta d\phi$ would behave as if the temperature of the system is T^*. However, the magnetic temperature which is defined through the thermal fluctuation remains to be T. An alternative understanding of this effective temperature is to rewrite the stationary solution by $P_s \propto \exp(-E^*/k_BT)$ where E^* is an effective activation energy

$$E^*=E\left(1-\frac{a_J}{a_c}\right). \tag{16}$$

In other words, we can state that the spin torque alters the magnetic energy and thus there will be an effective energy barrier associated with the spin current. Therefore, it is equivalent to think of the effect of the spin torque via the modification of the temperature or of the energy barrier.

To conclude this section, we have found a stationary solution P_s of the stochastic LL equation. Since the life time or the relaxation time τ is inversely proportional to the probability density P_s, we can write the generalized Néel-Brown formula below

$$\tau^{-1}=f_0\exp\left(-\frac{E_b(1-a_J/a_c)}{k_BT}\right), \tag{17}$$

where f_0 is an attempt frequency and E_b is the energy barrier.

IV. COMPARISON BETWEEN NUMERICAL AND ANALYTICAL RESULTS

The stationary solution, $P_s\propto\exp[-E(\mathbf{M})/k_BT^*]$, is based on the assumption of high-energy barrier. In general cases, one should start the calculation of magnetization dynamics from our generalized LL equation, Eq. (3). Since the stationary solution is simple and easy to use in analyzing experimental data, it would be necessary to establish the range of validity of Eq. (17) for various interesting experimental situations. Once its validity is established, we expect that our stationary solution would be serving as a starting point to understand various thermal agitation phenomena in the present of the current. In this section, we numerically integrate Eq. (3) and compare the result with P_s.

The standard white-noise spectrum, Eq. (4), where D is given by Eq (7), is used for the modeling of the temperature dependence of random fields. The calculation procedure is same as that for the standard LLG equation, except a spin torque is added to the equation of the motion. A magnetic layer, whose lateral size is 64 nm$\times$64 nm and whose thickness is 2.5 nm, is treated as a single macrospin. The in-plane uniaxial anisotropy field H_K is 500 Oe and the saturation magnetization is $4\pi M_s=12\,000$ Oe. These parameters are reasonably consistent with the experiments performed by the Cornell group.[8] The Gilbert damping constant was taken as $\alpha=0.03$ throughout the modeling. The magnetization of the free layer is initially saturated at $+x$ direction. At $t=0$, we apply a magnetic field at $-x$ direction whose magnitude is close to but less than the anisotropy field H_K. At the same time, a_J is also applied to the system. Equation (3) was numerically integrated in time using the stochastic Heun method with a 0.3 ps time step. A smaller time step has been tested and it yields nearly identical results in all the cases presented in the paper.

With above specified parameters and procedure, we first determine the probability $P(t)$ that the magnetic layer has been reversed within the waiting time t. By performing up to 5×10^4 independent runs for each set of parameters (each run starts at $t=0$ and ends at the time that the magnetization has either been just switched or ends at the time up to $t=5$ μs, whichever is smaller) and then by recording the number of them that the magnetization is switched at time interval $(t,\ t+\Delta t)$, we obtained a simulated $P(t)$ that is fitted by a simple exponential function, i.e., $P(t)=1-\exp(-t/\tau)$ where τ is the fitting parameter for the relaxation time. We have found that the fit works remarkably well for any values of a_J we have considered. In Fig. 1, we show the fitted relaxation time τ as a function of the temperature for a fixed applied field $H_{ext}=440$ Oe. Two features are immediately seen: first the data fall on a straight line for any fixed a_J; this indicates the thermally assisted reversal can be described by an activation process, i.e., $\ln\tau\propto1/k_BT$. Therefore, it is meaningful to introduce an effective activation energy, see Eq. (16), in accordance with the Néel-Brown law of thermal agitation. The second conclusion is that the slope, or the activation energy depends on STT: the positive a_J favors a lower-energy barrier. All these features are well described by Eqs. (15)–(17).

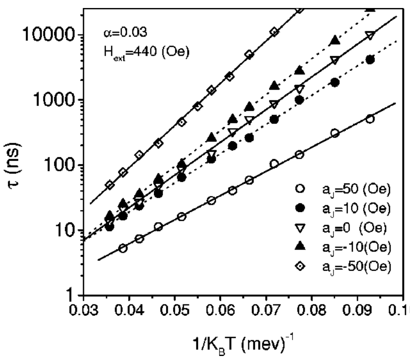

FIG. 1. Relaxation time τ as a function of the inverse of the temperature for several values of the spin-transfer torques. The external field of $H_{ext}=440$ Oe is applied along $-x$ direction.

In Fig. 2, the effective activation energies are shown to be linearly dependent on the current and they vanish at almost the same point a_c for different external fields [note that a_c is weakly dependent on the external field, see the definition of a_c after Eq. (15)]. In the insert of Fig. 2, we have shown the activation energy as a function of the magnetic field for several different STT. The activation energy can be fitted by

$$E_b^*=E_b(H_{ext})\left(1-\frac{a_J}{a_c}\right)=E_0\left(1-\frac{H_{ext}}{H_s}\right)^{\beta}\left(1-\frac{a_J}{a_c}\right), \tag{18}$$

where H_s is the switching field at zero temperature, E_0 is the energy barrier at zero magnetic field, and β is a constant,

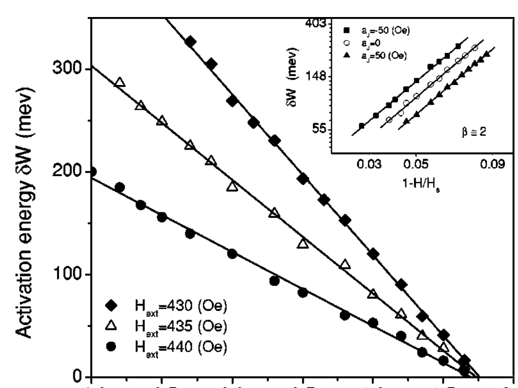

FIG. 2. Activation energy E_b^* as a function of a_J at three different external fields. Inset: E_b^* vs $1-H_{ext}/H_s$ for several different values of STT.

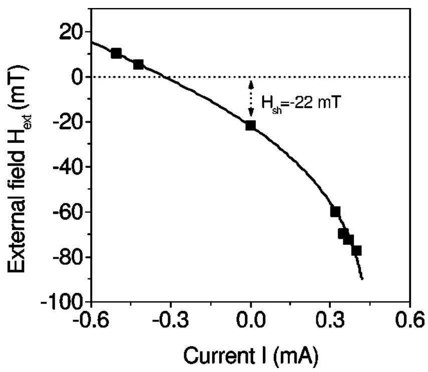

FIG. 3. H-I Phase boundary of equal dwell times $\langle \tau_{AP} \rangle = \langle \tau_P \rangle$. The coupling field is $H_{sh} = -22$ mT, the critical currents are $I_c^{AP} = -1.25$ mA for the transition from AP to P alignments and $I_c^P = 0.425$ mA from P to AP alignments. Line; Eq. (19) except H_{ext} being replaced by $H_{ext} + H_{sh}$; Square, experimental data (Refs. 11 and 12).

which has been argued to be 1.5 or 2. The exponent $\beta = 2$ for the external field applied parallel to the easy axis. These simulated results confirm our analytical result, Eq. (16).

V. COMPARISON WITH EXPERIMENTAL DATA

A number of experiments on spin torque induced thermal agitation had been carried out. It would be interesting to see whether our prediction, Eq. (17), agrees with these existing data. The phenomenon that we want to compare first is the so-called ''telegraph noises'' or dwell times. Experimentally, one simultaneously applies an external magnetic field and a spin current to a spin valve structure so that the magnetization direction of the free layer is fluctuating from one direction to another due to thermal agitation.[10,12] The dwell time τ_P (τ_{AP}) is defined as an average time the magnetization of the free layer is parallel (antiparallel) to that of the fixed layer. In general, $\tau_P \neq \tau_{AP}$. However, by adjusting the magnetic field or the spin current, one is able to obtain an equal dwell time for parallel and antiparallel states, $\tau_P = \tau_{AP}$. From Eq. (17) for the parallel and the antiparallel states, the condition of the equal dwell time is

$$\left(1 + \frac{H_{ext}}{H_s}\right)^{1.5}\left(1 - \frac{I}{I_c^{AP}}\right) = \left(1 - \frac{H_{ext}}{H_s}\right)^{1.5}\left(1 - \frac{I}{I_c^{P}}\right), \tag{19}$$

where I_c^{AP} and I_c^P are the critical currents for the magnetization switching from antiparallel to parallel alignments and vice versa; their magnitudes are not necessary the same, i.e., $I_c^{AP} \neq -I_c^P$ in a typical experimental geometry.[11,12] To compare our prediction, Eq. (19), with experiments, we plot the H-I phase diagram of equal dwell time $\tau_P = \tau_{AP}$ in Fig. 3. It is noted that we have shifted the external field by $H_{sh} = -22$ mT to take into account of the magnetic coupling between the free and fixed layers. The coupling may come from either the exchange or dipolar couplings. Since we assume that the magnetization of the fixed layer is held at the direction of $\mathbf{e}_x$, the free layer receives an effective coupling field that will be added to H_{ext} in Eq. (19). Evidently, the agreement between our results and the experimental data is excellent.[11,12]

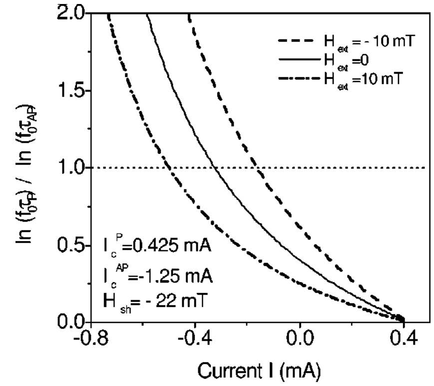

FIG. 4. Ratio of the relaxation times $\ln(f_0\tau_P)/\ln(f_0\tau_{AP})$. The parameters are same as those in Fig. 3.

Next we compare the ratio of the dwell times of antiparallel and parallel states for a fixed magnetic field. Again, from Eq. (17), we have

$$\frac{\ln(f_0\tau_P)}{\ln(f_0\tau_{AP})} = \frac{\left(1 - \frac{H_{ext}}{H_s}\right)^{1.5}\left(1 - \frac{I}{I_c^{P}}\right)}{\left(1 + \frac{H_{ext}}{H_s}\right)^{1.5}\left(1 - \frac{I}{I_c^{AP}}\right)}. \tag{20}$$

In Fig. 4, we show the ratio of the dwell times for a fixed magnetic field as a function of the spin torque by using the same set of experimental parameters as in Fig. 3. The results are consistent with experimental data (however, the data points in Refs. 11 and 12 are rather scattered so that we do not include those data in the figure).

Up till now, we have studied the thermal activation by *abruptly* introducing an external field and a STT at $t=0$. In experiments, there are ramping times, e.g., STT is gradually increasing at rate of, say 10^{-5} Oe/μs (Ref. 13) and care must be taken when one compares our theory, Eq. (17), with experiments. In the current ramping period, a_J is not a constant and thus the activation energy, Eq. (18), varies with time. In this case, one should utilize the differential form of the switching probability instead of $P(t) = 1 - \exp(-t/\tau)$,

$$\frac{dP(t)}{1 - P(t)} = \frac{dt}{\tau(t)}. \tag{21}$$

By assuming a linear ramping of STT, i.e., $C = da_J/dt$ is a constant and by placing Eqs. (17) and (18) into Eq. (21), we integrate Eq. (21) from $t=0$ to $t=t_0$ and find the average switching STT $a_c(T,C) \equiv a_J(t_0)$

$$a_c(T,C) \cong a_c\left[1 - \frac{k_B T}{E_b}\ln\left(\frac{f_0' k_B T a_c}{E_b C}\right)\right], \tag{22}$$

where $f_0' = -f_0 \ln[1 - P(t_0)]$. The variance of the switching STT is found as

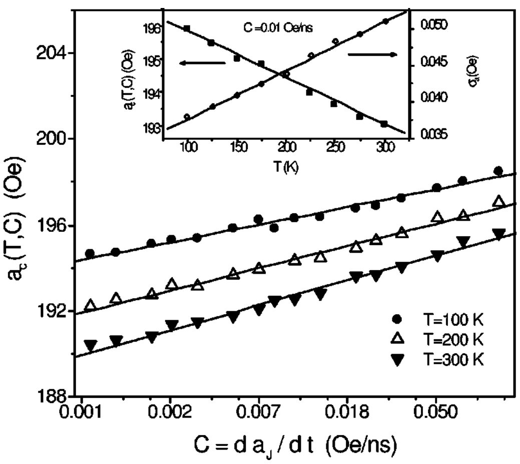

FIG. 5. Dependence of $a_c(T,C)$ on the sweeping rate C at finite temperature. Inset: $a_c(T,C)$ and σ_a as a function of temperature for the sweeping rate of STT at 0.01 Oe/ns.

$$\sigma_a \cong a_c \frac{k_B T}{E_b}. \tag{23}$$

These relations, Eqs. (22) and (23), are consistent with earlier studies on the similar energy barrier formalism for a very different physical system.[18]

Numerically, the finite ramping rate can be rather easily handled. In determining $a_c(T,C)$ at finite temperature, we ramp a_J with a fixed rate. At a certain time, the magnetization vector switches and we record the value of a_J. By repeating the above procedure 800 times, we are able to establish the switching a_J histograms from which the mean switching $a_c(T,C)$ and its standard deviation σ_a are obtained. At temperature between 100 K and 300 K, we calculated the mean and standard deviation of the distributions at a sweeping rate between 0.001 Oe/ns and 0.1 Oe/ns. As expected for a thermally activated process, $a_c(T,C)$ increases with decreasing temperature and with increasing sweeping rate. Figure 5 shows temperature and sweeping rate dependence of $a_c(T,C)$ and σ_a. A logarithmic dependence of $a_c(T,C)$ on the sweeping rate has been found, which is in a good agreement to Eq. (22). Moreover, the inset of Fig. 5 describes the temperature dependence of $a_c(T,C)$ and σ_a. We have verified that the temperature dependence of σ_a is a linear relationship and $a_c(T,C)$ monotonically decreases with increasing T.

Myers *et al.*[13] have discovered that the thermal activation driven by spin torque is qualitatively different from that driven by the magnetic field. They have suggested an activation energy whose form is similar to ours, except that they have postulated an arbitrary exponent, i.e., $\delta W \propto (1 - a_J/a_c)^{\xi}$. Although they have stated that ξ might be 1.5, most of experimental data shown in their paper can be used to determine the value of ξ. One set of data, Fig. 1(d) of Ref. 13, shows that $a_c(T,C)$ linearly increases with $\ln C$ as predicted by our Eq. (22). If one uses different scaling relation, e.g., $\delta W \propto (1 - a_J/a_c)^{1.5}$, one would obtain $a_c(T,C) \propto (\ln C)^{2/3}$ that would disagree with the experimental data. Therefore, the existing data support the linear scaling between the activation energy and the spin torque.

VI. CONCLUSIONS

In summary, we have extended the law of thermal agitation to include the spin-transfer torque driven by the spin-polarized current in magnetic multilayers. Although the concept of the energy barrier or the temperature in the Néel-Brown formula breaks down in the presence of the spin-transfer torque, we are able to reestablish the Néel-Brown formula by properly introducing an activation energy or an effective temperature to replace the true energy barrier or true lattice temperature. Our formalism is further supported by numerical solutions and is in agreement with experimental results.

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