Photoconductance of quantum wires in a magnetic field

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We have calculated the photoconductance of a parabolic quantum wire subject to a magnetic field. It is shown that the photoconductance has maxima in the vicinity of a threshold of ballistic conductance steps. Electromagnetic irradiation is found to decrease the resistance of the quantum wire. The dependence of the photoconductance on the radiation frequency, on the magnetic field, and on the electron energy is investigated.

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Quantum wires are widely used to study the conductance quantization in three-dimensional systems. As a rule the transport properties of such systems are studied ignoring the influence of a high-frequency electromagnetic field on the conductance. However, the use of microwave irradiation with an applied magnetic field may provide a convenient tool to study quantum electron states in nanostructures. Therefore, the influence of electromagnetic irradiation on the conductance is attracting a great deal of experimental[1–5] and theoretical[6–9] interest.

Note that the electric current is more favorable to measure than the absorption coefficient to study the electron states because a direct measurement of the absorption is restricted by the fact that the volume of the wire is much smaller than the total volume of the electromagnetic resonator.[8] The absorption of the electromagnetic field, polarized in the transverse direction (in this case there is no transfer of longitudinal momentum), can give a strong influence on the conductance due to electron transitions between different modes in the system.[9] Only electrons above the Fermi surface make a positive contribution to the photocurrent. In this case the states below the Fermi surface are occupied by the electrons from the reservoir. Hence the ordinary ballistic conductance, which is conditioned by the electrons below Fermi surface, does not change. Since only photoelectrons make a contribution to the photocurrent it is necessary to count the number of electrons above the Fermi surface to calculate the photoconductance. Thus one of the criteria of nonzero photocurrent is as follows: average time of electron transitions from the reservoir to the wire must be less than average time of electron transitions from the one reservoir to the other. Additionally, the condition for the conductance to be ballistic is as follows: the average transition time from the one reservoir to the other must be less than the average time of the phase-breaking process.[10] Thus in our case the total conductance of the system is a sum of the ordinary conductance and photoconductance. The latter is due to the photoelectron above Fermi surface (i.e., due to the electrons in excited levels).

The purpose of this work is to investigate the influence of a high-frequency electromagnetic field on the conductance quantization in the three-dimensional anisotropic quantum wire. To model the confinement of the quantum wire we use the parabolic potential. This potential is widely employed in theoretical investigations to study the physical properties of quantum wires.[11,12] Note also that in many experiments the confining potential is of the parabolic form to a very good approximation. The choice of the parabolic potential to describe electron properties of quantum wires is justified by a number of factors.[13] However, this potential has one more advantage, namely, even in the presence of an external magnetic field, a parabolic potential gives the quadratic Hamiltonian. This circumstance lets us reduce the quadratic Hamiltonian to the canonical form (in our case it is a sum of squares of momenta and positions) with the help of a linear canonical transformation of the phase space. The direct calculation of matrix elements of electromagnetic perturbation operators to find the photoconductance of the quantum wire is a complicated computational problem. Method of canonical transformation of the phase space lets us simplify our problem. It will be shown below that using this method we can bring the Hamiltonian of a quantum wire with magnetic field to the Hamiltonian of a quantum wire without magnetic field but with other frequencies named *hybrid frequencies*.

The spinless one-particle Hamiltonian of an electron in the three-dimensional anisotropic quantum wire has the form

$$H=\frac{1}{2m^*}\left(\mathbf{p}-\frac{e}{c}\mathbf{A}\right)^2+\frac{m^*}{2}(\Omega_x^2x^2+\Omega_z^2z^2), \tag{1}$$

where $\mathbf{A}$ is the vector potential of a magnetic field $\mathbf{B}$, Ω_i $(i=x,z)$ are the characteristic frequencies of the parabolic potential, and m^* is the effective electron mass.

In the case of a longitudinal magnetic field it is convenient to choose the following gauge for the vector potential:

$$\mathbf{A}=(\tfrac{1}{2}Bz,0,-\tfrac{1}{2}Bx).$$

By means of linear canonical transformation of the phase space, we find the new phase coordinates $(\mathbf{P},\mathbf{Q})$ such that H has the following canonical form:

$$H(\mathbf{P},\mathbf{Q})=\frac{1}{2m^*}(P_1^2+P_2^2+P_3^2)+\frac{m^*}{2}(\omega_1^2Q_1^2+\omega_2^2Q_3^2), \tag{2}$$

where ω_i $(i=1,2)$ are the hybrid frequencies. They have the form[14]

$$\omega_{1,2}=\tfrac{1}{2}\left[\sqrt{(\Omega_x+\Omega_z)^2+\omega_c^2}\pm\sqrt{(\Omega_x-\Omega_z)^2+\omega_c^2}\right], \tag{3}$$

where ω_c is the cyclotron frequency.

The spectrum of the Hamiltonian Eq. (2) has the form

$$E_{nmP_2}=\hbar\omega_1\left(n+\frac{1}{2}\right)+\hbar\omega_2\left(m+\frac{1}{2}\right)+\frac{P_2^2}{2m^*}, \quad (4)$$

where $n,m=0,1,\ldots$. The corresponding wave functions are as follows:

$$\Psi_{nmP_2}=\frac{1}{\sqrt{2\pi\hbar}}\exp(iP_2Q_2/\hbar)\Phi_n(Q_1)\Phi_m(Q_3), \quad (5)$$

where Φ_k are the oscillator functions.

The transition matrix $\Lambda=(a_{ij})$ $(i,j=1,\ldots,4)$ from the initial phase coordinates (p_x,p_z,x,z) to the new ones (P_1,P_3,Q_1,Q_3) has the form $(P_2=p_y,Q_2=y)$

$$\begin{pmatrix} p_1\\ p_3\\ q_1\\ q_3\end{pmatrix}=\Lambda\begin{pmatrix} P_1\\ P_3\\ Q_1\\ Q_3\end{pmatrix}, \quad (6)$$

where the matrix elements a_{ij} are

$$a_{13}=\frac{m^*\omega_1(\Omega_z^2+\omega_c^2-\omega_1^2)}{\sqrt{(\Omega_z^2-\omega_1^2)^2+\Omega_z^2\omega_c^2}}, \quad a_{14}=\frac{m^*\omega_2(\Omega_z^2+\omega_c^2-\omega_2^2)}{\sqrt{(\Omega_z^2-\omega_2^2)^2+\Omega_z^2\omega_c^2}}, \quad (7)$$

$$a_{21}=-\frac{1}{2}\frac{\omega_c}{\omega_1}\frac{(\omega_1^2+\Omega_z^2)}{\sqrt{(\Omega_z^2-\omega_1^2)^2+\Omega_z^2\omega_c^2}},$$

$$a_{22}=-\frac{1}{2}\frac{\omega_c}{\omega_2}\frac{(\Omega_z^2+\omega_2^2)}{\sqrt{(\Omega_z^2-\omega_2^2)^2+\Omega_z^2\omega_c^2}}, \quad (8)$$

$$a_{31}=\frac{1}{m^*\omega_1}\frac{(\Omega_z^2-\omega_1^2)}{\sqrt{(\Omega_z^2-\omega_1^2)^2+\Omega_z^2\omega_c^2}},$$

$$a_{32}=\frac{1}{m^*\omega_2}\frac{(\Omega_z^2-\omega_2^2)}{\sqrt{(\Omega_z^2-\omega_2^2)^2+\Omega_z^2\omega_c^2}}, \quad (9)$$

$$a_{43}=\frac{\omega_c\omega_1}{\sqrt{(\Omega_z^2-\omega_1^2)^2+\Omega_z^2\omega_c^2}}, \quad a_{44}=\frac{\omega_c\omega_2}{\sqrt{(\Omega_z^2-\omega_2^2)^2+\Omega_z^2\omega_c^2}}. \quad (10)$$

To first order in perturbation theory, the photoconductance is given by the following formula that is analogous to Refs. 8 and 9:

$$G^{ph}(\omega)=-\frac{4\pi e^2}{\hbar}\sum_{\alpha,\beta}\frac{\partial f_\alpha^0}{\partial\mu}|\langle\alpha|\hat V_\omega|\beta\rangle|^2[\delta(E_\alpha-E_\beta-\hbar\omega)-\delta(E_\alpha-E_\beta+\hbar\omega)], \quad (11)$$

where f_α^0 is the Fermi-Dirac distribution function, μ is the chemical potential, and ω is the phonon frequency. The operator $\hat V_\omega$ of the interaction of electrons with the high-frequency electromagnetic field has the form

$$V_\omega=\frac{e\varepsilon_\omega}{m^*\omega}\left(p_z-\frac{m^*}{2}\omega_c x\right), \quad (12)$$

where ε_ω is the amplitude of the applied electromagnetic field which is assumed to be polarized in the z direction (the y direction corresponds to the axis of the wire). The states $|\alpha\rangle$ and $|\beta\rangle$ are characterized by the mode numbers n,m and n',m', respectively.

The physical meaning of Eq. (11) is quite straightforward. It is clear that the photocurrent is determined by the transition probability from the state below the Fermi surface to the state above Fermi surface and the inverse process. Under the below assumptions and the temperature close to zero, the last process conditioned by emission of a quantum of electromagnetic field is small.

We calculate the matrix elements of V_ω in the new phase coordinates. In this case, using the transition matrix Λ, one can obtain the squares of the matrix elements of the operator V_ω on the analogy of Ref. 15:

$$|\langle nmp|\hat V_\omega|n'm'p'\rangle|^2=\frac{e^2\varepsilon_\omega^2\hbar}{2m^*\omega^2}\left\{X_1\left[\frac{n'}{2}\delta_{n,n'-1}-\frac{n'+1}{2}\delta_{n,n'+1}\right]\delta_{m,m'}+X_2\left[\frac{m'}{2}\delta_{m,m'-1}-\frac{m'+1}{2}\delta_{m,m'+1}\right]\delta_{n,n'}\right\}\delta_{p,p'}, \quad (13)$$

where

$$X_i=\frac{\Omega_z^4\omega_c^2}{\omega_i[(\Omega_z^2-\omega_i^2)^2+\Omega_z^2\omega_c^2]}, \quad i=1,2.$$

Substituting Eq. (13) into Eq. (11) and taking into account the smearing of the energy levels caused by collisions, we

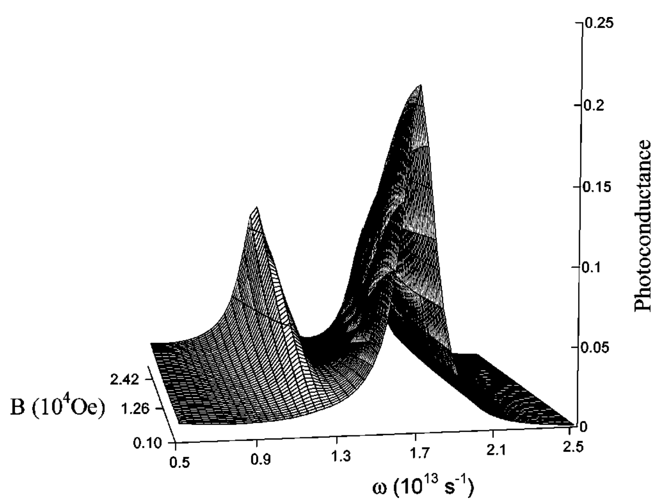

FIG. 1. Photoconductance as a function of the magnetic field B and the frequency of the electromagnetic radiation ω. $T=1$ K, $\Omega_x=1.2\times10^{13}$ s^{-1}, $\Omega_z=1.7\times10^{13}$ s^{-1}, $\varepsilon_\omega=100$ V/cm, and $\mu=0.41\times10^{-13}$ erg.

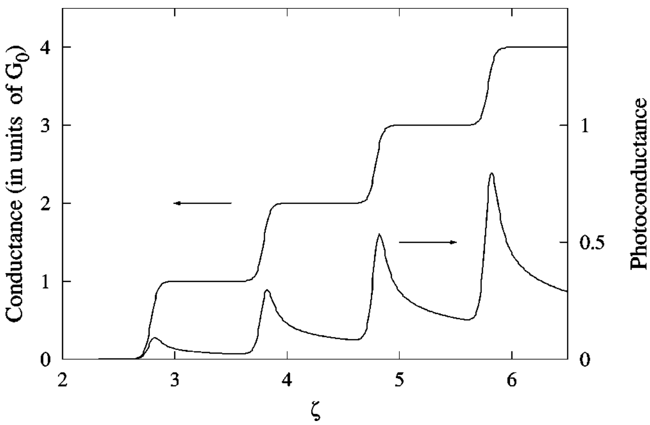

FIG. 2. Ballistic conductance and photoconductance as functions of the chemical potential ($\zeta=\mu/\hbar\Omega_z$). $T=2$ K, $B=0$, $\Omega_x=0.9\times10^{13}$ s^{-1}, $\Omega_z=4.12\times10^{13}$ s^{-1}, $\omega=0.9\times10^{13}$ s^{-1}, and $\varepsilon_\omega=100$ V/cm.

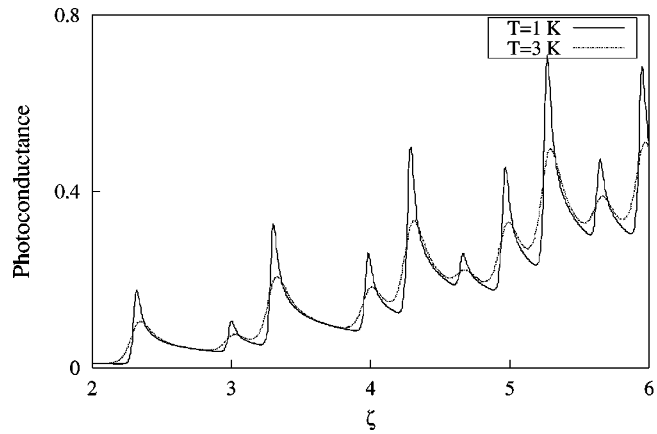

FIG. 3. Photoconductance as a function of the chemical potential ($\zeta=\mu/\hbar\Omega_z$). $B=10^4$ Oe, $\Omega_x=1.1\times10^{13}$ s^{-1}, $\Omega_z=1.8\times10^{13}$ s^{-1}, $\omega=1.082\times10^{13}$ s^{-1}, and $\varepsilon_\omega=130$ V/cm.

obtained the following expression for the photoconductance:

$$\frac{G^{ph}(\omega)}{G_0}=-\frac{\pi e^2\varepsilon_\omega^2\tau}{2m^*\omega^2}\sum_\alpha\frac{\partial f_\alpha^0}{\partial\mu}\left[\frac{X_1\left(n+\frac{1}{2}\right)}{1+(\omega+\omega_1)^2\tau^2}-\frac{X_1\left(n+\frac{1}{2}\right)}{1+(\omega-\omega_1)^2\tau^2}+\frac{X_2\left(m+\frac{1}{2}\right)}{1+(\omega+\omega_2)^2\tau^2}-\frac{X_2\left(m+\frac{1}{2}\right)}{1+(\omega-\omega_2)^2\tau^2}\right], \quad (14)$$

where G_0 is the conductance quantum.

As is clear from Eq. (14), there are two resonance peaks at points $\omega=\omega_{1,2}$ which correspond to the transition between the neighboring hybrid levels. Note that the amplitudes of the resonance peaks depend strongly on the magnitude of magnetic field and the characteristic frequencies (Fig. 1).

Let us consider the case of equal characteristic frequencies $\Omega_z=\Omega_x=\Omega$. In this case the frequencies $\omega_{1,2}$ have the form $\omega_{1,2}=(\sqrt{4\Omega^2+\omega_c^2}\pm\omega_c)/2$. Then the resonance peaks have a doublet structure in this case of a weak magnetic field ($\omega_c\ll\Omega$). We stress that the amplitudes of the peaks are approximately equal. The distance between the doublet components is equal to the cyclotron frequency ω_c [$\omega_{1,2}\simeq\Omega\pm\omega_c/2+O(\omega_c^2/\Omega)$]. In the case of a strong magnetic field ($\omega_c\gg\Omega$) the frequencies $\omega_{1,2}$ can be represented in the form $\omega_1\simeq\omega_c$, $\omega_2\simeq\Omega^2/\omega_c$. Note that the amplitude of the peak at the frequency Ω^2/ω_c is appreciably smaller than at the frequency ω_c. Hence, a strong absorption is observed at the cyclotron frequency in the case of strong magnetic quantization.

In the limit $B=0$ we have the following formula for the conductance of the quantum wire:

$$\frac{G^{ph}(\omega)}{G_0}=-\frac{4\pi e^2\varepsilon_\omega^2\hbar\tau}{m^{*2}\omega^2 l_1^2}\left[\frac{1}{1+(\omega+\Omega_z)^2\tau^2}-\frac{1}{1+(\omega-\Omega_z)^2\tau^2}\right]\sum_\alpha\frac{\partial f_\alpha^0}{\partial\mu}(n+1/2), \quad (15)$$

where $l_1=\sqrt{\hbar/m^*\omega_1}$. In this case there is only one resonance point $\omega=\Omega_z$ associated with electron transitions between neighboring levels $n\rightarrow n+1$. The transition accompanied by a change in the quantum number m is forbidden.

We stress that electromagnetic irradiation has a strong effect on the transport properties only in the neighborhood of the resonance points. Hence, it is interesting to consider the ballistic conductance and the photoconductance together. In Fig. 2 we display the conductance and the photoconductance as functions of the electron energy. As can be seen from Fig. 2 the photoconductance has maxima in the vicinity of a threshold of conductance steps. As is clear from Eqs. (14) and (15), and Fig. 2, electromagnetic irradiation always decreases the resistance of the parabolic quantum wire. The dependence of the conductance on the electron energy is shown in Fig. 3. It is seen from Fig. 3 that the photoconductance undergoes nonperiodic oscillations. The distance between oscillation peaks is determined by the relationship between characteristic frequencies as was shown above. Note that the temperature strongly smears the oscillating peaks (Fig. 3).

In conclusion, we have theoretically investigated the influence of electromagnetic irradiation on the transport properties of quantum wire. We have shown that irradiation has a strong effect on the transport properties only in the neighborhood of the resonance points; namely, electromagnetic irradiation always increases the conductance of systems. The amplitude and position of resonance peaks are found. It is shown that the resonance peaks have a doublet structure in the case of a weak magnetic field.

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