Possible isotope effect on the resonance peak formation in high-T_c cuprates

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Within effective *t-J* Hamiltonian we analyze the influence of electronic correlations and electron-phonon interaction on the dynamical spin susceptibility in layered cuprates. We find an isotope effect on the resonance peak in the magnetic spin susceptibility Im $\chi(\mathbf{q},\omega)$, seen by inelastic neutron scattering. It results from both the electron-phonon coupling and the electronic correlation effects taken into account beyond random-phase approximation scheme. We find at optimal doping the isotope coefficient $\alpha_{res}\approx 0.4$ which can be further tested experimentally.

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An understanding of the elementary and the spin excitations in high-T_c cuprates is of central significance. For example, it is known that the Cooper-pairing scenario via the exchange of antiferromagnetic spin fluctuations was quite successful in explaining the various features of superconductivity in hole-doped cuprates such as $d_{x^2-y^2}$-wave symmetry of the superconducting order parameter and its feedback on the elementary and spin excitatitions.[1] Most importantly, in this scenario the dynamical spin susceptibility $\chi(\mathbf{q},\omega)$ controls mainly the superconducting and normal-state properties of the layered cuprates.[1] One of the key experimental facts in the phenomenology of high-T_c cuprates is the occurrence of a so-called resonance peak in the inelastic neutron-scattering (INS) experiments.[2,3] It occurs below T_c in the dynamical spin susceptibility $\chi(\mathbf{q},\omega)$ at the antiferromagnetic wave vector $\mathbf{Q}=(\pi,\pi)$ and $\omega\approx\omega_{res}$ which is of the order of 40 meV in the optimally doped cuprates. Its feedback in various electronic properties such as optical conductivity, Raman response function, and elementary excitations has been observed experimentally by various techniques.[1] Furthermore, its successful explanation within spin-fluctuation-mediated Cooper-pairing together with $d_{x^2-y^2}$-wave symmetry of the superconducting order parameter favors this scenario as a basic one for superconductivity in the cuprates. On the other hand, recent experiments indicate that also electron-phonon interaction influences strongly their behavior.[4–8] In particular, in the observation of the relatively large isotope effect in various characteristics of cuprates such as penetration depth,[5] ''kink'' structure seen by angle-resolved photoemission spectroscopy[9] still raises a question: what is the role of phonons in determining the superconducting properties of cuprates?

Here, we derive an effective *t-J* Hamiltonian where both the hopping integral *t* and the superexchange interaction between neighboring spins, *J*, are renormalized by phonons. We analyze the influence of the electron-phonon interaction on the dynamical spin susceptibility in layered cuprates. In particular, we find an isotope effect on the resonance peak in the magnetic spin susceptibility Im$\chi(\mathbf{q},\omega)$. It results from both the electron-phonon coupling and the electronic correlation effects taken into account beyond random-phase approximation (RPA) scheme. We show that even if the superconductivity is driven by the magnetic exchange the characteristic energy features of cuprates can be significantly renormalized by the strong electron-phonon interaction.

Effective Hamiltonian We start from the atomic limit of the three-band *p-d* Hamiltonian

$$H_0=\sum\ \epsilon_d d^\dagger_{i\sigma}d_{i\sigma}+\sum\ \epsilon_p p^\dagger_{i\sigma}p_{i\sigma}+\sum\ U_d n_{d\uparrow}n_{d\downarrow}$$

$$+\sum\ U_p n_{p\uparrow}n_{p\downarrow}+\sum\ \hbar\omega_{\mathbf{q}} f^\dagger_{\mathbf{q}}f_{\mathbf{q}}, \qquad (1)$$

where ϵ_d and ϵ_p are the on-site energies of the copper and the oxygen holes, $n_{d\sigma}=d^\dagger_{i\sigma}d_{i\sigma}$ and $n_{p\sigma}=p^\dagger_{i\sigma}p_{i\sigma}$ are the copper $3d$ and oxygen $2p$ hole densities for site *i*, respectively. U_d and U_p refer to the on-site copper and oxygen Coulomb repulsion, respectively. $f^\dagger_{\mathbf{q}}$ denotes the phonon creation operator and $\hbar\omega_{\mathbf{q}}$ is a phonon energy dispersion. We consider the hopping term between copper and oxygen,

$$H_2=\sum_\sigma\ t_{pd}(d^\dagger_{a\sigma}p_{b\sigma}+\text{H.c.}) \qquad (2)$$

and the electron-phonon interaction

$$H_1=\sum_{l=d,p}\ g_l n_l(f^\dagger_{-\mathbf{q}}+f_{\mathbf{q}}) \qquad (3)$$

as a perturbation. Here, t_{pd} is a hopping term between copper and oxygen, and g_l is an electron-phonon coupling strength at the site *l*. This notation is similar to the simplified Holstein model where the migrating charge interacts locally with breathing phonon modes forming electron-vibrational states.

To derive an effective *t-J* Hamiltonian we employ the canonical Schrieffer-Wolf-like transformations $e^{-S}He^{S}$.[10,11] The matrix of the unitary transformation for the initial Hamiltonian is found by excluding the odd terms with respect to the hopping integral with an accuracy up to the sixth-order perturbation theory. Then the *S*-operator consists of the sum of five terms. Each of these is determined by the following iteration procedure:

$$[H_0S_1]=-H_2,\ \ [H_0S_2]=-[H_1S_1], \qquad (4)$$

$$[H_0S_3]=-[H_1S_2]-\tfrac{1}{3}[[H_2S_1]S_1],$$

$$[H_0S_4]=-[H_1S_3]-\tfrac{1}{3}[[H_2S_1]S_2]-\tfrac{1}{3}[[H_2S_2]S_1],$$

$$[H_0S_5]=-[H_1S_4]-\tfrac{1}{3}[[H_2S_1]S_3]-\tfrac{1}{3}[[H_2S_3]S_1]-\tfrac{1}{3}[[H_2S_2]S_2]+\tfrac{1}{45}[[[[H_2S_1]S_1]S_1]S_1].$$

The calculations are straightforward and their details will be given elsewhere. Note, in the second-order perturbation the effective hopping integral t_{ij} appears. It is further renormalized by the electron-phonon interaction in the fourth-order term where we introduce the average over the phonons. Similarly, the superexchange interaction occurs in the fourth-order perturbation theory and its renormalization takes place in the sixth-order term. Finally, the relevant effective Hamiltonian is given by

$$H=\sum_{ij} t_{ij}\Psi_i^{pd,\sigma}\Psi_j^{\sigma,pd}+\sum_{i>j} J_{ij}\left[(\mathbf{S}_i\cdot\mathbf{S}_j)-\frac{n_in_j}{4}\right]. \quad (5)$$

Note, in general case the effective Hamiltonian contains also the Coulomb interaction between doped holes and the interaction of quasiparticles via the phonon field. We dropped these terms here, because they do not contribute directly to the spin susceptibility. In Eq. (5) we use the projecting Hubbard-like operators $\Psi_i^{\alpha,\beta}=|i,\alpha\rangle\langle i,\beta|$ in order to satisfy no double occupancy constraint. The index pd corresponds to a Zhang-Rice singlet formation with one hole placed on the copper site whereas the second hole is distributed on the neighboring oxygen sites.[12] Note, $t_{ij}=t_{ij}^0e^{-\gamma E_i^*/\hbar\omega_i^*}$, where t_{ij}^0 is the effective hopping integral without taking into account electron-phonon interaction, $E_i=(g_i^*)^2/\hbar\omega_i^*$ is the so-called polaron stabilization energy of the copper-oxygen singlet state, and $0<\gamma<1$. Note, from the experimental data[13] the whole exponential factor was estimated to be $\gamma E_i^*/\hbar\omega_i^*\approx 0.92$ around the optimal doping and its value is increasing upon decreasing doping. We further assume that the lifetime of the Zhang-Rice singlet is much larger than the relaxation time of the local deformations.

Similarly the superexchange interaction between nearest copper spins is given by

$$J=J_0\left\{1+\frac{3\hbar}{\Delta_{pd}^2}\left[E_p\omega_p\coth\left(\frac{\hbar\omega_p}{2k_BT}\right)+E_d\omega_d\coth\left(\frac{\hbar\omega_d}{2k_BT}\right)\right]\right\}, \quad (6)$$

where $\Delta_{pd}=\epsilon_p-\epsilon_d+U_p-U_d$ is the energy transfer from copper to oxygen and is known to be of the order of 1.5 eV in the cuprates. Here, J_0 is the superexchange interaction of copper spins via the intermediate oxygen atom in the absence of phonons. We took the phonon frequency of the order of $\omega^*\approx\omega_p\approx\omega_d=50$ meV which roughly corresponds to the energy of the longitudinal optical (LO) bond stretching phonon mode in cuprates. According to the recent experiments[6,7] it may play an essential role in the physics of cuprates. The so-called polaronic stabilization energy $E^*\approx E_p\approx E_d$ was estimated to be of the order of 0.5 eV in accordance with the measurements of the isotope effect in cuprates.[14]

Dynamical spin susceptibility. To derive the dynamical spin susceptibility in the superconducting state we use the method suggested by Hubbard and Jain[15] that allows one to take into account strong electronic correlations. First we add the external magnetic field applied along c axis into the effective Hamiltonian

$$H_i=\mathrm{Re}\sum_{\mathbf{q}} h_{-\mathbf{q}}e^{i(\omega t-\mathbf{q}\cdot\mathbf{R}_i)}+h_{\mathbf{q}}e^{-i(\omega t-\mathbf{q}\cdot\mathbf{R}_i)}. \quad (7)$$

Then we write an equation of motion for the Ψ operators using the Roth-type of the decoupling scheme[16] and expanding

$$P_{pd}^{\sigma}=\{\Psi_i^{\sigma,pd}\Psi_i^{pd,\sigma}\}=[(1+\delta_i)/2]+\sigma\mathrm{Re}\Sigma_{\mathbf{q}}[S_{-\mathbf{q}}^z e^{-i(\mathbf{q}\cdot\mathbf{R}_j-\omega t)}+S_{\mathbf{q}}^z e^{i(\mathbf{q}\cdot\mathbf{R}_j-\omega t)}]$$

up to the first order in $S_{\mathbf{q}}=\chi^{zz}(\mathbf{q},\omega)h_{\mathbf{q}}$. In particular,

$$i\hbar\frac{\partial\Psi_{\mathbf{k}}^{-\sigma,pd}}{\partial t}=(\epsilon_{\mathbf{k}}-\mu)\Psi_{\mathbf{k}}^{-\sigma,pd}+\Delta_{\mathbf{k}}\Psi_{-\mathbf{k}}^{pd,\sigma}+\left[\left(\frac{J_{\mathbf{q}}}{2}-t_{\mathbf{k}-\mathbf{q}}\right)S_{\mathbf{q}}-\frac{h_{\mathbf{q}}}{2}\right]\Psi_{\mathbf{k}-\mathbf{q}}^{-\sigma,pd}e^{-i\omega t}+\left[\left(\frac{J_{-\mathbf{q}}}{2}-t_{\mathbf{k}+\mathbf{q}}\right)S_{-\mathbf{q}}-\frac{h_{-\mathbf{q}}}{2}\right]\Psi_{\mathbf{k}+\mathbf{q}}^{-\sigma,pd}e^{i\omega t}, \quad (8)$$

and the similar expression occurs for $\Psi_{-\mathbf{k}}^{pd,\sigma}$. Here, $\Delta_{\mathbf{k}}=(\Delta_0/2)(\cos k_x-\cos k_y)$ is $d_{x^2-y^2}$-wave superconducting gap, and $J_{\mathbf{q}}=J(\cos k_x+\cos k_y)$ is the Fourier transform of the superexchange interaction on a square lattice.

The expression for the longitudinal component of the dynamical spin susceptibility can be obtained from the relation

$$\langle\Psi_i^{pd,\uparrow}\Psi_i^{\uparrow,pd}\rangle-\langle\Psi_i^{pd,\downarrow}\Psi_i^{\downarrow,pd}\rangle=0, \quad (9)$$

and using the Bogolyubov-like transformations to the new quasiparticle states

$$X_{\mathbf{k}}^{-\tilde{\sigma},pd}=u_{\mathbf{k}}\Psi_{\mathbf{k}}^{-\sigma,pd}+v_{\mathbf{k}}\Psi_{-\mathbf{k}}^{pd,\sigma},$$

$$X_{-\mathbf{k}}^{pd,\tilde{\sigma}}=u_{\mathbf{k}}\Psi_{\mathbf{k}}^{pd,\sigma}-v_{\mathbf{k}}\Psi_{-\mathbf{k}}^{-\sigma,pd}. \quad (10)$$

Here, $u_{\mathbf{k}}^2=\frac{1}{2}[1+\epsilon_{\mathbf{k}}-\mu/E_{\mathbf{K}}]$ and $v_{\mathbf{k}}^2=\frac{1}{2}[1-(\epsilon_{\mathbf{k}}-\mu)/E_{\mathbf{K}}]$ are the Bogolyubov coefficients, μ is a chemical potential, and $E_{\mathbf{k}}=\sqrt{(\epsilon_{\mathbf{k}}-\mu)^2+\Delta_{\mathbf{k}}^2}$ is the energy dispersion in the superconducting state. Substituting Eq. (10) in Eq. (9) and using the equations of motion (8) one obtains the expression for the dynamical spin susceptibility in the form

$$\chi(\mathbf{q},\omega)=\frac{\chi_0(\mathbf{q},\omega)}{J_{\mathbf{q}}\chi_0(\mathbf{q},\omega)+\Pi(\mathbf{q},\omega)+Z(\mathbf{q},\omega)}. \quad (11)$$

This is the central result of our paper. Here, $\chi_0(\mathbf{q},\omega)$ is the usual BCS-like Lindhard response function, and $\Pi(\mathbf{q},\omega)$ and $Z(\mathbf{q},\omega)$ result from the strong electronic correlation effects. In the normal state the expression for $\Pi(\mathbf{q},\omega)$ has been obtained by Hubbard and Jain.[15] In the superconducting state it is given by

$$\Pi(\mathbf{q},\omega)=\frac{P_{pd}}{N}\sum_{\mathbf{k}} u_{\mathbf{k}}u_{\mathbf{k+q}}(u_{\mathbf{k}}u_{\mathbf{k+q}}+v_{\mathbf{k}}v_{\mathbf{k+q}})$$
$$\times\frac{t_{\mathbf{k}}f_{\mathbf{k}}-t_{\mathbf{k+q}}f_{\mathbf{k+q}}}{\omega+i0^{+}+E_{\mathbf{k}}-E_{\mathbf{k+q}}}+v_{\mathbf{k}}v_{\mathbf{k+q}}(v_{\mathbf{k}}v_{\mathbf{k+q}}$$
$$+u_{\mathbf{k}}u_{\mathbf{k+q}})\frac{t_{\mathbf{k}}(1-f_{\mathbf{k}})-t_{\mathbf{k+q}}(1-f_{\mathbf{k+q}})}{\omega+i0^{+}-E_{\mathbf{k}}+E_{\mathbf{k+q}}}$$
$$+u_{\mathbf{k}}v_{\mathbf{k+q}}(u_{\mathbf{k}}v_{\mathbf{k+q}}-u_{\mathbf{k+q}}v_{\mathbf{k}})$$
$$\times\frac{t_{\mathbf{k}}f_{\mathbf{k}}-t_{\mathbf{k+q}}(1-f_{\mathbf{k+q}})}{\omega+i0^{+}+E_{\mathbf{k}}+E_{\mathbf{k+q}}}+u_{\mathbf{k+q}}v_{\mathbf{k}}(v_{\mathbf{k}}u_{\mathbf{k+q}}$$
$$-u_{\mathbf{k}}v_{\mathbf{k+q}})\frac{t_{\mathbf{k}}(1-f_{\mathbf{k}})-t_{\mathbf{k+q}}f_{\mathbf{k+q}}}{\omega+i0^{+}-E_{\mathbf{k}}-E_{\mathbf{k+q}}}. \quad (12)$$

The function $Z(\mathbf{q},\omega)$ is written as follows:

$$Z(\mathbf{q},\omega)=\frac{1}{N}\sum_{\mathbf{k}}\frac{\omega+i0^{+}}{\omega+i0^{+}+\epsilon^{(1)}_{\mathbf{k}}-\epsilon^{(1)}_{\mathbf{k+q}}}. \quad (13)$$

Here, $f_{\mathbf{k}}$ is the Fermi distribution function, $\epsilon^{(1)}_{\mathbf{k}}=(1-\delta)/2t_{\mathbf{k}}$, $\epsilon_{\mathbf{k}}=P_{pd}t_{\mathbf{k}}$ is the energy dispersion in the normal state, and $t_{\mathbf{k}}=2t(\cos k_x+\cos k_y)+4t'\cos k_x\cos k_y+2t''(\cos 2k_x+\cos 2k_y)$ is the Fourier transform of the hopping integral on a square lattice including nearest-, next-nearest, and next-next-nearest-neighbor hopping, respectively. The origin of the terms $\Pi(\mathbf{q},\omega)$ and $Z(\mathbf{q},\omega)$ relates to the no double occupancy constraint. In particular, for the Coulomb repulsion $U=\infty$ and $J=0$ the dynamical spin susceptibility does not reduce to the standard Lindhard response function but is renormalized by the electronic correlation effects.[17] For $\Delta_{\mathbf{k}}=0$, Eq. (11) agrees with the normal-state result for the dynamical spin susceptibility.[15,18,19]

Results and Discussion. INS measurements probe directly the imaginary part of the dynamical spin susceptibility. Therefore, it is of interest to analyze the role played by the electronic correlations on the ''resonance'' peak formation seen by INS.[3] Using various approaches this feature was well understood mainly as a result of the spin-density-wave (SDW) collective mode formation at $\omega=\omega_{res}$, i.e., when the denominator of the spin susceptibility at the antiferromagnetic wave vector $\mathbf{Q}$ is close to zero.[20,21]

In Fig. 1 we show the results of our calculations for the $\mathrm{Im}\chi(\mathbf{Q},\omega)$ as a function of frequency in the normal and the superconducting state. Here, we use $t=200$ meV, $t'=-0.1t$, and $t''=0.02t$ at optimal doping. Clearly in the normal state the spin-fluctuation spectrum is characterized by a broad feature which starts around 10 meV and extends up to higher frequencies. In the superconducting state it strongly renormalizes due to presence of the $d_{x^2-y^2}$-wave gap $[\Delta_{\mathbf{k}}=(\Delta_0/2)(\cos k_x-\cos k_y)]$ that leads to a resonance peak formation similar to the RPA result.[20,21] However, due to a strong frequency dependence of $Z(\mathbf{Q},\omega)$ and $\Pi(\mathbf{Q},\omega)$ the spectral weight of the resonance peak is redistributed away from (π,π) (see the dashed curve for comparison) leading to a well-pronounced dispersion of the latter. This is illustrated

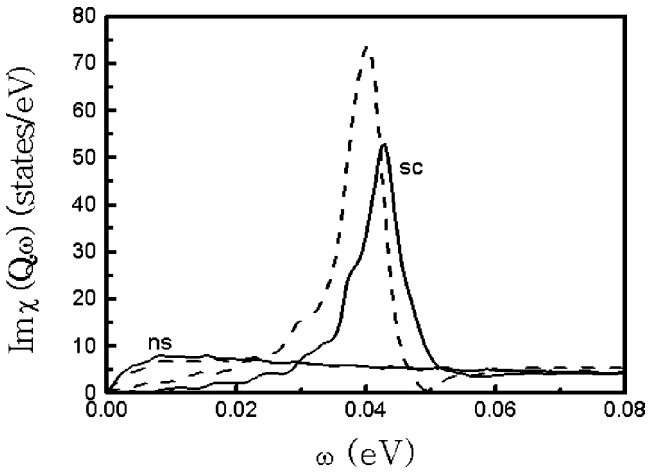

FIG. 1. Calculated imaginary part of the dynamical spin susceptibility $\mathrm{Im}\chi(\mathbf{Q},\omega)$ using Eq. (11) and $J=0.3t$ in the normal and superconducting state at optimal doping. Here, we use the superconducting gap $\Delta_0=0.14t$ (28 meV) and $T_c\approx 0.04t$ (90 K) for optimal doping from our earlier calculations of the mean-field phase diagram (Ref. 22). To illustrate the role of $Z(\mathbf{q},\omega)$ we also show the results for $Z(\mathbf{q},\omega)=0$ (dashed curve). Note, the damping was chosen $\Gamma=1.5$ meV.

in Fig. 2(a) where we show the calculated frequency and momentum dependence of Im $\chi(\mathbf{q},\omega)$ away from the antiferromagnetic wave vector $\mathbf{Q}=(\pi,\pi)$. The dispersion of the resonance excitations is clearly visible and is shown in Fig. 2(b) as a function of $q_x(q_y=\pi)$. As one sees there are well-pronounced dispersion curves $\propto\mathbf{q}^2$ in good agreement with experiment.[23,24] Note that in the RPA the dispersion of the resonance is much weaker (in particular the upper branch of the dispersion) due to a δ-function character of the resonance condition[21] while Z and Π terms, in particular, lead to a redistribution of the spectral weight away from $\mathbf{Q}=(\pi,\pi)$. Note, due to the tetragonal symmetry the same dispersion takes place for $q_y(q_x=\pi)$.

The position of the resonance peak is determined mainly by the magnitude of the $d_{x^2-y^2}$-wave superconducting gap Δ_0, the superexchange coupling constant, J, and by the proximity of the Fermi energy to the extended saddle point (the so-called Van Hove singularity) in the density of states determined by the ratio of t'/t. In order to illustrate this dependence we show in Fig. 3 the resonance peak position as a function of J/t and Δ_0/t at optimal doping. One can clearly see that it depends almost linearly on $2\Delta_0$. This follows from our Eq. (11). In particular, the superconducting gap determines mainly the position of the continuum of the spin excitations from $\mathrm{Im}\chi_0$ which is around $2\Delta_0$.[20] Thus, at the fixed value of J/t and t'/t this continuum influences also the resonant condition because the difference between the position of the resonance peak (SDW collective mode) and the continuum of the states has to remain the same.[20] On the other hand, for the fixed values of t'/t and Δ_0/t the resonance condition depends mainly on the value of J. Furthermore, its dependence is more complicated than in the case of Δ_0. At the relatively small values of the superexchange coupling the resonance lies close to the continuum and is more

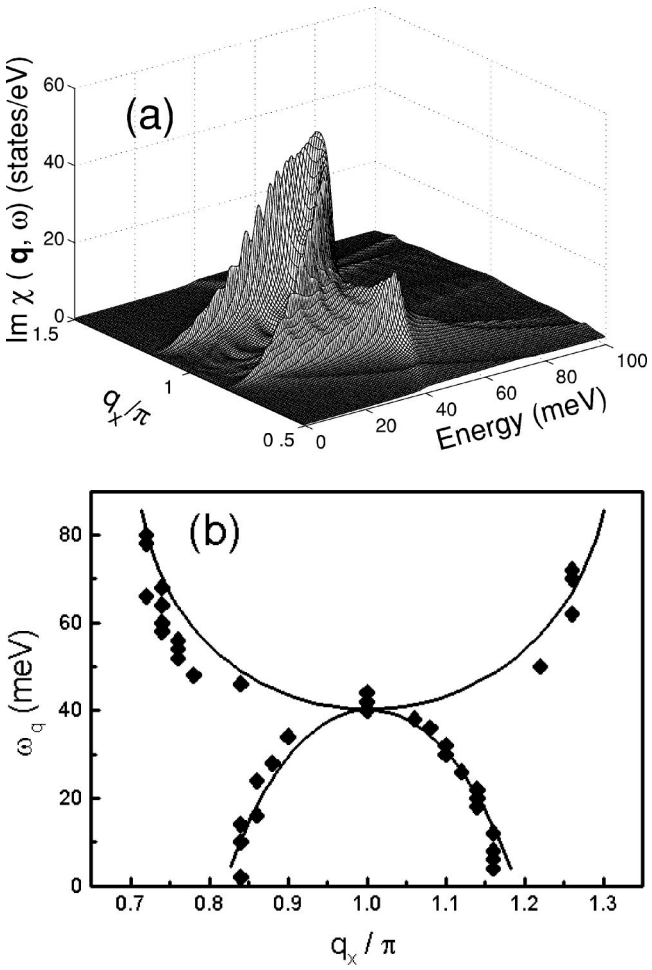

FIG. 2. (a) Calculated frequency and momentum dependence of $\mathrm{Im}\chi(\mathbf{q},\omega)$ away from $\mathbf{Q}=(\pi,\pi)$. (b) The dispersion of the resonance peak. Two branches of the dispersion curves are in good agreement with recent experimental data (Refs. 23 and 24). Note, the solid curve is a guide to the eye.

sensitive to the change of the superconducting gap magnitude than to the superexchange coupling constant. On the contrary, one could see from Fig. 3 that if the resonance peak is at small energies and lies relatively far from the continuum (which happens for large values of J/t) it will be most influenced by the change of J/t values. We note that the shift of the resonance peak to the lower frequency enhances its intensity and vice versa. In addition, the ratio of t'/t which determines the nesting of the Fermi surface and the position of Van Hove singularity influences also the resonance peak. Its influence is somewhat similar to J with one important difference. In particular, an increase of t'/t weakens the intensity of the resonance peak rather than changing the position itself (not shown). Note that our analysis agrees qualitatively with the previous ones.[20] Most importantly we find that by the slight variation of all parameters one could find another realistic set of Δ_0/t, J/t, and t'/t which would also correctly explain the position of the resonance peak at optimal doping.[25] This means that at present stage we could make only a qualitative analysis of the experimental data. Further, factors such as orthorhombic distortions may influ-

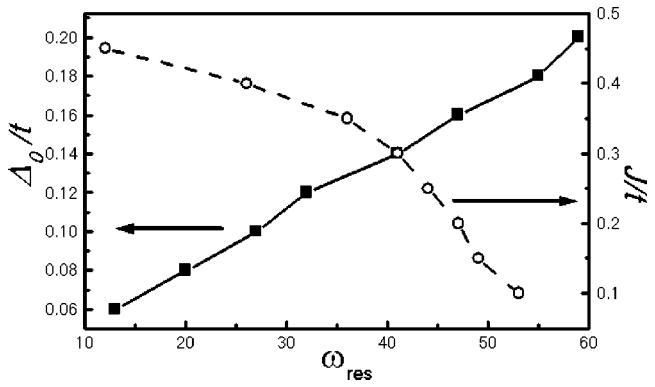

FIG. 3. Calculated dependence of the resonance frequency ω_{res} on the superexchange coupling constant J (open circles) and on the value of the $d_{x^2-y^2}$-wave superconducting gap Δ_0 (filled squares). Note, the lines are the guides to the eye.

ence the difference between the resonance peak intensity in $Bi_2Sr_2CaCu_3O_{8+x}$ (BSCCO) and $YBa_2Cu_3O_7$ (YBCO).

Finally, we discuss the influence of the electron-phonon interaction on the resonance peak formation by changing the isotope mass of ^{16}O by ^{18}O. This shifts the average frequency of the LO phonon mode and consequently renormalizes the hopping integral t and the superexchange coupling constant J. Most importantly, the electron-phonon interaction changes most dramatically the hopping integral t rather than the superexchange coupling J. In particular, as can be seen from Eq. (6) the superexchange coupling constant J changes less than 1% upon substituting the isotopes[26] which agrees well with experimental data.[27] Therefore, there is almost no influence of the isotope substitution on the resonance peak determined from RPA, since in this approximation its formation is determined mainly by J. In particular, we find *within RPA* no change in the ω_{res} value upon changing the isotopes. In the case of Eq. (11) the most important contribution to the isotope effect on the resonance peak appears due to $\Pi(\mathbf{q},\omega)\propto t_k$. In particular, using our estimation given above we find that at optimal doping the hopping integral changes by 6% upon replacing ^{16}O by ^{18}O. This results in the lowering of the resonance frequency at (π,π) from 41 meV for the ^{16}O isotope towards 39 meV for the ^{18}O sample. This leads to $\alpha_{res}=-d\ln\omega_{res}/d\ln M\approx 0.4$ for optimally doped cuprates. This effect is beyond the experimental error and can be further tested experimentally. Furthermore, in the underdoped cuprates one may expect larger isotope effect due to a larger value of $\gamma E_i^*/\hbar\omega_i^*$.[13] At the same time the superconducting transition temperature which is determined by J shows much weaker isotope effect and is around $\alpha_{T_c}\approx 0.05$.[26] Therefore, even if the superconductivity is driven by the magnetic exchange the resonance peak formation can be significantly renormalized by the strong electron-phonon interaction.

To summarize, we analyze the influence of the electronic correlations and the electron-phonon interaction on the dynamical spin susceptibility in layered cuprates. The electronic correlations taken beyond RPA redistribute the spectral weight of the resonance peak away from (π,π) leading to

the pronounced dispersion. This is in good agreement with recent INS data.[23,24] Furthermore, we find the isotope effect on the resonance peak due to strong coupling of the carriers to LO phonon mode. It results from both electron-phonon coupling and electronic correlation effects. In contrast to the small isotope effect on the superconducting transition temperature we find larger isotope coefficient on the resonance peak $\alpha_{res} \approx 0.4$ in optimally doped cuprates. We also would like to note that the value of the isotope coefficient depends strongly on the value of the exponential factor. Therefore, the experimental verification of our prediction is desirable. In particular, it would put a strong constraint on the ingredients the theory of cuprates must contain.

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