Linear conductance in Coulomb-blockade quantum dots in the presence of interactions and spin

Y. Alhassid,[1] T. Rupp,[1] A. Kaminski,[2] and L. I. Glazman[3]
[1]*Center for Theoretical Physics, Sloane Physics Laboratory, Yale University, New Haven, Connecticut 06520, USA*
[2]*Condensed Matter Theory Center, Department of Physics, University of Maryland, College Park, Maryland 20742-4111, USA*
[3]*Theoretical Physics Institute, University of Minnesota, Minneapolis, Minnesota 55455, USA*
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We discuss the calculation of the linear conductance through a Coulomb-blockade quantum dot in the presence of interactions beyond the charging energy. In the limit where the temperature is large compared with a typical tunneling width, we use a rate-equations approach to describe the transitions between the corresponding many-body eigenstates of a dot with N- and $(N+1)$-electrons. We consider both the elastic and rapid-thermalization limits, where the rate of inelastic scattering in the dot is either small or large compared with the elastic transition rate, respectively. In the elastic limit, we derive an implicit expression for the conductance, whose calculation requires the solution of a linear set of equations. In several special cases, including the case of a constant exchange interaction and the case where only ground-state to several-state transitions contribute to the conductance, we find an explicit closed solution. In the rapid-thermalization limit, a closed solution is possible in the general case. We show that the corresponding expressions for the linear conductance simplify for a Hamiltonian that is invariant under spin rotations.

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I. INTRODUCTION

In the Coulomb-blockade regime of quantum dots, the conductance of the dot exhibits peaks as a function of the gate voltage, where each peak corresponds to the tunneling of one more electron into the dot. Of particular interest are diffusive or ballistic chaotic dots, in which the mesoscopic fluctuations of the conductance peaks and their spacings reflect the statistical nature of the eigenfunction and eigenenergies of the isolated dot.[1,2] The simplest model of Coulomb blockade is the constant-interaction (CI) model, in which the electrons occupy single-particle levels in the dot and the interaction is described by an electrostatic Coulomb energy that is constant for a fixed number of electrons. In this model, the conductance near a Coulomb-blockade peak at temperatures that are large compared with a typical tunneling width can be derived in a closed form using a master-equation approach.[3,4] This regime of sequential tunneling is relevant to most Coulomb-blockade experiments involving weakly coupled dots.

At sufficiently low temperatures, the conductance through the dot is dominated by elastic processes. However, at finite temperature, the electrons in the dot may undergo inelastic scattering processes caused, for example, by electron-phonon or electron-electron interactions. In general, one may consider two limiting cases, depending on the relative magnitude of a typical elastic tunneling width $\Gamma_{\rm el}$ of an electron into the dot and a typical inelastic scattering width $\Gamma_{\rm in}$ of electrons in the dot. In the so-called elastic limit, $\Gamma_{\rm el}\gg\Gamma_{\rm in}$, and inelastic scattering processes in the dot can be ignored. In the opposite limit, $\Gamma_{\rm el}\ll\Gamma_{\rm in}$, inelastic scattering occurs on such a short time scale that electrons in the dot are effectively thermalized immediately after an electron tunnels in or out of the dot.[4] This limit will be called the rapid-thermalization limit, which in the literature is also sometimes referred to as the inelastic limit. The observed suppression of the weak-localization effect in weakly coupled dots at finite temperature[5] suggests that inelastic scattering in the dot becomes important already at temperatures comparable to the single-particle mean level spacing.[6] We note that since in weakly coupled dots $\Gamma_{\rm el}\ll\Delta/\hbar$, we still have $\Gamma_{\rm in}\ll\Delta/\hbar$ for $kT\sim\Delta$, and the above observed suppression of the weak-localization effect is not inconsistent with the predicted vanishing of the dephasing rate at low temperatures.[7]

In the CI model, the conductance peak heights and spacings are directly expressed in terms of the single-particle wave functions and energies. When the dot is chaotic, the fluctuation properties of these single-particle wave functions and energies are well described by random-matrix theory.[8] Statistical properties of the conductance peaks have been studied within the CI model in both the elastic[9,10] and rapid-thermalization[6,11,12] limits, as well as for intermediate situations.[13]

However, recent experiments in almost-isolated Coulomb-blockade dots clearly indicate that it is necessary to take into account electron-electron interactions beyond the CI model.[14] For such interactions, the eigenstates of the dot are no longer characterized by Slater determinants. An important question that arises is how to calculate the linear conductance through such a dot. In this work we provide an answer in the limit of sequential tunneling, generalizing the results of the CI model[4] to the interacting case. We assume that both the thermal energy and the typical excitation energies in the dot are small compared with the charging energy. This defines the regime of Coulomb blockade, where in the vicinity of the $(N+1)$st conductance peak only the manifolds of states with N and $N+1$ electrons in the dot contribute to the conductance (the manifolds with an electron number different from either N and $N+1$ are pushed away by the charging energy). We discuss the calculation of the linear conductance in terms of the transition widths between the many-body eigenstates in the N- and $(N+1)$-electron dot as an electron tunnels into the dot. This is done by introducing nonequilibrium probabilities of the dot to be in its various many-body

eigenstate with N and $N+1$ electrons. These probabilities satisfy a set of rate equations describing the transitions between the many-body eigenstates. We consider stationary solutions in the linear-response approximation in both limits where inelastic scattering of electrons in the dot is weak (elastic limit) and strong (rapid-thermalization limit).

In the elastic limit, we reduce the problem to a set of linear equations, and provide an expression for the conductance in terms of the solution of this linear set. There are several important cases where an explicit solution of these linear equations exists. Such cases include the situation of sufficiently low temperatures where only ground-state to ground-state, or ground-state to several-state transitions (of N and $N+1$ electrons, respectively) contribute to the conductance. The special case where only ground-state to ground-state transition is accounted for was derived in Ref. 15 using an ''intermediate state'' approach. However, the lowest triplet and singlet states in a quantum dot can be close in energy, in which case both states contribute to the conductance and it is necessary to use the ground-state to several-state formula derived here. Another case where the rate equations can be solved in closed form at any temperature corresponds to a Hamiltonian for which the occupations of the orbital single-particle states are good quantum numbers.[16] A particularly important such case corresponds to the universal Hamiltonian[2,17] in the limit of infinite Thouless conductance (ignoring a Cooper-channel term, which is allowed in the absence of an orbital magnetic field). This Hamiltonian includes a constant exchange interaction in addition to a constant charging energy.[17]

In the rapid-thermalization limit, we obtain a closed-form expression for the conductance in the most general case of arbitrary electron-election interactions and for any number of states in the dot with N and $N+1$ electrons.

A particularly important situation occurs when the Hamiltonian of the dot is spin-rotation invariant, as is the case of the Coulomb interactions in the absence of spin-orbit scattering. The expressions we find for the linear conductance can then be simplified by explicitly carrying out the sum over the magnetic quantum numbers (in both the elastic and rapid-thermalization limits).

The outline of this paper is as follows. In Sec. II we introduce the basic assumptions and notation. In Sec. III we discuss the rate equations satisfied by the probabilities to find the dot in one of its many-body eigenstates with N or $N+1$ electrons. We assume the elastic limit, where inelastic scattering of electrons in the dot is negligible. In Sec. IV we use linear-response theory to linearize these master equations in the stationary limit. A general expression for the linear conductance in terms of the solution to the rate equations is derived in Sec. V. We show that if detailed balance is satisfied for each pair of many-body eigenstates [in the N- and $(N+1)$-electron dots, respectively], then the master equations and consequently the conductance can be solved in closed form. Furthermore, the corresponding expressions are shown to be simplified for Hamiltonians that are invariant with respect to spin rotations. Physically relevant cases where such closed solutions exist are discussed in Sec. VI, and include the case of ground-state to several-state transitions (relevant for low temperatures), and the case of the universal Hamiltonian in the limit of infinite Thouless conductance. In Sec. VII we solve the rate equations and derive a closed form for the conductance in the case where the inelastic scattering width is much larger than the elastic width (rapid-thermalization limit). Finally, in Sec. VIII we summarize the main results of this paper.

II. MODEL

We consider an almost-isolated quantum dot described by a Hamiltonian $\hat{H}$ that includes a one-body part and a two-body interaction. The dot is weakly coupled to leads, and we assume the limit $kT \gg \hbar\Gamma$, where Γ is a typical transition rate of an electron from the leads into the dot. In this limit we can ignore the coherence between the dot and the leads and use a rate-equations approach to study the transport through the dot.[18] Furthermore, we assume the Coulomb energy e^2/C to be much larger than the thermal energy kT. In this limit, the linear conductance of the dot exhibits Coulomb-blockade peaks. When the gate voltage V_g is tuned in the vicinity of a Coulomb-blockade peak, the number of electrons in the dot can be either N or $N+1$. All manifolds with an electron number different from either N or $N+1$ are pushed away by the charging energy.

The rate equations have been solved in the CI model for a single spin-degenerate level[3] and for any number of single-particle levels.[4] In the CI model, the two-body interaction is modeled by a constant charging energy of an object with capacitance C and charge Ne. Here we derive and solve the rate equations in the presence of interactions beyond the charging energy, where the many-body states of the dots are no longer Slater determinants.

We denote the many-body eigenstates of the dot with N electrons by i, and their respective energies by $\varepsilon_i^{(N)}$. Similarly, the states of the dot with $N+1$ electrons are labeled as j and the corresponding energies are $\varepsilon_j^{(N+1)}$. M_N and M_{N+1} denote, respectively, the total number of the N- and $(N+1)$-electron states considered.

Of particular interest are cases where the dot's Hamiltonian is spin-rotation invariant. This requires the absence of a spin-orbit interaction such that the single-particle energies of the electrons in the dot ϵ_λ are spin-degenerate. In addition, for spin-rotation invariance to be valid, the matrix elements of the two-body interaction may depend only on the orbital states but not on the spin indices, as for the case of Coulomb interaction. We then have

$$[\hat{H},\hat{\mathbf{S}}^2]=0 \quad \text{and} \quad [\hat{H},\hat{S}_z]=0, \tag{1}$$

where $\hat{\mathbf{S}}$ is the total spin operator of the dot. The many-body eigenstates of the dot can be characterized by the good quantum numbers of the spin S and its projection $S_z=M$. The remaining quantum numbers, in addition to the total number of electrons N, will be labeled by α. Thus the eigenstates of the dot with n electrons are given by $i=(\alpha,S,M)$, and their respective energies $\varepsilon_{\alpha S}^{(N)}$ are independent of M. The states of the dot with $N+1$ electrons are similarly labeled by $j=(\alpha',S',M')$, and their corresponding energies are $\varepsilon_{\alpha' S'}^{(N+1)}$. An example of a spin-rotation invariant Hamiltonian is the recently proposed universal Hamiltonian.[2,17]

III. RATE EQUATIONS

We assume that a potential difference V is applied between the two leads (source and drain) at temperature T. Fractions $\eta_{\rm l}$ and $\eta_{\rm r}$ of this potential difference, with

$$\eta_{\rm l}+\eta_{\rm r}=1, \tag{2}$$

fall between the dot and the left lead, and the right lead and the dot, respectively. As a result a current I flows through the dot and various eigenstates of the dot can be occupied with different probabilities. We denote the nonequilibrium probability for the dot to be in a particular N-electron state i by $P_i^{(N)}$. Since we consider situations in which the dot can only be occupied by N or $N+1$ electrons, we require the normalization $\Sigma_i P_i^{(N)}+\Sigma_j P_j^{(N+1)}=1$.

We further assume that energy is conserved in the tunneling between the dot and the leads,[4] neglecting virtual transitions that are of higher order in the tunneling widths. Denoting the transition widths between the N-electron state i and the $(N+1)$-electron state j involving an electron tunneling into the dot from the left (right) lead by $\Gamma_{ij}^{\rm l}$ ($\Gamma_{ij}^{\rm r}$), we can write the following rate equations for the probabilities $P_i^{(N)}$ and $P_j^{(N+1)}$:

$$\frac{\partial P_i^{(N)}}{\partial t}=\sum_j P_j^{(N+1)}[(1-f_{ij}^{\rm l})\Gamma_{ij}^{\rm l}+(1-f_{ij}^{\rm r})\Gamma_{ij}^{\rm r}] - P_i^{(N)}\sum_j (f_{ij}^{\rm l}\Gamma_{ij}^{\rm l}+f_{ij}^{\rm r}\Gamma_{ij}^{\rm r}) \quad \text{for all } i, \tag{3a}$$

$$\frac{\partial P_j^{(N+1)}}{\partial t}=\sum_i P_i^{(N)}(f_{ij}^{\rm l}\Gamma_{ij}^{\rm l}+f_{ij}^{\rm r}\Gamma_{ij}^{\rm r})-P_j^{(N+1)}\sum_j [(1-f_{ij}^{\rm l})\Gamma_{ij}^{\rm l} +(1-f_{ij}^{\rm r})\Gamma_{ij}^{\rm r}] \quad \text{for all } j. \tag{3b}$$

The Fermi-Dirac function of an electron in the left (right) lead is evaluated at a suitable energy that can be found from energy conservation of the transition between states i and j, and is denoted by $f_{ij}^{\rm l}$ ($f_{ij}^{\rm r}$). In the presence of a gate voltage $V_{\rm g}$, the energy of the N-electron dot in state i is given by $\varepsilon_i^{(N)}-Ne\zeta V_{\rm g}$. Here $\zeta\equiv C_{\rm g}/C$, with $C_{\rm g}$ being the dot's capacitance with respect to the gate, and C being the full dot's capacitance. Thus the energy of an electron (relative to the Fermi energy $\epsilon_{\rm F}$ in the leads) in the left (right) lead that tunnels into an N-electron dot in state i to form an $(N+1)$-electron dot in state j is given by

$$\varepsilon_{ij}=\varepsilon_j^{(N+1)}-\varepsilon_i^{(N)}-\tilde{\epsilon}_{\rm F}, \tag{4}$$

where $\tilde{\epsilon}_{\rm F}\equiv e\zeta V_{\rm g}+\epsilon_{\rm F}$ is an effective Fermi energy. Taking into account the bias-potential drop between the dot and each lead, we have $f_{ij}^{\rm l}\equiv f(\varepsilon_{ij}+\eta_{\rm l}eV)$ and $f_{ij}^{\rm r}\equiv f(\varepsilon_{ij}-\eta_{\rm r}eV)$, where the Fermi-Dirac function at temperature T is defined by $f(x)=[1+\exp(\beta x)]^{-1}$ with $\beta=1/kT$.

We note that we can choose any values of $\eta_{\rm l}$ and $\eta_{\rm r}$ satisfying Eq. (2), but that the final result for the conductance (derived in Sec. V) must be independent of this choice. This independence must follow on physical grounds since the linear conductance is an intrinsic property of the dot and must be independent of the way that the source-drain potential difference is distributed. A rigorous proof of this statement is provided in the Appendix.

IV. LINEAR RESPONSE

To calculate the conductance, we are interested in finding stationary solutions of Eqs. (3), i.e., nonequilibrium probabilities $P_i^{(N)}$ and $P_j^{(N+1)}$ that satisfy $\partial P_i^{(N)}/\partial t=0$ and $\partial P_j^{(N+1)}/\partial t=0$ for all i and j, respectively. Furthermore, to find the linear conductance, it is sufficient to solve the rate equations in linear-response theory, i.e., linearizing Eq. (3) in V. To this end, we expand $P_i^{(N)}$ and $P_j^{(N+1)}$ around their equilibrium values $\tilde{P}_i^{(N)}$ and $\tilde{P}_j^{(N+1)}$

$$P_i^{(N)}=\tilde{P}_i^{(N)}[1+eV\beta\Psi_i^{(N)}],$$

$$P_j^{(N+1)}=\tilde{P}_j^{(N+1)}[1+eV\beta\Psi_j^{(N+1)}], \tag{5}$$

in terms of new variables $\Psi_i^{(N)}$ and $\Psi_j^{(N+1)}$. The equilibrium probabilities can be expressed explicitly in terms of the eigenenergies $\varepsilon_i^{(N)}$ and $\varepsilon_j^{(N+1)}$ of the dot with N and $N+1$ electrons,

$$\tilde{P}_i^{(N)}=\frac{e^{-\beta(\varepsilon_i^{(N)}-\tilde{\epsilon}_{\rm F}N)}}{Z}, \quad \tilde{P}_j^{(N+1)}=\frac{e^{-\beta[\varepsilon_j^{(N+1)}-\tilde{\epsilon}_{\rm F}(N+1)]}}{Z}, \tag{6}$$

with the partition sum

$$Z=\sum_i e^{-\beta(\varepsilon_i^{(N)}-\tilde{\epsilon}_{\rm F}N)}+\sum_j e^{-\beta[\varepsilon_j^{(N+1)}-\tilde{\epsilon}_{\rm F}(N+1)]}. \tag{7}$$

The equilibrium distribution is in principle grand-canonical with a chemical potential equal to the Fermi energy $\epsilon_{\rm F}$ (at temperature T) in the leads, but because of the presence of charging energy only two values of the electron numbers are allowed.

To find a linear set of equations for $\Psi_i^{(N)}$ and $\Psi_j^{(N+1)}$, we substitute the expansion (5) into the rate equations (3) and expand the Fermi-Dirac functions to first order in V, i.e., $f_{ij}^{\rm l}=f_{ij}+\eta_{\rm l}eVf'_{ij}$ and $f_{ij}^{\rm r}=f_{ij}-\eta_{\rm r}eVf'_{ij}$, where we have denoted $f_{ij}=f(\varepsilon_{ij})$ and f' is the derivative of f. Keeping only terms that are linear in the bias voltage V, and using the relation

$$\tilde{P}_j^{(N+1)}=\tilde{P}_i^{(N)}e^{-\beta\varepsilon_{ij}}, \tag{8}$$

we obtain[19]

$$\sum_j f_{ij}[(\Gamma_{ij}^{\rm l}+\Gamma_{ij}^{\rm r})(\Psi_j^{(N+1)}-\Psi_i^{(N)})+(\eta_{\rm l}\Gamma_{ij}^{\rm l}-\eta_{\rm r}\Gamma_{ij}^{\rm r})] =0 \quad \text{for all } i, \tag{9a}$$

$$\sum_i (1-f_{ij})[(\Gamma_{ij}^{\rm l}+\Gamma_{ij}^{\rm r})(\Psi_j^{(N+1)}-\Psi_i^{(N)})+(\eta_{\rm l}\Gamma_{ij}^{\rm l}-\eta_{\rm r}\Gamma_{ij}^{\rm r})] =0 \quad \text{for all } j. \tag{9b}$$

Equations (9) represent a system of M_N+M_{N+1} linear equations for M_N+M_{N+1} unknowns. However, only $M_N+M_{N+1}-1$ of these equations are linearly independent. Hence, the solutions for $\Psi_i^{(N)}$ and $\Psi_j^{(N+1)}$ are determined only up to an additive constant. This constant is fixed by imposing the constraint

$$\sum_i \tilde{P}_i^{(N)}\Psi_i^{(N)}+\sum_j \tilde{P}_j^{(N+1)}\Psi_j^{(N+1)}=0, \tag{10}$$

which follows from the normalization condition of the nonequilibrium probabilities. In general, it is necessary to solve Eqs. (9) numerically. These equations represent sets of detailed-balance equations. However, it is possible to find analytic solutions in cases where a detailed-balance equation holds for each pair of states i and j individually, i.e., in cases where each term in the square brackets of Eqs. (9) vanishes. This condition is equivalent to

$$\Psi_i^{(N)}-\Psi_j^{(N+1)}=\frac{\eta_{\rm l}\Gamma_{ij}^{\rm l}-\eta_{\rm r}\Gamma_{ij}^{\rm r}}{\Gamma_{ij}^{\rm l}+\Gamma_{ij}^{\rm r}} \quad \text{for any } i \text{ and } j. \tag{11}$$

We emphasize that there are M_NM_{N+1} equations (11) for only M_N+M_{N+1} unknowns, so in the general case it is not possible to satisfy Eq. (11). This is in contrast to the CI model where a closed solution always exists[4] because of the single-particle nature of the problem. Yet, there are a number of important cases where a closed solution of Eq. (11) does exist, as we discuss in Sec. VI. A solution to Eqs. (11), when it exists, does not depend on temperature and leads to an expression for the conductance as a sum of terms, in each of which the dependence on temperature and on decay widths factorizes [see Eq. (14)].

V. LINEAR CONDUCTANCE

The linear conductance G is defined as the ratio I/V in the limit $V\to 0$. The current through the dot can be expressed in terms of the quantities considered in the master-equation approach. In this framework, we can calculate, for example, the current I through the left tunneling contact, which equals the current through the dot and through the right contact,

$$I=\frac{e}{\hbar}\sum_{ij}\,[P_i^{(N)}f_{ij}^{\rm l}-P_j^{(N+1)}(1-f_{ij}^{\rm l})]\Gamma_{ij}^{\rm l}. \tag{12}$$

Using the expansion in Eq. (5) and expanding $f_{ij}^{\rm l}$ to first order in V, we find the elastic linear conductance

$$G_{\rm el}=\frac{e^2}{\hbar kT}\sum_{ij}\,\tilde{P}_i^{(N)}f_{ij}(\Psi_i^{(N)}-\Psi_j^{(N+1)}-\eta_{\rm l})\Gamma_{ij}^{\rm l}. \tag{13}$$

Although $\eta_{\rm l}$ appears explicitly in Eq. (13), the linear conductance is independent of $\eta_{\rm l}$. This must be so on physical grounds, and a formal proof is provided in the Appendix.

The complete solution for the (elastic) linear conductance in the general case of sequential tunneling is given by Eqs. (9) together with Eq. (13); we solve Eqs. (9) for $\Psi_i^{(N)}$ and $\Psi_j^{(N+1)}$ and substitute their values in Eq. (13).

However, in special cases where Eqs. (11) hold, the conductance is given by the closed form

$$G_{\rm el}=\frac{e^2}{\hbar kT}\sum_{ij}\,\tilde{P}_i^{(N)}f_{ij}\frac{\Gamma_{ij}^{\rm l}\Gamma_{ij}^{\rm r}}{\Gamma_{ij}^{\rm l}+\Gamma_{ij}^{\rm r}}, \tag{14a}$$

or equivalently

$$G_{\rm el}=\frac{e^2}{\hbar kT}\sum_{ij}\,\tilde{P}_j^{(N+1)}(1-f_{ij})\frac{\Gamma_{ij}^{\rm l}\Gamma_{ij}^{\rm r}}{\Gamma_{ij}^{\rm l}+\Gamma_{ij}^{\rm r}}. \tag{14b}$$

Notice that the parameters $\eta_{\rm l,r}$ no longer appear in the conductance formula. The independence of the conductance from the values of $\eta_{\rm l,r}$, which holds true in general (cf. Appendix), becomes apparent in this special case where an explicit formula can be given.

The conductance (14) is expressed as a sum over all pairs of many-particle states i and j of the N-electron and $(N+1)$-electron dots, respectively, in contrast to the CI model where the conductance is described as a sum over single-particle levels.[4] We observe that the temperature dependence of the contribution from each pair of many-particle states is independent from the transition widths $\Gamma_{ij}^{l,r}$.

So far, the spin symmetries have not been taken into account in the derivation of the rate equations and the conductance. If this is done, the sum over the many-body states i and j can be further simplified. For two many-body states $i=(\alpha,S,M)$ and $j=(\alpha',S',M')$, the associated partial decay widths $\Gamma_{ij}^{\rm l}$ and $\Gamma_{ij}^{\rm r}$ are given by

$$\Gamma_{ij}^{\rm l,r}=\Gamma_0^{\rm l,r}|\langle\alpha'S'M'|\psi_m^\dagger(\mathbf{r}_{\rm l,r})|\alpha SM\rangle|^2, \tag{15}$$

where the operator $\psi_m^\dagger(\mathbf{r}_{\rm l})$ $[\psi_m^\dagger(\mathbf{r}_{\rm r})]$ creates an electron with spin projection m at the left [right] point contact at $\mathbf{r}_{\rm l}$ $[\mathbf{r}_{\rm r}]$, and we have introduced overall coupling strengths $\Gamma_0^{\rm l,r}$ at the left and right point contact. The spin selection rules require $m=M'-M$ and $S'=|S\pm 1/2|$ for the matrix element in Eq. (15) not to vanish. For a Hamiltonian that is invariant with respect to spin rotations, we can use the Wigner-Eckart theorem to factorize the matrix element in Eq. (15) into a Clebsch-Gordan (CG) coefficient and a reduced matrix element that is independent of the spin-projection quantum numbers M, M', and m,

$$\langle\alpha'S'M'|\psi_m^\dagger(\mathbf{r})|\alpha SM\rangle=-\frac{1}{\sqrt{2S'+1}}(SM1/2m|S'M')\times(\alpha'S'\|\psi^\dagger(\mathbf{r})\|\alpha S). \tag{16}$$

For a given pair of many-body states i and j, the decay widths $\Gamma_{ij}^{\rm l}$ and $\Gamma_{ij}^{\rm r}$ involve the same CG coefficient. In Eq. (11), the CG coefficients in the ratio on the right-hand side cancel and we are left with

$$\Psi_i^{(N)}-\Psi_j^{(N+1)}=\frac{\eta_l\Gamma_0^l|(\alpha' S'\|\psi^\dagger(\mathbf{r}_l)\|\alpha S)|^2-\eta_r\Gamma_0^r|(\alpha' S'\|\psi^\dagger(\mathbf{r}_r)\|\alpha S)|^2}{\Gamma_0^l|(\alpha' S'\|\psi^\dagger(\mathbf{r}_l)\|\alpha S)|^2+\Gamma_0^r|(\alpha' S'\|\psi^\dagger(\mathbf{r}_r)\|\alpha S)|^2}. \tag{17}$$

Thus, if solutions for $\Psi_i^{(N)}$ and $\Psi_j^{(N+1)}$ exist that satisfy Eq. (11), they will be independent of the spin projection M or M', i.e., $\Psi_i^{(N)}=\Psi_{\alpha S}^{(N)}$ and $\Psi_j^{(N+1)}=\Psi_{\alpha' S'}^{(N+1)}$. Defining reduced decay widths

$$\tilde{\Gamma}^{l,r}_{\alpha S,\alpha' S'}=\Gamma_0^{l,r}|(\alpha' S'\|\psi^\dagger(\mathbf{r}_{l,r})\|\alpha S)|^2, \tag{18}$$

conditions (17) can be rewritten in a form in which the magnetic quantum numbers disappear,

$$\Psi_{\alpha' S'}^{(N+1)}-\Psi_{\alpha S}^{(N)}=\frac{\eta_l\tilde{\Gamma}^l_{\alpha S,\alpha' S'}-\eta_r\tilde{\Gamma}^r_{S,\alpha' S'}}{\tilde{\Gamma}^l_{\alpha S,\alpha' S'}+\tilde{\Gamma}^r_{\alpha S,\alpha' S'}}. \tag{19}$$

Using the reduced widths in the expression (14) for the linear conductance, and taking advantage of the unitarity of the CG, $\Sigma_{Mm}|(SM1/2m|S'M')|^2=1$, we can write the linear conductance in the form

$$G_{\rm el}=\frac{e^2}{\hbar kT}\sum_{\alpha S\alpha' S'}\tilde{P}_{\alpha S}^{(N)}f(\varepsilon_{\alpha S,\alpha' S'})\frac{\tilde{\Gamma}^l_{\alpha S,\alpha' S'}\tilde{\Gamma}^r_{\alpha S,\alpha' S'}}{\tilde{\Gamma}^l_{\alpha S,\alpha' S'}+\tilde{\Gamma}^r_{\alpha S,\alpha' S'}}. \tag{20}$$

Instead of using the reduced widths defined in Eq. (18), it is possible to express the conductance in terms of the widths for the maximally projected spin states (i.e., $M=S$ and $M'=S'$)

$$\Gamma^{l,r}_{\alpha S,\alpha' S'}\equiv\Gamma^{l,r}_{\alpha SS,\alpha' S' S'}. \tag{21}$$

The reduced matrix elements in Eq. (18) are nonzero only for $S'=S\pm1/2$ and, using the corresponding CG coefficients, we have

$$\tilde{\Gamma}^{l,r}_{\alpha S,\alpha' S'}=[2\max(S,S')+1]\Gamma^{l,r}_{\alpha S,\alpha' S'}. \tag{22}$$

A subject of current interest is the mesoscopic fluctuations of the conductance in chaotic or diffusive quantum dots with large Thouless conductance g_T.[1,2] The statistical properties of the single-particle levels in a band of width $\sim g_T$ around the Fermi energy are described by random-matrix theory. In the CI model, in which only charging energy is taken into account, the finite-temperature conductance through an almost-isolated dot can be calculated in terms of the single-particle spectrum and wave functions using the expression derived in Ref. 4. The statistical properties of the conductance peaks can then be inferred directly from the random-matrix properties of the single-particle Hamiltonian. However, a realistic description of an almost-isolated quantum dot requires additional interaction terms beyond the CI model. The generic form of the Hamiltonian of a chaotic dot is given by the universal Hamiltonian plus residual interaction.[2,17] The universal Hamiltonian includes, in addition to the charging energy term, an exchange interaction. The residual interaction terms fluctuate (because of the fluctuation of the single-particle wave functions) and are of order $1/g_T$.

The statistical properties of the tunneling widths Γ_{ij} between many-body eigenstates of the dot with N and $N+1$ electrons were studied in Ref. 20. Given a set of tunneling widths (and a given realization of the one-body Hamiltonian), the calculation of the conductance in the presence of exchange and residual interactions can no longer be done using the expressions derived in Ref. 4. In the limit $g_T\to\infty$ of the universal Hamiltonian, the orbital occupations are good quantum numbers and one can use the explicit solution derived in Sec. VI C below.[21] However, at finite g_T, it is necessary to include the residual interaction terms. If the temperature is very low, i.e., $kT\lesssim0.1\Delta$, where Δ is the single-particle mean level spacing (lowest temperatures attained in a typical experimental setup[14]), it is usually sufficient to take just the $S=1/2$ ground state in the dot with an odd number of electrons, while *both* the singlet ($S=0$) and triplet ($S=1$) states can contribute in the dot with even number of electrons. In this case we can use the explicit expression (20) for the conductance (see Sec. VI B below). However, at temperatures $kT\gtrsim0.1\Delta$, it is usually necessary to use the general solution described by Eqs. (9) and (13).

VI. EXPLICIT SOLUTIONS IN THE LIMIT OF ELASTIC SCATTERING

In general, it is not possible to find a closed solution for the conductance in the limit of elastic scattering, and Eqs. (9) have to be solved numerically. However, in the following we list four important cases where explicit solutions to Eqs. (17) or more generally to Eqs. (11) exist and the conductance is given by a closed expression.

A. Ground-state transition at low temperatures

At sufficiently low temperatures, only the transition between the ground states of the N- and $(N+1)$-electron dots $[(N,S)\to(N+1,S')]$ provides an important contribution to the conductance. It then sufficies to consider the two ground-state manifolds (N,S) and $(N+1,S')$ which are, respectively, $(2S+1)$- and $(2S'+1)$-fold degenerate. In this case, there is only one equation (17) and a solution can always be found;[22] the two variables $\Psi_S^{(N)}$ and $\Psi_{S'}^{(N+1)}$ are uniquely determined by

$$\Psi_{S'}^{(N+1)}=\Psi_S^{(N)}-\frac{\eta_l\tilde{\Gamma}^l_{S,S'}-\eta_r\tilde{\Gamma}^r_{S,S'}}{\tilde{\Gamma}^l_{S,S'}+\tilde{\Gamma}^r_{S,S'}} \tag{23}$$

and by the normalization condition (10). The linear conductance then yields

$$G_{\rm el}=\frac{e^2}{\hbar kT}\frac{[2\max(S,S')+1]f(\varepsilon_{S'}^{(N+1)}-\varepsilon_S^{(N)}-\tilde{\epsilon}_{\rm F})}{(2S+1)+(2S'+1)e^{-\beta(\varepsilon_{S'}^{(N+1)}-\varepsilon_S^{(N)}-\tilde{\epsilon}_{\rm F})}}\frac{\Gamma^{\rm l}_{S,S'}\Gamma^{\rm r}_{S,S'}}{\Gamma^{\rm l}_{S,S'}+\Gamma^{\rm r}_{S,S'}}, \tag{24}$$

where the widths $\Gamma^{\rm l,r}_{S,S'}$ correspond to the maximally projected spin states with $M=S$ and $M'=S'$ [cf. Eqs. (21) and (22)]. Equation (24) for ground-state to ground-state transition was first derived in Ref. 15 using an ''intermediate-state'' approach. Here we obtain this result as a special case within the master-equation approach. According to Eq. (24), the functional form of the conductance (versus the effective Fermi energy) does not depend on tunneling widths. In particular, $G_{\rm el}$ is maximized when the effective Fermi energy $\tilde{\epsilon}_{\rm F}\equiv e\zeta V_{\rm g}+\epsilon_{\rm F}$ is tuned to[15]

$$\tilde{\epsilon}_{\rm F,max}=\varepsilon_{S'}^{(N+1)}-\varepsilon_S^{(N)}-\frac{kT}{2}\ln\left(\frac{2S'+1}{2S+1}\right). \tag{25}$$

This result generalizes a known result for the CI model,[4] in which $S=0$ and $S'=1/2$ (or vice versa).

B. Ground-state to several-state transitions

We consider only a single state in either the N- or the $(N+1)$-electron system and allow for any number of states in the other system. In the case where the single state is associated with the N-electron dot, the variables associated with the $(N+1)$-electron states are chosen as

$$\Psi^{(N+1)}_{\alpha'S'}=\Psi^{(N)}_S-\frac{\eta_{\rm l}\tilde{\Gamma}^{\rm l}_{S,\alpha'S'}-\eta_{\rm r}\tilde{\Gamma}^{\rm r}_{S,\alpha'S'}}{\tilde{\Gamma}^{\rm l}_{S,\alpha'S'}+\tilde{\Gamma}^{\rm r}_{S,\alpha'S'}}, \tag{26}$$

and the variable $\Psi^{(N)}_S$ is then fixed by the normalization condition (10). Clearly, Eqs. (26) solve Eqs. (19) for all α' and the conductance has the form (20).

An example for which the case discussed here is useful corresponds to a dot in which the lowest states in the spin sectors $S=0$ and $S=1$ are close in energy (for an even number of electrons). The approximation of Sec. VI A may then be poor even at very low temperatures. A good approximation at low temperatures is then given by Eq. (20) with both transitions $(N,S=1/2)\to(N+1,S'=0)$ and $(N,S=1/2)\to(N+1,S'=1)$ included.

C. Systems with good orbital occupation numbers

We define $n_\lambda=n_{\lambda+}+n_{\lambda-}$ to be the total occupation of the (doubly degenerate) single-particle orbital λ, i.e., the sum of the number of spin-up and spin-down particles in that orbital. Both $n_{\lambda+}$ and $n_{\lambda-}$ can take the value 0 or 1, hence n_λ obtains the values 0, 1 or 2. We will show below that Eqs. (19) have a solution when all n_λ are good quantum numbers. For a given pair (α,S) and (α',S') only one term $\lambda=\lambda_0$ in $\psi^\dagger_\sigma(\mathbf{r})=\Sigma_\lambda\psi_\lambda(\mathbf{r})a^\dagger_{\lambda\sigma}$ will then contribute to the respective matrix element

$$(\alpha'S'\|\psi^\dagger(\mathbf{r})\|\alpha S)=\sum_\lambda\psi_\lambda(\mathbf{r})(\alpha'S'\|a^\dagger_\lambda\|\alpha S)$$
$$=\psi_{\lambda_0}(\mathbf{r})(\alpha'S'\|a^\dagger_{\lambda_0}\|\alpha S). \tag{27}$$

Clearly, if more than one orbital λ contributes to the sum in Eq. (27), then the occupations cannot be good quantum numbers in the final state α'. Since the dependence of the reduced matrix element in Eq. (27) on the point contact at $\mathbf{r}$ is only through the wave function $\psi_{\lambda_0}(\mathbf{r})$, we have

$$\frac{\eta_{\rm l}\tilde{\Gamma}^{\rm l}_{\alpha S,\alpha'S'}-\eta_{\rm r}\tilde{\Gamma}^{\rm r}_{\alpha S,\alpha'S'}}{\tilde{\Gamma}^{\rm l}_{\alpha S,\alpha'S'}+\tilde{\Gamma}^{\rm r}_{\alpha S,\alpha'S'}}=\frac{\eta_{\rm l}\Gamma^{\rm l}_{\lambda_0}-\eta_{\rm r}\Gamma^{\rm r}_{\lambda_0}}{\Gamma^{\rm l}_{\lambda_0}+\Gamma^{\rm r}_{\lambda_0}}, \tag{28}$$

where $\Gamma^{\rm l,r}_\lambda=\Gamma^{\rm l,r}_0|\psi_\lambda(\mathbf{r}_{\rm l,r})|^2$ are the single-particle decay widths.

We now argue that Eqs. (17) have a solution given by

$$\Psi^{(m)}_{\alpha S}=-\sum_\lambda\frac{\eta_{\rm l}\Gamma^{\rm l}_\lambda-\eta_{\rm r}\Gamma^{\rm r}_\lambda}{\Gamma^{\rm l}_\lambda+\Gamma^{\rm r}_\lambda}n_\lambda(\alpha,S)\quad\text{for}\quad m=N,N+1, \tag{29}$$

where $n_\lambda(\alpha,S)$ are the single-particle level occupation numbers of the states with quantum numbers α and S [note that the label α includes all the good quantum numbers $\{n_\lambda\}=(n_1,n_2,\ldots)$ plus any additional quantum numbers required to distinguish between states with the same occupations $\{n_\lambda\}$ and spin S]. Since two manifolds of states (α,S) and (α',S') can only be connected by changing the occupation of a particular orbital λ_0 by a single particle, we have

$$\Psi^{(N)}_{\alpha S}-\Psi^{(N+1)}_{\alpha'S'}=\sum_\lambda\frac{\eta_{\rm l}\Gamma^{\rm l}_\lambda-\eta_{\rm r}\Gamma^{\rm r}_\lambda}{\Gamma^{\rm l}_\lambda+\Gamma^{\rm r}_\lambda}[n_\lambda(\alpha,S)-n_\lambda(\alpha',S')]$$
$$=\frac{\eta_{\rm l}\Gamma^{\rm l}_{\lambda_0}-\eta_{\rm r}\Gamma^{\rm r}_{\lambda_0}}{\Gamma^{\rm l}_{\lambda_0}+\Gamma^{\rm r}_{\lambda_0}}. \tag{30}$$

Thus Eqs. (19) can be indeed satisfied by Eq. (29). For the conductance, we obtain the expression

$$G=\frac{e^2}{\hbar kT}\sum_{\substack{\alpha S\\ \alpha'S'}}\tilde{P}^{(N)}_{\alpha S}f(\varepsilon_{\alpha S,\alpha'S'})|(\alpha'S'\|a^\dagger_{\lambda_0}\|\alpha S)|^2\frac{\Gamma^{\rm l}_{\lambda_0}\Gamma^{\rm r}_{\lambda_0}}{\Gamma^{\rm l}_{\lambda_0}+\Gamma^{\rm r}_{\lambda_0}}, \tag{31}$$

where the orbital λ_0 depends on both the manifolds (α,S) and (α',S'). Of course, only manifolds that are connected by the addition of an electron to a single-particle orbital contribute to the sum in Eq. (31); i.e., the occupations $\{n_\lambda(\alpha',S')\}$ of the manifold (α',S') can only differ for one

orbital and by one unit from the occupations $\{n_\lambda(\alpha,S)\}$ of the manifold (α,S). An expression similar to Eq. (31) (but including explicit sums over the magnetic quantum numbers) was discussed in Ref. 16.

An important example of a Hamiltonian in the class discussed here is the so-called universal Hamiltonian in the limit of the Thouless conductance $g_T \to \infty$,

$$\hat{H} = \sum_{\lambda\sigma} \epsilon_\lambda a^\dagger_{\lambda\sigma} a_{\lambda\sigma} + \frac{e^2}{2C}\hat{n}^2 - J_S \hat{\mathbf{S}}^2, \tag{32}$$

where $\hat{\mathbf{S}} = \Sigma_\lambda \hat{\mathbf{S}}_\lambda$ with $\hat{\mathbf{S}}_\lambda = \Sigma_{\sigma\sigma'} a^\dagger_{\lambda\sigma} \boldsymbol{\sigma}_{\sigma\sigma'} a_{\lambda\sigma'}$ is the total-spin operator of the dot and $\boldsymbol{\sigma}$ is the vector of the three 2×2 Pauli matrices. Since $[\hat{n}_\lambda, \hat{\mathbf{S}}_\lambda] = 0$, the Hamiltonian (32) is invariant under spin rotations and characterized by the good quantum numbers $\{n_\lambda\}, S$, and M. The conductance through a weakly coupled dot with a Hamiltonian (32) is then given by Eq. (31).

D. Dots with degenerate ground states

In this subsection, we consider the case of low temperatures when only transitions between the ground states with N and $N+1$ electrons in the dot are allowed and these ground states are M_N- and M_{N+1}-fold degenerate, respectively. This case is more general than the similar case of Sec. VI A, since the degeneracy is not necessarily the usual spin degeneracy, and the transition widths $\Gamma^{\mathrm{l,r}}_{ij}$ can be arbitrary and are not necessarily related by a Wigner-Eckart theorem. Although we cannot write a closed solution for G_{el} in this more general case, the degeneracy of the levels participating in the transport process allows for significant simplification of the expression for G_{el} [as compared with the general expression given by Eqs. (9), (10), and (13)].

The level degeneracy allows us to reduce the rate equations (9) to

$$\sum_j \left[(\Gamma^{\mathrm{l}}_{ij} + \Gamma^{\mathrm{r}}_{ij})(\Psi^{(N+1)}_j - \Psi^{(N)}_i) + (\eta_{\mathrm{l}}\Gamma^{\mathrm{l}}_{ij} - \eta_{\mathrm{r}}\Gamma^{\mathrm{r}}_{ij})\right] = 0 \quad \text{for all } i, \tag{33a}$$

$$\sum_i \left[(\Gamma^{\mathrm{l}}_{ij} + \Gamma^{\mathrm{r}}_{ij})(\Psi^{(N+1)}_j - \Psi^{(N)}_i) + (\eta_{\mathrm{l}}\Gamma^{\mathrm{l}}_{ij} - \eta_{\mathrm{r}}\Gamma^{\mathrm{r}}_{ij})\right] = 0 \quad \text{for all } j. \tag{33b}$$

In the special case discussed here, the equilibrium probabilities for N and $N+1$ electrons are independent of the states i and j, respectively, and we denote $\tilde{P}^{(N)} = \tilde{P}^{(N)}_i$ for all i and $\tilde{P}^{(N+1)} = \tilde{P}^{(N+1)}_j$ for all j. In particular

$$\tilde{P}^{(N)} = \frac{1}{M_N + M_{N+1} e^{-\beta(\varepsilon^{(N+1)} - \varepsilon^{(N)} - \tilde{\epsilon}_{\mathrm{F}})}}, \tag{34}$$

where $\varepsilon^{(N)} = \varepsilon^{(N)}_i$ for all i and $\varepsilon^{(N+1)} = \varepsilon^{(N+1)}_j$ for all j.

Using Eq. (34), the general expression (13) for the conductance reduces to

$$G_{\mathrm{el}} = \frac{e^2}{\hbar k T} \frac{f(\varepsilon^{(N+1)} - \varepsilon^{(N)} - \tilde{\epsilon}_{\mathrm{F}})}{M_N + M_{N+1} e^{-\beta(\varepsilon^{(N+1)} - \varepsilon^{(N)} - \tilde{\epsilon}_{\mathrm{F}})}} \sum_{ij} (\Psi^{(N)}_i - \Psi^{(N+1)}_j - \eta_{\mathrm{l}}) \Gamma^{\mathrm{l}}_{ij}. \tag{35}$$

Here, $\Psi^{(N)}_i$ and $\Psi^{(N+1)}_j$ are a solution to Eqs. (33). These equations do not depend on the temperature, and we can therefore choose a private solution $\tilde{\Psi}^{(N)}_i$ and $\tilde{\Psi}^{(N+1)}_j$ that is temperature-independent and determined solely by the tunneling widths. The general solution of Eqs. (33) is given by $\Psi^{(N)}_i = \tilde{\Psi}^{(N)}_i + c$ and $\Psi^{(N+1)}_j = \tilde{\Psi}^{(N+1)}_j + c$, where c is a constant. The solution that satisfies the normalization condition (10) is the one with $c = -(\tilde{P}^{(N)} \Sigma_i \tilde{\Psi}^{(N)}_i + \tilde{P}^{(N+1)} \Sigma_j \tilde{\Psi}^{(N+1)}_j)$. While this constant c depends on temperature, it drops out in the final expression for the conductance, thus making the sum on the right-hand side of Eq. (35) temperature-independent.

The conductance in Eq. (35) factorizes into two contributions. The sum over i and j is completely determined by the tunneling widths $\Gamma^{\mathrm{l,r}}_{ij}$ and is independent of temperature and the effective Fermi energy (it is also independent of η_{l} and η_{r} as is shown in the Appendix). The prefactor of the sum on the right-hand side of Eq. (35) does not depend on the tunneling widths $\Gamma^{\mathrm{l,r}}_{ij}$ and contains the full dependence on the temperature and Fermi energy. This prefactor determines the functional dependence of the conductance peak on the gate voltage. Similar to Eq. (25) in Sec. VI A, the maximum of G_{el} is attained when the effective Fermi energy is tuned to

$$\tilde{\epsilon}_{\mathrm{F,max}} = \varepsilon^{(N+1)} - \varepsilon^{(N)} - \frac{kT}{2} \ln\left(\frac{M_{N+1}}{M_N}\right). \tag{36}$$

Equations (33) need to be solved numerically. An exception is the case discussed in Sec. VI A, in which the dot has spin-rotation symmetry and the degeneracy of the ground state corresponds to the various values of the spin projection M. In this case Eqs. (33) can be solved in closed form [see Eq. (23)], and the linear conductance (35) reduces then to Eq. (24).

VII. THE RAPID-THERMALIZATION LIMIT

The above derivation of the linear conductance assumes that elastic scattering dominates, a good approximation at sufficiently low temperatures. Although the exact temperature dependence of inelastic scattering events is not well understood, they should become more relevant with increasing temperature,[5] such that their width Γ_{in} will eventually be of comparable size to Γ_{el}. We therefore consider in this section the rapid-thermalization limit $\Gamma_{\mathrm{in}} \gg \Gamma_{\mathrm{el}}$. The rapid-thermalization conductance was calculated in Ref. 4 in the CI model using the single-particle occupation scheme. Here we derive a closed expression for this conductance in the presence of arbitrary interactions (beyond the charging energy).

We denote by $P(N)$ the probability of the dot to be in an N-electron state, and by $P(i|N) = P^{(N)}_i / P(N)$ the conditional probability of the dot to be in a particular many-body state i

given the dot is occupied by N electrons. In the rapid-thermalization limit, the conditional probabilities $P(i|N)$ and $P(j|N+1)$ are always given by their thermal equilibrium values $\tilde{P}(i|N)$ and $\tilde{P}(j|N+1)$, respectively. It is then only the probabilities $P(N)$ and $P(N+1)$ that obtain nonequilibrium values by a finite bias voltage V. These probabilities satisfy the following rate equations:

$$\frac{\partial P(N)}{\partial t}=P(N+1)\sum_{i,j}\tilde{P}(j|N+1)[(1-f^{\rm l}_{ij})\Gamma^{\rm l}_{ij}+(1-f^{\rm r}_{ij})\Gamma^{\rm r}_{ij}]$$
$$-P(N)\sum_{i,j}\tilde{P}(i|N)[f^{\rm l}_{ij}\Gamma^{\rm l}_{ij}+f^{\rm r}_{ij}\Gamma^{\rm r}_{ij}], \tag{37a}$$

$$\frac{\partial P(N+1)}{\partial t}=P(N)\sum_{i,j}\tilde{P}(i|N)[f^{\rm l}_{ij}\Gamma^{\rm l}_{ij}+f^{\rm r}_{ij}\Gamma^{\rm r}_{ij}]$$
$$-P(N+1)\sum_{i,j}\tilde{P}(j|N+1)[(1-f^{\rm l}_{ij})\Gamma^{\rm l}_{ij}$$
$$+(1-f^{\rm r}_{ij})\Gamma^{\rm r}_{ij}]. \tag{37b}$$

As in the elastic case, we are interested in stationary solutions, in which the rate of electron tunneling onto the dot $(N\to N+1)$ is equal to the rate of electron tunneling off the dot $(N+1\to N)$. As in the elastic case, these equations can be simplified in the linear response limit. In this limit we expand

$$P(N)=\tilde{P}(N)[1+eV\beta\Phi^{(N)}],$$
$$P(N+1)=\tilde{P}(N+1)[1+eV\beta\Phi^{(N+1)}], \tag{38}$$

where $\tilde{P}(N)$ and $\tilde{P}(N+1)$ are the corresponding equilibrium distributions, and $\Phi^{(N)}$ and $\Phi^{(N+1)}$ are unknown variables.

Collecting the terms that are independent of the bias voltage V, we obtain the usual detailed-balance equation at equilibrium. The terms that are linear in V lead to

$$\sum_{ij}\tilde{P}^{(N)}_i f_{ij}[(\Phi^{(N+1)}-\Phi^{(N)})(\Gamma^{\rm l}_{ij}+\Gamma^{\rm r}_{ij})-(\eta_{\rm l}\Gamma^{\rm l}_{ij}-\eta_{\rm r}\Gamma^{\rm r}_{ij})]$$
$$=0, \tag{39}$$

where we have used the equilibrium relations $\tilde{P}(N)\tilde{P}(i|N)=\tilde{P}^{(N)}_i$ and $\tilde{P}(N+1)\tilde{P}(j|N+1)=\tilde{P}^{(N+1)}_j$, together with Eqs. (8).

We observe that while in the elastic limit there were M_N+M_{N+1} unknown variables, in the inelastic limit we are left with only two unknown variables. Consequently, an explicit solution for $\Phi^{(N)}$ and $\Phi^{(N+1)}$ always exists and is given by equating to zero the expression in the square brackets of Eq. (39).

The rapid-thermalization limit of the linear conductance $G_{\rm therm}$ is given by Eq. (13) but with $\Phi^{(N)}$ and $\Phi^{(N+1)}$ replacing $\Psi^{(N)}_i$ and $\Psi^{(N+1)}_j$, respectively. Substituting the solution of Eq. (39), we then obtain a closed expression for the rapid-thermalization conductance

$$G_{\rm therm}=\frac{e^2}{\hbar kT}\frac{\left(\sum_{ij}\tilde{P}^{(N)}_i f_{ij}\Gamma^{\rm l}_{ij}\right)\left(\sum_{kl}\tilde{P}^{(N)}_k f_{kl}\Gamma^{\rm r}_{kl}\right)}{\sum_{rs}\tilde{P}^{(N)}_r f_{rs}(\Gamma^{\rm l}_{rs}+\Gamma^{\rm r}_{rs})}. \tag{40}$$

This expression generalizes an analogous expression for the CI model[4] to an interacting dot. We emphasize that, unlike Eq. (14) in the elastic case, Eq. (40) holds for the most general interaction and for any number of states in the dot. As in the elastic case, we can exploit the spin symmetries to perform explicitly the summation over the spin projections M and M'. Equation (40) can then be written in the form

$$G_{\rm therm}=\frac{e^2}{\hbar kT}\frac{\left(\sum_{\alpha S\alpha' S'}\tilde{P}^{(N)}_{\alpha S}f(\varepsilon_{\alpha S,\alpha' S'})\tilde{\Gamma}^{\rm l}_{\alpha S\alpha' S'}\right)\left(\sum_{\alpha S\alpha' S'}\tilde{P}^{(N)}_{\alpha S}f(\varepsilon_{\alpha S,\alpha' S'})\tilde{\Gamma}^{\rm r}_{\alpha S\alpha' S'}\right)}{\sum_{\alpha S\alpha' S'}\tilde{P}^{(N)}_{\alpha S}f(\varepsilon_{\alpha S,\alpha' S'})(\tilde{\Gamma}^{\rm l}_{\alpha S\alpha' S'}+\tilde{\Gamma}^{\rm r}_{\alpha S\alpha' S'})}. \tag{41}$$

VIII. SUMMARY AND CONCLUSION

We define

$$\langle\langle X_{ij}\rangle\rangle\equiv\sum_{ij}\tilde{P}^{(N)}_i f(\varepsilon_{ij})X_{ij}=\sum_{ij}\tilde{P}^{(N+1)}_j[1-f(\varepsilon_{ij})]X_{ij}, \tag{42}$$

where X_{ij} is a quantity that depends on the many-body states i and j of the N- and $(N+1)$-electron dots, respectively. The equilibrium probabilities $\tilde{P}^{(N)}_i$ and $\tilde{P}^{(N+1)}_j$ to find the dot in states i and j are given by Eqs. (6) and (7), while the energy difference ε_{ij} is defined in Eq. (4).

In the rapid-thermalization limit, the conductance of the almost-isolated dot is given in general by

$$G_{\rm therm}=\frac{e^2}{\hbar kT}\frac{\langle\langle\Gamma^{\rm l}_{ij}\rangle\rangle\langle\langle\Gamma^{\rm r}_{ij}\rangle\rangle}{\langle\langle\Gamma^{\rm l}_{ij}+\Gamma^{\rm r}_{ij}\rangle\rangle}, \tag{43}$$

where $\Gamma^{\rm l}_{ij}$ ($\Gamma^{\rm r}_{ij}$) are the partial transition widths between the states i and j involving the tunneling of an electron from the left (right) lead into the dot.

In the elastic limit, the conductance is calculated by solving the linear equations (9) for $\Psi^{(N)}_i$ and $\Psi^{(N+1)}_j$ and then substituting in the expression (13). There are several impor-

tant cases where a closed solution is possible, including the constant-exchange-interaction model (i.e., the universal Hamiltonian in the limit of infinite Thouless conductance) and the case of ground-state to several-state transitions with arbitrary electron-electron interactions. In these cases

$$G_{\rm el}=\frac{e^2}{\hbar kT}\left\langle\left\langle\frac{\Gamma^{\rm l}_{ij}\Gamma^{\rm r}_{ij}}{\Gamma^{\rm l}_{ij}+\Gamma^{\rm r}_{ij}}\right\rangle\right\rangle. \tag{44}$$

In general, expression (44) is valid when Eqs. (11) are satisfied.

When the Hamiltonian is invariant under spin rotations, these expressions can be simplified by carrying out explicitly the summation over the magnetic quantum numbers. The many-body levels of the N- and $(N+1)$-electron dots are now characterized by the quantum numbers α,S,M and α',S',M' and the respective energies are independent of M and M'. Equations (43) and (44) are now valid with the reduced widths $\tilde{\Gamma}^{\rm l,r}_{\alpha S\alpha' S'}$ replacing the widths $\Gamma^{\rm l,r}_{\alpha SM\alpha' S'M'}$, and the summation in Eq. (42) carried over αS and $\alpha' S'$ only (but not over M and M').

In conclusion, we have solved the rate equations and found the linear conductance in the presence of interactions in the dot (beyond the charging energy). In particular, we have taken into account the spin degrees of freedom of the dot and showed the simplifications that occur when the dot's Hamiltonian is invariant under spin rotations. Both the limits of dominantly elastic scattering and rapid thermalization were discussed. This work generalizes the results of Ref. 4, which were derived in the limit of non-interacting electrons (except for a constant charging energy).

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APPENDIX

Here we show that the linear conductance in Eq. (13) is independent of $\eta_{\rm l}$ and $\eta_{\rm r}$, the fractions of the bias potential difference between the dot and the corresponding leads. Defining the new variables $\Phi^{(N)}_i\equiv\Psi^{(N)}_i-\eta_{\rm l}$, we can rewrite Eqs. (9) in the form

$$\sum_j f_{ij}[(\Gamma^{\rm l}_{ij}+\Gamma^{\rm r}_{ij})(\Psi^{(N+1)}_j-\Phi^{(N)}_i)-\Gamma^{\rm r}_{ij})]=0 \quad \text{for all } i, \tag{A1a}$$

$$\sum_i (1-f_{ij})[(\Gamma^{\rm l}_{ij}+\Gamma^{\rm r}_{ij})(\Psi^{(N+1)}_j-\Phi^{(N)}_i)-\Gamma^{\rm r}_{ij})] = 0 \quad \text{for all } j. \tag{A1b}$$

The solution of Eqs. (A1) is determined up to an additive constant. Since $\eta_{\rm l}$ and $\eta_{\rm r}$ do not appear explicitly in Eqs. (A1), we can choose a private solution $\tilde{\Phi}^{(N)}_i$ and $\tilde{\Psi}^{(N+1)}_j$ that is independent of $\eta_{\rm l}$ and $\eta_{\rm r}$. The general solution is then given by $\Phi^{(N)}_i=\tilde{\Phi}^{(N)}_i+c$ and $\Psi^{(N+1)}_j=\tilde{\Psi}^{(N+1)}_j+c$, where c is a constant. In particular, the normalization condition (10) can be satisfied by choosing

$$c=-\left[\sum_i \tilde{P}^{(N)}_i(\tilde{\Phi}^{(N)}_i+\eta_{\rm l})+\sum_j \tilde{P}^{(N+1)}_j\tilde{\Psi}^{(N+1)}_j\right]. \tag{A2}$$

The constant c depends on $\eta_{\rm l}$, but disappears in the final expression for the conductance in Eq. (13),

$$G_{\rm el}=\frac{e^2}{\hbar kT}\sum_{ij}\tilde{P}^{(N)}_i f_{ij}(\tilde{\Phi}^{(N)}_i-\tilde{\Psi}^{(N+1)}_j)\Gamma^{\rm l}_{ij}. \tag{A3}$$

Expression (A3) for the conductance shows clearly its independence from $\eta_{\rm l}$ (and $\eta_{\rm r}$).

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