

Writing spin in a quantum dot with ferromagnetic and superconducting electrodes

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We propose an efficient mechanism for the operation of writing spin in a quantum dot, which is an ideal candidate for qubit. The idea is based on the Andreev reflection induced spin polarization (ARISP) in a ferromagnetic/quantum-dot/superconductor system. We find that on the resonance of Andreev reflection, the spin polarization of quantum dot *strongly* depends on the magnetization of ferromagnetic electrode, and the *sign* of the spin polarization is controllable by bias voltage. In the presence of intradot interaction, we show that ARISP effect can still survive as long as the charging energy U is comparable to the superconducting gap Δ . Detailed conditions and properties of ARISP are also discussed.

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Qubit is the basic unit in the achievement of quantum computing. Among many quantum two-level systems, the spin of quantum dot (QD) is an ideal candidate for such a purpose.¹ This is because QD fabricated in semiconductor two-dimensional electron gas (2DEG) can be well controlled by metallic gates; parameters such as resonant levels, tunnel barriers, and charging energy are experimentally tunable. In addition the spin coherence time is rather long in semiconductors (exceeding 100 ns at low temperature²), which is orders larger than charge coherence time. Moreover, due to the 0D nature of QD's, many spin-flip mechanisms are further suppressed, resulting in an even longer lifetime of spin in QD.³

One of the challenges to exploit the spin of QD as qubit is to efficiently control its local spin polarization. A natural idea is to apply a magnetic field and induce the spin polarization by Zeeman splitting.⁴ But this scheme has practical difficulties because the field is required to be of the order of tesla and confined within the scale of QD. More importantly, the sign of spin polarization in QD cannot be controlled electrically.

It is the purpose of this paper to propose an efficient mechanism for writing spin in a QD with ferromagnetic (F) and superconducting (S) electrodes. The idea is motivated by the fact that F can provide spin-polarized electrons while S can only accept Cooper pair which is spin singlet. We shall show below that on the resonance of Andreev reflection (AR),⁵⁻⁸ the spin polarization of QD strongly depends on the magnetization of F, and the sign of spin polarization is controllable by a bias voltage. The effect can survive even in the presence of strong Coulomb interaction, as long as the charging energy of QD is comparable to the superconducting gap in S.

Noninteracting case. To see the physics clearly, we start with the noninteracting ferromagnet/quantum-dot/superconductor (F-QD-S) system modeled by the following Hamiltonian, $H = H_F + H_S + H_D + H_T$, in which $H_F = \sum_{k\sigma} (\epsilon_k - \sigma h) a_{k\sigma}^\dagger a_{k\sigma}$ is for the F electrode, $H_S = \sum_{p\sigma} \epsilon_p b_{p\sigma}^\dagger b_{p\sigma} + \sum_p (\Delta b_{p\uparrow}^\dagger b_{-p\downarrow}^\dagger + \text{H.c.})$ the S electrode, $H_{dot} = \sum_{\sigma} E_{\sigma} c_{\sigma}^\dagger c_{\sigma}$ the noninteracting QD, and $H_T = \sum_{k\sigma} t_L a_{k\sigma}^\dagger c_{\sigma} + \sum_{p\sigma} t_R b_{p\sigma}^\dagger c_{\sigma} + \text{H.c.}$ the tunnel couplings be-

tween QD and electrodes. We define the Green functions \mathbf{G}^r , \mathbf{G}^a , and $\mathbf{G}^<$ in the Nambu representation.⁸ In the noninteracting case, these Green functions can be solved exactly, and the occupation of QD can be determined by $\mathbf{G}^<$.

Figure 1 shows the curves of the occupation number $\langle n_{\sigma} \rangle \equiv \langle c_{\sigma}^\dagger c_{\sigma} \rangle$ vs the resonant level $E_0 \equiv E_{\uparrow} = E_{\downarrow}$. (a), (b), and (c) correspond to the bias voltages $V > 0$, $V = 0$, and $V < 0$, respectively. For $V = 0$, there is a step from 0 to 1 in the curve, indicating an electron filling when E_0 sweeps the chemical potentials $\mu_F = \mu_S = 0$. For $V > 0$ ($V < 0$), the step is shifted to $E_0 = \mu_F = V$, and there emerges a resonant dip (peak) pinned at $E_0 = \mu_S = 0$. The results can be understood as follows [see the inset of (a)]: Because there is no single-particle state in the superconducting gap, electron filling to the QD is mainly determined by F, leading to a step linked to μ_F . When the resonant level lines up with μ_S , however, resonant AR may occur, in which a spin \uparrow and a spin \downarrow electron in QD can leak into S by forming a Cooper pair and vice versa. As a result, an Andreev dip (Andreev peak) is superposed on the steplike curve. The most remarkable features of AR dip (AR peak) are as follows: (1) The spin polarization of QD, $m \equiv \langle n_{\uparrow} \rangle - \langle n_{\downarrow} \rangle$, *strongly* depends on the magnetization of F; (2) the *sign* of m is controllable by the bias voltage, $m > 0$ for $V > 0$ and $m < 0$ for $V < 0$. Hereafter, the properties of AR resonance are referred to as Andreev reflection induced spin polarization (ARISP). Qualitatively, ARISP effect can be interpreted as follows: For $V > 0$ and $E_0 = \mu_S$, spin \uparrow and spin \downarrow electrons in the QD form Cooper pair and enter S. Since F can provide more spin \uparrow electron than spin \downarrow electron, the spin \downarrow electron will be depleted by the spin \uparrow electron in the process of AR, resulting in a strong spin polarization in QD. For $V < 0$ and $E_0 = \mu_S$, a Cooper pair is converted into a spin \uparrow and a spin \downarrow electron in QD. It is much easier for spin \uparrow electron than spin \downarrow electron to escape to the empty states of F, resulting in a reversed spin polarization.

For quantitative analysis, we evaluate $\langle n_{\sigma} \rangle$ near the AR resonance, obtaining $\langle n_{\uparrow} \rangle = 1 - n(p)$ and $\langle n_{\downarrow} \rangle = 1 - n(-p)$ for $V > 0$, $\langle n_{\uparrow} \rangle = n(p)$ and $\langle n_{\downarrow} \rangle = n(-p)$ for $V < 0$, where

$$n(p) \equiv \frac{1}{2} (1 - p) \frac{\Gamma_S^2}{4E_0^2(1 - p^2) + \Gamma_F^2(1 - p^2) + \Gamma_S^2}, \quad (1)$$

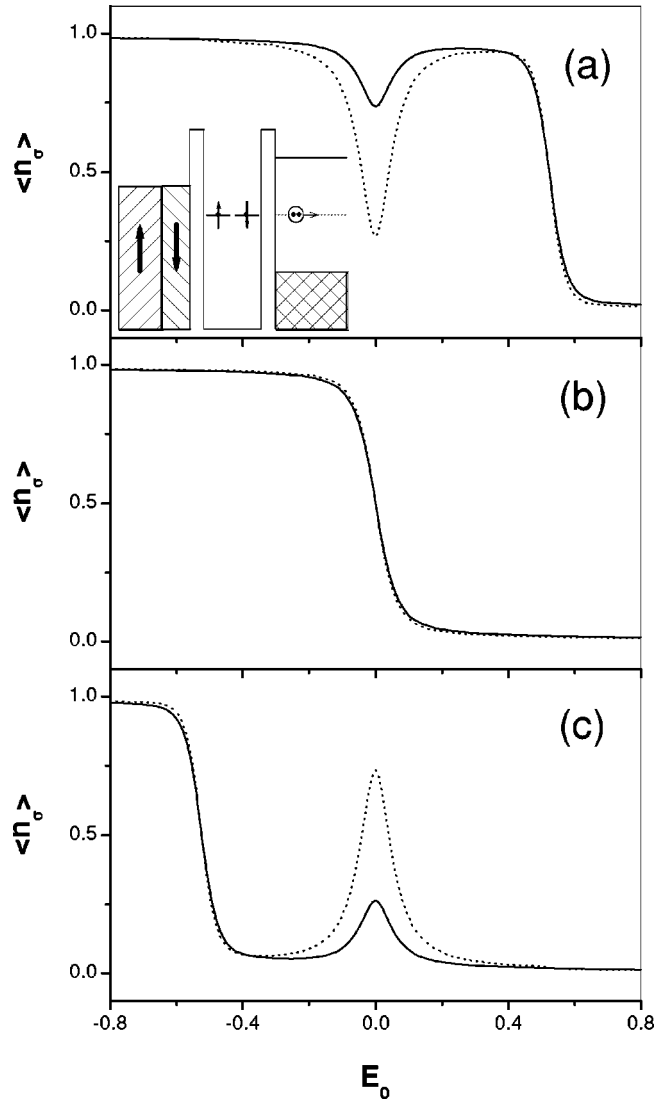


FIG. 1. The occupation number $\langle n_\uparrow \rangle$ (solid) and $\langle n_\downarrow \rangle$ (dotted) vs the resonant level E_0 in a noninteracting F-QD-S system. The superconducting gap $\Delta=1$ is set as energy unit, the bias voltage $V=0.5, 0,$ and -0.5 for (a), (b), and (c), respectively. Other parameters are $\Gamma_F=0.01$, $\Gamma_S=0.1$, $k_B T=0.02$, and $p=0.5$. The inset of (a) schematically shows the AR resonance for $V>0$.

in which Γ_S and $\Gamma_F \equiv (\Gamma_{F\uparrow} + \Gamma_{F\downarrow})/2$ are the coupling strengths between QD and electrodes, and $p \equiv (\Gamma_{F\uparrow} - \Gamma_{F\downarrow})/(\Gamma_{F\uparrow} + \Gamma_{F\downarrow})$ is the magnetization in F. Two features are noteworthy: (1) The AR resonance has the Lorentzian line shape with half-width $\sqrt{[\Gamma_F^2(1-p^2) + \Gamma_S^2]/4(1-p^2)}$, implying that the resonance is broadened with the increase of p ; (2) the spin polarization m reaches its maximum when $E_0 = \mu_S = 0$, and the maximum m_0 only depends on the ratio p and $r \equiv \Gamma_F/\Gamma_S$. ARISP effect is most pronounced when p is large and r is small. In the limit $r \rightarrow 0$, $m_0 = \frac{1}{2}(1 \pm p)$ and $m_0 = \frac{1}{2}(1 \mp p)$ (upper sign for $V>0$ and lower sign for $V<0$). This means that both F and S electrodes are important in the ARISP effect: F provides the asymmetry between two spin categories, while S enforces the asymmetry via AR process. Below we shall consider the interacting QD, to inves-

tigate whether ARISP effect can still survive in presence of intradot interaction. Due to the electron-hole symmetry, only the case of $V>0$ is discussed.

Interacting case. To include the Coulomb interaction, we add the term $Un_\uparrow n_\downarrow$ to H_D , which makes F-QD-S a strongly correlated system. Notice that in the limit $U \rightarrow \infty$, double occupation is forbidden in QD, and electron number cannot fluctuate by two. Hence AR is completely inhibited by the interaction in this limit. If, however, the charging energy U is comparable to the superconducting gap Δ , AR can still occur with the aid of bias voltage. Therefore, techniques for $U \rightarrow \infty$ limit (e.g., slave boson method) cannot be applied to the calculation of ARISP. Alternatively, we adopt the equation of motion (EOM) method, which is known to be reliable in the Coulomb blockade regime, and qualitatively correct for Kondo physics.^{9,10} Moreover, EOM solution becomes exact in the $U \rightarrow 0$ limit. In the EOM approach, one can derive the equation for the retarded Green function $\langle\langle A(t_1)|B(t_2) \rangle\rangle^r$ by differentiating with respect to t_1 or t_2 , and generate new Green functions. To close the equation chain, we make the truncation in those Green functions containing two electrode operators in a mean-field manner. After some algebra, the equation for \mathbf{G}^r can be obtained as

$$\mathbf{A}\mathbf{G}^r + \mathbf{G}^r\tilde{\mathbf{A}} = 2\mathbf{N} + \mathbf{B} + \tilde{\mathbf{B}}, \quad (2)$$

in which $\mathbf{B} \equiv (\mathbf{g}_1^{-1} - \Sigma_1^r)\mathbf{U}^{-1}$, $\mathbf{A} \equiv \mathbf{B}(\mathbf{g}_0^{-1} - \Sigma_0^r) + \Sigma_2^r$, $\tilde{\mathbf{A}}$ or $\tilde{\mathbf{B}}$ represents the transpose of \mathbf{A} or \mathbf{B} , \mathbf{U} , and \mathbf{N} are defined as

$$\mathbf{U} \equiv \begin{pmatrix} +U & 0 \\ 0 & -U \end{pmatrix}, \quad \mathbf{N} \equiv \begin{pmatrix} \langle n_\downarrow \rangle & 0 \\ 0 & \langle n_\uparrow \rangle \end{pmatrix}. \quad (3)$$

\mathbf{g}_0 and \mathbf{g}_1 are the bare Green functions for the resonances E_σ and $E_\sigma + U$,

$$\mathbf{g}_0 = \begin{pmatrix} \frac{1}{\omega - E_\uparrow} & 0 \\ 0 & \frac{1}{\omega + E_\downarrow} \end{pmatrix}, \quad (4)$$

$$\mathbf{g}_1 = \begin{pmatrix} \frac{1}{\omega - E_\uparrow - U} & 0 \\ 0 & \frac{1}{\omega + E_\downarrow + U} \end{pmatrix}.$$

Σ_0^r and Σ_1^r are the corresponding dressing self-energies due to tunnel coupling with electrodes. Σ_2^r is the self-energy contributed by the spin flip in the cotunneling process, which is related to Kondo physics. In the wideband limit, these self-energies can be evaluated analytically. Let $\Sigma_i^r = \Sigma_{Fi}^r + \Sigma_{Si}^r$,

$$\Sigma_{F0}^r = -\frac{i}{2} \begin{pmatrix} \Gamma_{F\uparrow} & 0 \\ 0 & \Gamma_{F\downarrow} \end{pmatrix}, \quad (5)$$

$$\Sigma_{F1}^r = -\frac{i}{2} \begin{pmatrix} \Gamma_{F\uparrow} + 2\Gamma_{F\downarrow} & 0 \\ 0 & \Gamma_{F\downarrow} + 2\Gamma_{F\uparrow} \end{pmatrix}, \quad (6)$$

$$\Sigma_{F2}^r = \begin{pmatrix} \Gamma_{F\downarrow} Q \left(\frac{-\omega_1 - V}{k_B T}, \frac{\omega_3 - V}{k_B T} \right) & 0 \\ 0 & \Gamma_{F\uparrow} Q \left(\frac{-\omega_3 - V}{k_B T}, \frac{\omega_2 - V}{k_B T} \right) \end{pmatrix}, \quad (7)$$

$$\Sigma_{S0}^r = \Gamma_S \begin{pmatrix} s_2(\omega_0) & -s_1(\omega_0) \\ -s_1(\omega_0) & s_2(\omega_0) \end{pmatrix}, \quad (8)$$

$$\Sigma_{S1}^r = \Gamma_S \begin{pmatrix} s_2(\omega_0) + s_2(\omega_1) + s_2(\omega_3) & s_1(\omega_3) \\ s_1(\omega_3) & s_2(\omega_0) + s_2(\omega_2) + s_2(\omega_3) \end{pmatrix}, \quad (9)$$

$$\Sigma_{S2}^r = \frac{\Gamma_S}{2} \begin{pmatrix} s_2(\omega_1) + s_2(\omega_3) + s_4(\omega_1) - s_4(\omega_3) & s_1(\omega_0) + s_1(\omega_3) + s_3(\omega_0) + s_3(\omega_3) \\ s_1(\omega_0) + s_1(\omega_3) - s_3(\omega_0) - s_3(\omega_3) & s_2(\omega_2) + s_2(\omega_3) + s_4(\omega_3) - s_4(\omega_2) \end{pmatrix}, \quad (10)$$

in which $s_1(\omega) \equiv -(1/\pi)J(\omega/\Delta)$, $s_2(\omega) \equiv -(1/\pi)J(\omega/\Delta) \times (\omega/\Delta)$, $s_3(\omega) \equiv -(1/\pi)I(\omega/\Delta)(\omega/\Delta)$, $s_4(\omega) \equiv -(1/\pi)J(\omega/\Delta)(\omega^2/\Delta^2)$, and $\omega_0 \equiv \omega$, $\omega_1 \equiv \omega - E_\uparrow - E_\downarrow - U$, $\omega_2 \equiv \omega + E_\uparrow + E_\downarrow + U$, $\omega_3 \equiv \omega - E_\uparrow + E_\downarrow$. The dimensionless functions I , J , Q are defined as

$$I(x) \equiv \begin{cases} \frac{\arcsin x}{x\sqrt{1-x^2}}, & |x| < 1 \\ \frac{i\pi}{2x\sqrt{x^2-1}} - \frac{1}{|x|\sqrt{x^2-1}} \ln(|x| + \sqrt{x^2-1}), & |x| > 1, \end{cases} \quad (11)$$

$$J(x) \equiv \begin{cases} \frac{\pi}{2} \frac{1}{\sqrt{1-x^2}}, & |x| < 1 \\ \frac{\pi}{2} \frac{i}{\sqrt{x^2-1}} \operatorname{sgn}(x), & |x| > 1, \end{cases} \quad (12)$$

$$Q(x, y) \equiv -\frac{i}{2} \left(\frac{1}{e^x + 1} + \frac{1}{e^y + 1} \right) + \frac{1}{2\pi} \operatorname{Re} \left[\psi \left(\frac{1}{2} + \frac{ix}{2\pi} \right) - \psi \left(\frac{1}{2} + \frac{iy}{2\pi} \right) \right], \quad (13)$$

with ψ being the digamma function.

As for $\mathbf{G}^<$, we invoke the stationary condition

$$\begin{pmatrix} \left\langle \frac{d}{dt} c_\uparrow^\dagger c_\uparrow \right\rangle & \left\langle \frac{d}{dt} c_\downarrow c_\uparrow \right\rangle \\ \left\langle \frac{d}{dt} c_\uparrow^\dagger c_\downarrow^\dagger \right\rangle & \left\langle \frac{d}{dt} c_\downarrow c_\downarrow^\dagger \right\rangle \end{pmatrix} = 0. \quad (14)$$

Using Heisenberg equation and neglecting high-order fluctuations,¹² we obtain

$$\left(\Sigma_0^r + \frac{1}{2} E_{\uparrow\downarrow} \sigma_z \right) \mathbf{G}^< - \mathbf{G}^< \left(\Sigma_0^a + \frac{1}{2} E_{\uparrow\downarrow} \sigma_z \right) = \mathbf{G}^r \Sigma_0^< - \Sigma_0^< \mathbf{G}^a, \quad (15)$$

in which $E_{\uparrow\downarrow} \equiv E_\uparrow + E_\downarrow + U$ and σ_z is the third Pauli matrix. This equation can be interpreted as ‘‘detailed’’ stationary condition, i.e., the observables in the energy interval ω and $\omega + d\omega$ are time invariant in the steady state. For comparison, we also try the Keldysh equation $\mathbf{G}^< = \mathbf{G}^r \Sigma^< \mathbf{G}^a$, and employ the commonly used Ng’s ansatz for $\Sigma^<$.^{10,11} The calculated results qualitatively agree with each other, but the numerical convergence is much poorer in the latter scheme. To sum up, Eq. (2) for \mathbf{G}^r and Eq. (15) for $\mathbf{G}^<$ will be applied to the numerical study of ARISP.

Before presenting the numerical results, we qualitatively analyze the physics in the problem. As seen in the noninteracting case, spin polarization occurs on the AR resonance. The conditions for AR resonance are as follows: (1) QD is occupied with even number electrons and (2) a pair of electrons can transfer freely between QD and S. In the noninteracting case, these conditions amount to $\mu_L > \mu_R$ and $E_0 = \mu_R$ for AR dip, or $\mu_L < \mu_R$ and $E_0 = \mu_R$ for AR peak. In the presence of intradot interaction, the occupation number of QD is quantized due to Coulomb blockade effect. Notice that the occupation of QD is mainly determined by F; it is energetically favorable that QD is empty when $E_0 - \mu_F > 0$, singly occupied when $-U < E_0 - \mu_F < 0$, and doubly occupied when $E_0 - \mu_F < -U$. AR resonance occurs in the evenly occupied valleys, and further requires $E_\uparrow + E_\downarrow + U = 2\mu_S$, meaning that the energy of QD is degenerate when moving two electrons between QD and S. Therefore the conditions for AR resonance are $E_0 - \mu_S = -U/2$ and $E_0 - \mu_F < -U$ for AR dip, $E_0 - \mu_S = -U/2$ and $E_0 - \mu_F > 0$ for AR peak. The key point is that electron filling is linked to μ_F while AR is linked to μ_S , and $\mu_F \neq \mu_S$ in nonequilibrium.

Figure 2 shows the curve of $\langle n_\sigma \rangle$ vs E_0 for $p = 0.5$. As expected, a spin-polarized AR dip is superposed on a steplike pattern. The steps from 0 to $\frac{1}{2}$ and $\frac{1}{2}$ to 1 are located around $E_0 - \mu_F = 0$ and $E_0 - \mu_F = -U$, and AR dip located around $E_0 - \mu_S = -U/2$, in agreement to the above analysis. $\langle n_\uparrow \rangle$ and $\langle n_\downarrow \rangle$ are nearly equal away from the AR resonance, but separated dramatically on the resonance, indicating ARISP effect. The inset (a) shows the detailed line shape of AR dip for several p . One can see that Eq. (1) derived in the nonin-

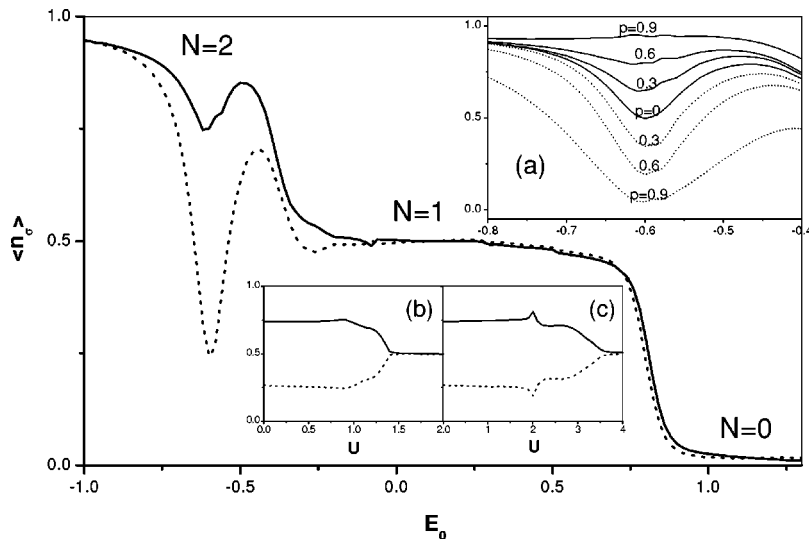


FIG. 2. The occupation number $\langle n_{\uparrow} \rangle$ (solid) and $\langle n_{\downarrow} \rangle$ (dotted) vs the resonant level E_0 in an interacting F-QD-S system. The bias voltage $V = 0.75$, the charging energy $U = 1.2$, other parameters are the same as Fig. 1. The inset (a) shows the detailed line shape of the Andreev dip for several p . The insets (b) and (c) show the curves of $\langle n_{\uparrow} \rangle$ (solid) and $\langle n_{\downarrow} \rangle$ (dotted) at $E_0 - \mu_S = -U/2$ vs the charging energy U , with the bias voltage $V = 0.5$ and $V = 1.5$, respectively.

interacting case is also applicable to the interacting case. In fact, the intradot interaction plays a role of quantizing the electron number of QD. Coulomb blockade is lifted under certain conditions, in which resonant AR occurs as if through a noninteracting QD. Next, we discuss the choice of charging energy U , which is determined by the size of QD. The insets (b) and (c) show the curves of $\langle n_{\sigma} \rangle$ vs U on the AR resonance ($E_0 - \mu_S = -U/2$). (b) is for the case of $V < \Delta$, in which all single-particle processes are forbidden and only AR dominates. One can see in the plot that the spin polarization is almost independent of U when $U < 2V$, but gradually suppressed when $U > 2V$. The reason is as follows: on the AR resonance, QD favors double occupation when $U < 2V$ and single occupation when $U > 2V$. For double occupation, ARISP is determined by the ratio p and r , and independent of other parameters. For single occupation, the effective S-QD coupling is greatly suppressed, and therefore ARISP vanishes. One tends to think that increasing the bias voltage V may help to overcome the charging energy U . This is true as long as $V < \Delta$. When $V > \Delta$, however, both single-particle processes and AR are allowed, and the situation is more complicated. (c) shows $\langle n_{\sigma} \rangle$ vs U for $V > \Delta$. It turns out that the spin polarization is still independent of U when $U < 2\Delta$, reaches a maximum at $U = 2\Delta$, suppressed when $2\Delta < U < 2V$, and gradually vanishes when $U > 2V$. The suppression in the range of $2\Delta < U < 2V$ can be attributed to

the onset of single-particle process. We note that EOM solution is valid for relatively small U , and the suppression of ARISP is probably underestimated when $U > 2\Delta$. Nevertheless, the maximum at $U = 2\Delta$ is reliable, which is related to the singularity in the density of states of S.

In conclusion, we have proposed an efficient mechanism for writing spin in a QD which is based on the ARISP effect in F-QD-S system. The scheme has the advantages that the magnetization of F is not required to be strong and the sign of spin polarization can be controlled in fully electric manner. Calculation shows that the optimal conditions for ARISP are $\Gamma_F \leq \Gamma_S$, $U = 2\Delta$, $E_0 - \mu_S = -U/2$, and $|V| > U/2$. The properties of ARISP can be described by Eq. (1). In practice, the resonant level E_0 can be tuned by gate voltage, and AR resonance can be monitored by the small tunnel current between F and S electrodes. In the context of intensive research and impressive progress in F/2DEG, S/2DEG, and F/S hybrid structures,¹³⁻¹⁶ the proposed F-QD-S system should be feasible with up-to-date nanotechnology. We are looking forward to hearing the relevant experimental response.

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