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Correlation functions of the magnetization in thin films

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We calculate the correlation functions of uniform magnetization in thin ferromagnetic films for small deviations from equilibrium using a functional formalism. To take account of dissipation and fluctuations consistently, the magnetization is coupled linearly to a bosonic heat bath. The correlation functions of the magnetization strongly depend on the density of states of the bath. Depending on what density of states we choose, we show how the recent results of Smith [J. Appl. Phys. **90**, 5768 (2001)] and Safonov and Bertram [Phys. Rev. B **65**, 172417 (2002)] obtained by macroscopic methods can be understood in light of the microscopic treatment presented here.

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I. INTRODUCTION

The problem of magnetic noise in nanosystems and in particular in giant-magnetoresistive read sensors is of considerable importance to the physics of magnetic recording. For macroscopic systems, the classical Landau-Lifshitz-Gilbert (LLG) equation has been used frequently to study thermal fluctuations in magnetic systems mainly through a stochastic approach.[1] Magnetic noise in thin anisotropic films with the magnetization close to equilibrium has been recently treated by at least a couple of different methods with different results.[2–4] The first work by Smith[2] is based on a linearized LLG and application of the fluctuation-dissipation theorem (FDT).[5] The second method by Safonov and Bertram[3] is also approximate and based on analogies with the harmonic oscillator and a seemingly different form of the FDT.[6]

To better understand this discrepancy, we study this particular problem of magnetic noise in thin films from a microscopic point of view. Since we are dealing with a magnetization slightly disturbed from equilibrium, we use bosonic degrees of freedom to describe the magnetization.[7] While this is a rather crude approximation, it is equivalent to the one used in Refs. 2 and 3. Moreover, we use the language of coherent states (CS) to describe the states of the magnetization since such a representation is the most natural for the semiclassical limit.[8] The dissipation is simulated by coupling the magnetization to a bosonic environment.[9] The noise spectrum is found by calculating the correlation functions of the magnetization. We carry out the calculations with and without use of the FDT. We find that both methods give the same result. However not using the FDT entails using functional methods not commonly used in recoding physics.[10,11] These methods are attractive because they are equally applicable to highly nonequilibrium situations which involve important problems such as the switching of the magnetization.

Two of us have already used functional methods in a recent paper that addressed the conditions under which a LLG equation can be recovered from a simple quantum model.[12] The results presented here complement those presented in Ref. 12. However this paper can be read independently of our previous work. The major result of this work is general expressions for the correlation functions of the transverse components of the magnetization from which we can recover both LLG-type correlation functions and oscillatorlike correlation functions for the simplified Hamiltonian. Currently experiments do seem to favor the LLG result, however, we will not address these questions here.[4,13] We simply show that for systems close to equilibrium and within the harmonic-oscillator approximation for the transverse components of the magnetization, we can have different theories for a large spin. However, it must be kept in mind that the results obtained are based on an artificial Hamiltonian and a very simple coupling between the magnetization and the bath. A more realistic coupling such as that of conduction electrons interacting with localized magnetic moments is treated elsewhere.[14]

The paper is organized as follows. In Sec. II, we first introduce the Hamiltonian for the system considered here. Then we write this Hamiltonian in a simplified approximate form valid for near-equilibrium cases. It is shown that if we use the same approximations as those used in Refs. 2 and 3, the spin Hamiltonian is exactly that of an oscillator. In Sec. III, we first derive the CS generating functional for this system. Then, we derive the general correlation functions for the magnetization without using the FDT. The linearized LLG result is shown to follow from the general result by assuming a bath for which the product of the density of states and the coupling constants is linear with frequency. This is the same condition recovered in Ref. 12. To get the Safonov-Bertram result,[3] we set the product of the coupling constants and the density of states to a constant. We find that in this particular case there is no need to introduce a tensor damping or use the rotating wave approximation to get our results. At the end of this section, we address questions[15] that put in doubt the correctness of the application of the FDT in deriving the correlation function in Ref. 3. We find that an equilibrium calculation produces the same results as those obtained by real-time nonequilibrium methods. Finally in Sec. IV, we summarize and discuss our results.

II. MICROSCOPIC MODEL

In this section, we introduce a microscopic model for a thin film and approximate the magnetization operator by a bosonic operator stressing the analogies with the harmonic oscillator (see, e.g., Ref. 16).

We consider the following system: A thin magnetic slab with easy axis along the z axis and a hard axis along the x axis which are both in plane. We assume that there is a large external magnetic field $\mathbf{H}$ along the easy axis that keeps the average magnetization in plane. We will be interested only in fluctuations around the equilibrium position of the magnetization, i.e., fluctuations in the x and y components of the magnetization. The Hamiltonian for such a system, coupled to a bosonic heat bath, has the general form ($\hbar = 1$)

$$\hat{\mathcal{H}} = -H\hat{S}_z - \frac{K_1}{2}\hat{S}_z^2 + \frac{K_2}{2}\hat{S}_x^2 + \sum_k \omega_k b_k^+ b_k + V(b_k, b_k^\dagger, \hat{\mathbf{S}}), \tag{1}$$

where K_1 and K_2 are the anisotropy constants and ω_k is the energy of the kth bath's oscillator. The spin-bath interaction V will be taken linear in $\hat{\mathbf{S}}$ and the bath variables b_k. A possible form for V is

$$V(b_k, b_k^\dagger, \hat{\mathbf{S}}) = \sum_k (\gamma_k^* \hat{S}_+ b_k + \gamma_k b_k^+ \hat{S}_-), \tag{2}$$

where

$$\hat{S}_+ = \hat{S}_x + i\hat{S}_y, \tag{3}$$

$$\hat{S}_- = \hat{S}_x - i\hat{S}_y. \tag{4}$$

γ_k's are the coupling constants, which can be time dependent. The bosonic-type operators for the bath oscillators, b_k, satisfy the usual commutation relations

$$[b_k, b_k^+] = \delta_{kk'}. \tag{5}$$

For the magnetization, $\hat{\mathbf{S}}$, we have the following commutation relation:

$$[\hat{S}_+, \hat{S}_-] = 2\hat{S}_z. \tag{6}$$

For precession around the equilibrium position, i.e., the z axis, S_z is usually assumed to be a constant[17] and the Hamiltonian expression for the spin part, $\hat{\mathcal{H}}_S$, can be simplified to be of the general form

$$\hat{\mathcal{H}}_S = \tfrac{1}{2}(A\hat{S}_x^2 + B\hat{S}_y^2), \tag{7}$$

where

$$A = K_1 + K_2, \quad B = K_1, \tag{8}$$

that is, the energy expression which was assumed in both calculations by Smith[2] and Safonov and Bertram.[3]

To account for thermal fluctuations, we need to calculate the two-point correlation functions of the transverse components of the magnetization. It is now more appropriate to define the following operators,[7]

$$a = \frac{1}{(2S)^{1/2}}\hat{S}_+ \tag{9}$$

and

$$a^+ = \frac{1}{(2S)^{1/2}}\hat{S}_-, \tag{10}$$

then, we have

$$\hat{S}_z \approx S - a^+ a, \tag{11}$$

which is the Holstein-Primakoff approximation at low temperature. However, here we insist on taking $\widehat{S_z}$ to be a constant operator with magnitude S. Then we can normalize by $2S$ the Hamiltonian of the system. The operators a and a^+ then behave as bosonic degrees of freedom, i.e., the magnetization behaves, in this approximation, like a harmonic oscillator (see Ref. 18 for a discussion of the validity of this approximation). If we rewrite the Hamiltonian in terms of these operators, we find

$$\hat{\mathcal{H}} = \Omega a^+ a + V(a, a^+) + \sum_k \omega_k b_k^+ b_k - \sum_k \gamma_k (a^+ b_k + b_k^+ a), \tag{12}$$

where

$$\Omega = H + K_1 + \tfrac{1}{2}K_2, \tag{13}$$

and the potential V is in this case equal to

$$V(a^+, a) = \tfrac{1}{4}K_2(aa + a^+ a^+). \tag{14}$$

In the following, we use this simplified Hamiltonian within a path-integral formalism in both real and imaginary time (i.e., in the nonequilibrium and equilibrium cases) to calculate the transition probabilities which are needed for the correlation functions. From this, we are able to identify the cause of the discrepancy between Refs. 2 and 3.

III. THE CORRELATION FUNCTIONS OF THE MAGNETIZATION IN THE HARMONIC-OSCILLATOR APPROXIMATION

To set the notation for what follows, we briefly review the coherent-state representation for the magnetization and the bath. This representation is used in a path-integral approach to define a functional generator for irreducible Green's functions. This functional is then used to calculate the symmetrized two-point correlation functions. The calculation is carried out using two different methods. One is based on the real-time approach, while the second is based on the imaginary-time (equilibrium) formalism. Both approaches are path-integral approaches. The equilibrium approach makes use of the FDT while the real-time one does *not* depend on it. Both methods are shown to give the same answer and hence the discrepancy in the results of Safonov and Smith is not due to a faulty use of the FDT.

A. The coherent-state representation : Equilibrium and nonequilibrium dynamics

Coherent states are the natural representation for semiclassical calculations. A Gaussian wave packet for a harmonic oscillator with minimum uncertainty is a coherent state. They are formally defined as eigenstates of the annihilation operator[8]

$$a|\alpha\rangle=\alpha|\alpha\rangle, \tag{15}$$

where α is a complex number. An important operator relation for a path-integral representation is the decomposition of the unit operator in terms of coherent projection operators

$$\int \frac{d\alpha^* d\alpha}{2\pi i} e^{-\alpha^*\alpha}|\alpha\rangle\langle\alpha|=\hat{1}, \tag{16}$$

which is used in the discretization of the path integral.[19] The coherent states form an over-complete basis.

The real-time formulation deals with nonequilibrium questions.[11] This is the method we adopt in the calculations of the correlation functions of the magnetization without use of the FDT. For a general operator $\mathcal{O}$, its average value at any time t is given in terms of the density matrix ρ,

$$\langle\mathcal{O}(t)\rangle=\mathrm{Tr}\langle\rho\mathcal{O}(t)\rangle. \tag{17}$$

The operator $\mathcal{O}$ is in the Heisenberg picture,

$$\mathcal{O}(t)=e^{i\hat{\mathcal{H}}t}\mathcal{O}e^{-i\hat{\mathcal{H}}t}. \tag{18}$$

Therefore, the average of the observable $\mathcal{O}$ at time t can be written in terms of that at $t=0$,

$$\langle\mathcal{O}(t)\rangle=\mathrm{Tr}(\rho e^{i\hat{\mathcal{H}}t}\mathcal{O}e^{-i\hat{\mathcal{H}}t}). \tag{19}$$

This latter average can be written in terms of path integrals as in the equilibrium case.[19] First we define the operators $\mathcal{K}$ and $\overline{\mathcal{K}}$ which are functions of external sources J and J^*. The operator $\mathcal{K}$ is a forward propagator and is defined as follows:

$$\begin{aligned}\mathcal{K}[J_1,J_1^*]&=\mathbf{T}\exp\left\{-i\int_{t_i}^{t_f}(\hat{\mathcal{H}}-J_1^*a-J_1a^+)dt\right\}\\&=\mathbf{T}\exp\left\{-i\int(\hat{\mathcal{H}}-F_1^xS_x-F_1^yS_y)dt\right\},\end{aligned} \tag{20}$$

where $\mathbf{T}$ is the time-ordering operator and $\mathbf{F}_1$ and $\mathbf{F}_2$ are real external fields which are coupled to the transverse components of the magnetization. $\overline{\mathcal{K}}$ is a backward operator and is therefore defined in terms of antiordered time operator $\overline{\mathbf{T}}$,

$$\begin{aligned}\overline{\mathcal{K}}[J_2,J_2^*]&=\overline{\mathbf{T}}\exp\left\{-i\int_{t_f}^{t_i}(\hat{\mathcal{H}}-J_2^*a-J_2a^+)dt\right\}\\&=\overline{\mathbf{T}}\exp\left\{-i\int(\hat{\mathcal{H}}-F_2^xS_x-F_2^yS_y)dt\right\}.\end{aligned} \tag{21}$$

Next, we define a generating functional

$$\mathbb{Z}[\mathbf{J},\mathbf{J}^*]=\mathrm{Tr}\{\rho[J_3,J_3^*]\overline{\mathcal{K}}[J_2,J_2^*]\mathcal{K}[J_1,J_1^*]\}. \tag{22}$$

$\mathbf{J}$ is now the three-vector (J_1,J_2,J_3). The density matrix ρ is assumed of the form

$$\begin{aligned}\rho[J_3,J_3^*]&=\mathbf{T}_I\exp\left\{-\int_0^\beta(\hat{\mathcal{H}}-J_3^*a-J_3a^+)d\tau\right\}\\&=\mathbf{T}_I\exp\left\{-\int_0^\beta(\hat{\mathcal{H}}-F_3^xS_x-F_3^yS_y)d\tau\right\},\end{aligned} \tag{23}$$

where $\mathbf{T}_I$ is now a time-ordering operator along the imaginary-time axis. Hence, all correlation functions can be obtained from the coefficients of the Taylor expansion of the functional $\mathbb{Z}[\mathbf{J},\mathbf{J}^*]$ around $\mathbf{J}=\mathbf{J}^*=0$ (or $\mathbf{F}_1=\mathbf{F}_2=\mathbf{F}_3=0$). For example, the average value of the x component of the magnetization at time t can be found by differentiating $\mathbb{Z}$ with respect to F_1^x at the same time t,

$$\left.\frac{1}{\mathbb{Z}}\frac{\delta\mathbb{Z}[\mathbf{J},\mathbf{J}^*]}{\delta F_1^x(t)}\right|_{F=0}=-\langle S_x(t)\rangle. \tag{24}$$

Next we define another functional $\mathbb{W}$ which at equilibrium becomes the thermodynamic potential of the system,

$$\mathbb{Z}[\mathbf{F}_{i=1,2,3}]=\exp\{i\mathbb{W}[F]\}. \tag{25}$$

The functional $\mathbb{W}$, as will be seen below, is the more appropriate functional to calculate and expand in powers of $\mathbf{J}$ and $\mathbf{J}^*$ (or $\mathbf{F}$). Therefore, we have for averages and two-point correlation functions,

$$\left.\frac{\delta\mathbb{W}}{\delta F_1^x(t)}\right|_{\mathbf{F}=0}=\langle S_x(t)\rangle \tag{26}$$

and

$$\left.\frac{\delta^2\mathbb{W}}{\delta F_1^x(t)\,\delta F_1^x(t')}\right|_{\mathbf{F}=0}=-i\langle\mathbf{T}(S_x(t)S_x(t'))\rangle. \tag{27}$$

Similar expressions hold when we differentiate $\mathbb{W}$ with respect to the sources $\mathbf{J}$ and $\mathbf{J}^*$. They are related to each other by the chain rule.

Next we give an explicit expression for the functional $\mathbb{W}$ in terms of coherent states and calculate all the associated propagators.

The generating functional $\mathbb{Z}$ is defined above, Eq. (22). Using coherent states, for both the bath and spin, this trace formula can be written in terms of path integrals over spin variables and bath variables,

$$\begin{aligned}\mathbb{Z}[\mathbf{J},\mathbf{J}^*]=&\int d\mu(\alpha_1)\int d\mu(\alpha_2)\int d\mu(\alpha_3)\int d\mu(\varphi_1)\\&\times\int d\mu(\varphi_2)\int d\mu(\varphi_3)\exp\{-|\alpha_1|^2-|\alpha_2|^2\\&-|\alpha_3|^2\}\exp\left\{-\sum_k(|\varphi_{1,k}|^2+|\varphi_{2,k}|^2+|\varphi_{3,k}|^2)\right\}\\&\times\langle\alpha_1,\varphi_1|\rho[J_3,J_3^*]|\alpha_2,\varphi_2\rangle\\&\times\langle\alpha_2,\varphi_2|\overline{\mathcal{K}}[J_2,J_2^*]|\alpha_3,\varphi_3\rangle\\&\times\langle\alpha_3,\varphi_3|\mathcal{K}[J_1,J_1^*]|\alpha_1,\varphi_1\rangle.\end{aligned} \tag{28}$$

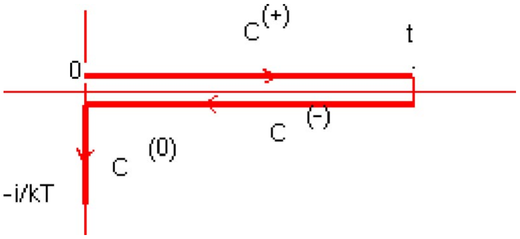

FIG. 1. Complex time path for the generating functional.

The $\alpha_{i=1,2,3}$ represent states of the spin system, while the $\varphi_{i=1,2,3}$ represent the bath states. This integral can be formally written as a path integral along the path in Fig. 1 with periodic boundary conditions similar to the equilibrium partition function calculations. This functional can be calculated exactly only in few cases in particular if the Hamiltonian is quadratic. Higher-order terms can be accounted for only approximately. This is best done through a graphical procedure such as the Feynman diagram technique. Here we have a quadratic Hamiltonian and hence we can solve for $\mathbb{Z}$, however, we will mention briefly what happens in the general case.

In our case, the bath degrees of freedom can be integrated out exactly and we can derive an exact effective action for the spin degrees of freedom. In the general case, the effective action can be derived perturbatively. From it, we calculate the correlation functions of **S**. We find

$$\begin{aligned}\mathbb{Z}[\mathbf{J},\mathbf{J}^*]=&\int d\mu(\alpha_1)\int d\mu(\alpha_2)\int d\mu(\alpha_3)\\&\times\exp\{-|\alpha_1|^2-|\alpha_2|^2-|\alpha_3|^2\}\int_{\alpha_1}^{\bar{\alpha}_3}d\mu(z_1)\\&\times\int_{\alpha_3}^{\bar{\alpha}_2}d\mu(z_2)\int_{\alpha_2}^{\bar{\alpha}_1}d\mu(z_3)\\&\times\exp\left\{\sum_{i=1}^{3}I_i[z_i,\bar{z}_i,J_i,J_i^*]\right\}\mathcal{F}(\mathbf{Z},\bar{\mathbf{Z}}),\end{aligned}\tag{29}$$

where $\mathcal{F}(\mathbf{Z},\bar{\mathbf{Z}})$ is the Feynman-Vernon functional for the spin-bath system given by

$$\ln\mathcal{F}(\mathbf{Z},\bar{\mathbf{Z}})=\int dt\int dt'\left[-\sum_k|\gamma_k|^2\bar{\mathbf{Z}}(t)\cdot G^k(t,t')\cdot\mathbf{Z}(t')\right].\tag{30}$$

The three-vector $\mathbf{Z}$ is related to the three branches of the curve C, Fig. 1,

$$\mathbf{Z}=\begin{pmatrix}z_1\\-z_2\\z_3\end{pmatrix}.\tag{31}$$

The time integrations are defined based on the path C:

$$t_i<t,t'<t_f,\quad t,t'\in C^{(+)},C^{(-)}$$

$$t_i<t,t'<t_i-i\beta,\quad t,t'\in C^{(0)}.$$

The Feynman-Vernon term is the only term which is dependent on the bath parameters. The functions $G_{ij}^k(t,t')$, nine in total, are propagators associated with the bath oscillators. Hence they can easily be calculated since the oscillator part of the Hamiltonian is quadratic and the spin can be considered as an external field.

In the following, we take account of the imaginary-time branch through the assumption that initially the system is in equilibrium. For a general potential V, the generating functional can be written in terms of that of a free system, $\hat{\mathcal{H}}_0=\Omega a^\dagger a$, interacting with the bath,

$$\mathbb{Z}[\mathbf{J},\bar{\mathbf{J}}]=\exp\left\{-i\int_C dtV\left[\frac{\partial}{\partial J_i(t)},\frac{\partial}{\partial\bar{J}_i(t)}\right]\right\}\mathbb{Z}_{SB}[\mathbf{J},\bar{\mathbf{J}}].\tag{32}$$

$\mathbb{Z}_{SB}$ is therefore the generating functional of a particle interacting with the bath only and with no external potential. This latter formula is valid in the general case and is the start of any perturbative calculations. The free action along the real-time trajectories is given by

$$iI_1^0=i\int dt\left[\frac{\dot{\bar{z}}_1z_1-\bar{z}_1\dot{z}_1}{2i}-\Omega\bar{z}_1z_1\right]\tag{33}$$

along the path $C^{(+)}$ and by

$$iI_2^0=-i\int dt\left[\frac{\dot{\bar{z}}_2z_2-\bar{z}_2\dot{z}_2}{2i}-\Omega\bar{z}_2z_2\right]\tag{34}$$

along the path $C^{(-)}$, Fig. 1. At $t_i\to-\infty$, the system is at equilibrium, then we can assume that the initial density matrix is thermal, with $J_3(t_i)=0$. Therefore we write that

$$\rho(t_i)=\frac{1}{Z(t_i)}e^{-\beta H(t_i)},\quad t_i\to-\infty.\tag{35}$$

Then, we observe that

$$\langle\alpha_1|\rho(-\infty)|\alpha_2\rangle=\int_{\alpha_1}^{\bar{\alpha}_2}d\mu(z_3)e^{iI_3^0[z_3,\bar{z}_3]}\mathcal{F}(z_3,\bar{z}_3),\tag{36}$$

where I_3^0 has the same expression as I_1^0 but with $t\to it$. Hence in this case, the initial density-matrix element is just another overall factor in the generating functional $\mathbb{Z}$,

$$\begin{aligned}\mathbb{Z}[\mathbf{J},\bar{\mathbf{J}}]=&\int d\bar{\mu}(\alpha_3)\int d\bar{\mu}(\alpha_1)d\bar{\mu}(\alpha_2)\langle\alpha_1|\rho(-\infty)|\alpha_2\rangle\\&\times\int_{\alpha_1}^{\bar{\alpha}_3}d\mu(z_1)\int_{\alpha_3}^{\bar{\alpha}_2}\Bigg\{d\mu(z_2)\\&\times\exp\left[-\int_{C^+}dtV\left(\frac{\partial}{\partial\mathbf{J}(t)},\frac{\partial}{\partial\bar{\mathbf{J}}(t)}\right)\right]\Bigg\}\\&\times\exp\Bigg\{iI_1^0[z_1,\bar{z}_1]+iI_2^0[z_2,\bar{z}_2]\\&+i\int dt(\mathbf{J}\cdot\bar{\mathbf{Z}}+\bar{\mathbf{J}}\cdot\mathbf{Z})\Bigg\}\mathcal{F}(\mathbf{Z},\bar{\mathbf{Z}}),\end{aligned}\tag{37}$$

with $\mathbf{Z}=(z_1,-z_2)$ and the measure is defined by

$$d\overline{\mu}(\alpha)=d\mu(\alpha)e^{-|\alpha|^2}. \tag{38}$$

Therefore we define a new generating functional $\hat{\mathbb{Z}}$,

$$\mathbb{Z}[\mathbf{J},\overline{\mathbf{J}}]=\int d\overline{\mu}(\alpha_3)\int d\overline{\mu}(\alpha_1)\times\left[\int d\overline{\mu}(\alpha_2)\langle\alpha_1|\rho(-\infty)|\alpha_2\rangle\times\hat{\mathbb{Z}}[\mathbf{J},\overline{\mathbf{J}}]\right].$$

We can now adopt a different notation that takes into account the path C implicitly by defining a scalar product and combine the different components into a single vector. The generating function becomes

$$\hat{\mathbb{Z}}[\mathbf{J},\overline{\mathbf{J}}]=\int d\mu(z_1)\int d\mu(z_2)\times\exp\left[-\int dtV\left(\frac{\partial}{\partial\mathbf{J}(t)},\frac{\partial}{\partial\overline{\mathbf{J}(t)}}\right)\right]\times\exp\left\{iI^0(\mathbf{Z},\overline{\mathbf{Z}})+i\int dt(\overline{\mathbf{Z}}\cdot\mathbf{J}+\overline{\mathbf{J}}\cdot\mathbf{Z})\right\}\mathcal{F}(\mathbf{Z},\overline{\mathbf{Z}}), \tag{39}$$

where now the vector $\mathbf{Z}$ is defined as

$$Z=\begin{pmatrix} z_1 \\ z_2 \end{pmatrix}, \tag{40}$$

and the free action is

$$I^0=\sum_{i,j}\sigma^{ij}I_j^0, \tag{41}$$

with

$$\sigma^{ij}=\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{42}$$

The complex scalar product is now defined by

$$\overline{\mathbf{Z}}\cdot\mathbf{J}=\sigma^{ij}\overline{z_j}J_j. \tag{43}$$

This notation makes it possible to take into account the closedness of the real-time path by just taking one branch of the curve C and doubling the components of the dynamical variables. The matrix σ_{ij} plays the role of a metric.

Now we turn to some properties satisfied by the functions G_{ij}^k. These properties are better displayed in Fourier space. The Fourier space representation of the Feynman propagator is given by

$$G_{11}^k(\omega)=\int dte^{i\omega t}[1+n(\omega_k)]\Theta(t)e^{-i\omega_k t}+\int dte^{i\omega t}n(\omega_k)\theta(-t)e^{-i\omega_k t} \tag{44}$$

$$=[1+n(\omega_k)]\frac{i}{\omega-\omega_k+i\epsilon}-n(\omega_k)\frac{i}{\omega-\omega_k-i\epsilon}, \tag{45}$$

where $\epsilon\rightarrow0^+$. $\mathcal{P}$ stands for the principal part of the integral. For the antitime-ordered propagator, we have

$$G_{22}^k(\omega)=\int dte^{i\omega t}n(\omega_k)\Theta(t)e^{-i\omega_k t}+\int dte^{i\omega t}[1+n(\omega_k)]\Theta(-t)e^{-i\omega_k t} \tag{46}$$

$$=n(\omega_k)\frac{i}{\omega-\omega_k+i\epsilon}-[1+n(\omega_k)]\frac{i}{\omega-\omega_k-i\epsilon}. \tag{47}$$

For the other remaining Green's functions, we have for positive ω,

$$G_{12}^k(\omega)=2\pi n(\omega_k)\delta(\omega-\omega_k) \tag{48}$$

and

$$G_{21}^k(\omega)=2\pi[1+n(\omega_k)]\delta(\omega-\omega_k). \tag{49}$$

These Green's functions are not all independent. We first observe that

$$G_{11}^k(\omega)+G_{22}^k(\omega)=G_{12}^k(\omega)+G_{21}^k(\omega). \tag{50}$$

This is an immediate result of their definition. Moreover, the term on the left-hand side is easily seen to be a symmetric sum of the product of two operators. Now it is not difficult to show from the above expressions of the Green's functions that we have

$$G_{11}^k(\omega)+G_{22}^k(\omega)=[1+2n(\omega)][G_{21}^k(\omega)-G_{12}^k(\omega)]. \tag{51}$$

The last factor on the right-hand side is an antisymmetric sum of two operators. Equation (51) is a statement of some form of the fluctuation-dissipation theorem.[20] These relations will be used in subsequent sections to calculate the correlation functions. In equilibrium, when the distribution functions are the Bose-Einstein functions

$$1+2n(\omega)=\coth\frac{\beta\omega}{2} \tag{52}$$

and Eq. (51) gives the usual form of the fluctuation-dissipation theorem.

B. A real-time calculation of the noise

After integrating out the bath degrees of freedom, we write the generating functional $\overline{\mathbb{Z}}$ in terms of the real operators $\hat{S}_x$ and $\hat{S}_y$. This doubling of the variables makes the remaining Gaussian integration slightly more complicated, since we will have to invert a 4×4 matrix. The bath contributes a term of the following form to the effective action:

$$
\begin{aligned}
S_{eff}(\mathbf{S}_1,\mathbf{S}_2)=&\int\frac{d\omega}{2\pi}|\gamma_k|^2G^k_{11}(\omega)[S^2_{1,x}+S^2_{1,y}+i\overline{S}_{1,x}S_{1,y}\\
&-i\overline{S}_{1,y}S_{1,x}]-\int\frac{d\omega}{2\pi}|\gamma_k|^2G^k_{22}(\omega)[S^2_{2,x}+S^2_{2,y}\\
&+i\overline{S}_{2,x}S_{2,y}-i\overline{S}_{2,y}S_{2,x}]+\int\frac{d\omega}{2\pi}|\gamma_k|^2G^k_{21}(\omega)\\
&\times[\overline{S}_{2,x}S_{1,x}+\overline{S}_{2,y}S_{1,y}+i\overline{S}_{2,x}S_{1,y}-i\overline{S}_{2,y}S_{1,x}]\\
&+\int\frac{d\omega}{2\pi}|\gamma_k|^2G^k_{12}(\omega)[\overline{S}_{1,x}S_{2,x}+\overline{S}_{1,y}S_{2,y}\\
&+i\overline{S}_{1,x}S_{2,y}-i\overline{S}_{1,y}S_{2,x}],
\end{aligned}
\tag{53}
$$

where the bar denotes the complex conjugate integration variables and S_1 (S_2) is the component along the path $C^{(+)}(C^{(-)})$, Fig. 1. Next, we define two new vectors **S** and **D**,

$$\mathbf{S}=\tfrac{1}{2}(\mathbf{S}_1+\mathbf{S}_2),\tag{54}$$

$$\mathbf{D}=\mathbf{S}_1-\mathbf{S}_2.\tag{55}$$

Similarly, we define

$$\mathbf{F}_d=\mathbf{F}_1-\mathbf{F}_2,\tag{56}$$

$$\mathbf{F}_a=\frac{1}{2}(\mathbf{F}_1+\mathbf{F}_2).\tag{57}$$

Finally, we make another definition. We define four-vectors **U** and **F**,

$$\mathbf{U}=(S_x,S_y,D_x,D_y),\tag{58}$$

$$\mathbf{F}=(\mathbf{F}_d,\mathbf{F}_a),\tag{59}$$

and write the generating functional in terms of these four-vectors along the path $C^{(+)}$ only. Since $\mathbf{U}(t)$ is real, then we have

$$\overline{U}(\omega)=U(-\omega),$$

and hence we should constrain the Fourier integration to positive frequencies only. The bath-independent part of the Hamiltonian then gives the following contribution to the phase of $\hat{\mathbb{Z}}$,

$$iI_1-iI_2=-\int_0^\infty\frac{d\omega}{\pi}\overline{U}_i(\omega)\mathcal{A}^{(0)}_{ij}(\omega)U_j(\omega),\tag{60}$$

where the matrix $\mathcal{A}^{(0)}$ is, in Fourier space,

$$\mathcal{A}^{(0)}_{ij}=\begin{bmatrix}0&0&iA&-\omega\\0&0&\omega&iB\\iA&-\omega&0&0\\\omega&iB&0&0\end{bmatrix}.\tag{61}$$

It is the inverse of the full matrix $\boldsymbol{\mathcal{A}}=\boldsymbol{\mathcal{A}}^{(0)}+\boldsymbol{\mathcal{A}}^{\mathrm{int}}$ that is needed to determine the correlation functions of the magnetization where $\boldsymbol{\mathcal{A}}^{\mathrm{int}}$ is the part that is due to the interaction with the bath. The determinant of $\boldsymbol{\mathcal{A}}$ determines the natural frequency of the system and any broadening due to interactions. The determinant of the free part is

$$\mathcal{D}_0=(\omega_0^2-\omega^2)^2,\tag{62}$$

where

$$\omega_0^2=AB,\tag{63}$$

is the ferromagnetic resonance (FMR) frequency of the system.

To recover dissipative behavior in the spin subsystem, we take the continuum limit in the number of oscillator modes. This limit guarantees that the probability of acquiring back any energy lost to the bath is almost zero. Because of the interaction with the bath, we expect that there will be a shift in the energy of the spin system accompanied by dissipation.

An explicit computation shows that in the continuum limit, i.e., converting the sum over k to an integral over the frequencies involving the density of states $\lambda(\omega_k)=dk/d\omega_k$, the correlation functions can be expressed in terms of the functions L_r and L_i:

$$L_r(\omega)=-i\int_0^\infty\frac{d\omega_k}{\pi}\pi\lambda(\omega_k)|\gamma(\omega_k)|^2(G^k_{11}-G^k_{22})(\omega_k),\tag{64}$$

$$L_i(\omega)=2\int_0^\infty\frac{d\omega_k}{\pi}\pi\lambda(\omega_k)|\gamma(\omega_k)|^2(G^k_{12}-G^k_{21})(\omega_k).\tag{65}$$

Using the definitions of the Green's functions, we find

$$L_r(\omega)=2\mathcal{P}\int_0^\infty\frac{d\omega_k}{\pi}\pi\lambda(\omega_k)|\gamma(\omega_k)|^2\frac{1}{\omega-\omega_k},\tag{66}$$

$$L_i(\omega)=-2\pi\lambda(\omega)|\gamma(\omega)|^2\theta(\omega).\tag{67}$$

Counterterms are needed to cancel ultraviolet divergences in L_r. For simplicity, we will assume that this is done via suitable subtractions. The effect of L_r is a redefinition of the given coefficients A and B. In principle, this redefinition changes the oscillation frequency. However, for a passive path, one neglects $L_r(\omega)$ and thus the frequency shift. In this approximation the coefficients A and B are kept unnormalized and all the physics is contained in $L_i(\omega)$. There is a subtlety here, since the expression (67) is not antisymmetric, whereas it has to be antisymmetric due to general properties of correlation functions. Therefore $L_i(\omega)$ has to be antisymmetrized. By noticing that $|\gamma(\omega)|^2$ is even in ω and extending $\lambda(\omega)$ to negative $\omega<0$ with a negative sign, the final result can be written in the form

$$\mathcal{A}=\begin{bmatrix} 0 & 0 & iA-\Delta(\omega) & -\omega \\ 0 & 0 & \omega & iB-\Delta(\omega) \\ iA+\Delta(\omega) & -\omega & \pi\lambda(\omega)|\gamma(\omega)|^2[1+2n(\omega)] & 0 \\ \omega & iB+\Delta(\omega) & 0 & \pi\lambda(\omega)|\gamma(\omega)|^2[1+2n(\omega)] \end{bmatrix}, \tag{68}$$

where $\Delta(\omega)$ is the odd function,

$$\Delta(\omega)=\frac{L_i(-\omega)-L_i(\omega)}{2}=\pi\lambda(\omega)|\gamma(\omega)|^2. \tag{69}$$

The determinant of this matrix is given by

$$\det\mathcal{A}=\mathcal{D}(\omega)=[\omega_0^2-\omega^2-\Delta(\omega)^2]^2+[\Delta(\omega)(A+B)]^2. \tag{70}$$

We also observe that for the functional integral to converge, we must have

$$\pi\lambda(\omega)|\gamma(\omega)|^2[1+2n(\omega)]>0.$$

This requires that the function $L_i(\omega)$ when extended to negative frequencies be an *odd* function which is consistent with the statement before Eq. (68).

To calculate the correlation functions, we need first to calculate the inverse matrix of $\mathcal{A}$. The cofactors needed for the correlation functions of the different components of the magnetization are for small couplings to the bath,

$$c_{11}=-\pi\lambda(\omega)|\gamma(\omega)|^2[1+2n(\omega)][B^2+\omega^2+\Delta(\omega)^2], \tag{71}$$

$$c_{12}=i\pi\lambda(\omega)|\gamma(\omega)|^2[1+2n(\omega)][(A+B)\omega], \tag{72}$$

$$c_{22}=-\pi\lambda(\omega)|\gamma(\omega)|^2[1+2n(\omega)][A^2+\omega^2+\Delta(\omega)^2], \tag{73}$$

$$c_{12}=-c_{21}=i\pi\lambda(\omega)|\gamma(\omega)|^2[1+2n(\omega)][(A+B)\omega]. \tag{74}$$

As will be seen below, the c_{11} (c_{22}) cofactor of the matrix $\mathcal{A}$ is associated with the $xx(yy)$ component of the magnetization while c_{12} is related to the xy component.

Next, we use these cofactors to calculate the correlation functions of the magnetization.

For a general operator $\mathcal{O}$, the average of the anticommutator $\{\mathcal{O}(t),\mathcal{O}(t')\}$ is found by differentiation of $\mathbb{W}[\mathbf{F}_a,\mathbf{F}_d]$ with respect to $\mathbf{F}_d$,

$$\frac{1}{2}\langle\mathcal{O}(t)\mathcal{O}(t')+\mathcal{O}(t')\mathcal{O}(t)\rangle=-i\frac{\delta^2\mathbb{W}[\mathbf{F}_a,\mathbf{F}_d]}{\delta\mathbf{F}_d(t)\,\delta\mathbf{F}_d(t')}. \tag{75}$$

Applying this procedure to the components of the magnetization, we find that for the x component

$$\frac{1}{2}\langle S_x(t)S_x(0)+S_x(0)S_x(t)\rangle=\int\frac{d\omega}{2\pi}\cos\omega t\,\frac{c_{11}(\omega)}{\mathcal{D}(\omega)}. \tag{76}$$

From Eq. (71), we then obtain

$$C_{xx}(t)=\int\frac{d\omega}{2\pi}\cos(\omega t)[1+2n(\omega)]\times\pi\lambda(\omega)|\gamma(\omega)|^2\frac{(\omega^2+B^2)+\Delta^2}{\mathcal{D}(\omega)}. \tag{77}$$

Now, we show how for different choices of the function $L_i(\omega)$, we can recover the correlation functions derived by Smith[2] and Safonov.[3]

1. Case 1: LLG

If we assume that the bath is defined such that

$$\pi\lambda(\omega)|\gamma(\omega)|^2=\alpha\omega, \tag{78}$$

i.e., $L_i(\omega)$ is odd, then in the limit of high temperature, $\beta\to 0$, the correlation function for the xx component takes the simple form

$$C_{xx}(t)=2\alpha kT\int\frac{d\omega}{2\pi}\cos(\omega t)\times\left[\frac{(1+\alpha^2)\omega^2+B^2}{[(1+\alpha^2)\omega^2-\omega_0^2]^2+[\alpha\omega(A+B)]^2}\right]. \tag{79}$$

This is the result that coincides with that derived from LLG.[2,12] This case also corresponds to a white-noise solution.[12] Moreover, we observe that the condition on the bath that gives LLG is similar to the one that gives the Langevin equation for the harmonic oscillator. In both cases the spectral density is linear in frequency.[21]

2. Case 2: Coherent Oscillator

This case is similar to the normal-mode result by Safonov.[3] We call it coherent oscillator because this case gives correlation functions similar to those of the collective operator c introduced by Safonov and Bertram in their normal-mode analysis. Here we choose $L_i(\omega)$ such that

$$\pi\lambda(\omega)|\gamma(\omega)|^2=\alpha,\quad \omega>0. \tag{80}$$

For $\beta\to 0$, we get the following expression for the xx component of the correlation functions:

$$C_{xx}(t)=2\alpha kT\int\frac{d\omega}{2\pi}\cos(\omega t)\left(\frac{1}{\omega}\right)\times\left[\frac{B^2+\omega^2+\alpha^2}{[\omega^2-\omega_0^2]^2+2\alpha^2(\omega^2+\omega_0^2)+\alpha^4}\right]. \tag{81}$$

The normal-mode result obtained by Safonov and Bertram is easily seen to follow by setting $A=B$ and $(1/\omega)$ by $(1/\omega_0)$ in the correlation functions. Therefore without this latter approximation, this model corresponds to a case of colored noise.[12] As $\omega\to 0$, the integral diverges. Therefore at small ω, the approximation in Eq. (80) is not applicable: $L_i(\omega)-L_i(-\omega)$ cannot be a constant but, for consistency with antisymmetry and analyticity, must vanish with ω at $\omega\to 0$.

In both cases above, we have taken the high-temperature limit to recover the results of Smith[2] and Safonov and Bertram.[3]. Given that the approximation of the magnetization degrees of freedom by that of an oscillator is only valid at low temperature, one might question the validity of this limit. However this is a well-known shortcoming of LLG and little attention has been paid to address this point.[22]

C. An equilibrium calculation of the noise

Here we show how the correlation functions derived in the real-time method can be also derived using the equilibrium imaginary-time formalism.[19] This is an equilibrium consistency check for our nonequilibrium computation and moreover it shows that the FDT was correctly applied in Ref. 3.

The basic idea in the equilibrium computation is to invoke the fluctuation-dissipation theorem (which is *not* assumed in the nonequilibrium computation) in order to derive the fluctuations from the dissipation, i.e., from the spectral density. The fluctuation-dissipation theorem says that in equilibrium the symmetric correlation functions can be written in terms of the spectral densities $\rho^{ij}(\omega)$ as

$$\langle\{S^i(t),S^j(0)\}\rangle=\int d\omega e^{i\omega t}\coth\frac{\beta\omega}{2}\rho^{ij}(\omega), \tag{82}$$

where i and j denote the indices x and y, respectively. From this definition it is immediate to see that the spectral densities must satisfy the relationships

$$\rho^{ij}(\omega)^*=\rho^{ji}(\omega), \quad \rho^{ij}(\omega)=-\rho^{ji}(-\omega). \tag{83}$$

In particular, ρ^{xx} and ρ^{yy} are real and antisymmetric:

$$\rho^{ii}(\omega)^*=\rho^{ii}(\omega), \quad \rho^{ii}(\omega)=-\rho^{ii}(-\omega). \tag{84}$$

Thus, one can extract the spectral densities $\rho^{ii}(\omega)$ from the spectral representation of the retarded self-energy,

$$D_R^{ij}(\omega)=\int d\omega'\frac{\rho^{ij}(\omega')}{\omega'-\omega+i\varepsilon}, \tag{85}$$

by taking the imaginary part

$$\rho^{ii}(\omega)=-\frac{1}{\pi}\mathrm{Im}\,D_R^{ii}(\omega). \tag{86}$$

Moreover, due to the behavior of the theory under time reflections $t\to -t$,

$$\langle\{S^x(t),S^y(0)\}\rangle=-\langle\{S^x(-t),S^x(0)\}\rangle. \tag{87}$$

we have that $\rho^{xy}(\omega)$ and $\rho^{yx}(\omega)$ are imaginary and symmetric:

$$\rho^{ij}(\omega)^*=-\rho^{ij}(\omega), \quad \rho^{ij}(\omega)=\rho^{ij}(-\omega), \quad i\neq j. \tag{88}$$

As a consequence, the $\rho^{ij}(\omega), i\neq j$ spectral densities can be extracted from the *real* part of the retarded self-energy:

$$\rho^{ij}(\omega)=\frac{i}{\pi}\mathrm{Re}D_R^{ij}(\omega), \quad i\neq j. \tag{89}$$

In order to compute the retarded propagators, one has to compute the Euclidean effective action obtained by integrating out the bath degrees of freedom in the Euclidean functional integral

$$e^{-S^E_{eff}(S^i)}=\int [db_k^* db_k]\exp\left[-\int_0^\beta d\tau L^E(S^i,b_k,b_k^*)\right], \tag{90}$$

where

$$L^E(S^i,b_k,b_k^*)=L^E_S(S^i)+L^E_R(b_k,b_k^*)+L^E_{SR}(S^i,b_k,b_k^*), \tag{91}$$

with

$$L^E_S(S^i)=S^x i\partial_\tau S^y+\tfrac{1}{2}A(S^x)^2+\tfrac{1}{2}B(S^y)^2, \tag{92}$$

$$L^E_R(b_k,b_k^*)=\sum_k b_k^*(\partial_\tau-\omega_k)b_k, \tag{93}$$

$$L^E_{SR}(S^i,b_k,b_k^*)=\sum_k b_k^*\gamma_k S_- + S_+\gamma_k^* b_k. \tag{94}$$

Since the integration on b_k and b_k^* is Gaussian, S_{eff} can be computed exactly and is quadratic in the spin fields:

$$S^E_{eff}(S^i)=\int_0^\beta d\tau\int_0^\beta d\tau'\,\tfrac{1}{2}S^i(\tau)D^E_{ij}(\tau-\tau')S^j(\tau'). \tag{95}$$

The Euclidean propagator is easily obtained in the Matsubara formulation by inverting the 2×2 matrix

$$D_E^{-1}(\omega_n)=\begin{pmatrix} A & \omega_n \\ -\omega_n & B\end{pmatrix}+\begin{pmatrix}\Pi_E(\omega_n) & 0 \\ 0 & \Pi_E(\omega_n)\end{pmatrix}, \tag{96}$$

where the first matrix is the inverse free propagator and the second matrix is the self-energy matrix

$$\Pi_E(\omega_n)=2\sum_k |\gamma_k|^2 G_E^k(\omega_n) \tag{97}$$

and where $\omega_n=2\pi nT$, $n=0,\pm 1,\pm 2,\ldots$ are the Matsubara frequencies. The inversion is trivial. The retarded propagator can be obtained with an analytic continuation $\omega_n\to i\omega$:

$$D_R(\omega)=\frac{1}{\mathcal{D}(\omega)}\begin{pmatrix} B+\Pi_R(\omega) & -i\omega \\ i\omega & A+\Pi_R(\omega)\end{pmatrix}, \tag{98}$$

where $\mathcal{D}(\omega)$ is the determinant

$$\mathcal{D}(\omega)=\omega_0^2-\omega^2+(A+B)\Pi_R(\omega)+\Pi_R^2(\omega). \tag{99}$$

In order to compute real and imaginary parts, it is convenient to split

$$\Pi_R(\omega)=\mathrm{Re}\Pi(\omega)+i\,\mathrm{Im}\Pi(\omega) \tag{100}$$

and to introduce the real quantities

$$\tilde{A}(\omega)=A+\mathrm{Re}\Pi(\omega),\quad \tilde{B}(\omega)=B+\mathrm{Re}\Pi(\omega). \tag{101}$$

Then the inverse determinant reads

$$\begin{aligned}\mathcal{D}^{-1}(\omega)&=\frac{1}{\omega_0^2-\omega^2-|\Pi_R|^2+(\tilde{A}+\tilde{B})\Pi_R}\\&=\frac{\omega_0^2-\omega^2-|\Pi_R|^2+(\tilde{A}+\tilde{B})(\mathrm{Re}\Pi-i\,\mathrm{Im}\Pi)}{|\mathcal{D}(\omega)|^2}\end{aligned} \tag{102}$$

with

$$\begin{aligned}|\mathcal{D}(\omega)|^2=&[\omega_0^2-\omega^2-|\Pi_R(\omega)|^2+\mathrm{Re}\Pi(\omega)(\tilde{A}+\tilde{B})]^2\\&+(\tilde{A}+\tilde{B})^2[\mathrm{Im}\Pi(\omega)]^2.\end{aligned} \tag{103}$$

The functions $\mathrm{Re}\Pi(\omega)$ and $\mathrm{Im}\,\Pi(\omega)$ are related to the previously defined functions $L_r(\omega)$, $L_i(\omega)$, and $\Delta(\omega)$. In particular

$$\begin{aligned}\mathrm{Im}\Pi(\omega)&=-2\pi\sum_k|\gamma_k|^2\delta(\omega-\omega_k)\\&=-\pi\lambda(\omega)|\gamma(\omega)|^2=-\Delta(\omega).\end{aligned} \tag{104}$$

Notice that the continuum limit has been taken by ensuring the antisymmetry of $\mathrm{Im}\Pi(\omega)$. The $\rho^{ii}(\omega)$ spectral densities are obtained by taking the imaginary part of the full retarded propagators $D_R^{ii}(\omega)$,

$$\rho^{xx}=-\frac{1}{\pi|\mathcal{D}|^2}[\omega_0^2-\omega^2-|\Pi_R|^2-B(\tilde{A}+\tilde{B})]\Delta, \tag{105}$$

$$\rho^{yy}=-\frac{1}{\pi|\mathcal{D}|^2}[\omega_0^2-\omega^2-|\Pi_R|^2-A(\tilde{A}+\tilde{B})]\Delta, \tag{106}$$

whereas the spectral densities $\rho^{ij}(\omega)(i\neq j)$ are obtained by taking the real part of $D_R^{ii}(\omega)$:

$$\rho^{xy}(\omega)=\frac{i}{\pi|\mathcal{D}|^2}[\omega(\tilde{A}+\tilde{B}]\Delta. \tag{107}$$

These results coincide with those derived by the real-time method. Therefore there is full consistency between the real-time and the imaginary-time formalism.

The LLG limit for small damping is recovered when $\mathrm{Re}\Pi\rightarrow\mathrm{const}$, $\mathrm{Im}\Pi\rightarrow\alpha\omega$, and the coherent oscillator is recovered in the region $|\omega|\sim\omega_0$ when $\mathrm{Re}\Pi\rightarrow\mathrm{const}$, $\mathrm{Im}\Pi\rightarrow\alpha\ \mathrm{sgn}\omega$. The term $\mathrm{Re}\Pi$ is usually set to zero after being absorbed in the definition of the FMR frequency.

Therefore, both equilibrium and nonequilibrium methods give the same correlation functions for the magnetization close to equilibrium. Below we give the full expressions for the symmetrized correlation functions for the transverse components of the magnetization. As far as our simple model is concerned, it is then concluded that the LLG result is a well behaved model of noise since it can be applied to the whole range of frequencies while the proposed modifications of the damping term in LLG by Safonov and Bertram rest on weak arguments. The general S_xS_x-correlation function is then given by

$$\begin{aligned}C_{xx}(t)=&\int\frac{d\omega}{2\pi}[1+2n(\omega)]\Delta(\omega)\cos(\omega t)\\&\times\left[\frac{\omega^2+B^2+\Delta(\omega)^2}{[\omega_0^2-\omega^2-\Delta(\omega)^2]^2+[\Delta(\omega)(A+B)]^2}\right].\end{aligned} \tag{108}$$

Similarly for the S_yS_y-correlation function, we have

$$\begin{aligned}C_{yy}(t)=&\int\frac{d\omega}{2\pi}[1+2n(\omega)]\Delta(\omega)\cos(\omega t)\\&\times\left[\frac{\omega^2+A^2+\Delta(\omega)^2}{[\omega_0^2-\omega^2-\Delta(\omega)^2]^2+[\Delta(\omega)(A+B)]^2}\right],\end{aligned} \tag{109}$$

and finally for the S_xS_y-correlation function, the correlation function is

$$\begin{aligned}C_{xy}(t)=&\int\frac{d\omega}{2\pi}[1+2n(\omega)]\Delta(\omega)\sin(\omega t)\\&\times\left[\frac{\omega(A+B)}{[\omega_0^2-\omega^2-\Delta(\omega)^2]^2+[\Delta(\omega)(A+B)]^2}\right],\end{aligned} \tag{110}$$

where

$$\Delta(\omega)=\pi\lambda(\omega)|\gamma(\omega)|^2. \tag{111}$$

Even though we have succeeded in showing the origin of the discrepancy between both calculations, the usefulness of these results is limited because of the harmonic approximation. Clearly more work is needed to understand the dissipation-noise problem in magnetic devices which goes beyond the simple approximations currently used to account for them.

IV. CONCLUSION

Starting from simple microscopic models which only differ in the density of states of the bath, we have been able to derive variant correlation functions for the magnetization close to equilibrium. We have limited ourselves only to linear-type couplings to the bath. Depending on the product of the coupling constants and the density of states of the bath, we showed how to obtain the different types of correlation functions: the classical LLG result obtained by Smith[2] and the normal-mode coupling solution of Safonov.[3] We have used two independent methods to get our solutions: a nonequilibrium method that does not make use of the FDT and an equilibrium method. The nonequilibrium formulation allowed us in particular to show that a generalized fluctuation-dissipation theorem (51) holds true even if the distribution functions are not exactly the Bose-Einstein ones. In principle, an analysis of this system when the distribution functions present strong differences from the thermal one is also possible in the general formalism we discussed here.

The LLG solution was obtained for a special type of density of states and coupling to the bath. The same condition was also obtained in Ref. 12 where in addition we were able to show that this choice gives the white-noise character in the stochastic formulation. The normal-mode solutions are, however, generally with memory unless we are interested in frequencies close to the FMR frequency. The assumption that damping is constant close to the FMR frequency makes the equations of motion Markovian.[23] The damping in the cases treated here is *independent* of the symmetries of the Hamiltonian spin system, the reason being that the dissipation kernel only depends on the coupling with the bath and the bath properties, but not on the spin Hamiltonian. For couplings other than linear, the damping is expected to depend on the symmetries of the full Hamiltonian, but, again, not at the leading order in perturbation theory. The point is that for nonlinear coupling, the effective Hamiltonian and therefore the correlation functions have to be computed perturbatively in terms of Feynman diagrams; in particular the spin propagator will enter in higher-order computations and since the spin propagator depends on the symmetry of the spin Hamiltonian, which could be isotropic $(A=B)$ or not $(A\neq B)$, we will have different results for $A=B$ and $A\neq B$. This does not happen at leading order in the nonlinear case, and does not happen *at all orders* in the linear case, where the exact result is shown here. For cases far from equilibrium, the harmonic approximation is no longer valid and we expect the damping to depend both on frequency and position and direction of the magnetization.

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