

# Critical susceptibility exponent measured from Fe/W(110) bilayers

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The critical phase transition in ferromagnetic ultrathin Fe/W(110) films has been studied using the magnetic ac susceptibility. A statistically objective, unconstrained fitting of the susceptibility is used to extract values for the critical exponent  $\gamma$ , the critical temperature  $T_c$ , the critical amplitude  $\chi_0$ , and the range of temperature that exhibits power-law behavior from individual experimental measurements of  $\chi(T)$ . This avoids systematic errors and generates objective fitting results. An ensemble of 25 measurements on many different films are analyzed. Those which permit a fitting range in reduced temperature extending lower than  $\approx 4.75 \times 10^{-3}$  give an average value  $\gamma = 1.76 \pm 0.01$ . Bilayer films give a weighted average value of  $\gamma = 1.75 \pm 0.02$ . These results are in agreement with the two-dimensional Ising exponent  $\gamma = \frac{7}{4}$ . Measurements that do not exhibit power-law scaling as close to  $T_c$  (especially films of thickness 1.75 monolayer) show a value of  $\gamma$  higher than the Ising value. Several possibilities are considered to account for this behavior.

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## I. INTRODUCTION

Experiments that seek to measure critical phase transitions are very important to physics at a fundamental level. Careful experiments can be used to test the theoretical models of universality and scaling. The true nature of the order parameter of a system, both in terms of dimensionality and degrees of freedom, is revealed at the transition and important physical insight is gained in the looking.

An ultrathin magnetic film closely approaches the physical realization of a truly two-dimensional magnetic system, and offers a better system for studying critical phase transitions in two dimensions than more traditional layered bulk materials such as  $\text{Rb}_2\text{CoF}_4$ ,<sup>1</sup> where interlayer interactions will always be present, even if only to a small degree. Bander and Mills<sup>2</sup> have shown that when ferromagnetic thin films have uniaxial anisotropy, the critical regime near the Curie temperature is described by the two-dimensional Ising model. For this reason, a great number of measurements of the static critical exponents of ultrathin ferromagnetic films have been reported. Almost all of this experimental work concentrates on the critical exponent of the magnetization,  $\beta$ .<sup>3–10</sup> To our knowledge, there are only a handful of reports in which the critical exponent of the magnetic susceptibility,  $\gamma$ , is investigated experimentally for an ultrathin magnetic film.<sup>4,10–14</sup> Unfortunately, almost all of these susceptibility studies have at least one of a number of deficiencies which call the results into question.

A common difficulty in the determination of critical exponents is the determination of  $T_c$ . Small variations in the assumed value of  $T_c$  have a profound effect on the fitted value of the critical exponent, and introduce confidence limits that are usually much larger than those derived from a simple two-parameter fit for the critical exponent and amplitude. The extreme sensitivity of the results to  $T_c$  implies that the same data must be used to determine both  $T_c$  and the critical parameters. This is particularly true for metastable ultrathin films, since very small shifts in the critical temperature are often introduced by temperature cycling and annealing, or by residual vacuum contamination. A second diffi-

culty is determining the temperature range where scaling is observed. Since real, finite systems do not show infinite divergences, the order parameter departs from power-law behavior close to  $T_c$  because of finite-size effects, dynamical effects (in ac measurements), a finite demagnetization factor, and so on. To fit the data properly, an objective, four-parameter power-law fit of the data is required. In addition to  $T_c$  and  $\gamma$ , the fit should find values for the critical amplitude  $\chi_0$  and the cutoff for power-law behavior near  $T_c$ . Finally, in order to demonstrate true systematic behavior, it is clear that the analysis of many films and measurements is necessary.

It is perhaps surprising that after two decades of investigating the critical properties of ultrathin ferromagnetic films, no published measurements of  $\gamma$  meet these criteria. An impressive study by Back *et al.*<sup>6</sup> on Fe/W(110) ultrathin films determines  $\beta$  and the exponent of the critical isotherm  $\delta$  using the dc magnetization, and then derives  $\gamma$  using the scaling relations between different exponents. The value of  $T_c$  is not fit, but rather taken to lie at the peak of the dc susceptibility for a particular experiment. The results agree with the predictions of the two-dimensional (2D) Ising model. This represents a check of the internal consistency of the data and scaling relations, but is not an independent measurement of  $\gamma$ . Elmers *et al.*<sup>4</sup> report dc-susceptibility results for a series of submonolayer films of Fe/W(110) and find  $\gamma = 2.8 \pm 0.2$ , significantly different than the 2D Ising value of 1.75.<sup>15</sup> It is not clear to what extent this finding is a result of using an incomplete film layer or if, as they suggest, the material is exhibiting the behavior of an anisotropic Heisenberg system. Other studies report results only for a single measurement from a single film.<sup>13</sup> Still others use questionable criteria for determining  $T_c$ , such as the disappearance of the imaginary component of the susceptibility in an ac measurement,<sup>12</sup> the peak of the real ac susceptibility,<sup>11,16</sup> or the presence of a “shoulder” above the peak of the susceptibility<sup>14</sup> under special circumstances.

This paper presents the results of a collection of 25 measurements of the ac magnetic susceptibility of Fe films between 1.5 and 2.0 ML (monolayer) grown epitaxially on W(110), and the values of  $\gamma$  derived from them using an

objective minimization of the statistical variance between the data and a power-law fit using four parameters:  $T_c$ ,  $\gamma$ , the amplitude  $\chi_0$ , and the low reduced-temperature cutoff  $t_x$  for fitting. Error estimates on  $T_c$  and  $\gamma$  are provided by  $1\sigma$  variations in the statistical  $\chi^2$ . The results fall into two distinct classes. Measurements exhibiting power-law behavior over a long range of reduced temperature extending down to a cutoff  $t_x < 4.75 \times 10^{-3}$  give an average critical exponent  $\gamma = 1.76 \pm 0.01$ . Measurements which exhibit power-law behavior down to larger values of  $t_x$  show a systematic trend to higher values of  $\gamma$  which depends rather linearly on  $\ln(t_x)$ . The possibility that films which give a high value of  $\gamma$  have a distribution of transition temperatures will be addressed to explain this unexpected result.

## II. THEORY

According to scaling theory, the real component of the intrinsic magnetic susceptibility ( $\chi_{int} = \partial M / \partial H$ ) above the Curie temperature of a critical phase transition is described by the power-law equation:

$$\chi'_{int}(t) = \chi_o t^{-\gamma}, \quad (1)$$

where  $\chi_o$  is the critical amplitude,  $\gamma$  is the static critical exponent for the susceptibility of the order parameter, and  $t$  is the reduced temperature above  $T_c$ , given as

$$t = \left( \frac{T - T_c}{T_c} \right). \quad (2)$$

For experimental measurements of the magnetic ac susceptibility, additional terms need to be added to account for both demagnetization and dynamical effects. The demagnetizing factor  $N$  is folded into the expression for the intrinsic susceptibility by augmenting the magnetic field by

$$H_{eff} = H - NM, \quad (3)$$

where  $H_{eff}$  is the effective field acting on the ferromagnet. This gives rise to an effective susceptibility of

$$\chi'_{eff}(T) = \frac{\chi'_{int}(T)}{1 + N\chi'_{int}(T)}. \quad (4)$$

It is easy to see that for a nonzero value of  $N$ , the susceptibility cannot diverge at  $T_c$ .  $N$  will “dampen” any experimental measurement of  $\chi$  as long as the value of the product  $N\chi_{int}$  is comparable to or greater than 1.

To accurately describe results from ac susceptibility, it is necessary to add the effect of the relaxation time of the magnetization to the effective susceptibility. In the linear-response approximation, for systems with an exponential relaxation time ( $\tau$ ),  $M(T) \propto \exp(-T/\tau)$  where  $T$  is time. Under the influence of an externally applied sinusoidal field, the real dynamic susceptibility ( $\chi'$ ) can be written as

$$\chi'(T) = \frac{\chi'_{eff}(T)}{1 + [\omega\tau(T)]^2}, \quad (5)$$

where  $\omega$  is the driving frequency of the magnetic field.<sup>33</sup> This final form of the magnetic susceptibility limits the ability of experiments to probe critical behavior very close to the transition. To observe any critical scaling in the experimental data, two requirements must be met: we must have  $N$  sufficiently small and we must have  $(\omega\tau)^2 \ll 1.0$ .

The extreme aspect ratio of ultrathin films leads to very small values of  $N$ . For systems that have their moments oriented in plane,  $N$  is proportional to first order to the thickness divided by the effective lateral dimension of the film.<sup>17</sup> For films that are one or two atomic layers thick and many thousands of lattice spacings wide,  $N$  will be extremely small. This is another reason why ultrathin films are ideal for studies of critical phenomenon in two dimensions. Previous studies of the susceptibility on ultrathin films have attempted to estimate  $N$  (and include the estimation in the power-law fits) by using the maximum value of the real susceptibility.<sup>12</sup> The argument proceeds by rearranging Eq. (4) as follows:

$$\frac{1}{\chi'_{eff}(T)} = \frac{1}{\chi'_{int}(T)} + N. \quad (6)$$

This leads one to the conclusion that at  $T_c$ , when  $\chi_{int}$  is infinite,  $N = 1/\chi_{max}$ . This simple treatment has several problems even for dc-susceptibility measurements (where  $\omega = 0$ ) in that it ignores other effects (finite field, saturated correlation length, etc.) that will saturate the susceptibility and will give a value for  $N$  that is artificially too high and is at best an upper limit.<sup>18</sup> If this limit of  $N$  is then used in the power-law analysis, the resulting quoted values for  $\gamma$  should be called into question.

Dynamic effects are only significant near  $T_c$  where critical slowing down will lead to a large relaxation time for the equilibration of the order parameter.<sup>19</sup> This can be less of a problem in dc measurements, but the increased signal-to-noise ratio that is achieved in ac measurements makes it worthwhile to deal with the dynamics problem. In fact, critical slowing effects should disappear once the temperature is increased more than a degree or two above  $T_c$ . Dynamic effects will change the temperature at which the susceptibility exhibits a maximum (depending on the measurement frequency used), making the evaluation of  $T_c$  by that method difficult if not impossible.

## III. EXPERIMENT

Fe/W(110) ultrathin films with high quality epitaxial layers can be grown at least up to 2 ML.<sup>20</sup> Previous studies of Fe/W(110) have shown that the magnetic properties of the films depend sensitively on the film thickness.<sup>21</sup> Some studies of films less than 1.5 ML show an interesting perpendicular magnetization due to the film structure which results from step-flow growth.<sup>22,23</sup> Pietzsch *et al.*<sup>24</sup> find perpendicular domains for narrow bilayer stripes on a Fe monolayer at low temperatures (about 16 K) grown on a miscut surface. For this study, we concentrate on the thickness range from 1.5 to 2.0 ML, where many studies have confirmed a large in-plane anisotropy for this system.

The experiments were performed in an UHV environment

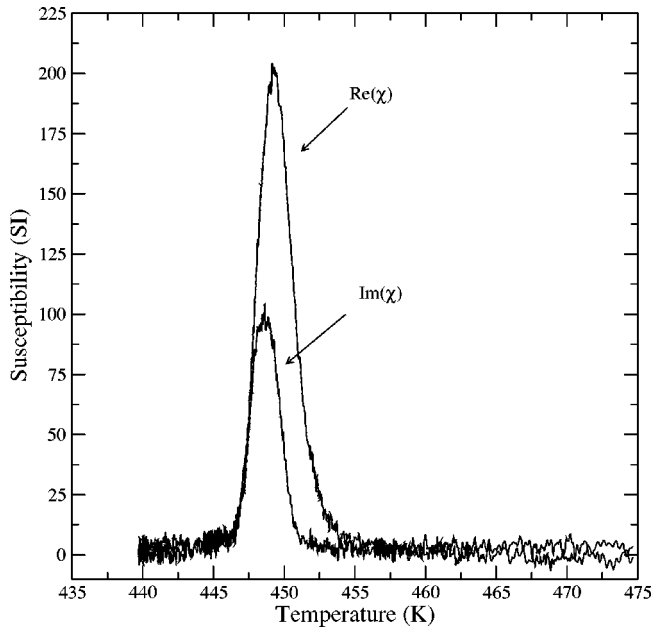


FIG. 1. Magnetic ac susceptibility measured from a 1.8 ML film of iron grown upon W(110). The real and imaginary components of the susceptibility were measured simultaneously.

with a base pressure of  $1 \times 10^{-10}$  torr. The films were grown by molecular-beam epitaxy from a 99.995% pure iron wire. The substrate was a tungsten single crystal that had been cut and polished to expose the [110] face. The cut is accurate to within  $0.2^\circ$ . The first Fe layer was deposited at room temperature and then annealed for 1 min to 500 K. This slight annealing produces increased sharpness of the resulting pseudomorphic low-energy electron diffraction (LEED) pattern. Further depositions were performed at room temperature with no annealing. The film growth, thickness, and quality were monitored by Auger electron spectroscopy and LEED.

The  $ac\text{-}\chi_m$  measurements were made via the surface magneto-optic Kerr effect using a focused He-Ne laser spot with a diameter of  $\approx 0.75$  mm. Small coils near the surface produced a sinusoidally oscillating magnetic field  $H$  which influences the moments in the paramagnetic film above  $T_c$ . The field was applied along the film's easy axis [110]. The surface magneto-optic Kerr effect produces a rotation of the polarization of the laser light reflected off of the magnetic surface. After the reflected light passes through a polarizer almost crossed with the incident polarization, the signal manifests itself in changes in the light intensity at the photodiode detector. The  $1f$  signal is read by a dual-phase lock-in amplifier that can simultaneously record both the in-phase (or real) susceptibility  $[\chi'(T)]$  and the out-of-phase (or imaginary) susceptibility  $[\chi''(T)]$ . The raw signal is calibrated to SI units and the entire signal can be represented as

$$\chi(T) = \chi'(T) + i\chi''(T). \quad (7)$$

Figure 1 shows a typical measurement of the complex sus-

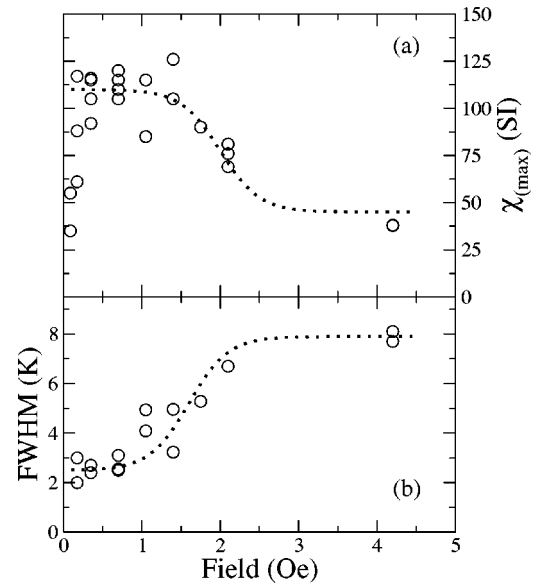


FIG. 2. (a) The maximum value of the magnetic susceptibility as a function of applied magnetic field. (b) FWHM of the real susceptibility peak plotted as a function of applied field amplitude. The minimum half-width is achieved for fields less than 1 Oe.

ceptibility measured from a 1.8 ML iron film. The measurement was made with an applied field amplitude of 0.7 Oe at a frequency of 400 Hz.

It would be best to use an infinitesimally small field, but of course this is not possible experimentally. A study of magnetic susceptibility peak shape as a function of field was conducted to see what value of the field would give the best compromise between signal and finite field effects. Figures 2(a) and 2(b) show the maximum value and full width at half maximum (FWHM) for the susceptibility peaks, respectively, as a function of the amplitude of the applied field. The trend below 1.0 Oe in both graphs is independent of field size, (except at extremely low fields where the signal itself disappears) but deviates for higher fields. Resulting measurements of a susceptibility peak measured in these small fields give a FWHM typically between  $2^\circ$  and  $3.5^\circ$ . In these measurements, smaller field amplitudes were accessible but this generally leads to a degradation of the signal-to-noise ratio.

Sample heating was accomplished by running ac current (no more than 1 A rms) through a small tungsten wire filament located behind the tungsten crystal. ac current at 60 Hz was used to reduce the effects of stray offset fields at the surface. It had been found in the past that a dc current introduced a 0.1 Oe offset field at the surface. The 0.1 Oe field caused by the heating filament is much less than the applied field used in the measurement (typically 0.7 Oe) and is much less than the field which increases the FWHM of the susceptibility peak (Fig. 2). Any questions about the effect of the heating current were answered by comparing data taken while increasing and decreasing the film temperature, respectively. The value of current used in the two methods differed by a factor of 3, and there was absolutely no difference in the final data. The temperature of the film was measured using a W/WRe thermocouple embedded in the tungsten crystal and the rate of temperature increase/decrease was in most cases

limited to  $0.2^\circ/\text{min}$ . This low rate more than adequately compensates for thermal variations in the crystal and permits even heating of the film over the entire surface ( $\approx 1 \text{ cm}^2$ ).

In a few cases, the susceptibility was also measured in the two directions orthogonal to the assumed easy axis:  $[001]$  (in plane) and  $[110]$  (perpendicular) in order to check that no perpendicular magnetization was present. These measurements showed zero signal, indicating no moments along those directions. This result does not necessarily contradict the findings of Pietzsch *et al.*<sup>24</sup> since the temperature and substrate step density are very different in the two experiments.

#### IV. DATA ANALYSIS

To fit the susceptibility data to Eq. (1), an objective, many-parameter fit was used to determine the best values for the Curie temperature  $T_c$ , the critical exponent  $\gamma$ , the critical amplitude  $\chi_o$ , and  $t_x$  which is the smallest value of the reduced temperature to show power-law scaling.

The fit is performed in double-logarithm space  $[\ln(\chi) \text{ vs } \ln(t)]$ , the slope of which will correspond to the critical exponent. Taking the logarithm of the susceptibility necessitated the removal of data points where  $\chi(T)$  goes to zero. Since these points are weighted the least, this “weeding” out of points does not adversely affect the final fit. A small range of temperatures close to the peak was chosen for possible values of  $T_c$  used in the reduced temperature. For each considered value of  $T_c$ , a weighted least-squares fit was performed on the data in the new  $\ln$ - $\ln$  data space from  $\ln(t_{\max})$  (which always corresponds to the data point measured at the highest temperature) to a cutoff value  $\ln(t_x)$ .

$t_x$  was itself varied over a range from just below  $t_{\max}$  to a value of  $t$  where the power-law scaling was obviously no longer valid. The variance of the fit was minimized for the best value of  $T_c$  and the cutoff  $t_x$ . The variance is the best test for a fit made in a many-parameter space<sup>25</sup> where the number of points does not remain constant. It is given by

$$s^2 = \sum_{i=t_{\max}}^{t_x} \frac{[\ln(\chi_i) - F(t_i)]^2}{\sigma_i^2} \bigg/ \sum_{i=t_{\max}}^{t_x} \frac{1}{\sigma_i^2}, \quad (8)$$

where  $\chi_i$  is the  $i$ th data point,  $F(t_i) = \ln(\chi_o) + \gamma \ln(t_i)$  is the fitted function, and  $\sigma_i$  is the error associated with the logarithm of each data point. Figure 3 shows data for which a contour plot of  $s^2$  as a function of  $T_c$  and  $\ln(t_x)$  is presented in Fig. 4. There is a global minimum at  $T_c = 455.84 \text{ K}$  and  $\ln(t_x) = -5.355$  (corresponding to a temperature of  $457.99 \text{ K}$ ). There are local minima exhibited in the graphs that have higher values for  $t_x$  than the global minimum. The fact that the global minimum fits the data closer to  $T_c$  increases its significance.

To get an error estimation on  $T_c$ , the fits were recalculated while keeping the optimum value of  $\ln(t_x) = -5.36$  to allow for a careful statistical  $\chi^2$  analysis for a consistent number of data points. According to statistics for a multivariable fit,<sup>25</sup> the 65% confidence range for a parameter is given by the parameter values that increase the unreduced  $\chi^2$  by 1.

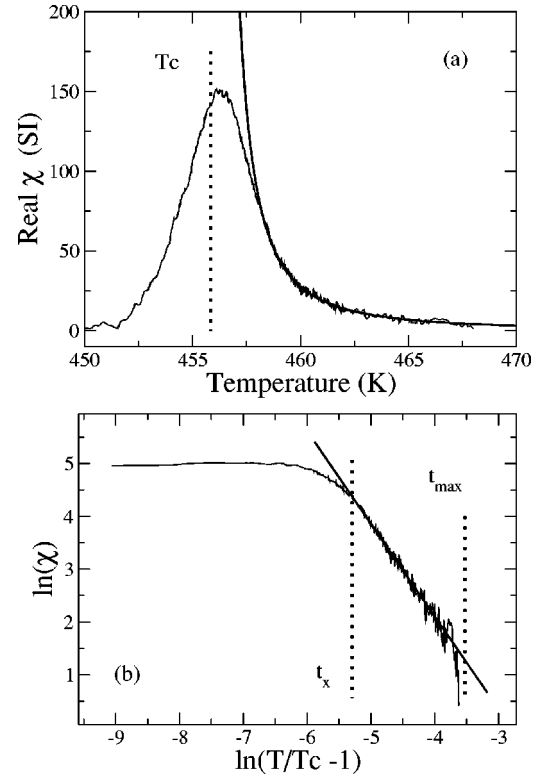


FIG. 3. Power-law fit for a typical susceptibility measurement. (a)  $\chi$  vs temperature. Solid line shows the fit in linear space and dotted line shows position of  $T_c$ . (b) Fit in log-log space, with dotted lines showing position of  $t_{\max}$  and  $t_x$ .  $t_{\max}$  always corresponds to the maximum temperature which was measured. The solid line represents the linear function fit in the double-log space.

Figure 5 shows  $\chi^2$  versus  $T_c$  for the data in Fig. 3. Due to the good signal-to-noise ratio of the data and the large number of points in the limited temperature range, the error for  $T_c$  is very small. The number of points in the fit used for Fig. 5 is 1905, which gives a reduced  $\chi^2$  for the fit of 1.8, signifying a very good fit to the data. The  $T_c$  value from this analysis is

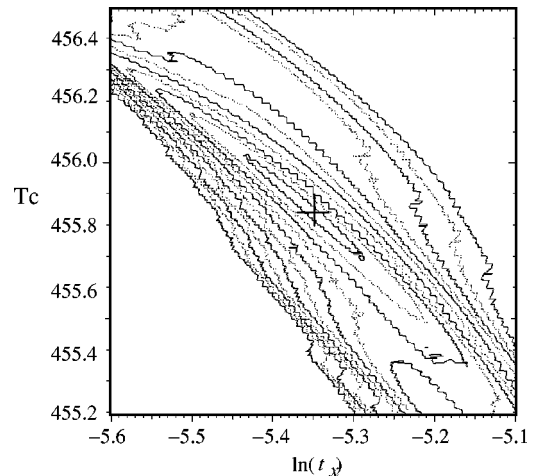


FIG. 4. Contour plot of  $s^2$  as a function of  $T_c$  and  $\ln(t_x)$ . The global minimum (indicated by the cross) shows the values of  $T_c = 455.84$  and  $\ln(t_x) = -5.355$  corresponding to the best fit.



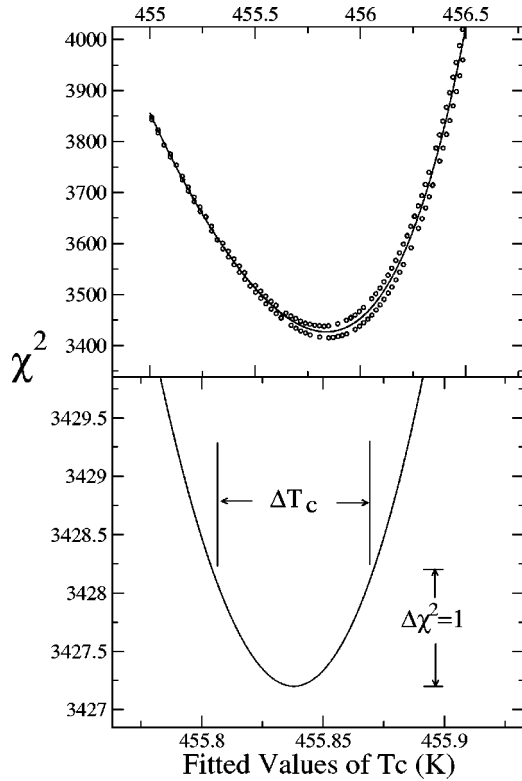


FIG. 5. Graphs of  $\chi^2$  vs value of  $T_c$  used in fit. (a) shows the minimum in  $\chi^2$  with a smooth function fit to the points. (b) shows the fitted curve with an indicated range corresponding to a change in  $\chi^2$  of 1.0. The value of  $T_c$  with error for this data set is  $T_c = 455.84 \pm 0.03$  K.

$455.84 \pm 0.03$ . While this range of  $T_c$  creates an uncertainty in  $\gamma$  on the order of the error from the least-squares analysis, the two effects should compound to increase the confidence limit on  $\gamma$  slightly. The value for the critical exponent from the particular data set in Fig. 3(a) is  $\gamma = 1.75 \pm 0.02$ . The fitted critical amplitude  $\chi_o$  is  $7.3 \pm 0.3 \times 10^{-3}$ .

It now becomes necessary to check for both dynamic and demagnetization effects in the data. It has already been remarked that a demagnetization factor equal to  $1/\chi_{max}$  provides an upper limit on the value of  $N$ . This assumption would lead to a value of  $N$  for the data in Fig. 3(a) to be about  $1/150$  or  $6.67 \times 10^{-3}$ . Numerical simulations show that once the value of the product  $N\chi$  approaches 0.05, the observed power-law behavior of the intrinsic susceptibility is lost [see Eq. (4)]. For this data set, this would occur at a temperature of 464.2 K, giving a value of  $\ln(t_x)$  approximately equal to  $-4.0$ . In other words, if we believe the above estimate for  $N$ , then no linear segment in double-log space would extend closer to  $T_c$  than this. The results in Fig. 3(b) clearly show the linear segment extending much lower than  $-4.0$ . The value of  $N$  must therefore be much smaller. The power-law behavior in fact deviates at a temperature of  $\approx 458.0$  K. If we take the “5% rule” a step further, the maximum value of  $N$  then becomes  $\approx 1/1632$  or  $6.1 \times 10^{-4}$ , a full order of magnitude lower than the previous estimate. This lower value is more in keeping with the value of  $N$  expected from geometric arguments and provides a new upper limit on  $N$ .

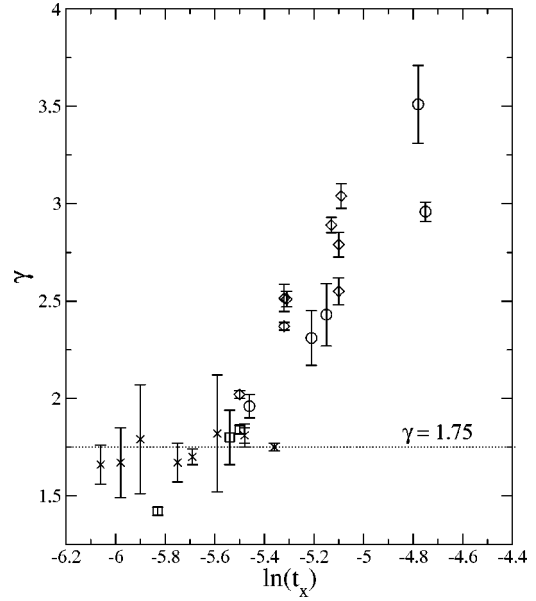


FIG. 6. Best-fit values of  $\gamma$  plotted as a function of reduced temperature cutoff,  $\ln(t_x)$ . Open circles represent films that are slightly less than 1.5 ML, squares are 1.5 ML films, diamonds are 1.75 ML, and X's are 2.0 ML.

Checking the saturation from dynamic effects requires a more definite knowledge of the time response of the moments as a function of temperature than these measurements currently allow. However, a simple calculation can be made on a theoretical basis. Near  $T_c$ , the relaxation time of the magnetization will undergo critical slowing down which, by theory, follows the formula

$$\tau(T) = \tau_o t^{-z\nu}, \quad (9)$$

where  $\nu$  is the critical exponent associated with the correlation length and  $z$  is the critical slowing down exponent. While there are very few experiments that measure the critical slowing down of the relaxation time  $\tau$  on ferromagnetic systems, theoretical simulations<sup>26,27</sup> suggest that the value of  $z$  should be  $\approx 2.2$  for the 2D Ising system. The value for  $\tau_o$  should be very small, on the order of inverse GHz to agree with ferromagnetic resonance frequencies.

To see no dynamic effect in  $\chi_{int}$  as per Eq. (5) requires  $(\omega\tau)^2 < 0.05$ . Using  $z\nu = 2.2$ ,  $\tau_o = 1 \times 10^{-9}$  s and  $\omega = (2\pi)150.0$  Hz, we find that  $\ln(t_x)$  will be  $-5.6$ . This is close to the fit value for  $\ln(t_x)$  and may be the reason for the saturation of the susceptibility. Better estimates of  $\tau_o$  and  $z\nu$  are required to pursue this question further.

## V. RESULTS FROM MANY FILMS

Critical power-law fitting was performed on a sample of 25 different measurements from many films grown between 1.5 and 2.0 ML. Figure 6 shows a plot of  $\gamma$  as a function of  $\ln(t_x)$  for all 25 measurements. For films with a small  $\ln(t_x)$ ,  $\gamma$  is consistently close to the 2D Ising value. For films with larger  $\ln(t_x)$ ,  $\gamma$  grows systematically larger. It is also apparent that the value of  $\gamma$  is correlated to the film thickness.

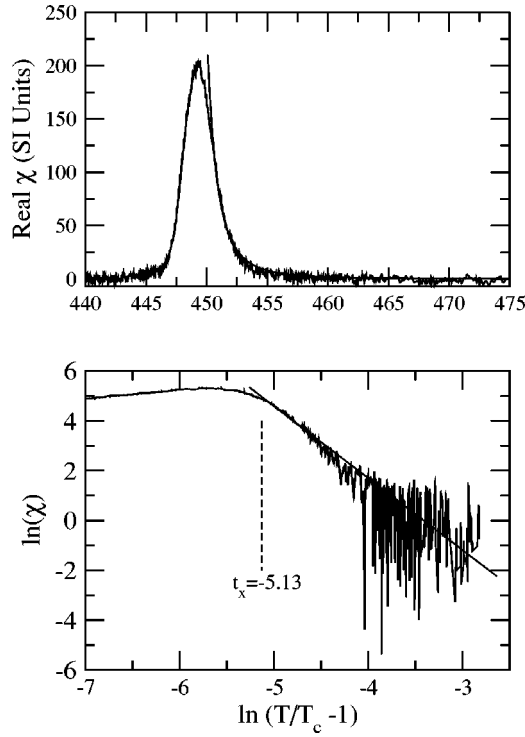


FIG. 7. Susceptibility measurement and fit for 1.75 ML film. Objective fitting algorithm fits a value of  $\gamma$  outside 2D Ising class. Solid line in both graphs represents the fit. Minimum in  $s^2$  occurs at  $T_c = 447.78$  K and  $\ln(t_x) = -5.13$  giving  $\gamma = 2.89 \pm 0.04$  and  $\chi_o = 5.3 \pm 0.5 \times 10^{-5}$ .

The following weighted and unweighted average results for  $\gamma$  can be given.

- (1) For bilayer films, the weighted average value of  $\gamma$  is  $1.75 \pm 0.015$  with an unweighted average of  $1.74 \pm 0.023$ .
- (2) For sesquilayer films, the weighted average is  $1.63 \pm 0.01$ . (This weighted average is suspect as there are only three data points with small individual error which do not overlap.) The unweighted average is  $1.68 \pm 0.13$ .
- (3) The weighted average value of  $\gamma$  for films with  $\ln(t_x)$  less than  $-5.35$  is  $1.76 \pm 0.01$ . The unweighted average is  $1.76 \pm 0.04$ . Most of the films with values of  $t_x$  in this last range are either 2.0 or 1.5 ML, but there is also one measurement at 1.75 ML and another just below 1.5 ML.

Films with a thickness of 2.0 ML and 1.5 ML consistently have the lowest values of  $\ln(t_x)$  and these are the films that give (on average) the 2D Ising result. If these data are reanalyzed by artificially increasing  $\ln(t_x)$ , the value of  $\gamma$  does not increase significantly. Figure 7 shows data for a 1.75 ML film which gives a non-Ising value of  $\gamma$ . It is clearly not meaningful to decrease  $\ln(t_x)$  for this data. It is thought that the higher values of  $t_x$  are an indicator of an as-yet not understood process that affects the power-law scaling when the film thickness is just below 1.5 ML or between 1.5 and 2.0 ML. It is possible that this process is also responsible for the high value of  $\gamma$  reported for films of 0.8 ML thickness.<sup>4</sup>

We have examined several possible explanations for this behavior. The first involved using corrections to scaling arguments<sup>28,29</sup> that should be taken into account for fitting data far away from  $T_c$ . If this is the case, then the effective

value of the exponent,  $\gamma_{eff}$ , is approximated by

$$\gamma_{eff} = \gamma - a\Delta|\bar{t}|^\Delta, \quad (10)$$

where  $a$  is a constant,  $\bar{t}$  is some “average temperature” representing the fitting range, and the exponent  $\Delta$  is close to 0.5 for Ising systems<sup>29</sup> regardless of the dimensionality of the system. If  $t_x$  is chosen as  $\bar{t}$ , the data in Fig. 6 can be reasonably described by Eq. (10) with  $\Delta \approx 3$ . The large discrepancy between the fitted and theoretical value of  $\Delta$  suggests that corrections to scaling are not the important factor here.

Another possibility for the rising value of  $\gamma$  is dimensional crossover from an Ising system to an anisotropic Heisenberg system as a function of temperature.<sup>30</sup> It is well known that true two-dimensional Heisenberg systems cannot support long-range magnetic order above  $T=0$ ,<sup>31</sup> but only a small amount of anisotropy is required to lift this restriction.<sup>2</sup> It is possible that the films that exhibit high values of  $\ln(t_x)$  are showing higher values of  $\gamma$  because they have a smaller anisotropy and the exponent is measured in a temperature range where it is still crossing over from one universality class to another. This explanation was also offered by Elmers *et al.*<sup>4</sup> for their 0.8 ML results, and it is interesting to note that the value of  $\ln(t_x)$  in their result would be  $-5.3$ , which is consistent with the onset of high  $\gamma$  values in this study. However, there are several arguments against this idea. First, none of the data with an Ising exponent show a crossover to larger  $\gamma$  when  $t_x$  is artificially increased. Second, none of the data used in this work, including those data sets that fit with a  $\ln(t_x)$  value less than  $-6$ , shows anything resembling a “break point” in the double-log slope indicating different critical power laws over different temperature ranges. Therefore, there is no clear indication that dimensional crossover is occurring. Finally, a reduced anisotropy should result in a change in the trend of the transition temperature as a function of thickness,<sup>32</sup> an effect which we do not observe.

The third possibility is that the films in the sensitive thickness range have a wider distribution of transition temperatures. It is easy to understand how this would affect the fitted slope. If some fractional area of the film undergoes a phase transition at a temperature slightly above the average “mean” value of  $T_c$  used to reduce the temperature for the logarithmic plot, then those areas will register as an artificially high slope in the fit. While the exact nature of the distribution is unknown, it is certain that any distribution with values above the  $T_c$  used in the fitting routine will increase the fitted exponent. To gauge the effect quantitatively, a series of data sets were modeled using a normalized Gaussian distribution of  $T_c$  and an intrinsic value of  $\gamma$  of 1.75. No significant increase is found to occur as long as the half-width is less than 0.25 K. A half-width in the distribution of  $T_c$  of just over 0.5 K to cause a 1% increase in  $\gamma$ , and a half-width of 1 K gives a fit exponent of 1.81, a 3.5% increase. To achieve a fitted value for  $\gamma$  of 3 (near the maximum fit value in the 25 measurements) requires a half-width of 2.5 K. It may be that the films less than 1.5 ML and between 1.5 and 2.0 ML are more sensitive to small structural inhomogeneities that give rise to a wider distribution of  $T_c$ . Films with a complete second monolayer will be more

homogeneous than films that are slightly thinner. We speculate that the distribution of transition temperatures may be related to the distribution of atoms that are located at step edges between the first and the incomplete second monolayer. For the complete 2 ML, the films should be very homogeneous and a narrow distribution may be expected. The 1.5 ML films have equal areas that are 1 ML and 2 ML thick, respectively, and as such present a uniform configuration of steps which have been shown<sup>22</sup> to give a correlated magnetic state. The 1.75 films are on the threshold of the percolation limit of the second monolayer and it is possible that slight structural deviations are more likely to cause a wider distribution of transition temperatures. This suggestion may also explain the high value of  $\gamma$  reported for 0.8 ML films.<sup>4</sup>

## VI. CONCLUSION

We report the results of fitting measurements of the magnetic ac susceptibility for critical power-law exponents. We

find the critical exponent for bilayer Fe/W(110) films to be  $1.75 \pm 0.02$  and for films in general with a value of  $t_x$  below  $4.75 \times 10^{-3}$ ,  $\gamma = 1.76 \pm 0.01$ . This result confidently places this system in the 2D Ising universality class. The fitting routine allows the simultaneous extraction of the critical exponent and the critical temperature from a single measurement of the susceptibility. There is evidence of another process which affects fitting of the susceptibility for certain thicknesses. This may be due to these films having a larger distribution of critical temperatures.

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<sup>33</sup>In these equations, the relaxation time of the magnetization is used, where  $dM/dt = -(1/\tau)(M - M_\infty)$ . Dissipation can also be expressed in terms of the relaxation of the effective field using the Landau-Lifshitz equation and the damping parameter  $\lambda$ . The two terms are related by  $\tau = \chi_{eff}/\lambda$ .