Symmetry of anisotropic exchange interactions in semiconductor nanostructures

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The symmetry of exchange interaction of charge carriers in semiconductor nanostructures (quantum wells and quantum dots) is analyzed. It is shown that the exchange Hamiltonian of two particles belonging to the same energy band can be universally expressed via pseudospin operators of the particles. The relative strength of the anisotropic exchange interaction is shown to be independent of the binding energy and the isotropic exchange constant.

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The reduced symmetry of semiconductor nanostructures suggests that the exchange interaction of charge carriers in such structures is not necessarily described by the isotropic (Heisenberg) spin Hamiltonian. In particular, the exchange interaction of electrons and holes in quantum wells and quantum dots is known to be extremely anisotropic, giving rise to a fine structure of nanostructure excitons.[1] Anisotropy of the electron-hole exchange interaction in the exciton is known also for bulk crystals where it is defined by the unit-cell symmetry and the exciton wave vector.[2] It has been shown[3] that the exchange interaction of conduction-band electrons is also anisotropic if the structure lacks inversion symmetry. The main term of the anisotropic exchange Hamiltonian in this case has the Dzyaloshinskii-Moriya form.[4] The electron-electron anisotropic exchange was subsequently widely discussed in relation to the quantum computing problem.[5–7] It has been recently detected experimentally via its contribution to the spin relaxation of donor-bound electrons in GaAs,[8] where it has been shown to put the upper boundary for the electron-spin lifetime at donor concentrations around 10^{16} cm^{-3}. However, it remained so far unclear whether or not the anisotropic spin Hamiltonian suggested in Ref. 3 is universal for all types of charge carriers, e.g., for two-dimensional holes. The issue of the dependence of the anisotropic exchange constant on the parameters of the localizing potential is also very sensitive, especially for the discussion on feasibility of quantum computation with solid-state spin systems.[6] The constant was so far calculated using approximate methods.[3,6] For instance, the applicability of the Heitler-London method, which is known to give an incorrect asymptotic expression for the isotropic exchange integral,[9,10] was recently questioned by Gor'kov and Krotkov.[11] Using the median-plane method,[9] they have obtained a correct asymptotic formula for the anisotropic exchange constant in a specific case of hydrogenlike centers in zinc blende semiconductors, different from that calculated earlier by the Heitler-London method.[3] However, their approach is not always applicable to coupled quantum dots, where the distance between quantum dots can be comparable to the quantum dot size. So far, no analytical expression for the anisotropic exchange constant has been obtained for this range of interdot distances, especially interesting for solid-state quantum computing.[12]

The collection of unsolved problems and blank spaces in the existing knowledge on the anisotropic exchange in semiconductor structures, given above, demonstrates the evident demand for a consistent theoretical analysis of the issue, based on a general approach. In this paper, we consider exchange interaction of two identical charge carriers localized in any symmetric double-well potential in a two-dimensional semiconductor structure. Using the pseudospin formalism allows us to obtain a universal spin Hamiltonian describing this class of systems.

Let us consider the exchange interaction of two identical charge carriers (electrons or holes), localized in two centrosymmetric potential hollows [further referred to as quantum dots (QD's)[13,14]] in a quasi-two-dimensional semiconductor structure [quantum well (QW)]. The QD's may be, for example, self-organized QD's;[14] otherwise, they can be induced by electrostatic potential of nanometer-sized gates[15] or impurity centers.[16] The distance between centers of the QD's will further be denoted as R_{12}. In quasi-two-dimensional structures, the fourfold degeneracy of the valence band, typical of cubic semiconductors, is lifted. The states at extremum points of two-dimensional subbands in absence of magnetic fields retain only the Kramers twofold degeneracy. Their wave functions can be written as $\Psi(\mathbf{r})u_\nu(\mathbf{r})$, where $\Psi(\mathbf{r}) = \varsigma(z)\psi(\rho)$ is an envelope function, ρ is the in-plane position vector, and $u_\nu(\mathbf{r})$ is a Bloch amplitude, $\nu = \pm 1/2$. The function $\varsigma(z)$ is defined by size quantization. The Bloch amplitudes $u_\nu(\mathbf{r})$ are two-component functions transformed into each other by the operator of time reversal:[17]

$$u_{+1/2}(\mathbf{r}) = A(\mathbf{r})\chi_{+1/2} + B(\mathbf{r})\chi_{-1/2},$$

$$u_{-1/2}(\mathbf{r}) = -i\hat{\sigma}_y u^*_{+1/2}(\mathbf{r}) = A^*(\mathbf{r})\chi_{-1/2} - B^*(\mathbf{r})\chi_{+1/2}, \quad (1)$$

where the Pauli operator $\hat{\sigma}_y$ acts upon spinors χ_μ, and $A(\mathbf{r})$ and $B(\mathbf{r})$ are functions of coordinates only.

This property allows to associate the Kramers index ν with an eigenvalue of a projection of a pseudospin operator $\mathbf{j}$ ($j = 1/2$) on some (generally fictitious) axis. The choice of basis functions for $\mathbf{j}$ is not unambiguous. It is limited only by the condition given by Eq. (1). In particular, for heavy holes with the projection of the angular momentum on the structure axis Z, equal to $J_z = \pm 3/2$, it is convenient to choose the functions as[1] $|j, +1/2\rangle = |J_z, -3/2\rangle$ and $|j, -1/2\rangle = |J_z, +3/2\rangle$. This choice allows to avoid phase multipliers which would otherwise appear at wave functions in the pseudospin

representation. For conduction-band electrons, the pseudospin coincides with the electron spin **s**. Linear transformations of pseudospin wave functions determined in the basis $\{u_{+1/2}, u_{-1/2}\}$ are equivalent to rotations of usual spinor functions:[17]

$$u_{+1/2}^{\alpha\beta\gamma} = \exp(i\gamma/2)[u_{+1/2}\exp(i\alpha/2)\cos(\beta/2) + u_{-1/2}\exp(-i\alpha/2)\sin(\beta/2)],$$

$$u_{-1/2}^{\alpha\beta\gamma} = \exp(-i\gamma/2)[-u_{+1/2}\exp(i\alpha/2)\sin(\beta/2) + u_{-1/2}\exp(-i\alpha/2)\cos(\beta/2)], \quad (2)$$

where α, β, and γ are analogs of Euler angles. Following the analogy, one can introduce the total pseudospin $\mathbf{I}=\mathbf{j}_1+\mathbf{j}_2$. Indeed, the Hilbert space of two-pseudospin wave functions $A_{\mu\nu}u_\mu(\mathbf{r}_1)u_\nu(\mathbf{r}_2)$ breaks into two subspaces invariant with respect to the simultaneous transformation of both pseudospins along Eq. (2) with the same α, β, and γ. The basis functions of these subspaces, $\xi_0=[u_{+1/2}(\mathbf{r}_1)u_{-1/2}(\mathbf{r}_2)-u_{-1/2}(\mathbf{r}_1)u_{+1/2}(\mathbf{r}_2)]/\sqrt{2}$ and ξ_{1M}, equal to $[u_{+1/2}(\mathbf{r}_1)u_{-1/2}(\mathbf{r}_2)+u_{+1/2}(\mathbf{r}_1)u_{-1/2}(\mathbf{r}_2)]/\sqrt{2}$ ($M=0$) or $u_{\pm1/2}(\mathbf{r}_1)u_{\pm1/2}(\mathbf{r}_2)$ ($M=\pm1$), are, obviously, eigenfunctions of the operators $\hat{I}^2$ and $\hat{I}_z$.

The general form of the one-particle Hamiltonian of the two-dimensional charge carrier in the pseudospin representation is

$$\hat{H}_1=\frac{\hbar^2}{2m}k^2+V(\boldsymbol{\rho})+\mathbf{h}(\mathbf{k})\cdot\mathbf{j}, \quad (3)$$

where $V(\rho)$ is an effective two-dimensional potential [averaged over z with the size-quantization wave function $\varsigma(z)$], m is the effective mass for the two-dimensional motion, and the ''spin-orbit field'' $\mathbf{h}(\mathbf{k})$ is a vector in the pseudospin space.[18] $\mathbf{h}(\mathbf{k})$ is an odd function of the components of the wave vector. It is not equal to zero if the structure lacks inversion symmetry (which is very typical for nanostructures). This is the case when either the crystal unit cell lacks inversion symmetry [bulk inversion asymmetry,[19] (BIA)] or the QW is asymmetric [structure inversion asymmetry[20] (SIA)]. The components of $\mathbf{h}(\mathbf{k})$ may be, or may not be, associated with certain Cartesian axes in the real space. In the two-dimensional case, $\mathbf{h}(\mathbf{k})$ is dominated by linear in $\mathbf{k}$ terms:[20–22]

$$h_\eta = A_{\eta\xi}k_\zeta, \quad (4)$$

where the matrix A is defined by the structure symmetry.

The problem we are going to solve is finding the fine structure of the ground state of the two-particle Hamiltonian (where SO represents spin orbit):

$$\hat{H}=\hat{H}_0+\hat{H}_{SO}, \quad (5)$$

where

$$\hat{H}_0=\frac{\hbar^2}{2m}k_1^2+\frac{\hbar^2}{2m}k_2^2+V_1(\rho_1)+V_2(\rho_2) + \int U_{12}(|\mathbf{r}_1-\mathbf{r}_2|)\varsigma^2(z_1)\varsigma^2(z_2)dz_1dz_1, \quad (6)$$

$$\hat{H}_{SO}=\mathbf{h}(\mathbf{k}_1)\cdot\mathbf{j}_1+\mathbf{h}(\mathbf{k}_2)\cdot\mathbf{j}_2, \quad (7)$$

and $U_{12}(|\mathbf{r}_1-\mathbf{r}_2|)$ is the operator of the Coulomb interaction between the two particles. Before tackling the effects of spin-orbit interaction in the form of Eq. (7) on the exchange interaction, we should reconsider the ground-state structure of the Hamiltonian H_0. It is indeed well known for vacuum electrons whose one-particle wave functions are $\Psi(\mathbf{r})\chi_\mu$, where the spinor χ_μ is not a function of coordinates. To the contrary, the Bloch amplitude u_ν does depend on coordinates, and, moreover, it may contain spinors with both $\mu=+1/2$ and $\mu=-1/2$. The exciton (an electron-hole pair) in a QD is a good example demonstrating that the exchange interaction of charge carriers may have a very different symmetry as compared to that of free electrons. The QD exciton fine structure[1] consists of two doublets, being thus quite different from the fine structure of a pair of vacuum electrons, i.e., the well-known singlet-triplet structure associated with the Heisenberg exchange.

In order to analyze the fine structure of H_0 for two particles belonging to the same subband, we first note that their behavior should be identical to that of bare electrons in all aspects but the Coulomb interaction. Indeed, although Bloch amplitudes are functions of coordinates within the unit cell, the one-particle operators of the kinetic energy and of the potential energy in Eq. (6) in the effective-mass approximation act upon envelope function, not Bloch amplitudes. Therefore, with respect to these one-particle operators, the Bloch amplitudes are just equivalent to spinors. To the contrary, calculating the Coulomb energy assumes taking integrals over the unit cell also. It is due to this fact that the symmetry of Bloch amplitudes of holes and electrons has an impact on the fine structure in the exciton.[1,23]

The fermionic wave functions of the two charge carriers can be written in the following form, similar to that of vacuum electrons:

$$\Psi_0(\mathbf{r}_1,\mathbf{r}_2)=[\Phi_0(\mathbf{r}_1,\mathbf{r}_2)+\Phi_0(\mathbf{r}_2,\mathbf{r}_1)]\xi_0,$$

$$\Psi_{1M}(\mathbf{r}_1,\mathbf{r}_2)=[\Phi_1(\mathbf{r}_1,\mathbf{r}_2)-\Phi_1(\mathbf{r}_2,\mathbf{r}_1)]\xi_{1M}, \quad (8)$$

where $\Phi_0(\mathbf{r}_1,\mathbf{r}_2)$ and $\Phi_1(\mathbf{r}_1,\mathbf{r}_2)$ are two-particle envelope functions defined so that each particle is most likely to be found near its ''home'' center, while $\Phi_0(\mathbf{r}_2,\mathbf{r}_1)$ and $\Phi_1(\mathbf{r}_2,\mathbf{r}_1)$ correspond to interchanged particle positions. To determine the structure of respective energy levels, we should recall a property of the Bloch amplitudes u_ν, which results from their symmetry with respect to time reversal, and is an equivalent formulation of the Kramers theorem. As follows from the Kramers theorem, the states symmetric with respect to time reversal remain degenerated unless magnetic field is applied. Mathematically, this means that matrix elements of any function of coordinates (not containing deriva-

tives or spin operators) between $u_{+1/2}(\mathbf{r})$ and $u_{-1/2}(\mathbf{r})$ are zero, while diagonal matrix elements are equal to each other:

$$\langle u_{+1/2}(\mathbf{r})|f(\mathbf{r})|u_{-1/2}(\mathbf{r})\rangle = \langle u_{-1/2}(\mathbf{r})|f(\mathbf{r})|u_{+1/2}(\mathbf{r})\rangle = 0,$$

$$\langle u_{+1/2}(\mathbf{r})|f(\mathbf{r})|u_{+1/2}(\mathbf{r})\rangle = \langle u_{-1/2}(\mathbf{r})|f(\mathbf{r})|u_{-1/2}(\mathbf{r})\rangle. \quad (9)$$

These equalities can be derived straightforwardly from Eq. (1).

Using Eq. (9), one can easily find that

$$\langle \Psi_0(\mathbf{r}_1,\mathbf{r}_2)|U_{12}(|\mathbf{r}_1-\mathbf{r}_2|)|\Psi_{1M}(\mathbf{r}_1,\mathbf{r}_2)\rangle = 0,$$

$$\langle \Psi_0(\mathbf{r}_1,\mathbf{r}_2)|U_{12}(|\mathbf{r}_1-\mathbf{r}_2|)|\Psi_0(\mathbf{r}_1,\mathbf{r}_2)\rangle \neq \langle \Psi_{1M}(\mathbf{r}_1,\mathbf{r}_2)|U_{12}(|\mathbf{r}_1-\mathbf{r}_2|)|\Psi_{1M'}(\mathbf{r}_1,\mathbf{r}_2)\rangle = \mathrm{const}\,(\delta_{MM'}). \quad (10)$$

Thus, the Coulomb interaction retains the singlet-triplet structure of the ground state of two identical charge carriers. Exactly like in the case of two bare electrons, two-particle states with the same total pseudospin I are degenerated. Consequently, the Hamiltonian of the exchange interaction in terms of pseudospin operators takes the Heisenberg form:

$$\hat{H}_S = -2\Delta(\mathbf{j}_1\cdot\mathbf{j}_2+1), \quad (11)$$

where Δ is a constant to be determined for each specific case.

Now we can consider the effect of the spin-orbit terms given by Eq. (7) on the exchange interaction. In the following, we will choose the axis X along the straight line connecting the localization centers (QD's). To handle the spin-orbit terms, we make use of a unitary transformation proposed by Levitov and Rashba[24] who used it to eliminate spin-orbit terms in the one-dimensional case. The matrix T defined as

$$T = \exp\left[i\frac{2m}{\hbar^2}\sum_\alpha A_{\alpha x}(\hat{J}_{1\alpha}x_1 + \hat{J}_{2\alpha}x_2)\right] \quad (12)$$

transforms the Hamiltonian [Eq. (5)] into the form

$$T\hat{H}T^{-1} = \hat{H}' = \hat{H}'_0 + \hat{H}'_{SO}, \quad (13)$$

where

$$\hat{H}'_0 = \frac{\hbar^2}{2m}k_1'^2 + \frac{\hbar^2}{2m}k_2'^2 + V_1(\rho'_1) + V_2(\rho'_2) + \int U_{12}(|\mathbf{r}'_1-\mathbf{r}'_2|)\varsigma^2(z)dz - \sum_\alpha \frac{m(A_{\alpha x})^2}{\hbar^2} \quad (14)$$

and

$$\hat{H}'_{SO} = \sum_\alpha A_{\alpha y}(k_{1y}\hat{J}'_{1\alpha} + k_{2y}\hat{J}'_{2\alpha}) \quad (15)$$

where $\mathbf{j}'_1 = T\mathbf{j}_1T^{-1}$, $\mathbf{j}'_2 = T\mathbf{j}_2T^{-1}$.

The Hamiltonian $\hat{H}'$ does not contain spin-orbit terms and therefore results in the exchange interaction in the form of Eq. (11):

$$\hat{H}'_S = -2\Delta(\mathbf{j}'_1\cdot\mathbf{j}'_2+1). \quad (16)$$

Due to the axial symmetry of the system, the matrix elements of k_{1y} and k_{2y}, calculated on the ground-state eigenfunctions of $\hat{H}'_0$ [they can be obtained from Eq. (8) by the transformation with the matrix T, which does not affect their dependence on y], are exactly equal to zero. The same is true for all the odd powers of k_{1y} and k_{2y}. Therefore, $\hat{H}'_{SO}$ does not affect the exchange interaction.

Finally, to obtain the exchange Hamiltonian in the nontransformed basis, one should substitute the expressions for $\mathbf{j}'_1$ and $\mathbf{j}'_2$ into Eq. (16). Since the transformation T is a rotation through the angle $(2m/\hbar^2)\sqrt{\Sigma_\nu A_{\nu x}A_{\nu x}}x$ around the vector $A_{\nu x}$ in the pseudospin space, $\hat{H}_S$ in the nontransformed basis is not unambiguously defined; it depends on the coordinates x_1 and x_2 at which we take the pseudospin operators $\mathbf{j}_1$ and $\mathbf{j}_2$. A natural choice is to define them at the centers of corresponding QD's; for instance, this definition allows us to write the Zeeman interaction in the usual form, $\hat{H}_Z = \mu_B g_{\alpha\beta} j_\alpha B_\beta$, where B is the magnetic field, μ_B is the Bohr magneton, and $g_{\alpha\beta}$ is a symmetric tensor g factor[25] whose principal directions do not depend on the envelope wave function of the localized particle. This way, we come to the expression for $\hat{H}_S$ obtained in Ref. 3:

$$\hat{H}_S = -2\Delta(1 + \mathbf{j}_1\cdot\mathbf{j}_2\cos\gamma + (\mathbf{d}\cdot\mathbf{j}_1)(\mathbf{d}\cdot\mathbf{j}_2)(1-\cos\gamma) + \mathbf{d}\cdot[\mathbf{j}_1\times\mathbf{j}_2]\sin\gamma), \quad (17)$$

where $\gamma = (2m/\hbar^2)\sqrt{\Sigma_\nu A_{\nu x}A_{\nu x}}R_{12}$ and $\mathbf{d}$ is a unit vector in the pseudospin space, defined so that $d_\nu = A_{\nu x}/\sqrt{\Sigma_\nu A_{\nu x}A_{\nu x}}$. The first anisotropic term has the form of pseudodipole interaction,[25] and the second one, of the Dzyaloshinskii-Moriya interaction.[4] At small γ, the Dzyaloshinskii-Moriya interaction dominates.

Equation (17) demonstrates a remarkable universality of the exchange interaction in two-dimensional semiconductor nanostructures; this form of the Hamiltonian holds for both electrons and holes, for any type of centrosymmetric localizing potentials. Equation (17) is valid for identical as well as for different QD's. Moreover, the angle γ characterizing the relative strength of the anisotropic exchange depends only on the distance between the QD's and the orientation of the pair of QD's with respect to the crystal axes. It is not sensitive to binding energies of the charge carriers in the QD's and to the value of the isotropic exchange constant Δ.

The value of γ can be now easily calculated for those structures where the components of the matrix A are known.

In [100] oriented GaAs quantum wells the dominating BIA terms are[22] $A_{yy} = -A_{xx} = [\alpha\hbar^3/(m\sqrt{2mE_g})]\langle k_z^2\rangle$ and $A_{xy} = A_{yx} = 0$, where $\alpha \approx 0.065$, Ref. 26 (here coordinates x and y are taken along the cubic crystal axes). This gives $\gamma = (2\alpha\hbar/\sqrt{2mE_g})\langle k_z^2\rangle R_{12}$. For example, in a 5-nm-wide GaAs QW with infinitely high barriers, $\gamma = (3$

$\times 10^5$ cm^{-1})R_{12}. For example, if the distance between centers of the QD's is 20 nm, we obtain $\gamma = 0.6$.

For Rashba terms, $A_{yy} = A_{xx} = 0$ and $A_{xy} = -A_{yx} = a$. In a single-side modulation-doped *n*-type Si/SiGe quantum well, the constant a of 1.1×10^{-12} eV cm was measured.[27] This gives $\gamma = (2ma/\hbar^2)R_{12} \approx (6.7 \times 10^2$ cm$^{-1})R_{12}$.

Bulk inversion asymmetry terms for holes in zinc blende semiconductors include both cubic and linear in k terms.[28] The cubic term H_{3V} has the same symmetry as the Dresselhaus term for electrons, with the constant $\alpha_V \hbar^3/m\sqrt{2mE_g}$, where m is the conduction-band electron mass and $\alpha_V \approx 0.1$ for GaAs. The linear term is given by the expression

$$H_{1V} = \frac{2}{\sqrt{3}} \varkappa (\mathbf{k} \cdot \mathbf{\Omega}), \tag{18}$$

where $\hat{\Omega}_z = \hat{J}_z(\hat{J}_x^2 - \hat{J}_y^2) + (\hat{J}_x^2 - \hat{J}_y^2)\hat{J}_z$ (other components of $\mathbf{\Omega}$ are obtained by cyclic permutation of indices), $\mathbf{J}$ is the hole spin operator ($J = 3/2$), and $\varkappa \approx 10^{-10}$ eV cm. Taking matrix elements of H_{1V} and H_{3V} within pairs of the states with $J_z = \pm 1/2$ (light holes) and $J_z = \pm 3/2$ (heavy holes), and going to the pseudospin notation, we obtain for a [100] QW:

$$\mathbf{h}_l(\mathbf{k}) = -\left(2\frac{\alpha_V \hbar^3}{m\sqrt{2mE_g}} \langle k_z^2 \rangle - \sqrt{3}\varkappa \right)(k_x \mathbf{e}_x - k_y \mathbf{e}_y),$$

$$\mathbf{h}_h(\mathbf{k}) = -2\sqrt{3}\varkappa (k_x \mathbf{e}_x - k_y \mathbf{e}_y), \tag{19}$$

where $\mathbf{h}_l(\mathbf{k})$ and $\mathbf{h}_h(\mathbf{k})$ are spin-orbit fields [see Eq. (3)] for light and heavy holes, respectively, $\mathbf{e}_x$ and $\mathbf{e}_y$ are unit vectors along X and Y, respectively. Consequently,

$$\gamma_l = \left(\frac{4\alpha_V \hbar m_l}{m\sqrt{2mE_g}} \langle k_z^2 \rangle - \frac{2\sqrt{3}m_l}{\hbar^2}\varkappa \right) R_{12}$$

for light holes, and $\gamma_h = (4\sqrt{3}m_h/\hbar^2)\varkappa R_{12}$ for heavy holes, where m_l and m_h are effective masses of the light and heavy holes, respectively, corresponding to their motion along the QW plane. For example, in a 10-nm-wide GaAs QW, $\gamma_l \approx (5 \times 10^5$ cm$^{-1})R_{12}$, $\gamma_h \approx (10^5$ cm$^{-1})R_{12}$.

The symmetry of exchange interactions has been discussed in relation to feasibility of quantum computation with spins of localized electrons in semiconductor nanostructures.[3,5,6] A necessary condition for practical quantum computing to become possible is that the error probability per quantum gate be less than a certain value (of the order of 10^{-5}).[29] As shown in Ref. 6, there exists a way of performing exchange-mediated quantum gates, which allows to avoid errors caused by the anisotropy, provided the direction of **d** remains constant when the isotropic exchange constant is changed. The above consideration shows that this is indeed the case as long as the direction of $\mathbf{R}_{12}$ and the matrix of spin-orbit terms A do not change. The latter may change, in principle, when the gate is performed by applying electric fields that can bring about additional SIA (Rashba) terms. In this specific situation, the anisotropy-induced errors can be still suppressed using time-symmetric gate pulses, as proposed recently by Stepanenko *et al.*[30] Since the anisotropic component of the exchange interaction is present in practically all types of semiconductor structures currently considered prospective for qubit implementation, and typical values of γ are, according to the above estimations, not small; measures to suppress the uncontrollable effect of anisotropy (for example, as proposed in Ref. 30) should be taken in any design of exchange-mediated quantum gates. On the other hand, large values of γ are favorable for quantum-computing algorithms employing anisotropy of the exchange interaction.[7]

Another issue that is worth mentioning in relation with the quantum-computing problem is a possibility to use hole pseudospins as qubits. Indeed, according to Eqs. (11) and (17), pseudospins of holes interact exactly the same way as spins of electrons. Consequently, all the quantum gates proposed to be implemented with electron spins can, in principle, be done with holes.[31] Recent experiments[32,33] have shown that quantum dot holes can have quite long spin lifetimes (of the order of nanoseconds) in zero or small magnetic fields. On the other hand, holes have an advantage of being not coupled with lattice nuclei by the contact Fermi interaction[18,34] (the remaining dipole-dipole interaction is several orders of magnitude weaker). The contact hyperfine interaction of electron spins with hundreds of thousands of nuclei within the QD results in a complex pattern of spin relaxation[35,36] and may present a formidable source of decoherence because of the high entropy of the nuclear-spin reservoir. Though it has been proposed to suppress the nuclear-spin-induced decoherence either with magnetic fields or using dynamical polarization of nuclear spins,[37] this would impose additional requirements to quantum-computer designs, while existing requirements are already extremely stringent. These considerations suggest that holes in quantum dots may be good candidates for modeling few-qubit systems virtually decoupled from the nuclear-spin environment (though building a scalable quantum computer with holes would face the same major difficulties[38] as all the other approaches discussed so far).

In conclusion, the exchange interaction of charge carriers (electrons or holes) localized in two-dimensional semiconductor structures is shown to be described by a universal Hamiltonian in terms of carriers' pseudospins. It has the Heisenberg form unless spin-orbit terms, linear in the carrier wave vector, are present in the total Hamiltonian of the system. In this latter case, anisotropic contributions having both Dzyaloshinskii-Moriya and pseudodipole form arise. The ''rotation angle'' γ, characterizing the relative strength of the anisotropic exchange, linearly depends on the distance between the localization centers and does not depend on binding energies of the carriers.

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