Quasiclassical approach to vortex-induced suppression of the superconducting electron density in *d*-wave superconductors

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Quasiclassical Eilenberger equations are solved numerically for the case of an isolated two-dimensional vortex in a *d*-wave superconductor. The asymptotical behavior of the amplitude and the phase of the pairing potential, superconducting current, and superconducting electron density at long distances from the vortex core are obtained. The local Doppler-shift (DS) method is found to work reasonably well for description of the superconducting electron density at distances longer than the coherence length at low temperatures. Nonlocal effects are important inside the vortex core and for a description of the effects of the fourfold vortex symmetry outside the core. It is also shown that at higher temperatures the DS method should be modified by including a pairing potential calculated self-consistently.

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I. INTRODUCTION

Finding a strong linear term in the temperature dependence of the magnetic penetration depth λ and the superconducting electron density n_s was a key development in identifying the *d*-wave symmetry of the order parameter of high-T_c superconductors.[1] The decrease of $n_s(T)$ with increasing temperature, caused by thermal excitations near the nodes of the *d*-wave order parameter, is insensitive to the direction of the superflow. Yip and Sauls[2] proposed that the field dependence of λ in the Meissner state provides a sensitive test of the order parameter symmetry. They argued that a pure *d*-wave superconductor should, at a sufficiently low temperature, also exhibit a linear magnetic field dependence of n_s, the magnitude of which depends on the relative orientation of the applied field and the nodes of the order parameter. This effect arises from Doppler energy shift (DS) of the quasiparticle states by the supercurrent. The *nonlinear* Meissner effect has been observed,[3] but with a magnitude and a temperature dependence of the field-dependent correction to λ which differ considerably from the predictions of the Yip-Sauls theory.[2]

The absence of the Yip-Sauls effect[2] could possibly be explained by the *nonlocality* of the supercurrent response in the vicinity of the gap nodes. In clean d_{x2-y2} superconductors the coherence length ξ_0, which is inversely proportional to the gap size, diverges at the nodes. Thus, near the nodes of the Fermi surface we have $\xi_0 > \lambda$ and the supercurrent at any point is obtained by averaging the field over a region of radius ξ_0. For a nonuniform field the nonlocal effects weaken the supercurrent response. Indeed, a calculation of $\delta\lambda(H)$, including both nonlocal and nonlinear effects, shows that the nonlocal effects drastically reduce $\delta\lambda(H)$ below the first critical field H_{c1}, rendering it practically unobservable in the Meissner state.[4–6]

As found in μ-SR investigations of the flux line lattice in the mixed state of YBaCuO single crystals[7–9] the currents induced by the vortices in a *d*-wave superconductor can suppress n_s far from the vortex core. Magnetic measurements on YBaCuO nanoparticles with trapped vortices also show modification of the $n_s(T)$ dependence at presence of the vortices.[10,11] Both in the mixed and Meissner states the main contributions to the suppression of n_s come from the nonlocal and nonlinear effects. The influence of the nonlocal term has been investigated,[12,13] obtaining the effective $n_s(B,T)$ dependence. But similar dependences were found,[14,15] where only the nonlinear effect in the DS approximation was taken into account.

A modification of the nonlocal theory taking into account the nonlinear effects has been done by a perturbative calculation,[12] predicting that in small fields both effects are of the same order but in fields >2 T the nonlocal effects are prevailing. It was also pointed out that such calculations are rather sensitive to the *form of the vortex core*, which can be described in the London theory only phenomenologically. Even for isotropic *s*-wave superconductors some conjectures have been made about the field dependence of the vortex core in the modified London equation.[12,16] For *d*-wave superconductors the vortex core is highly anisotropic due to core states extended in the gap node directions.[17–19]

In this paper we solve the quasiclassical Eilenberger equations numerically for the case of an isolated two-dimensional vortex in a *d*-wave superconductor. This theory includes the nonlocal, nonlinear, and core effects simultaneously. The Eilenberger equations can be obtained from a full quantum mechanical approach (the Bogoliubov–de Gennes equations) using an expansion in terms of α^{-1}, where $\alpha = v_F / v_\Delta$ is the Dirac cone anisotropy, v_F is the Fermi velocity, and v_Δ is the quasiparticle velocity tangential to the Fermi surface at the node. This expansion is quite reasonable for the description of high-T_c superconductors, where $\alpha = 14$ for YBaCuO and 20 for BiSrCaCuO.[20] The Eilenberger equations have been solved previously[17,21] in the vortex core region. Here we find the behavior of the amplitude and the phase of the pairing potential $\Delta(r)$ as well as the superconducting current $j(r)$ and $n_s(r)$ at long distances r from the vortex core. We also compare the exact solution for $n_s(r)$ with the local DS approximation. It is found that the DS method works reasonably well at distances $r \geqslant \xi_0$ at low temperatures. The nonlocal effects are important inside the core and for the description of effects of the fourfold vortex symmetry outside the core. It is also shown that at higher temperatures the DS method should be modified by including a pairing potential $\Delta(r)$ calculated self-consistently.

II. QUASICLASSICAL APPROACH

We consider an isolated two-dimensional vortex in a d-wave superconductor. The center of the vortex is taken as the origin. The Fermi surface is assumed to be isotropic and cylindrical. To obtain the quasiclassical Green functions we solve the quasiclassical Eilenberger equations for the pairing potential $\Delta(\theta,r)=\overline{\Delta}(r)\cos(2\theta)\exp(i\phi)$,[17,21] where θ is the angle between the $\mathbf{k}$ vector and the a axis (or x axis) and $\exp(i\phi)=(x+iy)/r$. It should be noted here that the spatial variation of the supercurrent and the d-wave order parameter induce small subdominant s and d_{xy} components in the pairing order parameter.[22] We are not considering these effects because they can be included in a straightforward way in our calculations. Throughout this paper, the energies and lengths are measured in units of the uniform gap Δ_0 at $T=0$ and the coherence length $\xi_0=v_F/\Delta_0$, respectively.

For numerical calculation it is convenient to parametrize the quasiclassical Green function via[21]

$$\overline{f}=\frac{2\overline{a}}{1+\overline{a}\overline{b}},\quad \overline{f}^{\dagger}=\frac{2\overline{b}}{1+\overline{a}\overline{b}},\quad g=\frac{1-\overline{a}\overline{b}}{1+\overline{a}\overline{b}},\tag{1}$$

where the anomalous Green functions $\overline{f}$ and $\overline{f}^{\dagger}$ are related to the usual notations as $f=\overline{f}\exp(i\phi)$ and $f^{\dagger}=\overline{f}^{\dagger}\exp(-i\phi)$. The functions $\overline{a}$ and $\overline{b}$ satisfy the independent nonlinear Ricatti equations

$$\partial_{\parallel}\overline{a}(\omega_n,\theta,\mathbf{r})=\overline{\Delta}(\theta,\mathbf{r})-\{2\omega_n+i\partial_{\parallel}\phi+\overline{\Delta}^{*}(\theta,\mathbf{r})\overline{a}(\omega_n,\theta,\mathbf{r})\}\overline{a}(\omega_n,\theta,\mathbf{r}),\tag{2}$$

$$\partial_{\parallel}\overline{b}(\omega_n,\theta,\mathbf{r})=-\overline{\Delta}(\theta,\mathbf{r})+\{2\omega_n+i\partial_{\parallel}\phi+\overline{\Delta}(\theta,\mathbf{r})\overline{b}(\omega_n,\theta,\mathbf{r})\}\overline{b}(\omega_n,\theta,\mathbf{r}),\tag{3}$$

where $\omega_n=(2n+1)\pi T$ is the fermionic Matsubara frequency, $\partial_{\parallel}=d/dr_{\parallel}$ and $\partial_{\parallel}\phi=-r_{\perp}/r^2$. Here we use the coordinate system $\hat{\mathbf{u}}=\cos\theta\hat{\mathbf{x}}+\sin\theta\hat{\mathbf{y}}$, $\hat{\mathbf{v}}=-\sin\theta\hat{\mathbf{x}}+\cos\theta\hat{\mathbf{y}}$. Thus a point $\mathbf{r}=x\hat{\mathbf{x}}+y\hat{\mathbf{y}}$ is denoted as $\mathbf{r}=r_{\parallel}\hat{\mathbf{u}}+r_{\perp}\hat{\mathbf{v}}$. Equations (2) and (3) include both nonlocal effects ($\partial_{\parallel}\overline{a}$ and $\partial_{\parallel}\overline{b}$ terms) and nonlinear effects ($\overline{a}$ and $\overline{b}$ are nonlinear functions of $\partial_{\parallel}\phi$). Since we consider an isolated vortex in extreme type-II superconductors the vector potential in Eqs. (2) and (3) can be neglected.

We solve Eqs. (2) and (3) along a trajectory where $r_{\perp}$ is constant. The initial values for $\overline{a}$ and $\overline{b}$ in the bulk superconductor have to be taken as

$$\overline{a}(-\infty)=\frac{\overline{\Delta}(-\infty)}{\omega_n+(\omega_n^2+|\overline{\Delta}(-\infty)|^2)^{1/2}},\tag{4}$$

$$\overline{b}(+\infty)=\frac{\overline{\Delta}^{*}(+\infty)}{\omega_n+(\omega_n^2+|\overline{\Delta}(+\infty)|^2)^{1/2}}.\tag{5}$$

The self-consistent condition for the pairing potential $\overline{\Delta}(\theta,\mathbf{r})=\overline{\Delta}(\mathbf{r})\cos(2\theta)$ is given by

$$\overline{\Delta}(\mathbf{r})=VN_0 2\pi T\sum_{\omega_n>0}\int_0^{2\pi}\frac{d\theta}{2\pi}\overline{f}(\omega_n,\theta,\mathbf{r})\cos(2\theta),\tag{6}$$

where V is the pairing interaction energy and N_0 is the density of states at the Fermi surface. The product VN_0 can be obtained from the expression

$$\frac{2}{VN_0}=\ln\frac{T}{T_c}+2\pi T\sum_{0<\omega_n<\omega_c}\frac{1}{|\omega_n|}.\tag{7}$$

The energy cutoff in this equation can be taken as $\omega_c=20T_c$.[17]

To solve Eqs. (2) and (3) we take the initial distribution $\overline{\Delta}_{in}(\mathbf{r})=\overline{\Delta}(T)\tanh r$, where $\overline{\Delta}(T)$ is the temperature-dependent uniform gap without the vortex, obtained from Eqs. (4)–(6). After that the right-hand side of Eq. (6) is calculated and a new distribution for $\overline{\Delta}(\mathbf{r})$ is obtained. Using this pair potential the Eilenberger equations (2) and (3) are solved again. At low temperatures we repeat this iteration procedure 20 times and obtain a self-consistent solution for $\overline{\Delta}(\mathbf{r})$. At $0.9T_c$ and $0.95T_c$ we make 200 iterations.

As has been shown[23] the solution of Eqs. (2) and (3) is quite stable: after integration over a length of a few ξ_0 it becomes almost independent of the initial values. This solution corresponds to a simple exponential relaxation of the functions $\overline{a}$ and $\overline{b}$ to their local "steady-state" values defined by the local values of the order parameter. Therefore, to find $\overline{\Delta}$ at a given point, one does not need the values of $\overline{\Delta}$ at distances larger than several ξ_0 along the trajectory. This peculiarity is used for integration of Eqs. (2) and (3) at long distances. First, we find some approximative solution at the distance of several ξ_0 from a given point and consider it as the boundary condition. Next, we make the integration up to a given point by the Runge-Kutta method with a variable step. To find this approximate boundary condition linear expansions $\overline{a}=a_0+a_1r_{\parallel}$, $\overline{b}=b_0+b_1r_{\parallel}$, and $\overline{\Delta}=\Delta_0+\Delta_1r_{\parallel}$ are used near the given point. Substituting this expansion into Eqs. (2) and (3) and equating the coefficients under the same power of $r_{\parallel}$, we obtain the set of equations for a_0, a_1, b_0, and b_1. Using the values Δ_0 and Δ_1 determined in the previous iteration and a_0 and b_0 from Eqs. (4) and (5) as the initial values, this set of equations can be solved with a few iterations.

In order to avoid computational artifacts, the mesh points are located on the cylindrical coordinates.[17] To decrease the amount of the mesh points under fixed accuracy, interpolation with fast Fourier transform coefficients and with polynoms between the points with $r=\text{const}$ and $\phi=\text{const}$ is used.

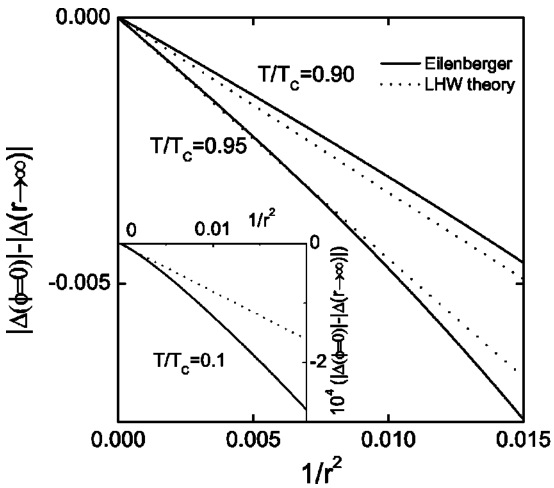

FIG. 1. Asymptotical behavior of the amplitude of the pairing potential $|\Delta|$ at long distances for $\phi=0$ and $T/T_c=0.9,0.95$ and at $T/T_c=0.1$ (inset).

The calculations[17] of $\overline{\Delta}(\mathbf{r})$ have been done only for $r<r_c=10$. Far from the vortex core, $r>r_c$, the initial distribution $|\Delta(r)|\propto\tanh r$ has been taken. To have the uniform distribution of the mesh points in the range of $0\leqslant r\leqslant\infty$ the reduced radius $0\leqslant q\leqslant 1$

$$r=4\frac{T}{T_c}\frac{q}{1-q^2} \tag{8}$$

is used. With the same method we reproduce the results in the core area[17] and obtain a new asymptotical behavior of $\overline{\Delta}(\mathbf{r})$ at $r\to\infty$.

Figure 1 shows the asymptotical behavior of the amplitude of the pairing potential at long distances for $\phi=0$, $T/T_c=0.9$ and 0.95 in the main panel, and $T/T_c=0.1$ in the inset. As can be seen from this figure $|\Delta|$ relaxes to its bulk value ($r\to\infty$) as $1/r^2$ with the power of r being independent of the temperature. The same law of the relaxation has been obtained in the numerical solution of the Bogoliubov-de Gennes equations[18] at $T=0$ K. This relaxation law is different from the law,[17] $|\Delta(r)|\propto\tanh(r)$.

The analytical expansion of the BCS solution near the infinity point has been obtained perturbatively by Li, Hirschfeld, and Wölfle (LHW theory).[22] In the Ginsburg-Landau regime near T_c they found

$$|\Delta(T,r)|-|\Delta(T)|=-\frac{1}{3/4+\epsilon_F m v_s^2(\mathbf{r})/\Delta^2(T)}\frac{\epsilon_F m v_s^2(\mathbf{r})}{\Delta^2(T)}, \tag{9}$$

where ϵ_F is the Fermi energy and $\mathbf{v_s}(\mathbf{r})$ is the superconducting electron velocity. At low temperatures and long distances $mv_sv_F\ll T\ll\Delta(T)$ the solution[22] is

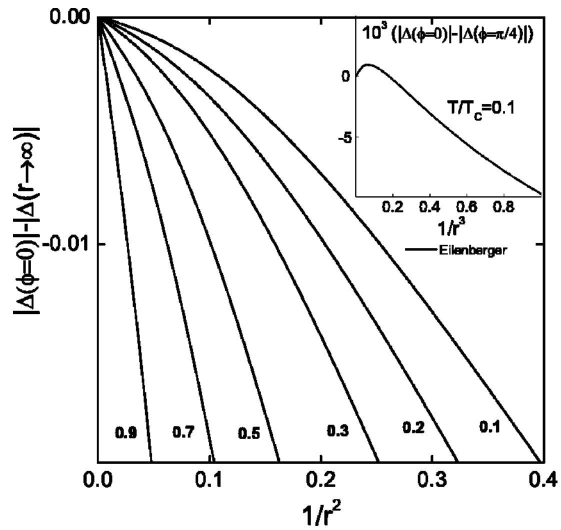

FIG. 2. Behavior of the amplitude of the pairing potential at intermediate distances for $\phi=0$ and six values of T/T_c between 0.1 and 0.9. The inset depicts the asymptotical behavior of the difference of the amplitude along 0 and $\pi/4$ directions, $|\Delta(\phi=0)|-|\Delta(\phi=\pi/4)|$ at $T/T_c=0.1$.

$$|\Delta(T,r)|-|\Delta(T)| =-\frac{2\ln 2}{1-[9\zeta(3)T^3-4\ln 2\,\epsilon_F T m v_s^2(\mathbf{r})]/\Delta^3(T)} \times\frac{\epsilon_F m v_s^2(\mathbf{r})}{\Delta^2(T)}\frac{T}{\Delta(T)}, \tag{10}$$

where $\zeta(3)$ is the Riemann function. The dotted lines in the main panel and the inset of Fig. 1 show the values $|\Delta(T,r)|-|\Delta(T)|$ obtained from Eq. (9) at $T/T_c=0.9$ and 0.95 and from Eq. (10) at $T/T_c=0.1$, respectively. As can be seen from this figure the slopes of asymptotics in LHW theory agrees well with those obtained numerically by us.

Figure 2 shows the behavior of the amplitude of the pairing potential at intermediate distances for $\phi=0$ and six values of T/T_c between 0.1 and 0.9. The behavior of $|\Delta(\phi=0)|-|\Delta(\phi=\pi/4)|$ at $T/T_c=0.1$ and the change of the sign of this quantity are clearly visible in the inset to Fig. 2. This was also observed in Ref. 17. Far from the vortex core one can use the results of the LHW theory.[22] At $T\ll mv_sv_F\ll\Delta(T)$ they found

$$|\Delta(T,r)|-|\Delta(T)| =-\frac{\sum_{l=\pm 1}\left|\cos\left(\theta+l\frac{\pi}{4}\right)\right|^3}{3-\sum_{l=\pm 1}\left|\cos\left(\theta+l\frac{\pi}{4}\right)\right|^3\left(\frac{mv_sv_F}{\Delta(T)}\right)^3} \times\frac{\epsilon_F m v_s^2(\mathbf{r})}{\Delta^2(T)}\frac{mv_sv_F}{\Delta(T)}. \tag{11}$$

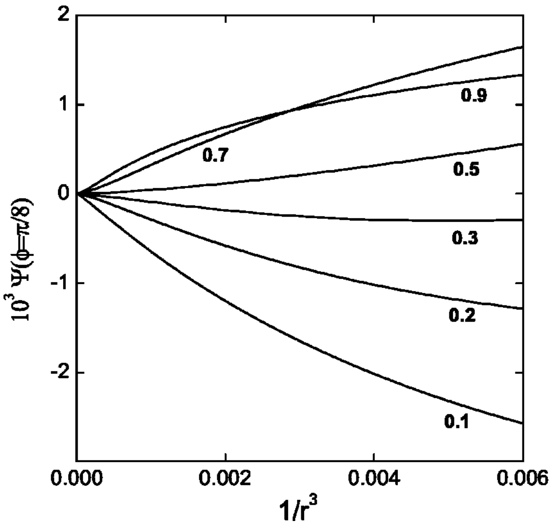

FIG. 3. Asymptotical behavior of the fourfold phase $\Psi = \arg \Delta - \phi$ at long distances for $\phi = \pi/8$ and $T/T_c = 0.1$–0.9.

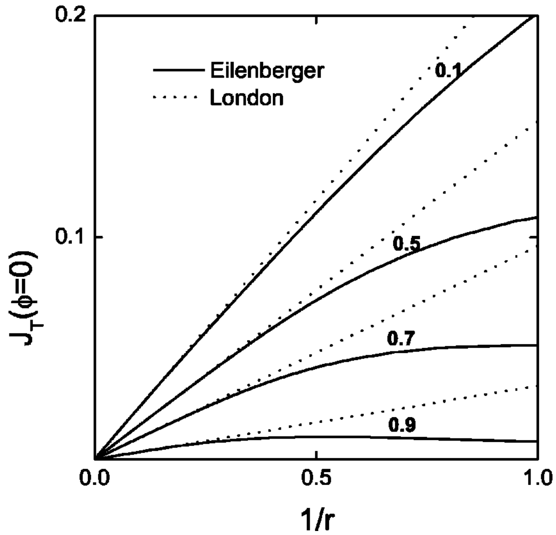

FIG. 4. Asymptotical behavior of the tangential component of the supercurrent $J_T(r)$ at long distances for $\phi = 0$ and $T/T_c = 0.1$–0.9 in units of $-2eN_0 v_F \Delta_0$.

This equation predicts the positive sign of $|\Delta(\phi=0)| - |\Delta(\phi=\pi/4)|$. Thus, our calculations correctly reproduce the behavior of the pairing potential at short and long distances.

As was shown[17] the phase of the pairing potential in the d-wave superconductor with a vortex differs from ϕ and exhibits fourfold symmetry. It has a maximum along the $\pi/8$ direction and decreases rapidly outside the vortex. As predicted,[22] the ratio $\mathrm{Im}\,\overline{\Delta}(r)/\mathrm{Re}[\overline{\Delta}(r) - \overline{\Delta}(r\to\infty)] \ll 1$. This is due to the fact that different terms in the free energy of the superconductor are responsible for the real and imaginary parts of $\overline{\Delta}(r)$, being the local DS term in the first case and the nonlocal term dependent on derivatives of the superconducting electron velocity in the latter case.

Figure 3 shows the asymptotical behavior of the phase $\Psi(r) = \arg \Delta - \phi$ at long distances, $\phi = \pi/8$ and $T/T_c = 0.1$–0.9. As can be seen from this figure Ψ relaxes to its bulk value as $1/r^3$, and changes the sign with the temperature. A change of the sign has also been obtained.[17] The relaxation laws $1/r^2$ and $1/r^3$ for the real and the imaginary parts of $\overline{\Delta}(r)$, respectively, indicate that the ratio $\mathrm{Im}\,\overline{\Delta}(r)/\mathrm{Re}[\overline{\Delta}(r) - \overline{\Delta}(r\to\infty)] \to 0$ for $r\to\infty$ in agreement with the LHW theory.[22] However, this limit is achieved only at very long distances. For example, even in the point with $r=10$, which is very far from the vortex core, this ratio is 5.2 at $T/T_c = 0.1$.

III. SUPERCONDUCTING ELECTRON DENSITY

The supercurrent around a vortex is given in terms of $g(\omega_n, \theta, \mathbf{r})$ by

$$\mathbf{J}(\mathbf{r}) = 2ev_F N_0 2\pi T \sum_{\omega_n>0} \int_0^{2\pi} \frac{d\theta}{2\pi} \frac{\hat{\mathbf{k}}}{i} g(\omega_n, \theta, \mathbf{r}). \quad (12)$$

In Fig. 4 we show the asymptotical behavior of the tangential component of the supercurrent J_T at $\phi = 0$ and $T/T_c = 0.1$–0.9 obtained from the solution of Eqs. (1)–(3) and (12) and from the London approximation. The London approximation can be obtained from the Eilenberger equations neglecting the terms with the derivative and keeping the terms linear in $\partial_\parallel \phi$ in the solution. Then the supercurrent acquires the well-known form

$$\mathbf{J}(\mathbf{r}) = n_s(T) e \mathbf{v_s}(\mathbf{r}), \quad (13)$$

where $\mathbf{v_s}(\mathbf{r}) = \hbar/(2mr)\hat{\boldsymbol{\theta}}$ is the superconducting electron velocity. The temperature dependent function $n_s(T)$ can be written in a compact form[24]

$$\frac{n_s(T)}{n} = \frac{1}{\pi}\int_0^{2\pi} d\theta \cos^2\theta \times \left\{ 1 - \frac{1}{2T}\int_0^\infty d\xi \frac{1}{\cosh^2\left(\frac{\sqrt{(\xi^2 + |\Delta(\theta)|^2)}}{2T}\right)} \right\}, \quad (14)$$

where n is the total density of the electrons.

As can be seen from Fig. 4 the vortex currents suppress n_s resulting in a nonlinear dependence $J_T(1/r)$. The nonlocal effect, the nonlinear effect, and the vortex core effect are not taken into account in the London theory. Neglecting the $\partial_\parallel \overline{a}$ and $\partial_\parallel \overline{b}$ terms in Eqs. (2) and (3) one can consider the nonlinear effects exactly. In this approximation the solutions are

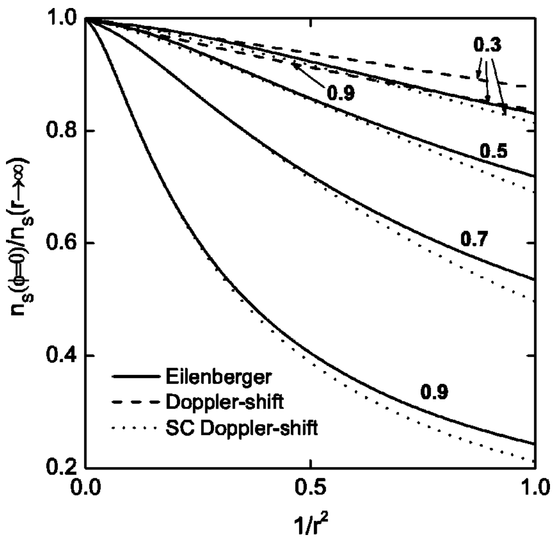

FIG. 5. Asymptotical behavior of the superconducting electron density $n_s(r)$ normalized to the London value $n_s(r\to\infty)$ at long distances at $\phi=0$ and $T/T_c=0.3-0.9$.

$$\overline{a}(r)=\frac{\overline{\Delta}(r)}{(\omega_n+i\partial_{\parallel}\phi/2)+[(\omega_n+i\partial_{\parallel}\phi/2)^2+|\overline{\Delta}(r)|^2]^{1/2}}, \tag{15}$$

$$\overline{b}(r)=\frac{\overline{\Delta}^*(r)}{(\omega_n+i\partial_{\parallel}\phi/2)+[(\omega_n+i\partial_{\parallel}\phi/2)^2+|\overline{\Delta}(r)|^2]^{1/2}}, \tag{16}$$

which differ from the solution for a homogeneous superconductor by the Doppler shift in the energy: $\omega_n\to\omega_n+i\partial_{\parallel}\phi/2$. We consider the following two approximations: (i) the Doppler-shift method, where $\Delta(r)=\Delta(r\to\infty)$ and the core effects are neglected; and (ii) the self-consistent (SC) Doppler-shift method, where $\Delta(r)$ has the local value obtained from the exact solution of Eqs. (1)–(3) and (6) taking the core effects into account.

In the exact solution and the DS approximation one can also determine the effective superconducting electron density $n_s(r)=J_T(r)/[ev_s(r)]$ with $J_T(r)$ obtained from Eq. (12) and $\overline{a}(r)$ and $\overline{b}(r)$ obtained from Eqs. (2) and (3) and Eqs. (15) and (16), respectively. Opposite to the London approach n_s now depends on the coordinate. Then the London approximation [Eq. (14)] can be obtained at the limit of $n_s(r\to\infty)$. In Fig. 5 we show the asymptotical behavior of $n_s(r,\phi=0)/n_s(r\to\infty)$ in the exact solution as well as in the DS and the SC DS approximations at $T/T_c=0.3$–0.9. It should be noted that the normalization constant is also strongly temperature dependent. Using the reduced radius q [Eq. (8)] one can present $n_s(r)$ in the whole range of r, including the vortex core area at $T/T_c=0.2,0.9$ (Fig. 6). As can be seen from Fig. 6 the DS method gives a reasonable approximation of the exact solution for >0.5 at $T=0.2T_c$. It is explained by the steep decrease of the vortex core on lowering the temperature (Pesch-Kramer effect[25]). It should be also noted that the considered case of the vortex in an extreme type-II superconductor is different from that of the flux line lattice in strong fields.[12,13] In the first case there is no screening length for the supercurrent (decreasing as the power law $1/r$) while in the latter case the variation of the current occurs in the intervortex distance, which is comparable to the coherence length in strong fields, resulting in an increase of the gradient of $\mathbf{v}_s(\mathbf{r})$.

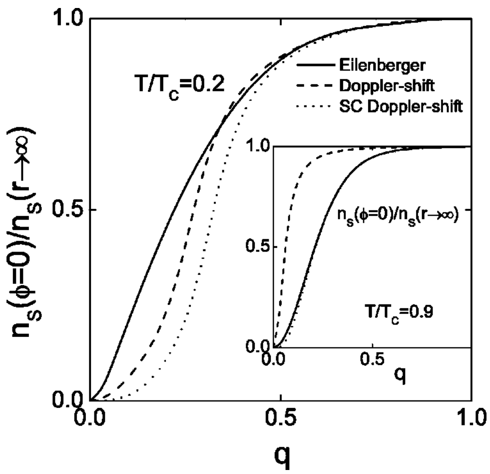

FIG. 6. Superconducting electron density $n_s(r)$ normalized to the London value $n_s(r\to\infty)$ in the whole range of the radius [reduced radius q is determined by Eq. (8)] at $\phi=0$ and $T/T_c=0.2$. The inset depicts $n_s(r)/n_s(r\to\infty)$ at $\phi=0$ and $T/T_c=0.9$.

In the core region $n_s(r)$, obtained from the exact solution, is considerably higher than that from the DS or the SC DS approximations (see Fig. 6). The discrepancy is due to quantum effects that average over the rapid variations in $\mathbf{v}_s(\mathbf{r})$ near the vortex core.

The DS method fails at higher temperatures (see Fig. 5) and the suppression of the pairing potential should be taken into account. This can be done in the SC DS method. As shown in Fig. 5 this method gives a good approximation of the exact solution. At $T/T_c=0.9$ the local SC DS method is extremely good in the whole coordinate range (see the inset to Fig. 6).

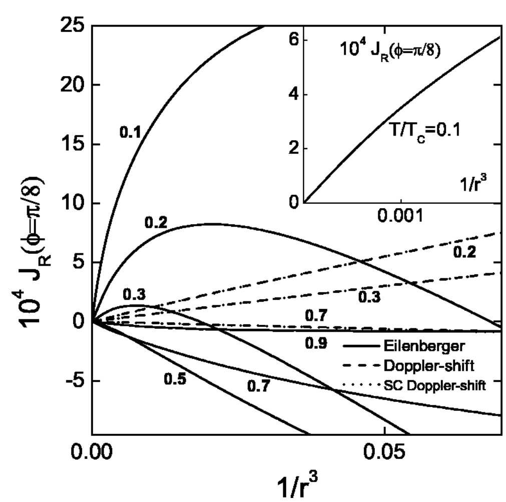

FIG. 7. Asymptotical behavior of the radial component of the supercurrent $J_R(r)$ at $\pi/8$ and $T/T_c=0.1$–0.9 in units of $-2eN_0v_F\Delta_0$. The inset demonstrates the $1/r^3$ relaxation law of the $J_R(r)$ asymptotic at $T/T_c=0.1$ on a large scale.

In the London theory the supercurrent has only a tangential component. As has been pointed out[17] the fourfold symmetry of the vortex results in the appearance of a radial component of the current J_R. Similar to Ψ, J_R has a maximum in the $\pi/8$ direction. Figure 7 shows the asymptotical behavior of $J_R(\pi/8)$ at $T/T_c=0.1$–0.9 obtained from the solution of Eqs. (1)–(3) and (12). The DS and SC DS methods are local approximations and fail to describe the fourfold anisotropy. This can be seen from Fig. 7, where the dependence $J_R(r)$ calculated in the DS approximation at $T/T_c=0.2$, 0.3, and 0.7 is shown. There is no agreement with the Eilenberger theory in this case. The dependence $J_R(r)$ at long distances, calculated in the SC DS approximation, coincides within the line thickness with that obtained in the DS method.

As can be seen from the inset to Fig. 7 J_R relaxes to its bulk value as $1/r^3$. As well as the phase $\Psi(r)$, J_R changes the sign and decreases strongly with the temperature. This behavior is consistent with that found previously.[17] The rapid relaxation of the anisotropy of $\Delta(r)$ and $\mathbf{J}(\mathbf{r})$ is in contrast with that of the local density of states (DOS). As was shown,[17,19] there are four sharp peaks in the DOS at $\phi=\pi/4$. These peaks are connected with the resonance scattering of the quasiparticles at the phase gradient around the vortex center, and can be interpreted as Andreev scattering resonances.[19] Moreover, due to the Aharonov-Bohm effect the resonances in the DOS appear to be nonzero even in the domain of $r>\lambda$ with a slow relaxation law $\propto 1/r$.[26]

IV. CONCLUSION

The quasiclassical Eilenberger equations are solved numerically for an isolated two-dimensional vortex in a *d*-wave superconductor in the whole coordinate range. In the core area our results reproduce those obtained previously.[17] An asymptotical behavior of the amplitude and the phase of the pairing potential, superconducting current, and the superconducting electron density at long distances from the vortex core are obtained. The Doppler-shift method fails to predict the dependence of n_s on r at high temperatures. Taking into account the suppression of the pairing potential as in the self-consistent Doppler-shift (SC DS) method, a good agreement with the exact calculation is observed over the whole range of the radius.

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