Negative refraction and omnidirectional total transmission at a planar interface associated with a uniaxial medium

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(Received 13 October 2003; published 4 March 2004)

The conditions for yielding negative refraction as well as omnidirectional total transmission at planar interfaces between uniaxial media have been analyzed. It is found that both phenomena can occur at a wide variety of interfaces associated with uniaxial media. In particular, when certain conditions are satisfied, even an interface between isotropic and uniaxial media can also exhibit negative refraction and omnidirectional total transmission, adding considerably to the flexibility in potential device applications.

DOI: 10.1103/PhysRevB.69.115402 PACS number(s): 78.20.Ci, 42.25.Gy, 41.20.Jb

The phenomena of reflection and refraction of light at the interface of two transparent media are the underlying mechanisms that are widely used for steering light in many optical devices.[1] Two phenomena—omnidirectional total reflection and omnidirection total refraction, where wave is completely reflected or transmitted for arbitrary incoming directions—are of particular interest. There has been much discussion on realizations of omnidirectional total reflection (without energy loss due to refraction), either by omnidirectional reflector (see, e.g., Refs. 2–5) or, maybe preferably, by photonic crystals.[6–8] The refraction, however, requires a refractive index mismatch, which seems to inevitably result in a finite reflection loss, except at particular oblique incidence like Brewster's angle.[1] A question arising naturally is whether there exists any type of interface that enables omnidirectional total transmission without any reflection while bending the direction of light propagation—namely, whether waves can be refracted without reflection, like the reflection counterpart on omnidirectional reflectors and photonic crystals.

Negative refraction, in which the tangential component of the time-averaged Poynting vector changes sign when refracted, has recently attracted an increasing amount of attention[9–17] due to its implication for realizing so-called subwavelength focusing[18,19] and many other extraordinary wave propagation phenomena. Although the achievability of negative refraction in existing metamaterials was once questioned,[20,21] the phenomenon has been examined and confirmed both experimentally and numerically by many researchers[13–15] after the first experimental observation using a metamaterial composed of wires and split ring resonators.[9] Besides occurring at the interface between a positive (conventional) medium, with both electric permittivity ϵ and magnetic permeability μ being positive, and a negative refractive index materials, where ϵ and μ are simultaneously negative,[18] negative refraction was also found to take place in photonic crystals without a negative refractive index.[16,17] In addition, it was also found that negative refraction can be achieved at an interface associated with a uniaxially anisotropic medium with only one of the four parameters of $\hat{\boldsymbol{\epsilon}}$ and $\hat{\boldsymbol{\mu}}$ tensors being negative.[22–24] This leads to the question whether negative refraction may arise at interfaces associated with a conventional uniaxial medium without any negative component of $\hat{\boldsymbol{\epsilon}}$ and $\hat{\boldsymbol{\mu}}$.

An intriguing finding by Zhang and co-workers[25] answers the two questions posed above. They demonstrated experimentally that a unique type of interface of a special category of twinning structures in uniaxial crystals serves as an example for achieving both negative refraction and total transmission (refraction without reflection). In this paper, we will present a systematic theoretical analysis of the conditions to realize negative refraction and omnidirectional total transmission at the interfaces associated with conventional uniaxial media that are readily available. It is found that negative refraction can be observed over a range of incident angle at a variety of interfaces between an isotropic medium and a uniaxial medium, as well as between two uniaxial media. Omnidirectional total transmission is by no means limited to the interface of twining structures[25] either. With a suitable arrangement of the optical axis of a uniaxial medium and appropriate tuning of the permittivity of an isotropic medium, omnidirectional total transmission can also occur at interfaces between isotropic and uniaxial media, adding considerably to the flexibility in potential device applications.

The system considered here is a generalization of the interface of twinning structure studied in Ref. 25. It is an interface between two nonmagnetic uniaxial media. The optical axes of both uniaxial media are coplanar with the normal of the interface. In our coordinate system, the planar interface is in the x-y plane, while the normal of the interface and both optical axes all lie in the x-z plane, with the interface normal along the z direction. A schematic plot of the system is shown in Fig. 1(a), with θ_L and θ_R being angles between the normal of the interface and the optical axis of the uniaxial medium on the left-hand side (denoted by subscript L) and right-hand side (denoted by subscript R) of the interface. In the principal coordinate system, the relative permittivity tensors are given by

$$\hat{\boldsymbol{\epsilon}}^{(0)}_{L(R)} = \begin{pmatrix} \epsilon_{L(R)} & 0 & 0 \\ 0 & \epsilon_{L(R)} & 0 \\ 0 & 0 & \epsilon'_{L(R)} \end{pmatrix} \tag{1}$$

for the the anisotropic medium on the left- and right-hand sides of the interface. Here the superscript (0) is used to imply an expression in the principal coordinate system. An isotropic medium is recovered by setting $\epsilon = \epsilon'$ on either

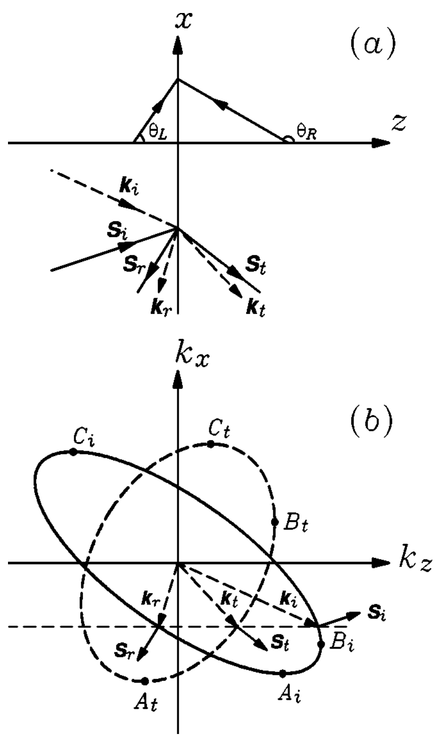

FIG. 1. (a) A schematic plot of interface between uniaxial media and their optical axes. The time averaged Poynting vectors $\mathbf{S}_i$, $\mathbf{S}_r$, and $\mathbf{S}_t$, together with the wave vectors $\mathbf{k}_i$, $\mathbf{k}_r$, and $\mathbf{k}_t$, correspond to those shown in (b) and show an example of negative refraction at interfaces with $S_{ix}>0$ and $S_{tx}<0$. (b) Diagram of the wave vectors of the incident, reflected, and transmitted extraordinary waves for an interface between two positive uniaxial media. The wave vector surfaces resemble ellipses. Also shown are examples of the time-averaged Poynting vectors $\mathbf{S}_i$, $\mathbf{S}_r$, and $\mathbf{S}_t$ and the wave vectors $\mathbf{k}_i$, $\mathbf{k}_r$, and $\mathbf{k}_t$, for incident, reflected, and transmitted waves. See text for the description of points *A*, *B*, and *C*.

side. If one sets $\theta_R=\pi-\theta_L$, $\epsilon_R=\epsilon_L$ and $\epsilon_R'=\epsilon_L'$, the system reduces to an interface between twinning structures studied by Zhang *et al.*,[25] hereinafter referred to as the symmetric case.

For an electromagnetic wave with electric field **E** polarized in the *y* direction, the system considered here reduces to the isotropic case with permittivity equal to ϵ_L (ϵ_R) on the left- (right-) hand side of the interface, since in this case the **E** field is orthogonal to both uniaxes, which is usually known as the ordinary wave. So in the following we concentrate on the extraordinary wave with the **E** field in the *x*-*z* plane. Owing to symmetry, we study the case with $0\leqslant\theta_L\leqslant 90°$ and $0\leqslant\theta_R\leqslant 180°$, without loss of generality. For simplicity of analysis, we limit our discussion to the case with incident wave vector $\mathbf{k}_i$ confined in the *x*-*z* plane.

Let us first focus on the problem of negative refraction. To analyze the conditions for yielding negative refraction, the diagram of wave vector **k** proves convenient.[26] In Fig. 1(b) we show the constant-ω contour of the wave vector **k** for both incident and transmitted waves for an example of positive uniaxial media ($\epsilon_{L(R)}'>\epsilon_{L(R)}$). The constant-$\omega$ contour (*k* surfaces) for both incident and transmitted waves are ellipses described by the dispersion relations

$$\frac{(k_{ix}\cos\theta_L-k_{iz}\sin\theta_L)^2}{\epsilon_L'}+\frac{(k_{ix}\sin\theta_L+k_{iz}\cos\theta_L)^2}{\epsilon_L}=\frac{\omega^2}{c^2},$$

$$\frac{(k_{tx}\cos\theta_R-k_{tz}\sin\theta_R)^2}{\epsilon_R'}+\frac{(k_{tx}\sin\theta_R+k_{tz}\cos\theta_R)^2}{\epsilon_R}=\frac{\omega^2}{c^2}, \tag{2}$$

where ω is the circular frequency, *c* is the speed of light in vacuum, and k_{ix} and k_{iz} (k_{tx} and k_{tz}) are, respectively, the *x* and *z* components of the wave vectors for incident (transmitted) wave. For simplicity, hereinafter we set the wave vector in vacuum $\omega/c=1$.

For a lossless medium, at any point on the *k* surface, the direction of the time-averaged Poynting vector, which defines the direction of the propagation of light, is normal to the surface at that point.[26] In Fig. 1(b), the time-averaged Poynting vectors for incident, reflected, and transmitted waves, denoted, respectively, by $\mathbf{S}_i$, $\mathbf{S}_r$, and $\mathbf{S}_t$, are shown together with the corresponding wave vectors $\mathbf{k}_i$, $\mathbf{k}_r$, and $\mathbf{k}_t$. Here the subscripts *i*, *r*, and *t* represent the incident, reflected and transmitted waves, respectively. Let A_i (C_i) denote the point at the *k* surface for incident waves which satisfies $\partial k_{ix}/\partial k_{iz}=0$ and $k_{ix}<0$ ($k_{ix}>0$) and B_i stand for the point where $\partial k_{iz}/\partial k_{ix}=0$ and $k_{iz}>0$, as shown in Fig. 1(b). Points A_t, B_t, and C_t are defined in the same way as A_i, B_i, and C_i, except they refer to the *k* surface for transmitted waves. A propagating incident (transmitted) wave vector should lie on the arc that goes from A_i (A_t) anticlockwise to C_i (C_t) such that $S_{iz}>0$ ($S_{tz}>0$), where S_{iz} (S_{tz}) is the *z* component of $\mathbf{S}_i$ ($\mathbf{S}_t$). Because point B_i satisfies $\partial k_{iz}/\partial k_{ix}=0$, if $\mathbf{k}_i$ lies on point B_i, then $S_{ix}=0$ [see Fig. 1(b)]. Here S_{ix} is the *x* component of $\mathbf{S}_i$. In addition, if $\mathbf{k}_i$ lies on the arc between points A_i and B_i, then one has $S_{ix}<0$, as can be seen from Fig. 1(b). In other words, an incident wave with wave vector $\mathbf{k}_i$ satisfying

$$k_{A_ix}<k_{ix}<k_{B_ix} \tag{3}$$

has $S_{ix}<0$, where k_{B_ix} (k_{A_ix}) denotes the *x* component of the wave vector **k** at point B_i (A_i) on the incident *k* surface. Vice versa, if $\mathbf{k}_i$ locates on the arc between B_i and C_i, as shown in Fig. 1(b), then $S_{ix}>0$; namely,

$$k_{B_ix}<k_{ix}<k_{C_ix} \tag{4}$$

leads to $S_{ix}>0$. A similar analysis applies to transmitted waves. Explicitly,

$$k_{A_tx}<k_{tx}<k_{B_tx} \tag{5}$$

guarantees $S_{tx}<0$ and

$$k_{B_tx}<k_{tx}<k_{C_tx} \tag{6}$$

results in $S_{tx}>0$. Here k_{A_tx}, k_{B_tx}, etc., are *x* components of **k** at corresponding points. S_{tx} is the *x* component of $\mathbf{S}_t$. The lower [upper] bound of (5) [(6)] is to eliminate the occurrence of evanescent waves due to the total internal reflection at the interface.

At the interface, the tangential component of the **k** vector is continuous—namely, $k_{tx}=k_{ix}$—so one concludes that the negative refraction, which is manifested by $S_{ix}S_{tx}<0$, can be achieved by requiring

$$k_{A_ix}<k_{ix}<k_{B_ix}, \tag{7a}$$

$$k_{B_tx}<k_{ix}<k_{C_tx} \tag{7b}$$

or

$$k_{B_ix}<k_{ix}<k_{C_ix}, \tag{7c}$$

$$k_{A_tx}<k_{ix}<k_{B_tx}, \tag{7d}$$

where Eq. (7a) yields negative S_{ix} and Eq. (7b) implies positive S_{tx}, whereas Eq. (7c) leads to $S_{ix}>0$ and Eq. (7d) gives rise to $S_{tx}<0$. Figures 1(a) and 1(b) show a case when Eqs. (7c) and (7d) are satisfied. A negative refraction is observed with $S_{ix}>0$ and $S_{tx}<0$. The explicit expressions for $\mathbf{k}_{A_i}$, $\mathbf{k}_{B_i}$, etc., can be worked out, based on the dispersion relations (2), as

$$k_{A_ix}=-\beta_L,\ k_{B_ix}=\gamma_L,\ k_{C_ix}=\beta_L,$$

$$k_{A_tx}=-\beta_R,\ \ k_{B_tx}=\gamma_R,\ \ k_{C_tx}=\beta_R, \tag{8}$$

which give rise to the negative refraction conditions in terms of optical parameters:

$$\max\{-\beta_L,\gamma_R\}<k_{ix}<\min\{\gamma_L,\beta_R\} \tag{9a}$$

for $\gamma_L>\gamma_R$ or

$$\max\{\gamma_L,-\beta_R\}<k_{ix}<\min\{\beta_L,\gamma_R\} \tag{9b}$$

for $\gamma_L<\gamma_R$ where α_L, β_L, etc., are given by

$$\alpha_{L(R)}=[\epsilon_{L(R)}\cos^2\theta_{L(R)}+\epsilon'_{L(R)}\sin^2\theta_{L(R)}]^{1/2},$$

$$\beta_{L(R)}=[\epsilon_{L(R)}\sin^2\theta_{L(R)}+\epsilon'_{L(R)}\cos^2\theta_{L(R)}]^{1/2},$$

$$\gamma_{L(R)}=\frac{[\epsilon_{L(R)}-\epsilon'_{L(R)}]\sin\theta_{L(R)}\cos\theta_{L(R)}}{\alpha_{L(R)}}. \tag{10}$$

In terms of incident angle, the conditions for yielding negative refraction become

$$\max\left\{-\frac{\pi}{2},\theta_0\right\}<\theta_i<\min\{0,\theta_-\} \tag{11a}$$

for $\gamma_L>\gamma_R$ or

$$\max\{0,\theta_+\}<\theta_i<\min\left\{\theta_0,\frac{\pi}{2}\right\} \tag{11b}$$

for $\gamma_L<\gamma_R$, where the propagation direction of wave is defined based on the time-averaged Poynting vector, with, e.g., incident and refraction angles, denoted by θ_i and θ_t, given by

$$\theta_i=\tan^{-1}\frac{S_{ix}}{S_{iz}},\quad \theta_t=\tan^{-1}\frac{S_{tx}}{S_{tz}}. \tag{12}$$

$\theta_\pm$ and θ_0 in Eqs. (11) are given by

$$\theta_\mp=\begin{cases}\tan^{-1}\left(\dfrac{\alpha_L^2-k_\pm^2}{\pm\beta_Rk_\pm-\gamma_L\alpha_L}\right), & \beta_R<\beta_L,\\[2ex] \pm\dfrac{\pi}{2}, & \beta_R>\beta_L,\end{cases} \tag{13}$$

$$\theta_0=\begin{cases}\tan^{-1}\left(\dfrac{\alpha_L^2-k_0^2}{\gamma_Rk_0-\gamma_L\alpha_L}\right), & |\gamma_R|<\beta_L,\\[2ex] -\dfrac{\pi}{2}, & \gamma_R<-\beta_L,\\[2ex] \dfrac{\pi}{2}, & \gamma_R>\beta_L,\end{cases} \tag{14}$$

with

$$k_\pm=\frac{[\epsilon_L\epsilon'_L(\beta_L^2-\beta_R^2)]^{1/2}\pm\beta_R\gamma_L\alpha_L}{\beta_L^2},$$

$$k_0=\frac{[\epsilon_L\epsilon'_L(\beta_L^2-\gamma_R^2)]^{1/2}+\gamma_R\gamma_L\alpha_L}{\beta_L^2}. \tag{15}$$

Notice that $\theta_+\geqslant 0$ and $\theta_-\leqslant 0$.

As can be seen from Eqs. (11), the occurrence of negative refraction does not require symmetry as proposed by Zhang *et al.*[25] Nor does it require that both sides be uniaxially anisotopic. It can actually arise over a range of incident angle at various interfaces associated with a uniaxial medium. The case proposed by Zhang *et al.*[25] is actually a particular example to realize the negative refraction. For an interface between two isotropic media, however, due to $\gamma_L=\gamma_R=0$, neither Eq. (9a) nor (9b) can be satisfied, which recovers the well-known fact that no negative refraction can be observed at interfaces between conventional isotopic media with positive ϵ and μ.

Figure 2 shows the range of incident angle θ_i at which negative refraction arises for a symmetric case (with $\theta_R=\pi-\theta_L$, $\epsilon_R=\epsilon_L$, and $\epsilon'_R=\epsilon'_L$) and a case where the medium on the left-hand side of the interface is isotropic ($\epsilon'_L=\epsilon_L$). It is found that negative refraction can be obtained over a range of incident angles in both cases. In Fig. 3, the range of θ_i where the negative refraction occurs is displayed as a function of anisotropy parameter u, defined by $\epsilon'=\epsilon(1+u)$. It is found that as the anisotropy increases, one can obtain a greater region for realizing negative refraction. Figure 4 shows the maximum bending angle $\theta_b^{\max}$ (when negative refraction arises) and the maximum incident angle $\theta_i^{\max}$ as a function θ_L for a symmetric case of positive uniaxial media. Here $\theta_i^{\max}$ represents the upper bound such that negative refraction occurs. The bending angle θ_b is defined by

$$\theta_b=\theta_t-\theta_i. \tag{16}$$

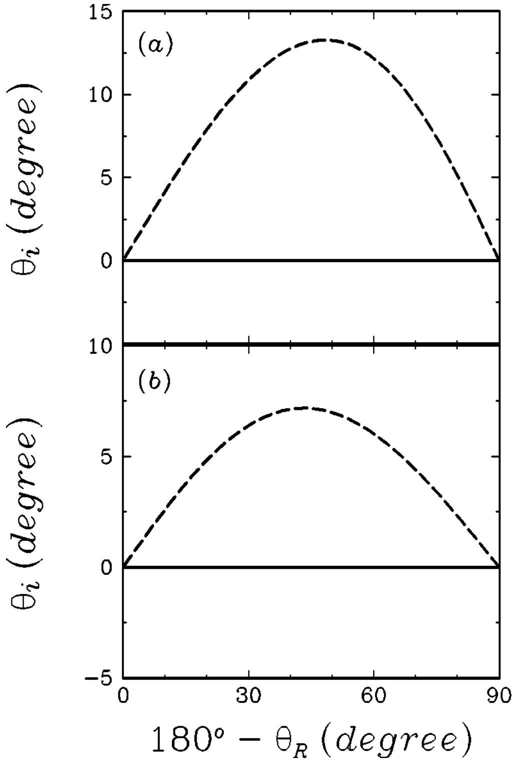

FIG. 2. Negative refraction occurs when the incident angle of light locates between the dashed and solid lines for (a) a symmetric case with $\epsilon_L=\epsilon_R=2^2$, $\epsilon_L'=\epsilon_R'=2.25^2$, and $\theta_L=180°-\theta_R$ and (b) a case with $\epsilon_L=\epsilon_L'=\epsilon_R=2^2$, and $\epsilon_R'=2.25^2$. The permittivities for uniaxial medium used here resemble those for YVO_4 (Ref. 25).

Given optical parameters and the direction of the optical axis, the maximum bending happens at $k_{ix}=0$ for the symmetric case, suggesting

$$\theta_b^{\max}=-2\tan^{-1}\left(\frac{\alpha_L\gamma_L}{\beta_L^2}\right). \tag{17}$$

From Fig. 4, it is found that the maximum values of both $-\theta_b^{\max}$ and $\theta_i^{\max}$ occur at θ_L away from 45°. The discrepancy increases as the anisotropic parameter u deviates away from 0.

The negative refraction discussed here finds its origin in the anisotropy. In an anisotropic medium, the direction of the time-averaged Poynting vector **S** deviates from that of the wave vector **k**. So, at the interface, the tangential components of **S** and **k**, denoted, respectively, by S_τ and k_τ, can have either the same or the opposite signs, depending on the value of k_τ as well as the orientation of the optical axis of anisotropic medium. If, under certain conditions, on the one side of the interface k_τ and S_τ have the same sign, while on the other side of the interface k_τ and S_τ have opposite signs, then because k_τ should be continuous and have the same sign on both sides, S_τ will exhibit opposite signs on different sides of the interface, leading to a manifestation of the negative refraction. Figure 1 shows an example of negative refraction at the interface, where k_{ix} and S_{ix} have opposite signs ($k_{ix}<0$ and $S_{ix}>0$) on the left-hand side, while $k_{tx}=k_{ix}$ and S_{tx} have the same sign on the right-hand side of the interface. From the analysis, it is concluded that the anisotropy serves as a mechanism for the negative refraction discussed here.

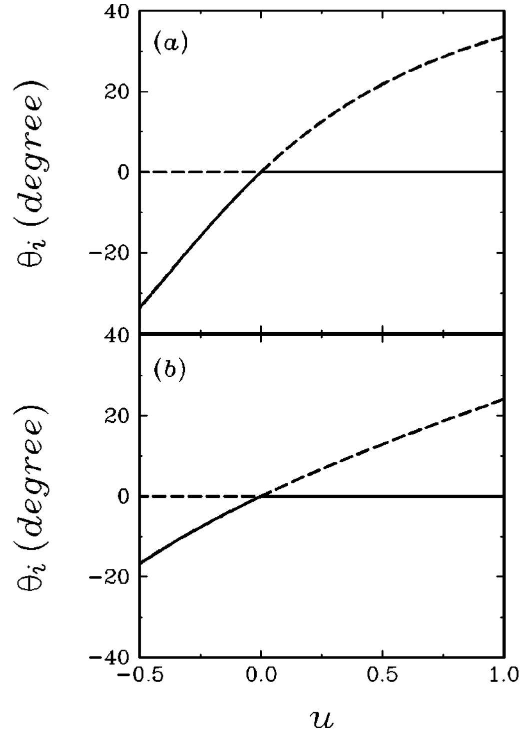

FIG. 3. Negative refraction arises when the incident angle of light locates between the dashed and solid lines for (a) a symmetric case with $\epsilon_L=\epsilon_R=2^2$, $\epsilon_L'=\epsilon_R'=\epsilon_L(1+u)$, and $\theta_L=180°-\theta_R=45°$ and (b) a case with $\epsilon_L=\epsilon_L'=\epsilon_R=2^2$, $\epsilon_R'=\epsilon_R(1+u)$, and $\theta_R=135°$.

Now we turn to the problem of total transmission. An extraordinary incident wave (with **E** field lying in the x-z plane) can be written as

$$\mathbf{E}_i=(E_{ix}\mathbf{e}_x+E_{iz}\mathbf{e}_z)e^{ik_{ix}x+ik_{iz}z-i\omega t}, \tag{18}$$

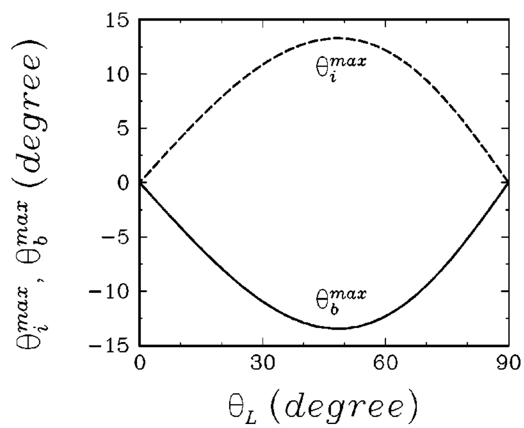

FIG. 4. The maximum incident angle $\theta_i^{\max}$ (which may cause negative refraction) and the maximum bending angle $\theta_b^{\max}$ as a function of θ_L for a symmetric case with $\epsilon_L=\epsilon_R=2^2$, $\epsilon_L'=\epsilon_R'=2.25^2$, and $\theta_R=180°-\theta_L$.

with $\mathbf{e}_x$ ($\mathbf{e}_z$) the unit vector in the x (z) direction. The wave equation

$$\nabla\times\nabla\times\mathbf{E}_i-\frac{\omega^2}{c^2}\hat{\boldsymbol{\epsilon}}\cdot\mathbf{E}_i=0 \tag{19}$$

serves to determine the dispersion relation (2) as well as the ratio $c_i=E_{iz}/E_{ix}$ for a given value of k_{ix}. As a result, the electric field $\mathbf{E}$ and magnetic field $\mathbf{H}$ for the incident wave are given by

$$\mathbf{E}_i=E_{ix}(\mathbf{e}_x+c_i\mathbf{e}_z)e^{ik_{ix}x+ik_{iz}z-i\omega t}, \tag{20a}$$

$$\mathbf{H}_i=\frac{E_{ix}}{\omega\mu_0}\mathbf{e}_y(k_{iz}-c_ik_{ix})e^{ik_{ix}x+ik_{iz}z-i\omega t}, \tag{20b}$$

where c_i is worked out to be

$$c_i=-\frac{-k_{iz}^2+\alpha_L^2}{k_{ix}k_{iz}-\gamma_L\alpha_L}. \tag{21}$$

The value of k_{iz} is uniquely determined by the dispersion relation (2) and the requirement $S_{iz}>0$:

$$k_{iz}=[\alpha_L\gamma_Lk_{ix}+\sqrt{\epsilon_L\epsilon_L'(\beta_L^2-k_{ix}^2)}]/\beta_L^2. \tag{22}$$

Similarly, the reflected and transmitted waves are

$$\mathbf{E}_r=E_{rx}(\mathbf{e}_x+c_r\mathbf{e}_z)e^{ik_{ix}x+ik_{rz}z-i\omega t}, \tag{23a}$$

$$\mathbf{H}_r=\frac{E_{rx}}{\omega\mu_0}\mathbf{e}_y(k_{rz}-c_rk_{ix})e^{ik_{ix}x+ik_{rz}z-i\omega t} \tag{23b}$$

and

$$\mathbf{E}_t=E_{tx}(\mathbf{e}_x+c_t\mathbf{e}_z)e^{ik_{ix}x+ik_{tz}z-i\omega t}, \tag{24a}$$

$$\mathbf{H}_t=\frac{E_{tx}}{\omega\mu_0}\mathbf{e}_y(k_{rz}-c_tk_{ix})e^{ik_{ix}x+ik_{tz}z-i\omega t}, \tag{24b}$$

where use has been made of $k_{tx}=k_{rx}=k_{ix}$, and c_r and c_t are given by

$$c_r=-\frac{-k_{rz}^2+\alpha_L^2}{k_{ix}k_{rz}-\gamma_L\alpha_L},\quad c_t=-\frac{-k_{tz}^2+\alpha_R^2}{k_{ix}k_{tz}-\gamma_R\alpha_R}, \tag{25}$$

with the value of k_{rz} (k_{tz}) uniquely determined by the dispersion relation (2) and $S_{rz}<0$ ($S_{tz}>0$):

$$k_{rz}=[\alpha_L\gamma_Lk_{ix}-\sqrt{\epsilon_L\epsilon_L'(\beta_L^2-k_{ix}^2)}]/\beta_L^2,$$

$$k_{tz}=[\alpha_R\gamma_Rk_{ix}+\sqrt{\epsilon_R\epsilon_R'(\beta_R^2-k_{ix}^2)}]/\beta_R^2. \tag{26}$$

Let $r=E_{rx}/E_{ix}$ and $t=E_{tx}/E_{ix}$. On applying the standard boundary conditions

$$(\mathbf{E}_i+\mathbf{E}_r)\times\mathbf{e}_z=\mathbf{E}_t\times\mathbf{e}_z, \tag{27a}$$

$$(\mathbf{H}_i+\mathbf{H}_r)\times\mathbf{e}_z=\mathbf{H}_t\times\mathbf{e}_z, \tag{27b}$$

one has

$$r-t+1=0, \tag{27c}$$

$$q_rr-q_tt+q_i=0, \tag{27d}$$

where Eq. (27c) originates from Eq. (27a), while Eq. (27d) comes from Eq. (27b). The coefficients q_i, q_r, and q_t are worked out to be

$$q_i=\sqrt{\frac{\epsilon_L\epsilon_L'}{\beta_L^2-k_{ix}^2}},$$

$$q_r=-\sqrt{\frac{\epsilon_L\epsilon_L'}{\beta_L^2-k_{ix}^2}},$$

$$q_t=\sqrt{\frac{\epsilon_R\epsilon_R'}{\beta_R^2-k_{ix}^2}}, \tag{28}$$

where use has been made of Eqs. (22) and (26).

The necessary and sufficient condition of total transmission ($r=0$) is therefore

$$q_i=q_t. \tag{29}$$

With the use of Eq. (28), the conditions for *omnidirectional* total transmission (total refraction irrespective of the value of k_{ix}) can be explicitly written as

$$\epsilon_L\epsilon_L'=\epsilon_R\epsilon_R',$$

$$\epsilon_L\sin^2\theta_L+\epsilon_L'\cos^2\theta_L=\epsilon_R\sin^2\theta_R+\epsilon_R'\cos^2\theta_R. \tag{30}$$

It is therefore concluded that the omnidirectional total transmission is not limited to the symmetric case. It can be obtained at a variety of interfaces between uniaxial media and, also, at the interface between the isotropic medium and uniaxial medium, since Eq. (30) can be satisfied with a suitable choice of $\epsilon_{L(R)}$, $\epsilon_{L(R)}'$, and $\theta_{L(R)}$. In the following, we discuss some simple cases.

When both media are isotropic, from $\epsilon_L=\epsilon_L'$, $\epsilon_R=\epsilon_R'$, $\beta_L^2=\epsilon_L$, and $\beta_R^2=\epsilon_R$, it follows that the condition for total transmission, Eq. (29), is satisfied if and only if

$$k_{tz}\epsilon_L-k_{iz}\epsilon_R=0, \tag{31}$$

which holds only when the angle of incidence is equal to Brewster's angle. Actually, the conditions for omnidirectional total transmission, Eqs. (30), require, for the interface between isotropic media, $\epsilon_L=\epsilon_R$, which reduces to the trivial homogeneous system without an interface. The wave is not refracted at all, although totally transmitted. Omnidirectional total refraction cannot occur at the interface between isotropic media.

The simplest case to realize omnidirectional total transmission is the symmetric case, as proposed in Ref. 25, where $\theta_R=\pi-\theta_L$, $\epsilon_R=\epsilon_L$, and $\epsilon_R'=\epsilon_L'$ lead straightforwardly to Eqs. (30) One thus realizes omnidirectional total refraction, as proposed and experimentally demonstrated in Ref. 25.

When the medium on the left-hand side of the interface is isotropic while that on the right-hand side is uniaxially anisotropic, one has $\epsilon_L'=\epsilon_L$, and the conditions for omnidirectional total refraction, Eqs. (30), reduce to

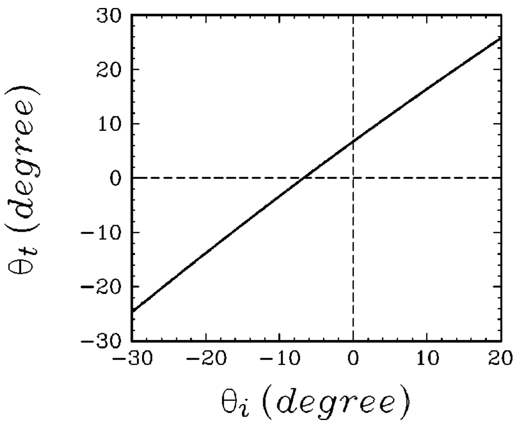

FIG. 5. Refraction angle θ_t vs incident angle θ_i for the case with $\epsilon_R=2^2$, $\epsilon_R'=2.25^2$, and θ_R, $\epsilon_L'=\epsilon_L$ given by Eq. (32), showing that besides omnidirectional total refraction, negative refraction may also occur, over a range of incident angle, at the interface between isotopic and uniaxial media.

$$\epsilon_L=\sqrt{\epsilon_R\epsilon_R'},\quad \cos 2\theta_R=\frac{\sqrt{\epsilon_R}-\sqrt{\epsilon_R'}}{\sqrt{\epsilon_R}+\sqrt{\epsilon_R'}}. \tag{32}$$

As a result, with a suitable arrangement of the optical axis of the uniaxial medium and selection of an isotropic medium with appropriate permittivity, omnidirectional total transmission can also occur at the interface between isotropic and uniaxial media. Figure 5 displays the refraction angle θ_t as a function of incident angle θ_i for the case with $\epsilon_R=2^2$, $\epsilon_R'=2.25^2$, and θ_R, $\epsilon_L=\epsilon_L'$ given by Eqs. (32). The permittivity of the uniaxial medium resembles those for YVO_4 as in Ref. 25. It is seen that apart from omnidirectional total refraction, negative refraction also arises, over a range of incident angle, at the interface between isotropic and uniaxial media.

To summarize, we have presented a systematic theoretical analysis of the conditions for yielding negative refraction as well as omnidirectional total transmission at planar interfaces between uniaxial media. It is found that both negative refraction and omnidirectional total refraction can occur at a wide variety of interfaces associated with a uniaxial medium. In particular, both phenomena can arise at the interface between an isotropic medium and a uniaxial medium, adding considerably to the flexibility in potential device applications in the field of, e.g., high-power optics, where steering light without reflection could be rather valuable. A frequency-selective omnidirectional total transmission device may also be derived based on the unique property by selecting a dispersive uniaxial medium and suitably arranging its optical axis such that Eqs. (32) are satisfied at a certain frequency. Finally, it is worth pointing out that the analysis here also applies (with some minor revisions) to ballistic electrons. With the conditions similar to Eqs. (11) and (30), one can design negative refraction and omnidirectional total transmission interfaces to provide bending, angular dispersion, energy filtering, and beam collimating for electrons in semiconductor ballistic electron devices.

Z.L. thanks Dr. Yong Zhang for making his paper available prior to publication. This work was supported in part by CNKBRSF and CNNSF.

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