Electron–longitudinal optical phonon interaction between Landau levels in semiconductor heterostructures

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Calculations of electron–longitudinal optical phonon scattering rates between Landau levels in semiconductor heterostructures with a magnetic field applied parallel to the direction of confinement are reported. We have found that the scattering rate shows strong oscillations as a function of the applied field, depending on the configuration of the energy states in the structure. In the limit $B\to 0$, the expressions for the intersubband scattering rate between Landau levels reduce to those obtained in the case of two-dimensional subbands at $B=0$ T.

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I. INTRODUCTION

The relaxation of carriers in zero-dimensional (0D) semiconductor systems has been extensively studied in recent years, both in quantum dots and in two-dimensional (2D) structures subjected to a strong magnetic field applied parallel to the confinement direction.[1–6] In particular, the interaction between electrons and phonons, which is a very efficient relaxation path for carriers in 2D systems, has been widely considered in 0D systems. In 0D systems, both theoretical and experimental results have shown that electrons and longitudinal optical (LO) phonons are in a strong coupling regime.[7–10] In particular, energy splittings between polaron states have been observed both in quantum dots and in 2D structures in strong magnetic fields.[9,10] We have calculated these energy splittings in a quantum cascade (QC) laser structure in a quantizing magnetic field and shown that, as expected, their value is only a few meV's (for GaAs-like parameters) even at very large fields ($B>40$ T). We believe this finding prevents the use of a polaron formalism for the present state of art GaAs-based QC lasers. We shall instead show that a quasi-weak-coupling description of the electron-LO phonon interaction is possible for a very broad class of imperfect QC structures. The central assumption of this model is that the actual output of a QC laser results among other parameters from the inhibition (or not) of the LO phonon assisted intersubband transitions. These intersubband transitions are envisioned as taking place in many microsamples (μ *samples*) which differ from one another by random values of one of their characteristic thicknesses. This model, which recalls the inhomogeneous broadening model of the optical transitions in a quantum well (QW) structure, allows an easy derivation of the population relaxation rate due to the irreversible escape of the carriers from the ground level of a Landau ladder to all the other levels at lower energy. To our knowledge, this is the first time that such a calculation is presented, since previous reports were limited to simplified cases (e.g., at the magnetophonon resonance or in the case where only the first electronic subband of the system is populated) (Refs. 11–18). We shall show that the population relaxation rate is an oscillating function of the magnetic field. We shall also show that our formalism reduces to the well-known expression of the $B=0$ relaxation rate at the edge of an excited subband due to the emission of one LO phonon to all the other 2D states of a lower subband. In Sec. II, we explain our model and in Sec. III detail the weak- and strong-field limits and discuss a structure where one should benefit from the quenching of the phonon—assisted transfer between subbands at high magnetic field.

II. MODEL

In the Landau gauge the one electron effective mass Hamiltonian reads

$$\hat{H}=\frac{p_x^2}{2\,m^*}+\frac{(p_y+eBx)^2}{2\,m^*}+\frac{p_z^2}{2\,m^*}+V(z), \tag{1}$$

where m^* is the carrier effective mass at energy ε and $V(z)$ is the confining potential in the growth direction. The spin Zeeman effect has been neglected because for GaAs-like materials the Landé g factor is so small that even at 40 T the spin Zeeman effect remains smaller than ≈ 1 meV; this value is significantly less than the effective electronic temperature broadening or the inhomogeneous broadening.

The eigenstates of Eq. (1) are factorized:

$$\psi_{n,p,k_y}(\vec{r})=\langle\vec{r}|E_n,p,k_y\rangle=\chi_n(z)\,\varphi_p(x+\lambda^2k_y)\frac{e^{ik_yy}}{\sqrt{L_y}}, \tag{2}$$

where φ_p is the pth Hermite function, L_y the sample size along the y direction, and $\lambda=\sqrt{\hbar/eB}$ is the cyclotron radius. The energies are additive for the longitudinal and transverse carrier motions:

$$\varepsilon_{n,p,k_y}=E_n+\left(p+\frac{1}{2}\right)\hbar\,\omega_c, \tag{3}$$

where E_n are the energies of the unperturbed system and $\hbar\omega_c=\hbar eB/m^*$ is the cyclotron energy.

Thus, the unperturbed Landau-level spectrum is discrete, similarly to that of a quantum dot. There exists a major difference between the two cases, which is the macroscopic degeneracy $L_xL_y/(2\pi\lambda^2)$ of each of the Landau energies given by Eq. (3).

The electron-LO phonon Hamiltonian is the sum of the interaction Hamiltonian with each phonon mode defined by its 3D wave vector $\vec{q}=(\vec{Q},q_z)$, where $\vec{Q}=(q_x,q_y)$ is its in-plane component:

$$\hat{H}_{e-ph}=\sum_{\vec{q}}\hat{H}_{e-ph}(\vec{q}),$$

$$\hat{H}_{e-ph}(\vec{q})=i\frac{g}{q}(e^{-i\vec{q}\cdot\vec{r}}a_{\vec{q}}^{+}-e^{i\vec{q}\cdot\vec{r}}a_{\vec{q}}), \tag{4}$$

where g^2 is the Frölich factor given by

$$g^2=2\pi\hbar\omega_{LO}\frac{e^2}{\varepsilon_p V},$$

$$\varepsilon_p=\frac{4\pi\varepsilon_0}{\varepsilon_\infty^{-1}-\varepsilon_s^{-1}}, \tag{5}$$

ω_{LO} is the energy of the dispersionless LO phonons which we consider bulklike. ε_s and ε_∞ are the static and high-frequency relative dielectric constants respectively and V is the sample volume.

A completely discrete spectrum implies a conceptual difficulty in computing the transition from one given level to the other ones, to the extent that there is no justification to apply the Fermi golden rule to compute the transition between the Landau levels due to the emission of LO phonon. Moreover, like in a quantum dot, the lack of electronic continuum leads us to expect the formation of mixed elementary excitations, the polarons, which are the stable exact eigenstates of the interacting electrons and LO phonons. The energy relaxation should subsequently be handled within this polaron formalism and this can be done by noting that the polarons are coupled to the phonon thermostat because of anharmonicity.[7,19] While there are now good evidences of robust electronic polarons in, e.g., InAs/GaAs quantum dots,[20] the Landau-level polarons are not as strong as in InAs dots for a similar coupling between the electrons and the LO phonons: anticrossings of the order of only ≈ 5 meV have been reported[21] in GaAs when (for the ground electronic subband) the cyclotron frequency ω_c is equal to the LO phonon frequency ω_{LO}. Since we are interested in evaluating inter-subband relaxation (where the z dependent wave functions are different in the initial and final electronic states), the polaronic effects are expected to be even smaller than those measured when $\omega_c=\omega_{LO}$, making the polarons prone to decay very quickly due to the imperfections of the materials. Consider the factorized electron-phonon states $|E_2,0,k_y\rangle\otimes|0\rangle$ and $|E_1,n_1,k'_y\rangle\otimes|1\rangle$. They comprise zero or one LO phonon. Assume the magnetic field is such that the zero-phonon state $|E_2,0,k_y\rangle\otimes|0\rangle$ has the same energy as the one phonon states of the lower subband $|E_1,n_1,k'_y\rangle\otimes|1\rangle$. This is the intersubband magnetopolaron resonance:

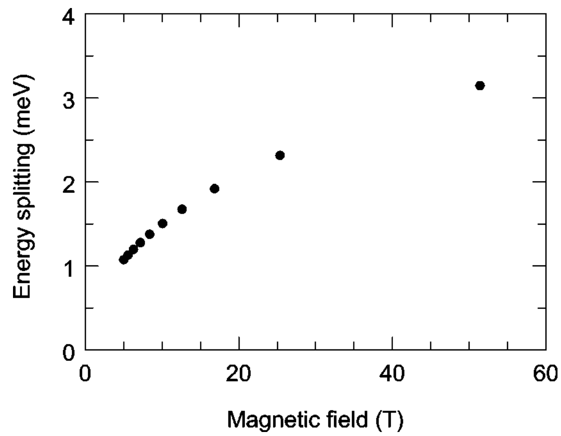

FIG. 1. Energy splitting as a function of the magnetic field at the polaron resonance in a 90 Å wide GaAs quantum well with $Al_{0.45}Ga_{0.55}As$ barriers, for $n_1=1-10$.

$$E_2=E_1+n_1\hbar\omega_c+\hbar\omega_{LO}. \tag{6}$$

The diagonalization of the electron-phonon Hamiltonian between these states leads to the formation of two polaron states whose energy splitting $\hbar\Omega$ between the states $|E_2,0,k_y\rangle\otimes|0\rangle$ and $|E_1,n_1,k'_y\rangle\otimes|1\rangle$ is given by

$$\hbar^2\Omega^2=\sum_{\vec{q}=(\vec{Q},q_z)}|I|^2g^2\frac{1+n_{\vec{q}}}{q^2}|\langle E_1|e^{iq_zz}|E_2\rangle|^2, \tag{7}$$

where the matrix element for the in-plane direction reads

$$|I|^2=\frac{(\lambda^2Q^2)^{n_1}}{2^{n_1}n_1!}\exp\left(\frac{-\lambda^2Q^2}{2}\right). \tag{8}$$

In Fig. 1 the $\hbar\Omega$ splitting is plotted as a function of the magnetic field at the polaron resonance in a 90 Å wide GaAs quantum well with $Al_{0.45}Ga_{0.55}As$ barriers for $n_1=1-10$. Its maximum value is of the order of 3 meV at $B=50$ T. Hence, in a real system, it is likely that the disorder induced by the impurities, interface defects, etc. will homogeneously or inhomogeneously broaden the Landau levels making the polaron effects hardly visible. In the case where the polaron description were necessary (i.e., for extremely low defects samples where the polaron splitting would largely exceed the Landau-level broadening), one would have to start from the polaron eigenstates and completely reconsider the problem of the energy relaxation, because no LO phonon emission or absorption could be further added to the model of stable mixed elementary excitations of the interacting electron and phonon system.[7,19] In the polaron framework, the energy relaxation can only be due to mechanisms which indirectly break the polaron: either a finite phonon lifetime (as, e.g., triggered by anharmonicity) or a finite electron lifetime (as, e.g., due to Auger effects or to recombination lifetime). In a quasiclassical description of these anharmonicity and/or Auger effects, the decay frequency of the polaron is equal to

$$\frac{1}{\tau_{pol}}=\frac{f_e}{\tau_e}+\frac{f_{ph}}{\tau_{ph}}, \tag{9}$$

where f_e and f_{ph} are the zero-phonon and one-phonon fractions in the polaron wave function, while τ_e and τ_{ph} are the electron and phonon lifetimes, respectively. This implies that the polaron lifetime is always longer than the shorter of the electron or phonon lifetimes, that is to say a few picoseconds in a GaAs-like material since τ_{ph} is itself a few picoseconds in these materials.

For state of the art materials, we believe that calculations performed in a strong coupling (polaronic) approach will not correctly describe the results of the experiments. This is because the anticrossing (at resonance) or, *a fortiori*, the difference between the polaron energy and the uncoupled electron-phonon energies (off resonance) are not much larger than the collision or inhomogeneous broadenings. Such a situation is in fact reminiscent of the one found in a low quality factor planar microcavity where the coupling between the material system and the photons is too weak compared to their dampings (due to defects, imperfections, losses, etc.) thereby preventing the formation of stable mixed modes (the polaritons in the microcavity case).

We shall now calculate the relaxation time τ_{n_2,n,k_y} of a given initial Landau state n centered at a given $-\lambda^2 k_y$ of a given subband n_2 due to the emission of a longitudinal optical phonon, which brings the electron in the various Landau states of different subbands n_m, $m<2$. We assume that the Landau levels are broadened and use the Fermi golden rule to write

$$\frac{1}{\tau_{n_2,n,k_y}}=\frac{2\pi}{\hbar}\sum_{n',k_y'}|\langle n_2,n,k_y|\hat{H}_{e-ph}|n_1,n',k_y'\rangle|^2 L_a[E_{n_2}-E_{n_1}+(n-n')\hbar\omega_c-\hbar\omega_{LO}], \tag{10}$$

where L_a is a normalized Lorentzian function characterized by the broadening parameter a or any function (e.g., a semiellipse) which represents a broadened delta function of the difference between Landau-level energies. A quick inspection of Eq. (10) shows that resonances in the scattering rate $1/\tau$ will take place when the LO phonon energy matches the energy difference between two Landau levels of two subbands, i.e., exactly at the intersubband magnetopolaron resonances. These resonances instead of occurring at discrete values of the magnetic field $B_{n,n'}$ will be smeared out and extend over finite segments around $B_{n,n'}$ because of the broadening effect. A self-consistent calculation of scattering rates in the case of highly singular density of states may be found in Ref. 22.

Actually, we shall not attempt to make such a calculation because the precise value of a or the precise shape of L_a are of little importance (provided a is small enough) to the extent that the central assumption of our model is that actual experiments involve an inhomogeneous broadening of the levels rather than an homogeneous one as it arises from microscopic scatterers. Our belief is that a macroscopic area of the sample is involved when measuring the output of a quantum cascade laser. Hence, we envision the light emitted by the sample as originating from a huge collection of similar samples which however differ from each others because one (or several) parameter, such as one (or several) of the well width(s) L, varies in a random fashion from one μ sample to the other. Depending on B and L, one such μ sample will be near the intersubband magnetopolaron resonance. Hence, its τ will be short and if n_2 corresponds to the subband to be populated to ensure laser action, this particular μ sample will be off duty. Instead for the same B value another μ sample characterized by a slightly different L will be off the intersubband magnetopolaron resonance. Its τ will be longer than the μ sample at resonance, and it will possibly participate to optical gain (provided the recombination wavelength matches the electromagnetic cavity requirement). Our model is thus similar to the well-known model of the "terrasselike interface defects."[23,24] These defects are assumed wide enough in the layer plane to leave the Landau quantization essentially unchanged. In other words, instead of assuming a global in-plane invariance, eventually perturbed by impurities, which would lead to extremely complicated self-consistent calculations, we add the response of N nearly ideal μ samples to evaluate Eq. (10). The properties of these N μ samples differ because of the random distribution of the parameter L. Since the number N is very large we can replace the discrete summation over the various μ samples by an integral whose integrand will display a probability density that the random variable L lays between L and $L+dL$. It is then clear that Eq. (10) will have to be replaced by

$$\left\langle\frac{1}{\tau_{n_2,n,k_y}}\right\rangle=\int_0^{\infty}P(L)\frac{1}{\tau_{n_2,n,k_y}(L)}dL. \tag{11}$$

In the following, we shall use a Gaussian probability density:

$$P(L)=\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{(L-L_0)^2}{2\sigma^2}\right), \tag{12}$$

and extend the lower bound of the integral in Eq. (11) to $-\infty$ because $\sigma\ll L_0$. Now, if the fluctuations of the energy difference $E_{n_2}-E_{n_1}$ due to the fluctuations of L are much larger than the a parameter, which determines the width of the Lorentzian, all the smearings out of the $1/\tau$ resonances versus B will effectively result from the parameter σ. This will allow us to go to the limit $a\to 0$, i.e., to replace the Lorentzian by a delta function.

Under such conditions, we get:

$$\left\langle\frac{1}{\tau_{n_2,n,k_y}}\right\rangle=\frac{2\pi}{\hbar}\sum_{k_y',n'}|\langle n_2,n=0,k_y|\hat{H}_{e-ph}|n_1,n',k_y'\rangle|^2 \times\int_{-\infty}^{+\infty}P(L)\,\delta[\Delta_{n_2,n_1}(L)-n'\hbar\omega_c-\hbar\omega_{LO}]dL, \tag{13}$$

where we have specialized to the ground Landau level ($n=0$) of the initial subband and where

$$\Delta_{n_2,n_1}(L)=E_{n_2}(L)-E_{n_1}(L). \tag{14}$$

In principle, there is also a variation of the squared matrix element with L. It is small however compared to those associated with the variations of the argument of the delta function. For simplicity we have neglected these variations. The energy difference Δ_{n_2,n_1} is a monotonously decreasing function of L. In order to keep analytical results as far as possible, we shall assume that around L_0 the energy difference Δ_{n_2,n_1} varies linearly with L:

$$\Delta_{n_2,n_1}(L)=\Delta_0-\gamma(L-L_0), \tag{15}$$

where $\gamma>0$ *a priori* depends on n_2 and n_1. With this (minor) assumption Eq. (13) simplifies as

$$\left\langle\frac{1}{\tau_{n_2,n,k_y}}\right\rangle=\frac{2\pi}{\hbar}\sum_{k'_y,n'}|\langle n_2,n=0,k_y|\hat{H}_{e-ph}|n_1,n',k'_y\rangle|^2 \times\frac{1}{\delta\sqrt{2\pi}}\exp\left(-\frac{(\Delta_0-n'\hbar\omega_c-\hbar\omega_{LO})^2}{2\delta^2}\right), \tag{16}$$

where $\delta=\gamma\sigma$ is the width of the Gaussian distribution of the energy differences Δ_{n_2,n_1}. Hence, as seen from Eq. (15), the inhomogeneous broadening model results in an expression of the average scattering rate which looks like the one obtained by replacing the deltalike peaks of the Landau-levels density of states by Gaussians. A typical value for linewidths in quantum cascade laser structures is ≈ 12 meV;[25] as a consequence in the following we will take $\delta=6$ meV.

Before discussing the limiting behavior of the scattering rate in both limits $B\to 0$ and $B\to\infty$ in Sec. III, let us mention that Eq. (16) shows that $\langle 1/\tau_{n_2,n,k_y}\rangle$ displays maxima whenever the following condition is satisfied:

$$\Delta_0-\hbar\omega_{LO}=n'\hbar\omega_c. \tag{17}$$

This means that a plot of the maxima versus $1/B$ will show peaks which are separated by $e\hbar/[m^*(\Delta_0-\hbar\omega_{LO})]$. Therefore if two series of peaks show up in the Fourier transform of $\langle\hbar/(2\pi\tau)\rangle$, the two periods will directly give access to the energy differences $E_{n_2}-E_{n_1}$ and $E_{n_2}-E_{n'_1}$.[14]

It is also important to note that the $\langle 1/\tau\rangle$ peaks occur at the intersubband magnetopolaron resonances. Hence, both models of a weak coupling or a strong coupling between electrons and LO phonons provide the same qualitative result as far as the variations of $\langle 1/\tau\rangle$ versus B are concerned. Thus, it is not the mere observation of the oscillatory behavior versus B which can discriminate between both descriptions of the electron-phonon coupling. Rather, we believe that the measurement of the absolute value of the scattering frequency can give us a hint on which approach is more likely to be correct since, in general, the strong coupling regime leads to a less efficient energy relaxation (by roughly a factor of 10) than the weak coupling one.

After some manipulations and using Eq. (8), we find

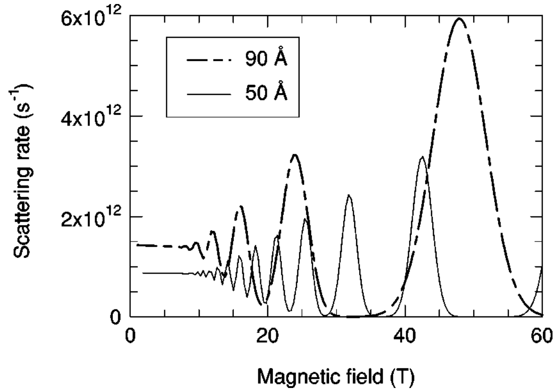

FIG. 2. Calculated scattering rates vs B for the LO-phonon emission between the ground Landau level of the E_2 subband to the Landau levels of the E_1 subband (GaAs well, $Al_{0.45}Ga_{0.55}As$ barriers). The Gaussian width δ is 6 meV.

$$\left\langle\frac{\hbar}{2\pi\tau_{n_2,0,k_y}}\right\rangle=g^2(1+n_{LO})\sum_{\vec{Q},q_z}\frac{|\langle\chi_{n_2}|e^{-iq_zz}|\chi_{n_1}\rangle|^2}{q_z^2+Q^2}S(Q),$$

$$S(Q)=\frac{e^{-\lambda^2Q^2/2}}{\delta\sqrt{2\pi}}\sum_{n'}\frac{1}{n'!}\left(\frac{\lambda^2Q^2}{2}\right)^{n'} \times\exp\left(-\frac{(\Delta_0-n'\hbar\omega_c-\hbar\omega_{LO})^2}{2\delta^2}\right), \tag{18}$$

where n_{LO} is the mean number of LO phonons. It is worth remarking that $1/\tau$ is k_y independent, which recalls the implicit in-plane translation invariance of the inhomogeneous broadening model. Thus, Eq. (18) represents also the average scattering rate of the population of the ground Landau level of the subband n_2. Equation (18) can be made a little more explicit by evaluating the summation over q_z. One readily finds:

$$\left\langle\frac{\hbar}{2\pi\tau_{n_2,0,k_y}}\right\rangle=(1+n_{LO})\frac{e^2\hbar\omega_{LO}}{8\pi\varepsilon_0}\left(\frac{1}{\varepsilon_\infty}-\frac{1}{\varepsilon_s}\right) \times\int_0^\infty f(Q)S(Q)dQ, \tag{19}$$

where $f(Q)$ is the form factor:

$$f(Q)=\int\int dzdz'\chi^*_{n_1}(z)\chi_{n_1}(z')\chi_{n_2}(z)\chi^*_{n_2}(z') \times\exp(-Q|z-z'|), \tag{20}$$

which vanishes both at $Q=0$ (like Q) and when $Q\to\infty$ (like $1/Q$).

We show in Fig. 2 the calculated scattering rates versus B for the LO phonon emission between the ground Landau level of the E_2 subband to the Landau levels of the E_1 subband for a GaAs well clad by $Ga_{0.55}Al_{0.45}As$ barriers. Two different well widths (50 Å and 90 Å, respectively), with $E_2 - E_1$ energies of 212 meV and 111 meV, respectively, have been considered.

III. THE WEAK- AND STRONG-FIELD LIMITS

In the weak-field limit, i.e., $\hbar\omega_c \to 0$, $S(Q)$ in Eq. (18) can after some manipulations be approximated by

$$S(Q) \approx G_\delta(Q) = \frac{1}{\delta\sqrt{2\pi}} \exp\left(-\frac{\left(\Delta_0 - \hbar\omega_{LO} - \frac{\hbar^2 Q^2}{2m^*}\right)^2}{2\delta^2} \right), \tag{21}$$

where m^* is the effective mass at the edge of the excited subband. We recognize in $G_\delta(Q)$ a broadened delta function of the energy difference between the edge of the initial subband and the total electron energy in the final subband. We note that the electron has gained in-plane kinetic energy because the global in-plane invariance of the interacting electron-phonon system forces the electron to acquire $\vec{Q}$ in the layer plane after the interaction with one LO phonon. Finally, the average population relaxation time reads

$$\left\langle \frac{\hbar}{2\pi\tau_{n_2,0}} \right\rangle = g^2(1+n_{LO}) \sum_{\vec{Q},q_z} \frac{|\langle \chi_{n_2}| e^{-iq_z z} |\chi_{n_1}\rangle|^2}{q_z^2 + Q^2} G_\delta(Q). \tag{22}$$

It is clear that Eq. (22) is the expression of the intersubband relaxation time for the emission of phonons from the edge of E_{n_2} to a lower subband E_{n_1} for an inhomogeneously broadened heterostructure. If, in addition, we neglect the inhomogeneous broadening we replace $G_\delta(Q)$ by $\delta[\Delta_0 - \hbar\omega_{LO} - \hbar^2 Q^2/(2m^*)]$ and Eq. (22) will coincide with the Fermi golden rule expression of the intersubband relaxation time.

Finally, let us discuss the strong magnetic-field regime and propose a structure which is meant to take benefit of the changes in the electron-phonon interaction in heterostructures by a magnetic field. It consists of a two-well active region sandwiched between injection/extraction regions. In this case, there is only one subband below the excited state and the change in its lifetime is maximized. We show in Fig. 3 the schematic conduction-band edge profile in this structure. The transition energy $E_2 - E_1$ will correspond to 137 meV. Figure 4 shows the changes in the calculated population relaxation time of the excited subband E_2 versus B. This lifetime is increased by a factor of 2 at $B = 15$ T and by a factor of 5 at $B = 19$ T. The study of the broadening of the Landau levels and the means to reduce it are thus an important issue in the perspective of exploiting the effects of low magnetic field on the mid-infrared devices. For longer wavelength, it is even possible to reach at high magnetic field a regime where the cyclotron energy exceeds the intersubband energies. In this case, one of the main cause of population transfer, the irreversible emission of LO phonons, will be quenched and the excited-state lifetime will become limited by other mechanisms. We think that in this case electron-electron interactions have an important role for relaxation: giving the necessary energy conservation, they allow electrons in a state where LO-phonon relaxation is impossible to reach another state in which relaxation is allowed. In the magnetic quantum limit, we can keep only the $n=0$ term in Eq. (18). In addition, we use a convenient interpolation of the form factor $f(Q)$, that is,

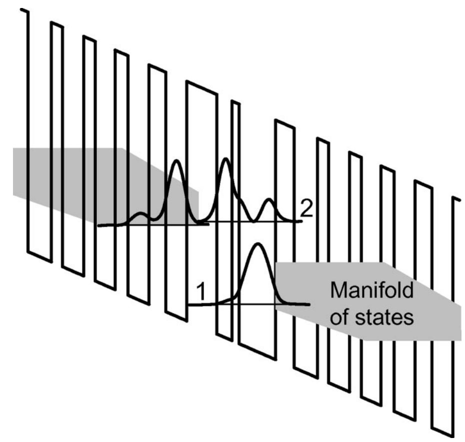

FIG. 3. Schematic conduction-band profile for a structure with a two-well active region.

$$f(Q) = \frac{-Q|z-z'|_{n_1,n_2}}{1 + \alpha Q + \beta Q^2}, \tag{23}$$

$$\beta = \frac{-|z-z'|_{n_1,n_2}}{2\int\int |\chi_{n_1}(z)\chi_{n_2}(z')|^2 dz\, dz'}, \tag{24}$$

where α is a fitting parameter [known e.g., by requiring that Eq. (24) has the right slope at $Q=0$] and

$$|z-z'|_{n_1,n_2} = \int\int \chi_{n_1}(z)\chi^*_{n_2}(z)|z-z'| \times \chi^*_{n_1}(z')\chi_{n_2}(z')\, dz\, dz'. \tag{25}$$

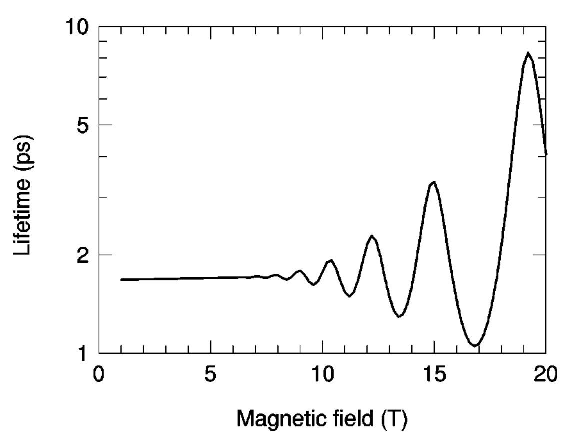

FIG. 4. Population relaxation time of the excited E_2 subband vs magnetic field for the structure shown in Fig. 3. The Gaussian width δ is 6 meV.

Therefore, in the strong-field regime, the intersubband scattering rate associated with the LO phonon emission behaves like

$$\left\langle \frac{1}{\tau_{n_2,0,k_y}} \right\rangle = (1+n_{LO}) \frac{e^2\omega_{LO}}{4\varepsilon_0} \left(\frac{1}{\varepsilon_\infty} - \frac{1}{\varepsilon_s} \right) \times \frac{e^{-(\Delta_0-\hbar\omega_{LO})^2/(2\delta^2)}}{\delta\sqrt{2\pi}} \left(\frac{-2eB\,|z-z'|_{n_1,n_2}}{\hbar} \right) \times \int_0^\infty \frac{x e^{-x^2}}{1+\alpha x\sqrt{\frac{2eB}{\hbar}} + \beta x^2 \left(\frac{2eB}{\hbar} \right)} dx. \quad (26)$$

It is worth remarking that Eq. (26) shows that once the $n=1$ level of the E_{n_1} phonon replica is beyond a few meV's from the $n=0$ level of the E_{n_2} subband, the scattering time reaches very large value (because the Gaussian has a very large and negative argument). At even larger field, we find a small decrease of $\tau_{n2,n1}$. This is because the inhomogeneous broadening model is, in essence, B independent, leading to a Gaussian contribution which does not vary with B. However, a very different behavior would arise if, e.g., a homogeneous broadening, *a priori* B dependent, of the level were taken into account. In fact, in the magnetic quantum limit, the intersubband relaxation will be very sensitive to disorder effects (hence, experimentally, it will vary strongly from sample to sample) and will be governed by transitions in the tail states of the lowest Landau level. A satisfactory theory of LO phonon emission between such singular densities of states remains to be built.

IV. CONCLUSION

In conclusion, we have calculated the population relaxation time of an excited subband due to the electron-LO phonon interaction for inhomogeneously broadened semiconductor heterostructures subjected to a quantizing magnetic field. In spite of the discrete density of states, we have shown that a weak-coupling description of the electron-LO phonon interaction remains possible. As a result, we have shown that the population relaxation rate is an oscillating function of the magnetic field. In addition, our formalism reduces to the well known expressions for this relaxation rate when $B \to 0$. Finally, we have shown that a quantizing magnetic field can be a very powerful tool to quench the LO phonon assisted transfer in carefully designed heterostructures. The case of heterostructures violating the weak-coupling regime for the electron-phonon interaction remains to be investigated more thoroughly both on the experimental and theoretical sides.

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