Magnetoresistance of nondegenerate quantum electron channels formed on the surface of superfluid helium

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Transport properties of quasi-one-dimensional nondegenerate quantum wires formed on the surface of liquid helium in the presence of a normal magnetic field are studied using the momentum balance equation method and the memory function formalism. The interaction with both kinds of scatterers available (vapor atoms and capillary wave quanta) is considered. We show that unlike classical wires, quantum nondegenerate channels exhibit strong magnetoresistance which increases with lowering the temperature.

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I. INTRODUCTION

Electrons trapped on the free surface of liquid helium form a nondegenerate two-dimensional electron system (2DES)[1] whose properties are complementary to the properties of the degenerate 2DES created in semiconductor structures. Considerable experimental and theoretical research has been performed on the quantum magnetotransport in such an almost pure and highly correlated 2DES (for a recent review, see Ref. 2). In the presence of a strong magnetic field applied in the normal direction to the system the electron energy spectrum is squeezed into the set of Landau levels slightly broadened due to the interaction with scatterers, if Coulomb forces are disregarded. The effect of Coulomb forces can be described in terms of the quasiuniform fluctuational electric field E_f (Refs. 3–5) which causes fast drift velocities of electron orbit centers. In local reference frames moving along with an electron orbit center the broadening of Landau levels is even reduced because of the fast drift velocities.[6] For surface electrons (SE) on liquid helium, the broadening of Landau levels is extremely narrow, usually much smaller than temperature, which is the origin of the unconventional Hall effect observed in this system under different experimental conditions.[7–9]

The SE can be confined in quasi-1D channels, when the helium surface is curved by capillary forces in the presence of a specially constructed dielectric substrate.[10,11] The conducting channels are formed in the valleys of the helium relief because of the strong holding electric field $E_\perp$ applied normally to the surface. These channels can be considered as the nondegenerate version of quantum wires created in semiconductor structures. In the presence of a strong normal magnetic field, the electron states in the channels[12] resemble the current-carrying edge states of the quantum Hall effect systems.[13] There is also an interesting evidence for the self-organized current filaments in the helium microchannels.[14]

The confining potential affects crucially the energy spectrum of electrons subject to the magnetic field B, because it removes the degeneracy of Landau levels. For example, in the 2DES the Landau spectrum does not depend on the orbit center coordinate $Y=-l_B^2 k_x$, and it is purely discrete $\varepsilon_{n,k_x}=\hbar\omega_c(n+1/2)$, where $\hbar k_x$ is the electron momentum along the x direction, $n=0,1,2,\ldots$, $l_B=(\hbar c/eB)^{1/2}$ is the magnetic length, and $\omega_c=eB/m_e c$ is the cyclotron frequency defined in terms of the free-electron mass m_e. In contrast, for the parabolic confining potential $U(y)=m_e\omega_0^2 y^2/2$, the electron energy spectrum under the magnetic field has a continuous term depending on the electron momentum along the channel:[12]

$$\varepsilon_{n,k_x}=\frac{\hbar^2 k_x^2}{2m_B}+\hbar\Omega\left(n+\frac{1}{2}\right), \tag{1}$$

where

$$m_B=m_e\left(1+\frac{\omega_c^2}{\omega_0^2}\right), \quad \Omega=\sqrt{\omega_0^2+\omega_c^2}. \tag{2}$$

The orbit center coordinate $Y_{k_x}=-(\omega_c/\Omega)l_y^2 k_x$ (here $l_y^2=\hbar/m_e\Omega$ is the typical electron localization length across the channel) is determined by the interplay of the magnetic field and the confining potential. Thus, the magnetic field shifts the electron wave function in the channel $\varphi_n(y-Y_{k_x})$ to the left or the right depending on the sign of k_x, as shown in Fig. 1. It increases also the frequency of the discrete part of the electron spectrum ($\omega_0\to\Omega$) and the effective mass of charge carries ($m_e\to m_B\propto\Omega^2$). Such an unusual behavior of

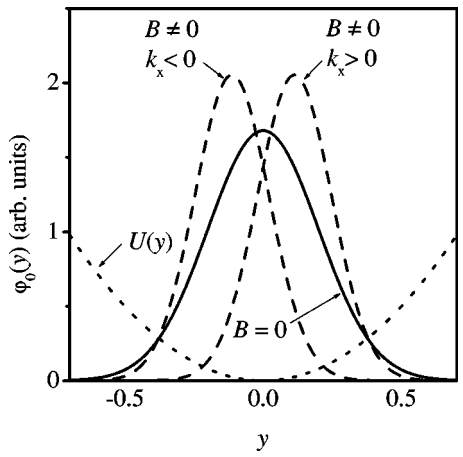

FIG. 1. Profiles of the electron wave function across the channel for $B=0$ (solid line) and for a characteristic $B\neq 0$ (dashed lines). The confining potential $U(y)$ is indicated by the dotted line.

the electron effective mass is very important for channel magnetotransport, localization effects, and the polaronic transition (for a review on the last two topics, see Ref. 15).

In this work we report the theory of quantum magnetotransport in quasi-1D electron channels under the condition that the electron-electron collision rate ν_{ee} is much higher than the effective collision frequency ν due to available scatterers. In this regime, usually realized for SE on helium, the memory function formulation for the electron conductivity[16] and the momentum balance equation approach leads to the same conductivity equation. The important advantage of these methods is that the electron conductivity along the channel can be quite generally expressed in terms of the electron dynamical structure factor (DSF) $S(\mathbf{q},\omega)$, which allows us to track the origin of the strong dependence of the electron mobility on the magnetic field. We find that, in contrast with classical wires, the resistivity of the nondegenerate quantum wires formed on the surface of liquid helium is highly affected by the magnetic field. This effect depends strongly on the temperature. The results obtained here are compared with those found for the pure 2D case.

II. BASIC RELATIONS

A. Electron channel states

In the presence of a strong holding electric field $E_\perp$ directed along the z axis the electrons in the channel are gathered near the minimum of the channel profile which we describe by the semicircular form $z(y)=R(1-\sqrt{1-y^2/R^2})\simeq y^2/2R$ if $y\ll R$ (here R is the curvature radius of the helium surface which usually ranges from 10^{-3} to 10^{-4} cm). As a result, the potential energy of an electron across the channel can be approximated by the parabolic potential $U(y)=m_e\omega_0^2y^2/2$, where $\omega_0^2=eE_\perp/m_eR$. As the magnetic field B is applied in the direction parallel to the holding electric field, we use the Landau gauge for the vector potential $\mathbf{A}=(-By,0,0)$ to take the advantage of the translational invariance along the channel (x axis). The electron motion along the y direction is quantized and the electron energy spectrum has the form given by Eq. (1).

Because the curvature radius is much larger than l_y, we restrict our study to the model of a 2DES subject to the oscillatory confining potential $U(y)$. The electron wave function in the channel is defined as

$$\psi_{n,k_x}(x,y)=\frac{1}{\sqrt{L_x}}\exp(ik_xx)\varphi_n[(y-Y_{k_x})/l_y], \qquad (3)$$

where $\varphi_n(x)$ is the Hermite functions, L_x is the linear dimension of the channel, and, as stated above, $Y_{k_x}=-(\omega_c/\Omega)l_y^2k_x$.

In general, we have to include the interaction of the electron spin with the magnetic field. This leads to the energy spectrum

$$\varepsilon_{n,k_x}=\frac{\hbar^2k_x^2}{2m_B}+\hbar\Omega\left(n+\frac{1}{2}\right)+\hbar\omega_c\sigma,$$

where $\sigma=\pm 1/2$ is the spin projection eigenvalue. In the pure 2DES ($\Omega=\omega_c$), the inclusion of spin causes an additional degeneracy of the electron states: the energy levels with $n=0$, $\sigma=1/2$, and $n=1$, $\sigma=-1/2$ coincide. For electrons in channels, these levels are split because $\Omega>\omega_c$. Because the interaction with scatterers does not involve the electron spin, we shall disregard it when calculating the scattering matrix elements.

The magnetic field does not modify the nature of the electron motion along the channel, and the continuous part of the electron spectrum has the usual form $\varepsilon_{k_x}=\hbar^2k_x^2/2m_B$. Still, the magnetic field increases strongly the effective mass m_B, and, for electrons with a fixed density (as it is indeed for SE on helium), it reduces the Fermi energy. At low electron densities we can consider only the lowest level with $n=0$. Then, in the absence of the magnetic field, the Fermi momentum $k_F=\pi n_{ch}/2$ (here n_{ch} is the linear electron density) and the 1D Fermi energy $\varepsilon_F\equiv\varepsilon_0$, where $\varepsilon_0=\pi^2\hbar^2n_{ch}^2/8m_e$. The magnetic field splits the lowest level, and for a fixed channel density, the 1D Fermi energy and the total energy of an electron at the Fermi level depend strongly on the energy parameter $\varepsilon_B=\pi^2\hbar^2n_{ch}^2/8m_B$ which decreases with B because of the mass enhancement. The total energy at the Fermi level can be written as

$$\begin{aligned}\varepsilon_{total}&=4\varepsilon_B+\hbar(\Omega-\omega_c)/2 \quad \text{if } \hbar\omega_c>4\varepsilon_B,\\ &=\varepsilon_B+\hbar\Omega/2+\hbar^2\omega_c^2/16\varepsilon_B \quad \text{if } \hbar\omega_c<4\varepsilon_B. \qquad (4)\end{aligned}$$

It should be noted that the field dependence of ε_{total} is nonmonotonous: the total energy defined above increases at low fields ($\hbar\omega_c<4\varepsilon_B$), and decreases steadily at high fields ($\hbar\omega_c>4\varepsilon_B$). For SE's on liquid helium the Fermi energy is much smaller than temperature and it is not a topical parameter. For other possible channel systems with a fixed density, the decrease of the Fermi energy and the total energy with B means that for finite temperatures and in the limit of high fields even a degenerate electron channel eventually becomes a nondegenerate system, whose transport properties are discussed in this paper.

B. Dynamical structure factor

As stated in the Introduction, the equilibrium electron DSF $S(\mathbf{q},\omega)$ plays an important role in the quantum transport theory of electrons with a short autocorrelation time. We define it as

$$S(\mathbf{q},\omega)=\frac{1}{N_e}\int_{-\infty}^{\infty} e^{i\omega t}\langle n_{\mathbf{q}}(t)n_{-\mathbf{q}}(0)\rangle dt, \tag{5}$$

where $n_{\mathbf{q}}=\Sigma_e \exp(-i\mathbf{q}\cdot\mathbf{r}_e)$ is the density fluctuation operator, $\mathbf{q}$ is the 2D wave vector, and $\mathbf{r}_e$ is the 2D position of an electron. For the energy spectrum given by Eq. (1), we can evaluate the average $\langle\cdots\rangle$ straightforwardly to obtain

$$S(\mathbf{q},\omega)=\frac{2\pi}{N_e}\sum_{k_x,k_x'}\sum_{n,n'} f(\varepsilon_{n,k_x})[1-f(\varepsilon_{n',k_x'})] \times|\langle n,k_x|e^{-i\mathbf{q}\cdot\mathbf{r}}|n',k_x'\rangle|^2\delta(\varepsilon_{n,k_x}-\varepsilon_{n',k_x'}+\hbar\omega), \tag{6}$$

where $f(\varepsilon)$ is the Fermi distribution function.

The SE on liquid helium form usually a nondegenerate system. Therefore we can disregard $f(\varepsilon_{n',k_x'})$ as compared to 1 in Eq. (6). Then, introducing the notation

$$J_{n,n'}(q_x,q_y)=\int_{-\infty}^{\infty} e^{iq_y l_y\tau}\varphi_n(\tau)\varphi_{n'}(\tau-q_x l_y\omega_c/\Omega)d\tau, \tag{7}$$

the DSF can be found in the form

$$S(\mathbf{q},\omega)=\hbar(1-e^{-\hbar\Omega/T})\sqrt{\frac{\pi}{\varepsilon_{q_x}T}}\sum_{n,n'} e^{-\hbar\Omega n/T}|J_{n,n'}|^2 \times\exp\left\{-\frac{[\varepsilon_{q_x}-\hbar\omega-\hbar\Omega(n-n')]^2}{4\varepsilon_{q_x}T}\right\}, \tag{8}$$

with

$$|J_{n,n'}(x_{\mathbf{q}})|^2=\frac{\min(n,n')!}{\max(n,n')!}e^{-x_{\mathbf{q}}}x_{\mathbf{q}}^{|n'-n|}[L^{|n'-n|}_{\min(n,n')}(x_{\mathbf{q}})]^2$$

and

$$x_{\mathbf{q}}=\left(q_y^2+q_x^2\frac{\omega_c^2}{\Omega^2}\right)\frac{l_y^2}{2}.$$

Here $L_n^m(x)$ are the associated Laguerre polynomials.

The matrix elements $|J_{n,n'}(x_{\mathbf{q}})|^2$ restrict differently the wave numbers q_x and q_y. For the lowest level $n=n'=0$, we have

$$|J_{0,0}(q_x,q_y)|^2=\exp\left(-\frac{q_y^2l_y^2}{2}-\frac{q_x^2l_y^2}{2}\frac{\omega_c^2}{\Omega^2}\right). \tag{9}$$

In the extreme case $B=0$ this equation restricts only transverse wave numbers q_y owing to the channel confining potential. For high magnetic fields $\Omega\simeq\omega_c$ both wave numbers q_x and q_y enter into these matrix elements in the same way, which is usual for the quantum magnetotransport in the 2DES. An additional restriction on q_x appears because of the factor $\exp(-\varepsilon_{q_x}/4T)$, but it becomes less important for strong fields when $m_B\gg m_e$.

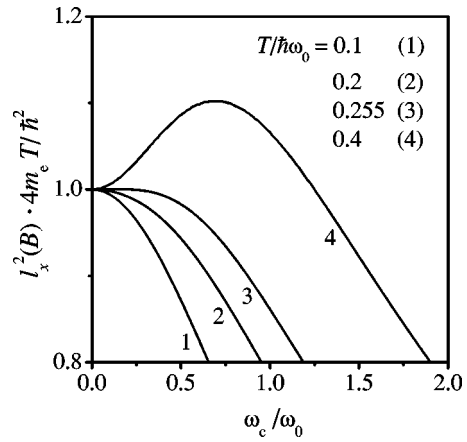

FIG. 2. The magnetic-field dependence of normalized l_x^2 which determines the momentum exchange along the channel in scattering events $q_x\sim 1/l_x$.

The channel DSF $S(\mathbf{q},\omega)$ exhibit features which are typical for both the free-electron gas under zero magnetic field and the 2DES in the presence of a strong perpendicular magnetic field. We first pay attention to the singularity $S(\mathbf{q},\omega)\propto 1/|q_x|$ and the factor $\exp(-\varepsilon_{q_x}/4T)$ which are inherent for free electrons. They concern the q_x component only, reflecting the free-electron motion along the channel. In the ultraquantum limit $T\ll\hbar\Omega$, Eq. (8) can be approximated by the terms with $n=0$. Then the channel DSF is given as a sum of Gaussian terms broadened by the width parameter $\Gamma^*=2\sqrt{\varepsilon_{q_x}T}$, which exhibit the resonant behavior as $\omega-n'\Omega\to 0$. This result is similar to that obtained for the 2D Coulomb liquid under a normal magnetic field.[2] In the limiting case $\Gamma^*\to 0$, the DSF is a sum of δ functions reflecting the singular nature of the 2DES under the magnetic field B.[17]

The broadening parameter of the Gaussians in Eq. (8) $\Gamma^*=2\sqrt{\varepsilon_{q_x}T}$ can be estimated combining two exponents proportional to q_x^2 by

$$\frac{q_x^2l_y^2}{2}\frac{\omega_c^2}{\Omega^2}+\frac{\varepsilon_{q_x}}{4T}\equiv\frac{q_x^2l_x^2}{2},$$

where we have defined

$$l_x^2=l_y^2\frac{\omega_c^2}{\Omega^2}+\frac{\hbar^2}{4m_BT}. \tag{10}$$

Then the typical electron wave numbers $q_x\sim\sqrt{2}/l_x$. In general $l_x^2(B)$ is a nonmonotonous function of the magnetic field, as shown in Fig. 2, where we have used the dimensionless parameters $T/\hbar\omega_0$ and ω_c/ω_0. We shall see that this behavior affects the channel magnetotransport.

At low temperatures ($T/\hbar\omega_0<0.255$), l_x decreases steadily with B, and when $l_x\approx l_y\omega_c/\Omega$ we have Γ^*

$\sim 2\sqrt{T\hbar\Omega}\,\omega_0/\omega_c \equiv \sqrt{2}eE_{\rm ch}l_y$, where $eE_{\rm ch} = \sqrt{2m_eT}\Omega\omega_0/\omega_c$, with $E_{\rm ch}$ being a characteristic electric field. For strong fields $\Omega \simeq \omega_c$, $eE_{\rm ch} \simeq \omega_0\sqrt{2m_eT}$ is the force acting on an electron at $y=\sqrt{2T/m_e\omega_0^2}$ due to the confining potential. It is interesting that in this limiting case, the temperature and field dependencies of the broadening of the channel DSF $\Gamma^* \propto \sqrt{T/B}$ are the same as that induced by strong internal forces in the 2D Coulomb liquid.[2,3,18] The conditions under which the fluctuational electric field in the channel can be disregarded will be discussed in Sec. III.

C. Channel magnetotransport

In this work we primarily consider the dc magnetotransport, which means that the frequency of the driving electric field is zero or negligibly low. Therefore, the current across the channel is assumed to be zero ($J_y=0$ and the Lorentz force is balanced by the field of the confining potential). Then for electrons with a short autocorrelation time, the electrical current along the wire J_x can be easily found by balancing the force of the driving electric field N_eeE_x and the frictional force $F_{\rm scat}$ acting on the electron system due to the scatterers.[2] In the linear transport regime the absolute value of the frictional force is proportional to the current or the average electron velocity in the channel $u_{\rm av}$, and can be written as $F_{\rm scat}=-N_em_e\nu_{\rm eff}(B)u_{\rm av}$. The proportionality factor $\nu_{\rm eff}(B)$ is called the effective collision frequency, which can be found by evaluating the momentum loss of the electron system per unit time. Then the current along the channel is given by

$$J_x \equiv eN_eu_{\rm av} = N_ee\mu E_x, \quad \mu = e/(m_e\nu_{\rm eff}), \tag{11}$$

where μ is the channel mobility. It is well known that for classical thin wires $\nu_{\rm eff}(B)$ does not depend on B (it coincides with the conventional collision rate) and the theory gives zero magnetoresistance. In the following we shall see that there is a strong magnetoresistance for nondegenerate quantum wires.

In order to find $F_{\rm scat}$ we can calculate the momentum absorbed by scatterers per unit time $F_{\rm scat}=-\dot{P}$. For SE on helium, the only scatterers are helium vapor atoms and capillary wave quanta (ripplons). At low temperatures $T \lesssim 0.5$ K the electrons are scattered predominantly by ripplons, because the vapor atom density decreases with cooling at an exponential rate. In this case the interaction Hamiltonian is proportional to the electron-density fluctuation operator $n_{\mathbf{q}}$ and given by

$$H_{\rm int} = \frac{1}{\sqrt{S_{\rm A}}}\sum_{\mathbf{q}} V_q\xi_{\mathbf{q}}n_{-\mathbf{q}},$$

where $S_{\rm A}$ is the surface area, V_q is the electron-ripplon coupling, and $\xi_{\mathbf{q}}$ is the Fourier component of the surface-displacement operator $\xi(\mathbf{r})$. In the limit of strong holding electric fields, the coupling parameter V_q does not depend on the wave number $V_q \simeq eE_\perp$. In general we have to include the polarization term $V_q=e(E_\perp+E_q)$, where E_q has a quite complicated dependence on the 2D wave number q.[19,20]

Following Refs. 2 and 21, the frictional force can be quite generally expressed in terms of the equilibrium electron dynamical structure factor

$$F_{\rm scat} = \frac{N_e}{\hbar S_{\rm A}}\sum_{\mathbf{q}} q_xV_q^2Q_q^2N_{\mathbf{q}}^{(\rm r)}[1-e^{\hbar q_xu_{\rm av}/T}]S_0(\mathbf{q},\omega_q-q_xu_{\rm av}), \tag{12}$$

where $\omega_q=\sqrt{\alpha/\rho}q^{3/2}$ and $N_{\mathbf{q}}^{(\rm r)}$ are the spectrum and distribution function of ripplons, $Q_q^2=\hbar q/2\rho\omega_q$, α and ρ are the surface tension and the liquid-helium mass density, respectively. Equation (12) is strictly valid for electrons with a short autocorrelation time ($\nu_{ee} \gg \nu_{\rm eff}$) which allows us to describe them by the the equilibrium DSF $S_0(\mathbf{q},\omega)$ in the center-of-mass reference frame moving along the channel. In the laboratory frame the frequency argument acquires the Doppler shift: $S(\mathbf{q},\omega) \simeq S_0(\mathbf{q},\omega-q_xu_{\rm av})$. The frictional force of Eq. (12) can also be found employing the momentum-balance equation method.[22,23]

According to Eq. (12) the linear transport regime is valid when the correction $\hbar q_xu_{\rm av}$ is much smaller than the temperature and the broadening of the electron DSF as a function of frequency. In the 2DES the latter condition is more restrictive because the broadening of the DSF is small. The DSF for channel electrons is much broader. Anyway, under the conditions of the linear transport regime $\hbar q_xu_{\rm av} \ll T$ and $\hbar q_xu_{\rm av} \ll \Gamma^*$ the effective collision frequency of channel electrons

$$m_e\nu_{\rm eff}^{(\rm r)}(B) = \frac{1}{S_{\rm A}T}\sum_{\mathbf{q}} V_q^2Q_q^2q_x^2N_q^{(\rm r)}S_0(\mathbf{q},\omega_q) \tag{13}$$

is similar to that obtained previously for an isotropic 2D electron liquid.[2,21] Thus the anisotropy of the channel system is hidden in its DSF: in contrast with the isotropic 2D case, the DSF of the electron channel $S_0(\mathbf{q},\omega_q)$ depends strongly on the direction of the wave vector $\mathbf{q}$, as discussed in Sec. II B. It is worthy to remark that the effective mass of electrons in the channel m_B enters into the frictional force only by means of the channel DSF [the free-electron mass m_e in the left-hand part of Eq. (13) is chosen as a convenient proportionality factor].

The same expression for the effective collision frequency could be obtained by means of the memory function formalism introduced by Götze and Wölfle.[24] In this approach the quantum conductivity equation looks like an extension of the classical Drude formula, in which the imaginary part of the conductivity relaxation kernel [the memory function $M(\omega)$] plays the role of the effective collision frequency. It should be noted that the approximation for the memory function frequently used in quantum transport equations is actually a high-frequency approximation ($\omega \gg \nu$), even though it usually gives correct results in the whole frequency range. Platzman *et al.*[25] were the first to apply this approach for the analysis of the effect of electron correlations. The important point is that the high-frequency approximation for the conductivity relaxation kernel and the dc approach based on the

condition $\nu_{ee} \gg \nu$ employed here yield the same equations for the effective collision frequency as it is in the semiclassical kinetic equation method.

If the temperature is relatively high ($T \gtrsim 0.7$ K), we have to consider the possibility of electron scattering by helium vapor atoms. Even though the helium vapor atoms represent a sort of impuritylike scatterers, we can disregard quantum localization effects because the electron-electron collision rate is extremely high ($\nu_{ee} \gg \nu$). For the interaction Hamiltonian, given in Ref. 26, in the quasielastic approximation the above described treatment leads to the following correction to the effective collision frequency induced by vapor atoms:

$$m_e \nu_{\mathrm{eff}}^{(\mathrm{a})}(B) = \frac{3 n_{\mathrm{a}} V_{\mathrm{a}}^2 \gamma}{16 T S_{\mathrm{A}}} \sum_{\mathbf{q}} q_x^2 S_0(\mathbf{q},0), \tag{14}$$

where n_{a} is the density of vapor atoms, γ is the parameter describing the wave function of SE states $\psi_1(z) \propto z \exp(-\gamma z)$, and V_{a} describes the interaction of a free electron with a single helium atom $V(\mathbf{R}-\mathbf{R}_a) = V_{\mathrm{a}} \delta(\mathbf{R}-\mathbf{R}_a)$. Even though the interaction parameter is usually written in the form $V_{\mathrm{a}} = 2\pi\hbar^2 s_0 / m_e$ containing the scattering length s_0 and m_e, the effective mass of electrons in the channel m_B appears only in the channel DSF.

III. RESULTS AND DISCUSSION

It is instructive to consider the ultraquantum limit $\hbar\Omega \gg T$. In this case, the electron DSF can be approximated by the term with $n = n' = 0$:

$$S(\mathbf{q},0) \simeq \hbar \sqrt{\frac{\pi}{\varepsilon_{q_x} T}} \exp\left(-\frac{q_y^2 l_y^2}{2} - \frac{q_x^2 l_x^2}{2} \right), \tag{15}$$

where the parameter l_x was defined in Eq. (10). We consider only the dc case, and disregard the frequency argument because $\hbar\omega_q / T$ is quite small and the relevant parameter $\hbar\omega_q / \Gamma^*$ is small if B is not too high. In this approximation Eq. (13) turns out to be

$$m_e \nu_{\mathrm{eff}}^{(\mathrm{r})} = \frac{2 m_B^{1/2}}{(2\pi)^{3/2} \alpha T^{1/2}} \int_0^\infty dq_x q_x e^{-q_x^2 l_x^2/2} \int_0^\infty dq_y \times [V_q^2 / (q_x^2 + q_y^2)] e^{-q_y^2 l_y^2/2}. \tag{16}$$

We point out that, in contrast with the pure 2D case where ν_{eff} and σ_{xx} are proportional to $1/\sqrt{T}$, the channel effective collision frequency [Eq. (16)] behaves differently because l_x^2 contains the temperature-dependent term according to Eq. (10). For a pure 2DES ($l_x = l_y = l_B$), Eq. (16) also results in $\nu_{\mathrm{eff}}^{(\mathrm{r})} \propto 1/\sqrt{T}$. For the electron channel, this behavior is limited to the temperature range where the temperature-dependent term of l_x^2 is small.

In general V_q is a very complicated function of the 2D wave number q, and Eq. (16) should be evaluated numerically for given channel parameters. If the electron channel is formed by applying a strong holding electric field $E_\perp$, i.e., $E_\perp \gg E_q$ we can use the approximation $V_q \simeq e E_\perp$. Then, the effective collision frequency of the electron channel can be found analytically as

$$m_e \nu_{\mathrm{eff}}(B) = \frac{2 m_e \nu^{(0)}(B)}{\pi \sqrt{1 - 4T/\hbar\Omega}} \times \arctan\left[(\hbar\omega_0^2 / 4\Omega T)^{1/2} \sqrt{1 - 4T/\hbar\Omega} \right], \tag{17}$$

where

$$m_e \nu^{(0)}(B) = \left(\frac{m_B}{m_e} \right)^{1/2} \frac{(e E_\perp)^2}{2\alpha l_y^2 \omega_0}. \tag{18}$$

We can see that only the increase of the effective mass described by $m_B = m_e \Omega^2 / \omega_0^2$ cannot explain the whole magnetic-field dependence of the electron mobility. The field dependence of the effective collision frequency is determined by the interplay of the mass enhancement and the field dependencies of $l_x(B)$ and $l_y(B)$. For example, an additional increase of the channel magnetoresistance appears because of the factor $1/l_y^2 \propto \Omega$, which comes from the scattering matrix elements. Recalling the definitions of m_B and l_y^2 given above we can find that the low-temperature limit $\nu^{(0)}(B)$ of the effective collision frequency $\nu_{\mathrm{eff}}(B)$ increases with B as

$$\nu^{(0)}(B) = \left(1 + \frac{\omega_c^2}{\omega_0^2} \right) \frac{(e E_\perp)^2}{2\alpha\hbar}. \tag{19}$$

For zero magnetic field, $\nu^{(0)}(B)$ tends to $\nu_0 = (e E_\perp)^2 / 2\alpha\hbar$.

Under the condition $4T\Omega / \hbar\omega_0^2 \ll 1$, Eq. (17) gives the following asymptote for the channel mobility

$$\mu \simeq \frac{2\alpha\hbar}{m_e e E_\perp^2} \left(\frac{\omega_0}{\Omega} \right)^2 \left[1 + \frac{4}{\pi} \left(\frac{T\Omega}{\hbar\omega_0^2} \right)^{1/2} \right]. \tag{20}$$

For $B = 0$, this equation reproduces the result found previously in Ref. 27. It is remarkable that the zero-temperature term of Eq. (20), as a function of B, agrees with the formal mass replacement $m_e \to m_B = m_e \Omega^2 / \omega_0^2$ in the mobility equation $\mu = 2\alpha\hbar / m_e e E_\perp^2$ obtained for electron in the channel at $B = 0$. We also note that the temperature correction increases with B and affects strongly the magnetoresistivity of the electron channel.

At high magnetic fields the condition $4T\Omega / \hbar\omega_0^2 \ll 1$ breaks down and we have to use the more general expression given by Eq. (17). For different temperatures the magnetic-field dependence of this expression is shown in Fig. 3 using the normalized units $T/\hbar\omega_0$ and ω_c / ω_0. We can see that even at relatively low temperatures ($T/\hbar\omega_0 = 0.1$), $\nu_{\mathrm{eff}}(B)/\nu_0$ deviates strongly from the zero-temperature asymptote. It is remarkable that, for $T/\hbar\omega_0 > 0.84$, electrons on the lowest channel level ($n = 0$) show a negative magnetoresistance in the region of low fields. The origin of this unusual behavior is the nonmonotonous field dependence of $l_x^2(B)$ discussed above and shown in Fig. 2. Still, later we shall show that at such high temperatures the average contribution from all terms with $n, n' \geq 0$ compensates this effect and channel electrons show only positive magnetoresistance.

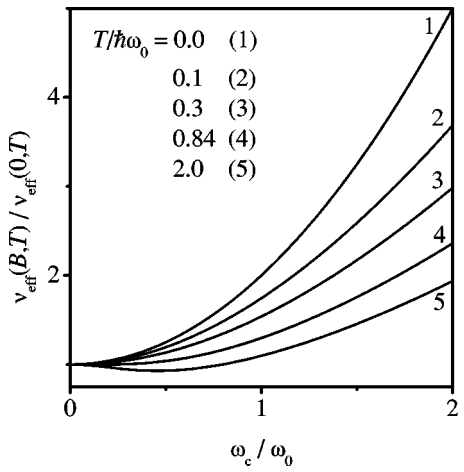

FIG. 3. Magnetic-field dependence of the effective collision frequency of electrons on the lowest channel level due to ripplons given in Eq. (17) for different temperatures.

Figure 4 shows how the magnetic field affects the temperature dependence of the resistivity of the electrons in the channel. For zero magnetic field (line 0), the temperature dependence is relatively weak. The magnetic field makes the temperature dependence sharper in the low-temperature range (lines representing $\omega_c/\omega_0=1$, 2, and 3), which is consistent with the singular nature of the 2DES subject to the normal magnetic field because at high fields ($l_y \to l_B$) the channel becomes effectively a quasi-2DES. Physically, the effective collision frequency increases due to the multiple electron scattering. The number of multiple scattering events is limited by the electron velocity which decreases strongly with B.

At high fields and low temperatures the effective broadening of the channel DSF $\Gamma^* \approx \sqrt{2}eE_{\rm ch}l_B$ decreases as $\Gamma^* \propto \sqrt{T/B}$. In this regime the electron channel becomes similar to the 2DES in which the broadening parameter of the DSF $\Gamma_{n,n'}$ is determined by the interaction with scatterers. In the self-consistent Born approximation $\Gamma_{n,n'}=\sqrt{(\Gamma_n^2+\Gamma_{n'}^2)/2}$,[2] where Γ_n is the collision broadening of the Landau levels.

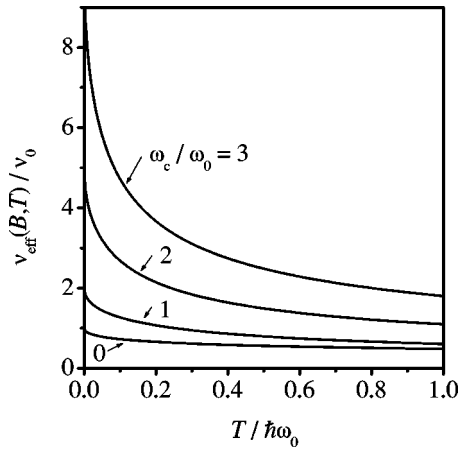

FIG. 4. Temperature dependence of the electron effective collision frequency due to ripplons given in Eq. (17) for different magnetic fields.

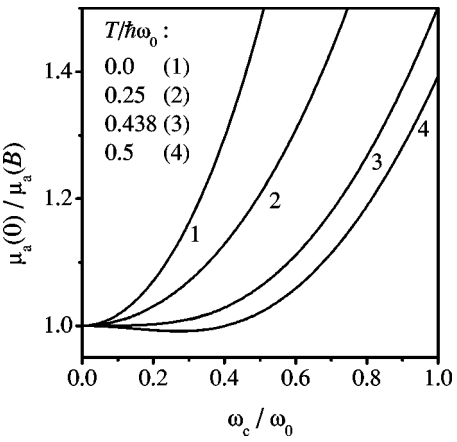

FIG. 5. The inverse mobility of electrons on the lowest channel level ($n=0$) vs magnetic field for different temperatures under the condition that electrons are predominantly scattered by short-range impuritylike scatterers (vapor atoms).

The collision broadening usually increases with B. Therefore, the above results are valid under the condition $\sqrt{2}eE_{\rm ch}l_B > \Gamma_0$. For the electron-ripplon interaction, the collision broadening has the same temperature dependence ($\Gamma_0 \propto \sqrt{T}$), and the breakdown of the channel magnetotransport equations occurs when the magnetic field is increased beyond the value given by the above condition.

For electron scattering by vapor atoms in the ultraquantum conditions, Eq. (14) can be evaluated analytically

$$m_e \nu_{\rm eff}^{(a)}(B) = \frac{3n_a V_a^2 \gamma m_B^{1/2}}{16\pi T^{3/2} l_y l_x^2}. \tag{21}$$

Using the dimensionless units $\Delta_c=\omega_c/\omega_0$ and $\tau=T/\hbar\omega_0$, the corresponding mobility of the electrons on the lowest channel level ($n=0$) can be written as

$$\frac{\mu_a(B)}{\mu_a(0)} = \frac{1+4\tau\Delta_c^2/\sqrt{1+\Delta_c^2}}{(1+\Delta_c^2)^{7/4}}. \tag{22}$$

In this case the negative magnetoresistance originating from the field dependence of $l_x^2(B)$ becomes more prominent for such short-range impuritylike scatterers, as shown in Fig. 5. It should be noted that the negative magnetoresistance of the electrons occupying the lowest level ($n=0$) resembles that observed for quantum localization effects in nondegenerate 2DES[28,29] under the condition $\nu_{ee} \ll \nu$, although in the transport regime $\nu_{ee} \gg \nu$ considered here quantum localization effects can be disregarded. However, under conditions $\tau \equiv T/\hbar\omega_0 > 0.44$ the contributions from other terms of the DSF ($n,n' > 0$) are important and the system behaves like the 2DES exhibiting positive magnetoresistance at $\nu_{ee} \gtrsim \nu$. In this case we have to perform numerical calculations.

Typical field dependencies of the channel mobility evaluated numerically using Eqs. (13) and (14) for different $n_{\rm max}=n'_{\rm max}$ which restrict the sums over n and n' are depicted in Fig. 6. For $\hbar\omega_0/T=0.46$, the electrons predominantly occupy the high-energy levels. We observe clearly the negative magnetoresistance only when $n_{\rm max}=0$ and $n_{\rm max}=1$. The in-

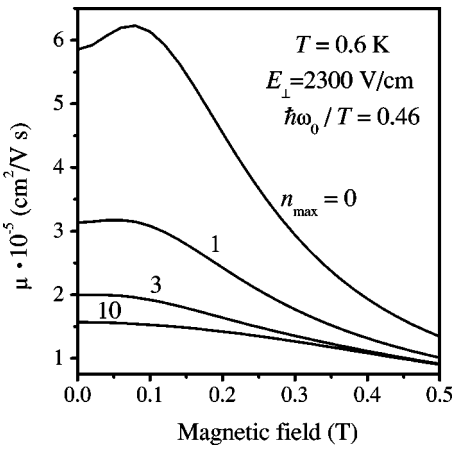

FIG. 6. Mobility of electrons in the channel vs magnetic field under the condition that channel levels with $n>0$ are occupied ($T/\hbar\omega_0 \simeq 2.2$). The succession of solid curves show how the inclusion of higher channel levels with $n,n' \leq n_{\max}$ diminishes the negative magnetoresistance in the region of weak fields.

clusion of high levels suppresses the negative magnetoresistance of the electrons in the channel, and in the limit $n_{\max} \gg 1$ the electron channel shows only a positive magnetoresistance because, as expected, it becomes effectively a quasi-2DES.

For fixed values of the magnetic field, typical temperature dependencies of the channel mobility evaluated numerically including high levels ($n,n' \gg 1$) are shown in Fig. 7. The solid lines of the figure represent the contributions from both scattering mechanisms which interplay at $T \approx 1$ K, while the dotted and dashed lines show the separate contributions from scattering by vapor atoms and ripplons correspondingly. For $T<1$ K electrons are predominantly scattered by ripplons. In this regime at high magnetic fields, there is a range where the electron mobility $\mu \propto \sqrt{T}$. Nevertheless, at lower temperatures μ approaches a finite value which decreases strongly with B.

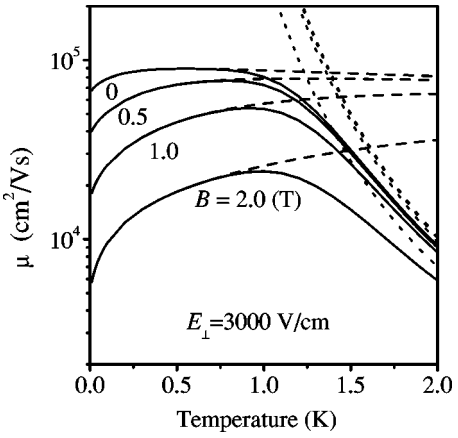

FIG. 7. The mobility of electrons in the channel vs temperature evaluated including all channel levels: solid curves show the effect of both scattering mechanisms, while other curves represent electron scattering by ripplons (dashed curves) and vapor atoms (dotted curves) solely.

It should be noted that in the limit of high B, the inelastic effect of the electron-ripplon interaction cannot be disregarded. In this case the expression in the integrand appearing in Eq. (16) contains an additional exponential factor $\exp[-(\hbar\omega_q/\Gamma^*)^2]$ because of the finite frequency argument of the electron DSF $S_0(\mathbf{q},\omega_q)$. Typical ripplon wave numbers and frequencies involved in scattering events increase strongly with B, while the effective broadening of the DSF of the electron channel $\Gamma^* = 2\sqrt{\varepsilon_{q_x} T}$ decreases (it decreases also by lowering T). Eventually, the energy exchange $\hbar\omega_q$ at a collision becomes comparable with Γ^* and the effective collision frequency decreases with cooling the system.

For the pure 2DES realized on the surface of liquid helium such an inelastic effect was discussed in Ref. 30. It was shown that for $T>0.1$ K, the decrease of $\nu_{\rm eff}^{(r)}$ due to inelastic effects becomes important only if $B>2$ T (actually it was observed at $B=6.4$ T). In this paper we consider lower magnetic fields to keep ω_c comparable with the confinement frequency ω_0. Additionally, Γ^* is assumed to be larger than the collision broadening of the Landau levels. In this range inelastic effects on the channel magnetotransport can be disregarded.

The quantum magnetotransport of a 2D nondegenerate electron liquid in the regime of weak and intermediate magnetic fields is strongly affected by the internal electric field $E_{\mathbf{f}}$ of fluctuational origin.[3] Therefore it is necessary to discuss how this Coulombic effect can affect the results obtained here for channel electrons. The average fluctuational field $E_{\rm f} \approx 3\sqrt{T} n_s^{3/2}$ depends on temperature and the areal electron density n_s. Under the condition $\Gamma_{\rm C} = \sqrt{2} e E_{\rm f}^0 l_B \ll T$ the fluctuational electric field $E_{\rm f}$ can be considered as quasiuniform. Such fields cause fast drift velocities of electron orbit centers which reduce the number of multiple-scattering events in the field. At the same time, in local reference frames moving along with orbit centers the single-electron wave functions remain the same.[6] Obviously, this kind of Coulombic effect is a purely 2D feature and it is suppressed in 1DES's. There are two important points supporting this conclusion.

We note first that the confining electric field of a 1D system (which is assumed to be the strongest) itself plays the same role as the fluctuational electric field in 2DES's. Therefore at high temperatures, when the channel system behaves similar to a 2D nondegenerate electron system, the results obtained here are limited by the condition $E_{\rm f} \ll E_{\rm ch} \simeq \omega_0\sqrt{2 m_e T}/e$. Since the both fields $E_{\rm f}$ and $E_{\rm ch}$ have the same dependence on T (the corresponding broadening parameters of the electron DSF $\Gamma_{\rm C}$ and Γ^* have the same dependence on B), the above given condition restricts only the electron density, which is quite obvious. From the other side our treatment is limited by the condition $\nu_{\rm eff} \ll \nu_{ee}$. The electron autocorrelation frequency ν_{ee} is usually estimated as the characteristic plasma frequency in zero magnetic field $\omega_{\rm p} \sim \sqrt{2\pi e^2/m_e a^3}$, where a is the electron spacing. One can see that even for such low areal electron densities $n_s \sim 10^7$ cm^{-2} such that the Coulombic effect can be disre-

garded in the 2DES, the electron autocorrelation frequency $\nu_{ee} \sim 7 \times 10^9$ sec^{-1} is much higher than the collision frequency due to ripplons.

At low temperatures, when the channel width $\langle y \rangle$ is much shorter than the average distance between electrons $a = 1/n_{\mathrm{ch}}$, the fluctuational electric field is mostly directed along the channel and cannot cause any fast drift velocity of an electron in the perpendicular direction because of the confining potential. In this case the quasiuniform electric field can only affect the electron population of the channel momentum states k_x which obviously can be described by the equilibrium distribution function. Therefore at low temperatures the validity range of the results obtained here is limited by the condition that the electron DSF is only weakly affected by internal forces. We expect that this condition is approximately the same as in 2D:[3] the frequency of electron vibrations ω_{p} is lower than $T/\hbar$. According to estimations given above, for typical densities $n_s \sim 10^7$ cm^{-2} and $T > 0.1$ K this condition is fulfilled.

In conclusion, we have shown that nondegenerate quantum wires may constitute a remarkable laboratory for testing the quantum transport theory. In contrast with classical wires, they exhibit strong magnetoresistance (the channel mobility decreases with the field intensity) which is a result of the interplay between the mass enhancement of channel carriers m_B and the magnetic-field-induced increase of the momenta exchange ($\hbar q_x$ and $\hbar q_y$) in scattering events. For the electron gas with a short autocorrelation time ($\nu_{ee} \gg \nu$), we have obtained the general relation between the channel mobility and the dynamical structure factor of such an anisotropic electron system. Evaluations performed for particular cases indicate that the effect of a normal magnetic field on the channel mobility is very strong for conducting channels formed on the surface of liquid helium under usual experimental conditions.

ACKNOWLEDGMENTS

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