

Massive spin collective mode in a quantum Hall ferromagnet

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It is shown that the collective spin rotation of a single Skyrmion in a quantum Hall ferromagnet can be regarded as precession of the entire spin texture in the external magnetic field, with an effective moment of inertia which becomes infinite in the zero g -factor limit. This low-lying spin excitation may dramatically enhance the nuclear spin relaxation rate via the hyperfine interaction in the quantum well slightly away from filling factor $\nu=1$.

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Nuclear spin dynamics in semiconducting heterojunctions under the conditions of the odd integer Quantum Hall (QH) effect is strongly influenced by the two-dimensional (2D) electron gas in the quantum well through the hyperfine interaction with the electronic spins. This effect was first demonstrated in a set of experiments, reported in Refs. 1, 2, where the ^{71}Ga Knight shift K_S and spin lattice relaxation time T_1 in GaAs multiple quantum well (MQW) structure under perpendicular magnetic field were detected by means of the optically pumped NMR (OPNMR) technique. K_S was found to reduce dramatically as the Landau level filling factor was shifted slightly away from $\nu=1$, indicating that the injection of a single charge into the 2D electron system is followed by reversal of many electronic spins. In the same interval of the filling factor, T_1 was found to drop by several orders of magnitude. Both effects are considered as strong evidence for the creation of Skyrmionic spin texture³⁻⁵ in the electronic spin distribution as ν shifts slightly away from unity, and indicate the crucial importance of the hyperfine interaction in controlling the nuclear spin dynamics.^{6,7}

At filling factor $\nu=1$ the ground state of the 2D electron gas is ferromagnetic even in the limit of zero Zeeman energy.³ Flipping nuclear spins in this state through the hyperfine interaction is followed by the creation of spin excitons.^{8,9} The extremely long T_1 observed by Barrett *et al.*^{1,2} may be due to the energy gap existing in the exciton spectrum (see below, however). In actual heterojunctions the gap is usually much smaller than the theoretical value. It can be suppressed by the combined effect of quantum confinement and external pressure¹⁰ or external electric fields.¹¹ Furthermore, in 2D electron gas under strong optical pumping the electronic Zeeman energy can be strongly suppressed by the magnetic hyperfine field.¹² In both cases the effective g factor is spatially inhomogeneous due to the presence of long-range impurity potential fluctuations.⁷

Microscopic calculations, based on Hartree-Fock (HF) approximation for a single, isolated Skyrmion,¹³ have found a family of low-energy excitations, with an approximately quadratic relation between the energy and the number of flipped spins K , which can be associated with the kinetic rotational energy of the entire spin texture. However, except for the

special case where K is a half integer, the spectrum has an excitation gap, which was estimated to be some fraction of the large Coulomb energy scale. To account for the observed enhancement in $1/T_1$, these authors have suggested¹⁴ that at $\nu \neq 1$, where there is a finite density of Skyrmions, the ground state is a Skyrme crystal, for which the spin waves spectrum is gapless due to the breakdown of the global spin rotation symmetry. The existence of such an ordered lattice at $|\delta\nu| \leq 0.1$, where $\delta\nu \equiv \nu - 1$, is hard to reconcile, however, with the small radius $R \leq 2l_B$ of the Skyrmions reported in Ref. 15, as the average distance $(8/|\delta\nu|)^{1/2}l_B$ between Skyrmions in this filling factors region is larger than $9l_B$. Unfortunately, the theoretical interpretation of these experiments is difficult due to the relatively large value of the reported g factor ($|g| \approx 0.4$).

In the present paper we propose to focus on different experimental situations, similar to the ones investigated in Refs. 10, 12, where the effective g factor can become much smaller than the bulk GaAs value ($|g| \approx 0.4$). We restrict the study to the filling factor region $|\delta\nu| \leq 0.05$, so that the collective rotations of individual Skyrmions found experimentally in Refs. 10, 12 (i.e., with $R \sim 7l_B$), may be regarded as independent. This is a reasonable assumption since the orientation-dependent interaction between neighboring Skyrmions decays exponentially with their relative distance beyond the characteristic length $(\sqrt{2\pi}/8g)^{1/2}l_B$,¹⁶ which is approximately $3l_B$ at $g=0.03$ (see below). We show that the excitation gap in the collective rotational spectrum of such a single Skyrmion diminishes quite sharply when the Skyrmion radius increases, so that for $R \sim 7l_B$ the spin excitation gap becomes comparable to the nuclear Zeeman energy under consideration. Since the collective spin rotation is coupled, through the hyperfine interaction, to nuclear spins, the nuclear spin relaxation rate should be dramatically enhanced.

We start our analysis by considering the Hamiltonian for nuclear spins interacting with 2D electron gas at $\nu=1$ in MQW structure $\hat{H} = \hat{H}_n + \hat{H}_e + \hat{H}_{en}$, where $\hat{H}_n = -\hbar \gamma_n \sum_j \hat{\mathbf{I}}_j \cdot \mathbf{B}_0$ is the nuclear Zeeman energy $\hat{H}_e = -\hbar \gamma_e \int d^2r \hat{\mathbf{S}}(\mathbf{r}) \cdot \mathbf{B}_0 + \hat{H}_{ee}$ is the electronic Hamiltonian, with \hat{H}_{ee} the electron-

electron interaction, and $\hat{H}_{en} = A \sum_j \hat{\mathbf{S}}(\mathbf{r}_j) \cdot \hat{\mathbf{I}}_j$ is the Fermi contact hyperfine interaction between the electron and nuclear spins. Here $\hat{\mathbf{I}}_j$ is the nuclear spin operator located at \mathbf{r}_j , $\hat{\mathbf{S}}(\mathbf{r})$ is the electronic spin density operator, \mathbf{B}_0 is the external magnetic field, which is perpendicular to the 2D electron gas ($\mathbf{B}_0 = B_0 \mathbf{z}$), $\gamma_n = g_n \mu_n / \hbar$ and $\gamma_e = g_e \mu_B / \hbar$ the nuclear and electronic gyromagnetic ratios, respectively, and $A = (8\pi/3) g_n \mu_n g_0 \mu_B |u_0(0)|^2$ is the Fermi contact hyperfine coupling constant. In this expression $u_0(0)$ is the periodic part of the Bloch wave function at the nucleus, and g_0 is the g factor of a free electron.

Within the HF approximation, and to leading order in a gradient expansion, the electronic Hamiltonian \hat{H}_e can be written as a functional of a unit vector field $\mathbf{n}(\mathbf{r})$,

$$\hat{H}_e = \frac{\varepsilon_C}{32\pi} \int d^2r (\nabla \cdot \mathbf{n})^2 - \frac{\varepsilon_{sp}}{4\pi l_B^2} \int d^2r (\mathbf{z} \cdot \mathbf{n}) - \frac{\varepsilon_C}{2} Q, \quad (1)$$

where $\varepsilon_{sp} = |g| \mu_B B_0$ is the Zeeman splitting energy, $l_B = \sqrt{c\hbar/eB_0}$ is the magnetic length, $\varepsilon_C = \sqrt{\pi/2} (e^2/\kappa l_B)$ is the Coulomb energy, κ is the dielectric constant, and $Q = (1/4\pi) \int d^2r (\mathbf{n} \cdot [\partial_x \mathbf{n} \times \partial_y \mathbf{n}])$ is the Skyrmion winding number (or topological charge). The unit vector field is proportional to the expectation value of the electron spin density, i.e., $\mathbf{n}(\mathbf{r}) = 4\pi \mathbf{S}(\mathbf{r})$, with $\mathbf{S}(\mathbf{r}) \equiv \langle \hat{\mathbf{S}}(\mathbf{r}) \rangle$.

The dynamics of the Skyrmionic spin texture can be studied by examining the corresponding effective Lagrangian. This should contain first order time derivative to account for the spin wave excitations (or spin-excitons) obtained within the microscopic HF scheme at $\nu=1$. It has the form $-(1/4\pi l_B^2) \int d^2r (\hbar \partial_t \mathbf{n}) \cdot \mathbf{A}(\mathbf{n})$, where $\mathbf{A}(\mathbf{n})$ is a vector potential of a unit magnetic monopole sitting at the origin in spin space.⁵ Since we are interested here in a rigid rotation of the entire spin texture, we may restrict the analysis, for the sake of simplicity, to the spatial region far away from the Skyrmion core, where \mathbf{n} is very close to \mathbf{z} , so that $\mathbf{n} \approx (n_x, n_y, 1 - n_x^2/2 - n_y^2/2)$, and $\mathbf{A}(\mathbf{n}) \approx \frac{1}{2} (-n_y, n_x, 0)$. Separating the collective spin rotational motion, described by the variable $\varphi(t)$, from all the other degrees of freedom in spin space, by writing $\psi(\mathbf{r}, t) \equiv n_x(\mathbf{r}, t) + i n_y(\mathbf{r}, t) = \tilde{\psi}(\mathbf{r}, t) e^{i\varphi(t)}$ and then adding a kinetic energy term $\frac{1}{2} m_s \psi^* \psi (\partial\varphi/\partial t)^2$, associated with the collective rotation angle $\varphi(t)$ with m_s being a phenomenological mass, the effective Lagrangian density is written in the form

$$\mathcal{L}\{\tilde{\psi}, \tilde{\psi}^*; \varphi\} = \frac{1}{4\pi l_B^2} \left\{ \begin{array}{l} \varepsilon_{sp} \left(1 - \frac{1}{2} \tilde{\psi}^* \tilde{\psi} \right) \\ + \frac{i}{4} \left[\tilde{\psi}^* \left(\hbar \frac{\partial \tilde{\psi}}{\partial t} \right) - \tilde{\psi} \left(\hbar \frac{\partial \tilde{\psi}^*}{\partial t} \right) \right] \\ - \frac{1}{2} \tilde{\psi}^* \tilde{\psi} \left(\hbar \frac{\partial \varphi}{\partial t} \right) \end{array} \right\} - \frac{1}{32\pi} \varepsilon_C (\nabla \tilde{\psi}^* \cdot \nabla \tilde{\psi}) + \frac{1}{2} m_s \tilde{\psi}^* \tilde{\psi} \left(\frac{\partial \varphi}{\partial t} \right)^2. \quad (2)$$

Note that in deriving this expression all terms of order higher than $o(|\psi|^2)$ are neglected. Furthermore, we omit here the last term appearing on the right-hand side of Eq. (1), which is topological invariant (i.e., under any continuous deformation of \mathbf{n}), and so does not influence the equation of motion. Also note that Eq. (2) is consistent with the effective Lagrangian derived from the microscopic Hamiltonian in the HF approximation by Apel and Bychkov.¹⁷

It should be stressed that the degrees of freedom in spin space, described by the field $\tilde{\psi}(\mathbf{r}, t)$, are assumed to be massless. This assumption is consistent with the lowest Landau level approximation, usually exploited in this context. The Euler-Lagrange equations, derived from the effective Lagrangian density (2), with respect to $\tilde{\psi}^*$ and φ are

$$-i\hbar \frac{\partial \tilde{\psi}}{\partial t} + \varepsilon_{sp} \tilde{\psi} - \frac{1}{4} \varepsilon_C l_B^2 \nabla^2 \tilde{\psi} + \left(\hbar \frac{\partial \varphi}{\partial t} \right) \tilde{\psi} - 4\pi l_B^2 m_s \left(\frac{\partial \varphi}{\partial t} \right)^2 \tilde{\psi} = 0, \quad (3)$$

$$\frac{d}{dt} \left[\tilde{\psi}^* \tilde{\psi} - 2\tilde{m}_s \tilde{\psi}^* \tilde{\psi} \left(\frac{d\varphi}{dt} \right) \right] = 0. \quad (4)$$

Since Eqs. (3) and (4) describe the dynamics of spins in the tail of the Skyrmion (where the spin polarization is nearly complete) the spin-wave form $\psi(\mathbf{r}, t) = \tilde{\psi}(\mathbf{r}, t) e^{i\varphi_0} = \exp(i(\mathbf{k} \cdot \mathbf{r} - \omega t + \varphi_0))$, where φ_0 is a constant, should be a solution. Indeed, substituting this form into Eqs. (3) and (4) the former equation reduces to the well-known spin-wave dispersion relation in the long wave-length limit $\hbar\omega = \varepsilon_{sp} + \frac{1}{4} \varepsilon_C l_B^2 k^2$ (whereas the latter equation is trivially solved).

The desired collective mode can be described by $\psi(\mathbf{r}, t) = \tilde{\psi}_0(\mathbf{r}) e^{i\varphi(t)}$, where $\tilde{\psi}_0(\mathbf{r})$ is time independent. It turns Eqs. (3) and (4), respectively, to

$$\left[\varepsilon_{sp} \tilde{\psi}_0 - \frac{1}{4} \varepsilon_C l_B^2 \nabla^2 \tilde{\psi}_0 \right] + \tilde{\psi}_0 \left[\left(\hbar \frac{\partial \varphi}{\partial t} \right) - 4\pi l_B^2 m_s \left(\frac{\partial \varphi}{\partial t} \right)^2 \right] = 0, \quad \left(\frac{\partial^2 \varphi}{\partial t^2} \right) = 0. \quad (5)$$

The first equation can be separated into purely spatial and temporal equations

$$\varepsilon_{sp} \tilde{\psi}_0 - \frac{1}{4} \varepsilon_C l_B^2 \nabla^2 \tilde{\psi}_0 = 0, \quad \left(\hbar \frac{\partial \varphi}{\partial t} \right) - 4\pi l_B^2 m_s \left(\frac{\partial \varphi}{\partial t} \right)^2 = 0$$

with the nontrivial solution for the time-dependent equation

$$\left(\frac{\partial \varphi}{\partial t} \right) = \frac{\hbar}{4\pi l_B^2 m_s} = \frac{e B_s}{m_s c}, \quad (6)$$

where $B_s \equiv (1/4\pi) B_0$. The solutions for the static, space dependent equation $\tilde{\psi}_0(r, \phi) \propto K_m(r/l_{sk}) e^{im\phi}$, with $K_m(x)$ a modified Bessel function of integer order m , coincide with the asymptotic form of the static Skyrmion solutions with winding number m , found in Ref. 4. Here l_{sk} is the length

scale corresponding to the Skyrmion's tail, $l_{\text{Sk}}^{-2} = 4\varepsilon_{\text{sp}}/\varepsilon_C l_B^2 = 2\sqrt{(2/\pi)}|g|\tilde{a}/l_B^3$, and $\tilde{a} = \kappa\hbar^2/m_0e^2$ is the effective Bohr radius (m_0 being the free electron mass).

Equation (6) yields an effective Larmor frequency for precession of the entire spin texture in a magnetic field B_s . The remarkable feature of this result is that B_s is, up to a constant, identical to the external magnetic field B_0 .

The collective mass m_s remains unknown within the phenomenological approach described above. It may be determined from the variational Skyrmion energy, derived in Ref. 4, by exploiting the connection⁵ between the total number K of flipped spins in the Skyrmion and the eigenvalues of \hat{L}_z —the canonical angular momentum conjugate to φ . The variational Skyrmion energy was calculated in the HF approximation near filling factor $\nu=1^4$ by adding to the Hamiltonian \hat{H}_e , consisting of the Zeeman energy and the leading exchange energy term in the gradient expansion, the Coulomb self-energy repulsion. The latter represents, in addition to the leading, topological invariant term, higher order corrections in the gradient expansion, resulting in the following expression for the total Skyrmion energy:

$$E_{\text{tot}}(R) = \frac{3\pi^2 e^2}{2^6 \kappa R} + \frac{1}{4} \varepsilon_C \left(\frac{R}{l_{\text{Sk}}} \right)^2 \ln \left(\frac{2l_{\text{Sk}}}{R} \right), \quad (7)$$

where R is a variational parameter describing the Skyrmion core radius. Identifying L_z with the number of flipped spins in the Skyrmion,⁵ the Zeeman energy is $\Delta E_Z = \varepsilon_{\text{sp}} \tilde{L}_z = \frac{1}{4} \varepsilon_C (R/l_{\text{Sk}})^2 \ln(2l_{\text{Sk}}/\sqrt{\bar{e}R})$, where $\tilde{L}_z \equiv L_z/\hbar$ and \bar{e} stands for the natural logarithm base, so that $\tilde{L}_z = (R/l_B)^2 \ln(2l_{\text{Sk}}/\sqrt{\bar{e}R})$. Minimization with respect to R yields for the equilibrium core radius $3\pi^2 e^2/2^6 \kappa R_{\text{eq}}^3 = \varepsilon_{\text{sp}}(2/l_B^2) \ln(2l_{\text{Sk}}/R_{\text{eq}})$, whereas $[(\partial^2/\partial R^2)E_{\text{tot}}]_{\text{eq}} \approx (6\varepsilon_{\text{sp}}/l_B^2) \ln(2l_{\text{Sk}}/R)$, so that $U = (\partial^2 E_{\text{tot}}/\partial \tilde{L}_z^2)_{\text{eq}} \approx \varepsilon_{\text{sp}}(l_B/R_{\text{eq}})^2 3 \ln(2l_{\text{Sk}}/R)/2 \ln^2(2l_{\text{Sk}}/\sqrt{\bar{e}R})$. An equivalent expression for U was derived in Ref. 16 in the context of a field theoretical study of Skyrme lattices. Its far reaching physical consequences for the spin dynamics of single Skyrmions, discussed here, have not been revealed there, however.

Expanding the energy (7) up to second order in \tilde{L}_z about its equilibrium value K , that is, writing $E_{\text{tot}}(\tilde{L}_z) = E_{\text{tot}}(K) + \frac{1}{2}U(\tilde{L}_z - K)^2 + \dots$, the second term on the right-hand side corresponds to the ‘‘classical’’ rotational energy of the entire spin texture about its symmetry axis. At the classical level any deviation of \tilde{L}_z from its equilibrium value K corresponds to a continuous spatial deformation of the Skyrmion with respect to its equilibrium configuration, thus conserving its topological charge Q but increasing the Skyrmion energy with respect to its equilibrium value. Quantization of this rotational motion can be achieved by replacing $L_z \rightarrow (\hbar/i)(\partial/\partial\varphi)$, which yields $\hat{H}_{\text{rot}} = \frac{1}{2}U[(1/i)(\partial/\partial\varphi) - K]^2$. Now, the rotational energy⁵ $E_{\text{rot}} = (\hbar^2/2U)(d\varphi/dt)^2$, may be equated to the lowest eigenvalue of the rotational Hamiltonian \hat{H}_{rot} , namely, $\sim \frac{1}{2}U$, to find the angular velocity

$$\left(\frac{d\varphi}{dt} \right) \sim U/\hbar = \frac{eB_0}{M_s c}, \quad M_s = 4 \left(\frac{\tilde{a}}{l_B} \right) |\tilde{g}|^{-5/3} m_0 \quad (8)$$

and $\tilde{g} \equiv g(\tilde{a}/l_B)$. This remarkable result is consistent with the Larmor frequency found in Eq. (6), provided the phenomenological mass is $m_s = M_s/4\pi$. The resulting effective mass (8) diverges with a vanishing g factor such as $|\tilde{g}|^{-5/3}$, reflecting the macroscopic moment of inertia associated with the collective rotation of a Skyrmion.

As indicated above, local g factors should fluctuate significantly in the space of the QW, due, e.g., to lattice strains, and long range fluctuating electric field, which always exists in the QW as a result of the ionized impurities located in the barriers. In fact for a QW width $l \sim 70 \text{ \AA}$,¹⁰ and typical electric field of about 10^{-3} V/\AA , the corresponding fluctuation in the g factor can be of the order of the bulk g factor itself,¹¹ so that local values can become very small. The size of the corresponding large Skyrmions is expected, however, to be limited by disorder potential.^{18,19,15} A lower bound on the values of $|g|$, below which the Skyrmion radius does not change, can be estimated from the experimental data reported in Ref. 10. This yields $g \sim 0.03$ (or $\tilde{g} \sim 0.002$) for which the corresponding mass ratio is $M_s/m_0 \sim 10^4$, and the collective Larmor frequency $\omega_{\text{Sk}} = eB_0/M_s c$, is comparable to the nuclear ⁷¹Ga Zeeman frequency $\omega_n \approx 10^{-4} \gamma_e B_0$. It should be stressed, however, that despite the huge enhancement of M_s , the radius $R \approx 7l_B$ of such a large spin texture, remains smaller than half of the average distance between neighboring Skyrmions, as long as $|\delta\nu| \leq 0.05$. Thus, within this filling factor region our picture of independently rotating Skyrmions in spin space should be valid.

Our theoretical tools are not yet sufficiently developed, however, to enable us sufficiently accurate evaluation of the desired relaxation rates. The best we can do at present is to set some reasonable bounds on these rates. We follow the theory developed in Refs. 21, 20 to estimate the decay of the deviation $\delta I_z(\mathbf{r}, t)$ of the nuclear spin polarization from its equilibrium value, i.e., $\delta I_z(\mathbf{r}, t) = \delta I_z(\mathbf{r}, 0) e^{-\Gamma(\mathbf{r}, t)}$, where $\Gamma(\mathbf{r}, t) = \text{Re} \int_0^t dt' \xi(\mathbf{r}, t')$, $\xi(\mathbf{r}, t) = (\alpha^2/2) \int_0^t d\tau e^{i\omega_n(\tau-t)} \times \langle \{ \hat{S}_+(\mathbf{r}, t), \hat{S}_-(\mathbf{r}, \tau) \} \rangle$, and the average is performed over the electronic states. Restricting the analysis to positions \mathbf{r} far away from the Skyrmion core, where $S_+(\mathbf{r}, t) \approx (1/4\pi) \tilde{\psi}_0(\mathbf{r}) e^{i\varphi(t)}$, we may rewrite $\xi(\mathbf{r}, t) \approx 2(\alpha |\tilde{\psi}_0(\mathbf{r})|/8\pi)^2 \int_0^t d\tau e^{i\omega_n(\tau-t)} \langle \{ e^{i\hat{\varphi}(t)}, e^{-i\hat{\varphi}(\tau)} \} \rangle$, where $e^{i\hat{\varphi}(t)} \equiv e^{i\hat{H}_{\text{rot}}/\hbar} e^{i\varphi} e^{-i\hat{H}_{\text{rot}}/\hbar}$. A straightforward algebra yields $\Gamma(\mathbf{r}, t) = (\alpha |\tilde{\psi}_0(\mathbf{r})|/4\pi)^2 \{ 1 - \cos[(U\delta K/\hbar - \omega_n)t]/(U\delta K/\hbar - \omega_n)^2 \}$, where $\delta K \equiv [K] - K$ and $[K]$ is the half integer closest to K . This expression, which describes a highly idealized situation, may be used to estimate a lower bound for T_1 by considering the limit $(U\delta K/\hbar - \omega_n) \rightarrow 0$. The corresponding nuclear spin relaxation is Gaussian, $\delta I_z(\mathbf{r}, t) \sim e^{-[\alpha |\tilde{\psi}_0(\mathbf{r})|/4\pi]^2 t^2}$, with characteristic local relaxation time $T_1^{\text{Sk}}(\mathbf{r}) \sim 4\pi/\alpha |\tilde{\psi}_0(\mathbf{r})|$, determined by the local transverse spin density. Typical values are therefore of the order of $1/K_S$, which is roughly 10^{-3} s for GaAs MQW. This extremely small result (as compared to typical values T_1

$\sim 10^3$ s, observed at $\nu=1$) indicates the great sensitivity of the nuclear relaxation time T_1 to the filling factor in the close vicinity of $\nu=1$.

It is interesting to make a similar lower bound estimate of the corresponding relaxation time at $\nu=1$, where the number of Skyrmions vanishes and the nuclear spin dynamics is controlled by the coupling to the spin waves. In this case,²⁰ $\Gamma(\mathbf{r}, t) = \Gamma(t) = 2(hK_S)^2 \int_0^\infty \tilde{k} d\tilde{k} e^{-\tilde{k}^2/2} [1 - \cos([\varepsilon_{\text{ex}}(\tilde{k})/\hbar - \omega_n]t)] / [\varepsilon_{\text{ex}}(\tilde{k}) - \hbar\omega_n]^2$, where $\varepsilon_{\text{ex}}(\tilde{k}) \approx \varepsilon_{\text{sp}} + \frac{1}{4}\varepsilon_C \tilde{k}^2$, for $\tilde{k} = kl_B \ll 1$. Thus in the limit $\varepsilon_{\text{ex}}(\tilde{k}) - \hbar\omega_n \rightarrow 0$, we find for $t \gg \hbar/\varepsilon_C$: $\Gamma(t) \rightarrow [2(2\pi)^2 K_S^2 / \varepsilon_C / h] t$, so that the relaxation of the nuclear spin polarization is a simple exponential, with a characteristic relaxation time $T_1^{\text{ex}} = (1/8\pi^2)(\varepsilon_C/hK_S)(1/K_S)$. For GaAs MQW this expression yields $T_1^{\text{ex}} \sim 4 \times 10^2$ s, which is of the same order of magnitude as the experimentally measured T_1 at $\nu=1$. The very long T_1 obtained in this case is due to the large (Coulomb) energy scale of the spin-wave excitations.

In contrast, the energy scale of the collective spin rotations of all possible Skyrmions with different g factors is always a small fraction of the electronic Zeeman energy, so that the corresponding T_1 is by several orders of magnitude shorter than T_1^{ex} .

In conclusion, we have shown that for a QH ferromagnet, realized in QW's with sufficiently suppressed effective g factor,^{10,12} one expects dramatic enhancement of the nuclear spin-lattice relaxation rate, while shifting the filling factor slightly away from $\nu=1$. This effect is associated with nearly gapless excitation of almost free rotations in spin space of individual Skyrmions.

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¹S. E. Barrett, G. Dabbagh, L. N. Pfeiffer, K. W. West, and R. Tycko, Phys. Rev. Lett. **74**, 5112 (1995).

²R. Tycko, S. E. Barrett, G. Dabbagh, L. N. Pfeiffer, and K. W. West, Science **268**, 1460 (1995).

³S. L. Sondhi, A. Karlhede, S. A. Kivelson, and E. H. Rezayi, Phys. Rev. B **47**, 16 419 (1993).

⁴Yu. A. Bychkov, T. Maniv, and I. D. Vagner, Phys. Rev. B **53**, 10 148 (1996); Yu. A. Bychkov, A. V. Kolesnikov, T. Maniv, and I. D. Vagner, J. Phys.: Condens. Matter **10**, 2029 (1998).

⁵For a review see S. M. Girvin, *The Quantum Hall Effect: Novel Excitations and Broken Symmetries*, in Les Houches Summer School 1998 (Springer Verlag, Paris, 1999).

⁶I. D. Vagner and T. Maniv, Phys. Rev. Lett. **61**, 1400 (1988).

⁷For a review see I. D. Vagner and T. Maniv, Physica B **204**, 141 (1995).

⁸Yu. A. Bychkov, S. V. Iordanskii, and G. M. Eliashberg, JETP Lett. **33**, 143 (1981).

⁹C. Kallin and B. I. Halperin, Phys. Rev. B **31**, 3635 (1984).

¹⁰D. K. Maude, M. Potemski, J. C. Portal, M. Henini, L. Eaves,

G. Hill, and M. A. Pate, Phys. Rev. Lett. **77**, 4604 (1996).

¹¹E. L. Ivchenko, A. A. Kiselev, and M. Willander, Solid State Commun. **102**, 375 (1997).

¹²I. V. Kukushkin, K. v. Klitzing, and K. Eberl, Phys. Rev. B **60**, 2554 (1999).

¹³H. A. Fertig, L. Brey, R. Cote, A. H. MacDonald, A. Karlhede, and S. L. Sondhi, Phys. Rev. B **55**, 10 671 (1997).

¹⁴R. Cote, A. H. MacDonald, L. Brey, H. A. Fertig, S. M. Girvin, and H. T. C. Stoof, Phys. Rev. Lett. **78**, 4825 (1997).

¹⁵P. Khandelwal, A. E. Dementyev, N. N. Kuzma, S. E. Barrett, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. **86**, 5353 (2001).

¹⁶M. Abolfath and M. R. Ejtehadi, Phys. Rev. B **58**, 10 665 (1998).

¹⁷W. Apel and Yu. A. Bychkov, Phys. Rev. Lett. **78**, 2188 (1997).

¹⁸D. Lilliehöök, K. Lejnell, A. Karlhede, and S. L. Sondhi, Phys. Rev. B **56**, 6805 (1997).

¹⁹A. J. Nederveen and Yu. V. Nazarov, Phys. Rev. Lett. **82**, 406 (1999).

²⁰T. Maniv, Yu. A. Bychkov, I. D. Vagner, and P. Wyder, Phys. Rev. B **64**, 193306 (2001).

²¹W. Apel and Yu. A. Bychkov, Phys. Rev. B **63**, 224405 (2001).