

High- T_c superconductivity, skyrmions, and the Berry phaseB. Basu,* S. Dhar,[†] and P. Bandyopadhyay[‡]*Physics and Applied Mathematics Unit, Indian Statistical Institute, Calcutta 700108, India*

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It is here pointed out that the antiferromagnetic spin fluctuation may be associated with a gauge field which gives rise to the antiferromagnetic ground-state chirality. This is associated with the chiral anomaly and Berry phase when we consider the two-dimensional (2D) spin system on the surface of a 3D sphere with a monopole at the center. This realizes the resonating valence bond (RVB) state where spinons and holons can be understood as chargeless spins and spinless holes attached with magnetic flux. The attachment of the magnetic flux to a charge carrier suggests that this may be viewed as a skyrmion. The interaction of a massless fermion representing a neutral spin with a gauge field along with the interaction of a spinless hole with the gauge field enhances the antiferromagnetic correlation along with the pseudogap at the underdoped region. As the doping increases the antiferromagnetic long-range order disappears for the critical doping parameter δ_{sc} . In this framework, the superconducting pairing may be viewed as caused by skyrmion-skyrmion bound states.

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I. INTRODUCTION

It is now well known that there exists an interplay between antiferromagnetism and d -wave superconductivity in cuprate materials. Indeed, on doping with holes, these insulating compounds develop into superconductors even for low concentration of holes. This implies that the antiferromagnetic spin fluctuation plays a significant role in the development of high- T_c superconductivity in these materials and the d -wave superconducting phase is a nearly antiferromagnetic Fermi liquid. In this context Monthoux, Balatsky, and Pines¹ have considered spin fluctuation driven pairing for the cuprates near optimal doping. Rantner and Wen,² in the framework of U(1) gauge fluctuations, have studied the underdoped cuprates where the spin behavior shows the peculiar competition between antiferromagnetic order and singlet formation as is evidenced by pseudogap observed in NMR and neutron scattering. The spin pseudogap can be well explained in terms of the RVB state as proposed by Anderson.³ It is argued that the effect of the preformed spin singlets present in the RVB picture on the doped holes can be described in terms of the fact that the spin of the doped holes becomes an excitation whereas the charge remains tied to the empty site. This leads to the chargeless spin excitations (spinons) and spinless charge excitations (holons). Superconductivity arises when coherence is established after spin-charge recombination.⁴ However, underdoped cuprates have a peculiar property which is apparently very puzzling. As the doping is lowered both the pseudogap and the antiferromagnetic correlation increases. Naively, it is expected that the larger the pseudogap stronger is the spin singlet formation and weaker the antiferromagnetic correlation. However, in the underdoped region the scenario is different and both the pseudogap and antiferromagnetic correlation increase.

In a study⁵ of high- T_c cuprates in the underdoped region from a gauge theoretical point of view it is shown that gauge-field fluctuations effectively remove the deficiencies of the mean-field theories in explaining the antiferromagnetic correlations as observed in experiments. It has been argued that gauge theory with an additional coupling to holons helps

to enhance the antiferromagnetic correlations.

A model is proposed^{6,7} for high- T_c superconductors which includes both the spin fluctuations of the Cu^{++} magnetic ions and of the spins of O^{--} doped holes (spinons). The charge of the doped holes (holons) is associated to quantum skyrmion excitations of the Cu^{++} background. The quantum skyrmion effective interaction potential is evaluated as a function of doping and temperature indicating that Cooper pair formation is determined by the competition between these two types of spin fluctuations. The superconducting transition occurs when the effective potential allows for skyrmion bound states.

In a recent paper⁴ we have also proposed a mechanism of high- T_c superconductivity from the view point of chirality and Berry phase. It is observed that the spin pairing and charge pairing is caused by a gauge force generated by magnetic-flux quanta attached to them. Different phase structures associated with high- T_c superconductivity have been studied from an analysis of the renormalization group equation involving the Berry phase factor μ which corresponds to the monopole strength associated with the magnetic-flux quanta. It is found that there are two crossovers above the superconducting temperature T_c , one corresponding to the glass phase and the other representing the spin gap phase. However, the spin gap temperature T_2^* is found to be dependent on T_c and T_2^*/T_c shows a universal behavior with respect to the hole doping δ/δ_0 with δ_0 being the optimal doping rate.

In this note we shall study the topological excitations of high- T_c superconductivity in cuprates in this framework and shall show that the charge carriers appear as skyrmion excitations of the Cu^{++} spin background. The enhancement of antiferromagnetic correlations along with pseudogap in the underdoped region is explained. The superconducting pairing caused by spin-charge recombination may be viewed as a consequence of formation of skyrmion-skyrmion bound state.

In Sec. II we shall discuss spin fluctuation and RVB theory from the view point of chirality and Berry phase. In

Sec. III we shall discuss skyrmion excitations and the enhancement of antiferromagnetic correlation and pseudogap in the underdoped region. In Sec. IV we shall derive the critical doping parameter δ_{sc} for the destruction of the Neel order. In Sec. V we shall discuss superconducting pairing in terms of skyrmions.

II. SPIN FLUCTUATION, RVB STATE, AND BERRY PHASE

We start with a spin system which is antiferromagnetic in nature. In terms of Schwinger bosons we may write the localized spin \vec{S}_j at site j as

$$\vec{S}_j = \frac{1}{2} (z_{j\uparrow}^\dagger, z_{j\downarrow}^\dagger) \vec{\sigma} \begin{pmatrix} z_{j\uparrow} \\ z_{j\downarrow} \end{pmatrix}. \quad (1)$$

Here $z_{j\sigma}^\dagger$ and $z_{j\sigma}$ represent Schwinger bosons at site j and obey boson commutative relations $[z_{i\sigma}, z_{j\sigma'}^\dagger] = \delta_{ij} \delta_{\sigma\sigma'}$ and $[z_{i\sigma}, z_{j\sigma'}] = [z_{i\sigma}^\dagger, z_{j\sigma'}^\dagger] = 0$. We have also the constraint $\sum_{\sigma} z_{j\sigma}^\dagger z_{j\sigma} = 1$ for $S=1/2$. The Hamiltonian for the localized spin system is given by

$$H = -\frac{1}{2} |J| \sum_{i < j} F_{ij}^\dagger F_{ij}, \quad (2)$$

where $|J| > 0$ and $F_{ij} = \sum_{\sigma} z_{i\sigma}^\dagger z_{j\sigma}$.

If a hole is doped in this spin system an appreciable amount of spin fluctuations may arise which may be represented by Q_{ij} where $\langle Q_{ij} \rangle = \sum_{\sigma} z_{i\sigma} z_{j\sigma}$. We may note that the spin fluctuation Q_{ij} consists of the phase fluctuation and the amplitude fluctuation. However, as the latter is effectively a high-energy mode, so we may concentrate on the phase fluctuation which is connected with the local gauge transformation of $\bar{z}_{j\sigma}$ and $z_{j\sigma}$ at each site given by

$$z_{j\sigma} \rightarrow z_{j\sigma} \exp(-i\theta_j). \quad (3)$$

This suggests that the transformation in the phase of Q_{ij} can be described by a gauge field A_{ij} .

To visualize the spin fluctuation in a two-dimensional (2D) antiferromagnetic system we consider the Heisenberg model with nearest-neighbor interaction represented by the Hamiltonian

$$H = J \sum (S_i^x S_j^x + S_i^y S_j^y + S_i^z S_j^z), \quad (4)$$

where S_i is a spin operator of an electron at site i and $J > 0$. The ground state of antiferromagnetic system in two dimensions on a lattice which allows frustration is characterized by the chirality operator⁸

$$W(C) = \text{Tr} \prod_{i \in C} \left(\frac{1}{2} + \vec{\sigma} \cdot \vec{S}_i \right), \quad (5)$$

where σ are Pauli matrices and C is a lattice contour. The topological order parameter $W(C)$ acquires the form of a lattice Wilson loop

$$W(C) = e^{i\phi(C)}, \quad (6)$$

which may be associated with the flux represented by the gauge field A_{ij} . Indeed, we may represent the chirality operator in terms of A_{ij} so that

$$W(C) = \prod_C e^{iA_{ij}}, \quad (7)$$

where A_{ij} represents a magnetic flux which penetrates through a surface enclosed by the contour C . We may associate this A_{ij} with the phase fluctuation associated with the spin fluctuation caused by the doped hole when we have doping induced frustration in the system. As A_{ij} represents the Berry phase related to chiral anomaly when we describe the system in three dimensions we may write⁴

$$W(C) = e^{i2\pi\mu}, \quad (8)$$

where μ represents the monopole strength ($\hbar = c = e = 1$). In view of this when a two-dimensional frustrated spin system on a lattice is taken to reside on the surface of a three-dimensional sphere of a large radius in a radial magnetic field, we can associate the chirality with the Berry phase. Eventually this will give rise to RVB state.⁹

It may be remarked here that when a chiral current interacts with a gauge field, we have the anomaly which is related to the Berry phase through the relation¹⁰

$$q = 2\mu = -\frac{1}{2} \int \partial_\mu J_\mu^5 d^4x = \frac{1}{16\pi^2} \text{Tr} \int^* F_{\mu\nu} F_{\mu\nu} d^4x, \quad (9)$$

where J_μ^5 is the axial-vector current $\bar{\psi} \gamma_\mu \gamma_5 \psi$, $F_{\mu\nu}$ is the field strength, and $^*F_{\mu\nu}$ is the Hodge dual. Evidently $q = 2\mu$ represents the Pontryagin index.

To study the spin system leading to a RVB state we consider a generalized nearly antiferromagnetic spin model with nearest-neighbor interaction as

$$H = J \sum (S_i^x S_j^x + S_i^y S_j^y + \Delta S_i^z S_j^z), \quad (10)$$

where $J > 0$ and the anisotropy parameter $\Delta = (2\mu + 1/2)$.¹¹ The Berry phase factor μ can take the values $\mu = 0, \pm 1/2, \pm 1, \pm 3/2, \dots$. It is noted that $\Delta = 1$ corresponds to $\mu = 1/2$ and represents the isotropic Hamiltonian which is SU(2) invariant. For $\Delta \rightarrow \infty$, it corresponds to an Ising system. When $\Delta = 0$ ($\mu = -1/2$) we have the XX model. For a frustrated spin system, this corresponds to the singlets of spin pairs which eventually represents the RVB state giving rise to a nondegenerate quantum liquid.

In a recent paper⁴ we have studied the different phases associated with superconductivity in cuprates through the renormalization group analysis involving the factor μ . It is noted that μ takes the usual discrete values of $0, \pm \frac{1}{2}, \pm 1, \pm \frac{3}{2}, \dots$ at fixed points of the renormalization group (RG) flows where μ is stationary and represents the Berry phase factor μ^* of the theory. In terms of energy scale, it is found that as energy increases (decreases) μ also increases (decreases). So to study a critical phenomenon, we can associate

a critical temperature with a standard discrete value of μ corresponding to the Berry phase factor μ^* which represents a fixed point of the RG flows. To study the crossover, it is noted that for $0 \leq |\Delta| < 1$ there are three critical values corresponding to $\mu=0$, $\mu=-\frac{1}{2}$, and $\mu=-1$ which represent the fixed points of the RG flow. We associate three critical temperatures T_1^* , T_2^* , and T_c with fixed values of $\mu=0$, $\mu=-1/2$, and $\mu=-1$, respectively. However, in a frustrated spin system, the chirality demands that μ should be nonzero. So the critical value $\mu=0$ is not achieved and as such there will be random coupling around the value $\mu=0$. This will then represent the cluster glass phase at this critical temperature T_1^* . In this situation, after doping, holes will form a glass of stripes. The next crossover will be at $\mu=-\frac{1}{2}$ corresponding to the pseudogap (spin gap) phase. As $\mu=-\frac{1}{2}$ corresponds to $\Delta=0$, the spin chain will represent the system of spin singlets leading to RVB phase. The spin-charge separation here describes the spin gap (pseudogap) phase. Finally, we arrive at the superconducting transition temperature T_c at $\mu=-1$ corresponding to $\Delta=-1/2$. At this point, the Ising part coupling constant is $(-1/2)J$ with a sign change which represents an attractive force causing the superconducting pair formation.

The concentration of doped holes may be parametrized by a length scale L . In view of this, we may consider μ as a function of δ at a fixed temperature. The doped holes will suppress the $U(1)$ gauge fluctuation describing the antiferromagnetic spin fluctuation. At zero doping, we have the Heisenberg antiferromagnet. The Neel temperature T_N is reduced upon doping and at a critical doping $T_N(\delta_c)=0$. As the doping is increased, the magnetic long range order is destroyed. However, as the doping is lowered both the pseudogap and the antiferromagnetic correlation is increased. This aspect will be discussed in the following section.

III. SKYRMIONS, ANTIFERROMAGNETIC CORRELATION, AND PSEUDOGAP

To study the spinon and holon excitations in our model^{4,12,13} let us consider a single spin down electron at a site j surrounded by an otherwise featureless spin liquid representing a RVB state. Due to the chirality caused by the gauge fluctuation we may consider the system such that a monopole represented by $\mu=-1/2$ is in the background leading to RVB ground state. As a result, the single spin will be characterized by $|\mu|=1$ formed by the single spin state characterized by $\mu=-1/2$ coupled with the orbital spin $\mu=-1/2$ caused by the monopole in the background. This neutral spin attached with magnetic-flux quanta given by $|\mu|=1$ will appear as an excitation and represent the spinon. Now when a doped hole interacts with this spinon, it will give rise to a spinless charged excitation called holon. Thus holons may also be represented by $|\mu|=1$ characterized by a flux $\phi_0=hc/2e$. The residual spinon will then correspond to $\mu_{eff}=0$ which is realized when the unit of magnetic flux characterized by $\mu=-1/2$ associated with the single down spin in the RVB liquid forms a pair with another up spin having $\mu=+1/2$ associated with the hole. Again the holon having $|\mu_{eff}|=1$ will also eventually form a pair each char-

acterized by $|\mu|=1/2$. Indeed for any integer μ the Berry phase may be removed to the dynamical phase and the geometric phase is realized when a pair is formed.¹⁴ Thus the spinon and holon may be viewed as if a neutral spin as well as a charged spinless hole is attached with a magnetic-flux quantum characterized by $|\mu|=1/2$ and these appear in a pair.

Now it is noted that when a spinless hole is dressed with a magnetic-flux quantum given by $|\mu|=1/2$, this will represent a skyrmion. Indeed, the magnetic-flux quantum has its origin in the background chirality which is associated with the chiral anomaly and Berry phase. Indeed, from Eq. (9), we note that the Berry phase factor μ is associated with ${}^*F_{\mu\nu}F_{\mu\nu}$ and we can write

$$\begin{aligned} q &= 2\mu \\ &= -\frac{1}{16\pi^2} \int \text{Tr} {}^*F_{\mu\nu}F_{\mu\nu} d^4x \\ &= \int d^4x \partial_\mu \Omega_\mu, \end{aligned} \quad (11)$$

where

$$\Omega_\mu = -\frac{1}{16\pi^2} \epsilon^{\mu\nu\alpha\beta} \text{Tr}(A_\nu F_{\alpha\beta} + \frac{2}{3} A_\nu A_\alpha A_\beta) \quad (12)$$

is the Chern-Simons secondary characteristic class. In case we have $F_{\alpha\beta}=0$ we can write

$$A_\mu = g^{-1} \partial_\mu g, \quad g \in SU(2) \quad (13)$$

and Ω_μ will represent a topological current J_μ given by

$$J_\mu = \frac{1}{24\pi^2} \epsilon^{\mu\nu\alpha\beta} \text{Tr}(g^{-1} \partial_\nu g)(g^{-1} \partial_\alpha g)(g^{-1} \partial_\beta g). \quad (14)$$

This may be written in terms of chiral fields $\pi_a(a=0,1,2,3)$ as

$$J_\mu = \frac{1}{12\pi^2} \epsilon^{\mu\nu\alpha\beta} \epsilon^{abcd} \pi_a \partial_\nu \pi_b \partial_\alpha \pi_c \partial_\beta \pi_d. \quad (15)$$

Now representing a hole by a Dirac fermion field ψ we may consider the doped hole coupling with the magnetic flux associated with the chirality in terms of the interaction given by the Lagrangian

$$L = \bar{\psi}[i\hat{D} + im(\pi_0 + i\gamma_5 \vec{\pi} \vec{\tau})]\psi, \quad (16)$$

where $\hat{D} = \gamma_\mu(\partial_\mu - iA_\mu)$ following the constraint $\pi_0^2 + \vec{\pi}^2 = 1$.

The Dirac fermion may be viewed as if it has flavor N so that for polarized and unpolarized state we have $N=1$ and 2 , respectively. Now integrating for fermions, we can write the action

$$\begin{aligned}
W &= -\ln \int \exp(-Ld^4x) D\psi D\bar{\psi} \\
&= -N \ln \text{Det}(i\hat{D} + img^{\gamma_5}) \\
&= iN \int d^4x A_\mu J_\mu + i\pi N H_3 + NM^2 \\
&\quad \times \int d^4x \text{Tr}(\partial_\mu g^{-1} \partial_\mu g). \tag{17}
\end{aligned}$$

Here $g^{\gamma_5} = (1 + \gamma_5/2)g + (1 - \gamma_5/2)g^{-1}$. M is a coupling constant having dimension of mass. H_3 is a topological invariant of the map of the space time into the target space S^3 . There are only two homotopy classes $\pi_4(S^3) = \mathbb{Z}_2$, so that $H_3 = 0$ or 1 . In fact the term $i\pi H_3$ is the geometric phase and represents the θ term. Thus we see that the charge carriers dressed with magnetic flux can be represented by a nonlinear σ model and may be treated as skyrmions.

To study the underdoped region of cuprates in this framework, we note that spinon-holon interaction through the gauge force effectively leads to a spin pair characterized by $\mu_{eff} = 0$ where the isolated down spin in the background with $\mu = -1/2$ forms the pair with the up spin of the hole with $\mu = +1/2$. Indeed this may be taken to represent as a spinon-antispinon bound state. This essentially corresponds to the SF flux phase as suggested by Rantner and Wen.² Indeed we can visualize a spin as a massless fermion and this picture of spinon-holon interaction may correspond to a massless fermion coupled to U(1) gauge field along with the holons coupled with the gauge field. The pair formed by massless fermions (spins) dressed with magnetic flux may be viewed as a spinon-antispinon bound state. This spinon-antispinon bound state present in the nearly antiferromagnetic chain will enhance the antiferromagnetic correlation of the system. The simultaneous presence of spin singlet state will lead to the pseudogap (spin gap). Thus in the underdoped region we will have the enhancement of the antiferromagnetic correlation along with the pseudogap. As mentioned earlier, as doping increases, the antiferromagnetic long-range order is destroyed.

IV. SKYRMIONS, CRITICAL DOPING, AND THE DESTRUCTION OF THE ANTIFERROMAGNETIC ORDER

In the present framework, superconductivity arises with the charge spin recombination when a phase coherence is established. Indeed, prior to spin-charge recombination, a spinless holon may be viewed as if a spinless hole is moving in the background of a monopole. This eventually causes the hole pair formation each having a magnetic-flux quantum characterized by $|\mu| = 1/2$. When the spin-charge recombination occurs a spin pair each having unit magnetic flux interact with each other through a gauge force and a phase coherence is established. As we have pointed out in the earlier section that the charge carrier attached with a magnetic flux corresponds to a skyrmion, we may view the superconducting pair as a skyrmion-skyrmion bound state. Indeed, the skyrmion excitation is created at each position of the carriers

and plays a role of magnetic field for the carriers. Because of the magnetic field around a carrier, the Lorentz force acts on another carrier. Due to this Lorentz force an attractive interaction is induced between carriers and leads to Cooper pair formation.

It is noted that the mechanism suggests a d -wave pairing. As already pointed out by Kotliar and Liu¹⁵ in the RVB theory spinons form the d -wave pairing. Now in the superconducting pair, the spin-charge recombination occurring through spinon-holon interaction along with the phase coherence suggests the charge carriers also have d -wave pairing. Indeed, the fact that superconductivity occurs in the vicinity of antiferromagnetic long-range order, the Cooper pair is d wave.

It is known that skyrmion topological defects which are introduced by doping are responsible for the destruction of the antiferromagnetic order and their energy may be used as an order parameter.^{6,7} Indeed, in two spatial dimensions the nonlinear sigma field n^a may be expressed in the CP^1 language in terms of a doublet of complex scalar fields z_i , $i = 1, 2$ with the component $z_i^\dagger z_i = 1$ as

$$n^a = z_i^\dagger \sigma_{ij}^a z_j, \tag{18}$$

where σ^a are Pauli matrices. In this language the continuous field theory corresponding to the Heisenberg antiferromagnet is described by the Lagrangian density in $2+1$ dimensions

$$L_{ns} = (D_\mu z_i)^\dagger (D_\mu z_i), \tag{19}$$

where $D_\mu = \partial_\mu + iA_\mu$ and $\mathcal{A}_\mu = iz_i^\dagger \partial_\mu z_i$. This possesses solitonic solutions called skyrmions and charge is defined as

$$Q = \int d^3x J^0, \tag{20}$$

where J^0 is the zeroth component of the topological current $J^\mu = (1/2\pi) \epsilon^{\mu\alpha\beta} \partial_\alpha \mathcal{A}_\beta$. It is noted that Q is nothing but the magnetic flux of the field \mathcal{A}_μ indicating that skyrmions are vortices and represent defects in the ordered Neel state.

Now the following Lagrangian density may be proposed for describing the dopants and their interaction with the background lattice in $2+1$ dimensions with the topological θ -term

$$\begin{aligned}
L_{z,\psi} &= (D_\mu z_i)^\dagger (D^\mu z_i) + i\bar{\psi}_a \partial_\mu \gamma_\mu \psi_a - m^* v_F \bar{\psi}_a \psi_a \\
&\quad - \bar{\psi}_a \partial^\mu \psi_a \mathcal{A}_\mu + L_H, \tag{21}
\end{aligned}$$

where the hole dopants are represented by a two-component Dirac field ψ_a , m^* and v_F are, respectively, the effective mass and Fermi velocity of dopants. Here L_H is the Hopf term given by

$$L_H = \frac{\theta}{2} \epsilon^{\mu\alpha\beta} \mathcal{A}_\mu \partial_\alpha \mathcal{A}_\beta. \tag{22}$$

It should be mentioned here that long ago it was shown that¹⁶ antiferromagnetic spin correlation do not produce a Hopf term on a 2D square lattice. In fact, these authors have pointed out that the presence of the nontrivial Hopf term may

come from something else other than the spin itself. In this case, the Hopf term arises from the doped holes which will be revealed later.

It is noted that the dopant dispersion relation is given by

$$\epsilon(k) = \sqrt{k^2 v_F^2 + (m^* v_F^2)^2}, \quad (23)$$

which is valid for YBCO ($\text{YBa}_2\text{Cu}_3\text{O}_{6+\delta}$) where the Fermi surface has an almost circular shape which is centered at $\mathbf{k} = 0$. For LSCO ($\text{La}_{2-\delta}\text{Sr}_\delta\text{CuO}_4$) the Fermi surface is different⁷ which corresponds to a dispersion relation of the form

$$\epsilon(k) = \sqrt{\left[\left(k_x \pm \frac{\pi}{2} \right)^2 + \left(k_y \pm \frac{\pi}{2} \right)^2 \right] v_F^2 + (m^* v_F^2)^2}. \quad (24)$$

Now following Marino⁷ the doping parameter δ is introduced by means of a constraint in the fermion integration measure

$$D[\bar{\psi}_a, \psi_a] = D\bar{\psi}_a D\psi_a \delta(\bar{\psi}_a \gamma_\mu \psi_a - \Delta^\mu), \quad (25)$$

where $\Delta^\mu = 4 \delta \int_{x,L}^\infty d\xi^\mu \delta^3(z - \xi)$ for a dopant at the position x and varying along the line L . Here the factor 4 corresponds to the degeneracy of the representation (4-component) for the Fermi fields. This yields the partition function

$$\begin{aligned} Z = & \int D(\bar{z}_0, z, \mathcal{A}, \bar{\psi}, \psi) \delta(\bar{z}z - 1) \delta(\bar{\psi} \gamma_\mu \psi - \Delta^\mu) \\ & \times \exp \left\{ \int_0^\infty d^3x \left[2\rho_s (D_\mu \bar{z}_i^\dagger D_\mu z_i) \right. \right. \\ & \left. \left. + \bar{\psi} \left(i \partial_\mu \gamma_\mu - \frac{m^* v_F}{\hbar} - \gamma^\mu \mathcal{A}_\mu \right) \psi + L_H \right] \right\}, \quad (26) \end{aligned}$$

where ρ_s is the spin stiffness and L_H is the Hopf term.

Upon integration over the fields z , \bar{z} , $\bar{\psi}$, ψ the resulting equation of motion for the zeroth component \mathcal{A}_0 yields the result

$$\theta \epsilon^{ij} \partial_t A_j = 4 \delta \delta^2 [\mathbf{z} - \mathbf{x}(t)], \quad (27)$$

where $\mathbf{x}(t)$ is the dopant position at a time t . If B is the magnetic flux or vorticity of \mathcal{A}_μ then this equation becomes

$$\theta B = 4 \delta \delta^2 [\mathbf{z} - \mathbf{x}(t)] \quad (28)$$

For the skyrmion $B = \delta^2 [\mathbf{z} - \mathbf{x}(t)]$ indicates that the skyrmion topological defect configuration coincides with the dopant position at any time and⁶

$$\pi \theta = 2 \delta. \quad (29)$$

When we translate this result in the 3+1 dimensional formalism where the 2D spin system is considered to reside on the surface of a 3D sphere with a monopole at the center, we note that in the Lagrangian (21), apart from μ being a four-dimensional index, we have to replace the Hopf term by the topological Pontryagin term given by

$$P = - \frac{1}{16\pi^2} {}^* \mathcal{F}_{\mu\nu} \mathcal{F}_{\mu\nu}, \quad (30)$$

where

$${}^* \mathcal{F}_{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} \mathcal{F}_{\alpha\beta}. \quad (31)$$

It is noted that in the partition function (26) when $\int L_H d^3x$ is replaced by $\int P d^4x$, the latter integral just represents the Pontryagin index q related to the monopole strength μ through the relation $q = 2\mu$ as given by Eq. (11).

From dimensional hierarchy, the relation between topological terms suggests that in 3+1 dimensions, when L_H is replaced by L_P , the coefficient θ is related to μ . Indeed replacing L_H by the Chern-Simons Lagrangian

$$L_{CS} = \frac{k}{4\pi} \epsilon^{\mu\alpha\beta} \mathcal{A}_\mu \partial_\alpha \mathcal{A}_\beta, \quad (32)$$

we note that the current is given by

$$J_\mu = \frac{k}{2\pi} \epsilon^{\mu\alpha\beta} \partial_\alpha \mathcal{A}_\beta \quad (33)$$

and the zeroth component corresponds to

$$J_0 = k \frac{B}{2\pi}. \quad (34)$$

So from the relations (22), (32), and (29) we find

$$\pi \theta = \frac{k}{2} = 2 \delta. \quad (35)$$

It is noted that if we take $\delta = 0$ which represents the pure undoped quantum antiferromagnet we do not have the Hopf term which is consistent with the observation of Fradkin and Stone.¹⁶

It has been shown in Ref. 11 that the Chern-Simons coefficient k is related to the monopole strength μ in 3+1 dimensions by the relation $k = 2\mu$. This implies $\mu = 2\delta$. As in the previous section we have noted that each charge carrier in the superconducting pair is associated with the skyrmion topological defect which is caused by the magnetic-flux quantum having $|\mu| = 1/2$, superconductivity occurs at $T = 0$ for the critical doping parameter δ_{sc} given by $|\mu| = 1/2 = 2\delta_{sc}$ yielding $\delta_{sc} = 0.25$ for YBCO. When the doping parameter x we have the relation $\delta = x - 0.18$ so that we have $x_{sc} = 0.43$, which is in good agreement with the experimental value $x_{sc} = 0.41 \pm 0.02$.^{17,7} For LSCO, the Fermi surface has four branches and this yields $\delta_{sc} = x_{sc} = 0.06$ which is to be compared with the experimental result $x_{sc} = 0.02$.¹⁸ It is noted that δ_{sc} is a universal constant depending only on the nature of the Fermi surface.

We have pointed out earlier that in 3+1 dimensions chiral anomaly leads to the realization of fermions represented by doped holes interacting with chiral boson fields π_i , with the constraint $\vec{\pi}_0^2 + \vec{\pi}^2 = 1$. The mapping of the space-time manifold on the target space leads to the homotopy $\pi_4(S^3)$

$=Z_2$ which takes the values 0 or 1 and leads to the θ term representing the geometric phase. The third term in Eq. (17) gives rise to the solitonic solution such that the charge carrier appears as a skyrmion. However in 3+1 dimensions, the stability of the soliton is not generated by this term alone as rescaling of the scale variable $x \rightarrow \lambda x$ may lead to shrinking it to zero size. However, in the present framework, the attachment of magnetic field with the charge carrier will prevent it from shrinking it to zero size.

Indeed this gives rise to a gauge theoretic extension of the extended body so that the position variable may be written as

$$Q_\mu = q_\mu + iA_\mu, \quad (36)$$

where q_μ is the mean position. As $\mu = -1/2$ and $+1/2$ corresponds to vortices in the opposite direction we may consider A_μ as $SU(2)$ gauge field when the field strength is given by

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu], \quad (37)$$

where A_μ is a $SU(2)$ gauge field. When $F_{\mu\nu}$ is taken to be vanishing at all points on the boundary S^3 of a certain volume V^4 inside which $F_{\mu\nu} \neq 0$, in the limiting case towards the boundary, we can take

$$A_\mu = g^{-1} \partial_\mu g, \quad g \in SU(2). \quad (38)$$

This helps us to write the action incorporating the θ term as

$$\begin{aligned} S = & \frac{M^2}{16} \int \text{Tr}(\partial_\mu g^{-1} \partial_\mu g) d^4x + \frac{1}{32\eta^2} \\ & \times \int \text{Tr}[\partial_\mu g g^{-1}, \partial_\nu g g^{-1}]^2 d^4x + \frac{i\pi}{24\pi^2} \\ & \times \int_{S^3} dS_\mu \epsilon^{\mu\nu\lambda\sigma} \text{Tr}[(g^{-1} \partial_\nu g)(g^{-1} \partial_\lambda g)(g^{-1} \partial_\sigma g)], \end{aligned} \quad (39)$$

where M is a constant having the dimension of mass and η is a dimensionless coupling constant. Here the first term is related to the gauge noninvariant term $M^2 A_\mu A^\mu$, the second term (Skyrme term) is the stability term which arises from the term $F_{\mu\nu} F^{\mu\nu}$, and the third term is the θ term given by $*F_{\mu\nu} F^{\mu\nu}$ which is related to the chiral anomaly and Berry phase.

Marino and Neto⁷ have pointed out that at the critical doping δ_{sc} , the energy of the skyrmion vanishes. When we compute the energy of the skyrmion from the action (39), we find the expression for the minimum energy¹⁹ as

$$E_{min} = \frac{12\pi^2 M}{\eta} \quad (40)$$

and the size for E_{min} as

$$R_0 = \frac{1}{2M\eta}. \quad (41)$$

Taking M and η as a function of δ , we note that for the vanishing energy we have $M(\delta_{sc}) = 0$ which corresponds to the fact that the spin stiffness vanishes. From the relation for R_0 , it is evident that the skyrmion size is infinite. However, we can have the vanishing energy for finite nonzero $M(\delta)$ when η is infinite. This suggests that at this point $R_0 = 0$. This implies that for finite M , the vanishing energy suggests that the skyrmion shrinks to the zero size. So apart from energy, we can take the size of the skyrmion also as an order parameter.

V. DISCUSSION

It has been pointed out here that the antiferromagnetic spin fluctuation gives rise to a gauge field which determines the antiferromagnetic ground-state chirality. This is related to the Berry phase and helps us to realize the RVB state where spinons and holons can be understood as chargeless spins and spinless holes attached with magnetic flux. The attachment of the magnetic flux of the charge carrier suggests that this may be viewed as a skyrmion. The interaction of a massless fermion representing a neutral spin with a gauge field along with the interaction of a spinless hole with the gauge field enhances the antiferromagnetic correlation along with the pseudogap at the underdoped region. The superconducting pairing may be viewed as caused by skyrmion-skyrmion bound states. This effectively leads to topological superconductivity. It is also shown that the destruction of antiferromagnetic order is at the critical doping parameter δ_{sc} which is a universal constant depending on the nature of the Fermi surface.

Abanov and Wiegman^{20,21} have pointed out that topological superconductivity in 3+1 dimensions and 2+1 dimensions has its roots in the 1D Peierls-Fröhlich model which suggests that the 2π phase solitons of the Fröhlich model²² are charged and move freely through the system making it an ideal conductor. In spatial dimension greater than one this corresponds to superconductivity when the solitonic feature of a charge carrier is attributed to the attachment of a magnetic flux to it. It may be remarked here that in 1+1 dimensions we will have a nonlinear sigma model with the Wess-Zumino term when the target space is S^3 which is the $O(4)$ nonlinear sigma model. In the Euclidean framework, however, this geometrically corresponds to the attachment of a vortex line to the two-dimensional sheet which is topologically equivalent to the attachment of a magnetic flux.²³ This suggests that the topological feature of ideal conductivity visualized by Fröhlich in 1+1 dimensions and that of superconductivity in 2+1 and 3+1 dimensions have a common origin.

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¹P. Monthoux, A. Balatsky, and D. Pines, Phys. Rev. Lett. **67**, 3448 (1991); see also A.V. Chubukov and J. Schmalian, Phys. Rev. B **57**, R11 085 (1998); J. Schmalian, D. Pines, and B.P. Stojković,

- Phys. Rev. Lett. **80**, 3839 (1998); T. Moriya, K. Ueda, and Y. Takahashi, J. Phys. Soc. Jpn. **49**, 2905 (1990); T.S. Nunner, J. Schmalian, and K.H. Bennemann, Phys. Rev. B **59**, 8859 (1999).
- ²W. Rantner and X.G. Wen, Phys. Rev. B **66**, 144501 (2002).
- ³P.W. Anderson, Science **235**, 1196 (1987).
- ⁴B. Basu, P. Bandyopadhyay, and D. Pal, Int. J. Mod. Phys. B **17**, 293 (2003).
- ⁵D.H. Kim and P.A. Lee, Ann. Phys. (N.Y.) **272**, 130 (1999).
- ⁶E.C. Marino, Phys. Lett. A **263**, 446 (1999).
- ⁷E.C. Marino and M.B. Neto, Phys. Rev. B **64**, 092511 (2001).
- ⁸P. Wiegmann, Prog. Theor. Phys. Suppl. **101**, 243 (1992); *Topological Mechanism of Superconductivity in Field Theory, Topology and Condensed Matter Physics*, edited by H. B. Geyer (Springer, Berlin, 1999).
- ⁹B. Basu, D. Pal, and P. Bandyopadhyay, Int. J. Mod. Phys. B **13**, 3393 (1999).
- ¹⁰D. Banerjee and P. Bandyopadhyay, J. Math. Phys. **33**, 990 (1992).
- ¹¹P. Bandyopadhyay, Int. J. Mod. Phys. A **15**, 1415 (2000).
- ¹²D. Pal, B. Basu, and P. Bandyopadhyay, Phys. Lett. A **299**, 304 (2002).
- ¹³D. Pal and B. Basu, Europhys. Lett. **56**, 99 (2001).
- ¹⁴B. Basu, D. Banerjee, and P. Bandyopadhyay, Phys. Lett. A **236**, 125 (1997).
- ¹⁵G. Kotliar and J. Liu, Phys. Rev. B **38**, 5142 (1988).
- ¹⁶E. Fradkin and M. Stone, Phys. Rev. B **38**, 7215 (1988).
- ¹⁷J. Rossat-Mignod *et al.*, in *Dynamics of Magnetic Fluctuations in High-Temperature Superconductors*, edited by G. Reiter, P. Horsch, and G. C. Psaltakis (Plenum, New York, 1991).
- ¹⁸F. Borsa, P. Carreta, J. H. Cho, F. C. Chou, Q. Hu, D. C. Johnston, A. Lascialfari, D. R. Torgeson, R. J. Gooding, N. M. Salem, and K. J. E. Vos, Phys. Rev. B **52**, 7334 (1995).
- ¹⁹B. Basu, S. Dhar, and P. Bandyopadhyay, cond-mat/0208426, Int. J. Mod. Phys. B (to be published).
- ²⁰A.G. Abanov and P.B. Wiegmann, Phys. Rev. Lett. **86**, 1319 (2001).
- ²¹A.G. Abanov and P.B. Wiegmann, Nucl. Phys. B **570**, 685 (2000).
- ²²G. Fröhlich, Proc. R. Soc. London, Ser. A **223**, 296 (1954).
- ²³A. Roy and P. Bandyopadhyay, J. Math. Phys. **33**, 1178 (1992).