High-field transport of electrons and radiative effects using coupled force-balance and Fokker-Planck equations beyond the relaxation-time approximation

Danhong Huang, T. Apostolova, P. M. Alsing, and D. A. Cardimona
Air Force Research Laboratory, Space Vehicles Directorate, Kirtland Air Force Base, New Mexico 87117, USA
(Received 3 September 2003; revised manuscript received 17 October 2003; published 27 February 2004)

The dynamics of a many-electron system under both dc and infrared fields is separated into a center-of-mass and a relative motion. The first-order force-balance equation is employed for the slow center-of-mass motion of electrons, and the Fokker-Planck equation is used for the ultrafast relative scattering motion of degenerate electrons. This approach allows us to include the anisotropic energy-relaxation process which has been neglected in the energy-balance equation in the past. It also leads us to include the anisotropic coupling to the incident infrared field with different polarizations. Based on this model, the transport of electrons is explored under strong dc and infrared fields by going beyond the relaxation-time approximation. The anisotropic dependence of the electron distribution function on the parallel and perpendicular kinetic energies of electrons is displayed with respect to the dc field direction, and the effect of anisotropic coupling to an incident infrared field with polarizations parallel and perpendicular to the applied dc electric field is shown. The heating of electrons is more accurately described beyond the energy-balance equation with the inclusion of an anisotropic coupling to the infrared field. The drift velocity of electrons is found to increase with the amplitude of the infrared field due to a suppressed momentum-relaxation process (or frictional force) under parallel polarization but decreases with the amplitude due to an enhanced momentum-relaxation process under perpendicular polarization.

DOI: 10.1103/PhysRevB.69.075214 PACS number(s): 72.20.Ht, 61.82.Fk, 61.80.Az, 72.10.Di

I. INTRODUCTION

High-field transport of electrons is usually referred to when the electric field applied to the system is no longer small and the current-voltage relation deviates from linear-response theory. There are many theories that have been proposed to describe this phenomenon since 1930. However, the semiclassical Boltzmann transport equation with a drift term still seems to be the only one amenable to practical calculations. In practice, however, solution of the Boltzmann transport equation also becomes a laborious task when the system goes beyond the linear regime. Fröhlich and Paranjape[1] first utilized a displaced Maxwellian distribution function to describe the electron transport in insulators and semiconductors. Later, Arai[2] applied a similar model to describe electron transport in metals at low temperatures but by means of a Fermi-Dirac distribution function, in which a finite electron temperature was predicted under an applied field even when the lattice temperature became zero. Shortly after that, Lei and Ting[3] proposed the coupled first-order force-balance and energy-balance equations to describe electrons in semiconductors and metals at both low and high temperatures by assuming an isotropic quasi-thermal-equilibrium (Fermi-Dirac) distribution for hot electrons with a temperature different from the lattice temperature.

The effect of acceleration of electrons under a dc field can be included in the Boltzmann transport equation using a field-induced drift term after an acceleration ansatz has been employed for uncorrelated scatters.[4] This approach was recently generalized to study the drifting motion of electrons under a low-frequency ac bias.[5] There appears to be much less justification for treating electron transport with an ac field oscillating at optical frequencies by directly adding a drift term to the Boltzmann transport equation even though the resulting conductivity is inversely proportional to the square of the optical frequency, since the time-average velocity of an electron under an optical field is in fact zero between two successive scattering events. For weak external fields, linear-response theory[6] or a linear-field expansion of the Boltzmann transport equation[4] could be applied. In this weak-field limit, a complete approach for treating electron-phonon drag effects of any degree under a magnetic field was proposed by simultaneously solving Boltzmann equations for both electrons and phonons.[7] However, these theories fail to produce correct results for high-field electron transport.

It has been known for quite a long time that the drift of electrons under a dc field can be regarded as a field-driven center-of-mass (collective) motion of many electrons.[8] The scattering of electrons by the lattice or by impurities can then be considered as a motion relative to this center of mass. A uniform external optical field is also known to couple only to the center-of-mass motion. The ions on the lattice and ionized impurity atoms in the material system remain stationary with respect to the moving center of mass. Therefore, the center-of-mass motion couples to the relative scattering motion. This causes a frictional force to act on each drifting electron due to lattice or impurities.[3] As a result, the acceleration of electrons under the dc field will be opposed by this friction. The classically measurable frictional force is a quantum-statistical average of the friction acting on each electron, involving a many-electron relative scattering motion, and therefore depends on the occupation of electrons in different states. On the other hand, the distribution of electrons in various states is determined by the relative elastic and inelastic scattering motions of electrons, including Coulomb scattering, phonon scattering, and impurity scattering.

The existence of a dc or an optical field couples the center-of-mass motion of electrons with their relative scattering motion. In this situation, the dc drift modifies the energy conservation in electron scattering with phonons by adding a

Doppler shift along the dc field direction, causing the scattering process to be anisotropic.[3] The time-dependent drift, on the other hand, anisotropically modifies the strength of the electron scattering as well as the energy conservation at the same time.[9,10] For a spatially uniform optical field incident on semiconductors, direct photon absorption is forbidden when the photon energy is smaller than the band gap. However, the incident photons can still be absorbed by electrons with the help of their scattering with phonons, which provides momentum to the electrons.

The simplest way to treat a steady-state distribution of electrons is the introduction of an electron temperature for a Fermi-Dirac function of hot electrons. The value of the electron temperature, different from the lattice temperature, can be obtained by employing a first-order energy-balance equation for the relative motion of electrons.[3] However, the assumption of an isotropic Fermi-Dirac function for hot electrons cannot be justified even in steady state when the external field exists for all time. In the limits of strong optical field and zero drift velocity, the exact solution of the Boltzmann scattering equation including the electron-electron interaction shows the existence of multiple side peaks in the electron distribution as a function of electron kinetic energy.[10] Thus, the electron distribution is nonequilibrium and the energy-balance formalism fails.[11] After the optical field has been turned off, the electron distribution quickly relaxes into a quasiequilibrium Fermi-Dirac function through electron-electron interaction, bringing the energy-balance equation back into validity. The energy-balance equation cannot be justified when either the drift velocity is large (the Doppler shift is comparable to the phonon energy) or the optical field is strong. In addition, the energy-balance equation excludes the effect of electron-electron scattering on the electron temperature. A more rigorous method involves the use of the Boltzmann scattering equation without a drift term for anisotropic scattering of electrons in relative motion. In the presence of an intense optical field only, this approach has been successfully applied to study laser damage in dielectric and semiconductor materials.[9,10,12] However, the incoherent Boltzmann scattering theory cannot be applied to study the dynamics of electrons if a dipole coupling of electrons to the external optical field exists when either the photon energy becomes greater than the band gap or a spatially nonuniform optical field is incident on the system after diffraction by a surface grating. In this case, a full coherent density-matrix theory[13] with off-diagonal matrix elements is required. One advantage of the current approach is that it includes a generalization of the definition of electron temperature for non-equilibrium electrons. In addition, the extension of the current approach to one that incorporates coherent density-matrix equations[13] for the relative motion of electrons is straight forward when pump-laser-induced interband excitation exists.

In this paper, we use a first-order force-balance equation to describe electron drift due to a slow center-of-mass motion under a dc electric field. At the same time, we adopt the Boltzmann scattering equation without a drift term for the ultrafast process of anisotropic scattering of electrons with phonons and impurities in a relative motion by assuming that no dipole coupling to the external optical field exists in the system. In order to highlight the physics and simplify the calculation of high-field electron transport in the presence of both dc and optical fields, we expand the Boltzmann scattering equation up to second order for degenerate electrons to obtain the Fokker-Planck equation including Joule heating and antidiffusion terms.[12] In this way, the anisotropic distribution of electrons and the anisotropic scattering of electrons under an incident optical field with different polarizations can be fully incorporated into our model. In addition, the heating of electrons can be more precisely described beyond the isotropic Fermi-Dirac function of hot electrons used in a first-order energy-balance equation.

The organization of the paper is as follows. In Sec. II, we introduce our model and derive the coupled force-balance and Boltzmann scattering equations by separating the center-of-mass and relative motions of electrons. After the expansion of the Boltzmann scattering equation we obtain the Fokker-Planck equation to simplify the calculation of high-field electron transport beyond the relaxation-time approximation. Analytical expressions for the expansion coefficients are presented in Sec. III. Numerical results are displayed in Sec. IV for the dependence of anisotropic distributions of electrons on the parallel and perpendicular kinetic energies with respect to the dc field direction, as well as for the dependence of drift velocities and heating of electrons on polarizations of the incident optical field parallel and perpendicular to an applied dc electric field. The paper is briefly concluded in Sec. V.

II. MODEL

Let us consider a bulk semiconductor, e.g., GaAs, which is doped with electron concentration σ_{3D} to form a three-dimensional (3D) electron gas. In this section, we first separate the dynamics of a many-electron system into a center-of-mass motion plus a relative motion under both dc and infrared fields. The equation for the center-of-mass motion of electrons is built after a quantum-statistical average is taken to obtain a classical frictional force. The relative motion of electrons is studied by using the Boltzmann scattering equation including anisotropic scattering of electrons with phonons and impurities beyond the relaxation-time approximation. After that, the Fokker-Planck equation for degenerate electrons including Joule heating and antidiffusion terms is derived by expanding the Boltzmann scattering equation to second order, which significantly simplifies the calculation of high-field electron transport.

A. Center-of-mass and relative motions

In the presence of a spatially uniform infrared field with a vector potential $\vec{A}(t)$ and a dc field $\vec{E}_{dc}$, the Hamiltonian of an *interacting* three-dimensional electron gas can be written as

$$\mathcal{H}=\frac{1}{2m^*}\sum_i(\hat{\vec{p}}_i-e\vec{A})^2+\sum_{i<j}\frac{e^2}{4\pi\epsilon_0\epsilon_r|\vec{r}_i-\vec{r}_j|}$$
$$-e\sum_i\vec{r}_i\cdot\vec{E}_{dc}+\sum_{i,a}U^{imp}(\vec{r}_i-\vec{R}_a)$$
$$-\sum_{i,\ell}\vec{u}_\ell\cdot\vec{\nabla}_{\vec{r}_i}U^{ion}(\vec{r}_i-\vec{R}_\ell), \quad (1)$$

where $i=1,2,\ldots,N_e$ is the index of N_e electrons, $a=1,2,\ldots,N_a$, is the index for N_a impurity atoms, $\ell=1,2,\ldots,N_\ell$ is the index for N_ℓ lattice ions, $\vec{r}_i$ is the position vector for the ith electron, $\vec{R}_a$ and $\vec{R}_\ell$ are the position vectors of impurity atoms and lattice ions, $\vec{u}_\ell$ represents the ion displacement from the thermal equilibrium position, m^* is the effective mass of electrons, and ϵ_r is the relative dielectric constant of the host material. The single-electron momentum operator is $\hat{\vec{p}}_i=-i\hbar\vec{\nabla}_{\vec{r}_i}$, and both the impurity potential $U^{imp}(\vec{r}_i-\vec{R}_a)$ and the ion potential $U^{ion}(\vec{r}_i-\vec{R}_\ell)$ are included. We first define the center-of-mass momentum and position vectors by

$$\widehat{\vec{P}^c}=\sum_{i=1}^{N_e}\hat{\vec{p}}_i, \quad \vec{R}^c=\frac{1}{N_e}\sum_{i=1}^{N_e}\vec{r}_i, \quad (2)$$

and those for the relative motion by

$$\widehat{\vec{p}'}_i=\hat{\vec{p}}_i-\frac{1}{N_e}\widehat{\vec{P}^c}, \quad \vec{r}_i{}'=\vec{r}_i-\vec{R}^c. \quad (3)$$

By using the center-of-mass and relative momentum and position vectors defined above, we can separate the total Hamiltonian, including the Hamiltonians of electrons and phonons, into one center-of-mass Hamiltonian $\mathcal{H}_{cm}$ and another relative Hamiltonian $\hat{\mathcal{H}}_{rel}$, given by

$$\mathcal{H}_{cm}=\frac{(\widehat{\vec{P}^c})^2}{2N_em^*}+\frac{N_ee^2A^2}{2m^*}-\frac{e}{m^*}\vec{A}\cdot\widehat{\vec{P}_c}-N_ee\vec{E}_{dc}\cdot\vec{R}^c, \quad (4)$$

$$\hat{\mathcal{H}}_{rel}=\sum_{\vec{k},\sigma}\varepsilon_k\hat{a}^\dagger_{\vec{k}\sigma}\hat{a}_{\vec{k}\sigma}+\sum_{\vec{q},\lambda}\hbar\omega_{q\lambda}\hat{b}^\dagger_{\vec{q}\lambda}\hat{b}_{\vec{q}\lambda}$$
$$+\frac{1}{2}\sum_{\vec{k},\vec{k}',\sigma,\sigma'}\sum_{\vec{q}}\frac{e^2}{\epsilon_0\epsilon_r q^2\mathcal{V}}\hat{a}^\dagger_{\vec{k}+\vec{q}\sigma}\hat{a}^\dagger_{\vec{k}'-\vec{q}\sigma'}\hat{a}_{\vec{k}'\sigma'}\hat{a}_{\vec{k}\sigma}$$
$$+\sum_{\vec{k},\sigma}\sum_{\vec{q},\lambda}C_{q\lambda}(\hat{b}_{\vec{q}\lambda}+\hat{b}^\dagger_{-\vec{q}\lambda})e^{i\vec{q}\cdot\vec{R}^c}\hat{a}^\dagger_{\vec{k}+\vec{q}\sigma}\hat{a}_{\vec{k}\sigma}$$
$$+\sum_{\vec{k},\sigma}\sum_{\vec{q},a}U^{imp}(q)e^{i\vec{q}\cdot(\vec{R}^c-\vec{R}_a)}\hat{a}^\dagger_{\vec{k}+\vec{q}\sigma}\hat{a}_{\vec{k}\sigma}, \quad (5)$$

where the infrared field is treated classically, $\mathcal{V}$ is the volume of the system, $\hbar\omega_{q\lambda}$ is the phonon energy with wave number q for mode λ (totally three modes), $\varepsilon_k=\hbar^2k^2/2m^*$ is the kinetic energy of electrons with wave number k, and the index $\sigma=\pm1$ is for up and down spin states for electrons. Here, we use $\hat{a}^\dagger_{\vec{k}\sigma}$ ($\hat{a}_{\vec{k}\sigma}$) to represent the creation (annihilation) operator of electrons and $\hat{b}^\dagger_{\vec{q}\lambda}$ ($\hat{b}_{\vec{q}\lambda}$) to denote the creation (annihilation) operator of phonons. $U^{imp}(q)$ is the Fourier transform of the impurity potential and $C_{q\lambda}$ is the electron-phonon coupling constant (given in Sec. II B). The existence of the infrared field causes the ions in the lattice (or ionized impurity atoms) to oscillate with time relative to the electrons. This driven oscillation transfers kinetic energy from the lattice to the electrons through the electron-phonon coupling. The quantum mechanics for the electron system gives rise to the following operator commutations[14] with $N_e\gg1$:

$$[\vec{r}_i{}',\widehat{\vec{P}^c}]=[\vec{R}^c,\widehat{\vec{p}'}_i]=0,$$
$$[R^c_\alpha,\hat{P}^c_\beta]=i\hbar\delta_{\alpha\beta},$$
$$[r'_{i\alpha},\hat{p}'_{j\beta}]=i\hbar\delta_{\alpha\beta}\left[\delta_{ij}-\frac{1}{N_e}\right]\approx i\hbar\delta_{\alpha\beta}\delta_{ij}, \quad (6)$$

where $\alpha,\beta=x,y,z$. By using these operator commutations, we get two Heisenberg equations for the center-of-mass motion of electrons:

$$\frac{d}{dt}\widehat{\vec{P}^c}=\frac{1}{i\hbar}[\widehat{\vec{P}^c},\mathcal{H}_{cm}+\hat{\mathcal{H}}_{rel}]$$
$$=N_ee\vec{E}_{dc}-i\sum_{\vec{q},\lambda}C_{q\lambda}\vec{q}e^{i\vec{q}\cdot\vec{R}^c}(\hat{b}_{\vec{q}\lambda}+\hat{b}^\dagger_{-\vec{q}\lambda})\hat{\rho}_{\vec{q}}$$
$$-i\sum_{\vec{q},a}U^{imp}(q)\vec{q}e^{i\vec{q}\cdot(\vec{R}^c-\vec{R}_a)}\hat{\rho}_{\vec{q}}, \quad (7)$$

$$\hat{\vec{u}}=\frac{d}{dt}\vec{R}^c=\frac{1}{i\hbar}[\vec{R}^c,\mathcal{H}_{cm}+\hat{\mathcal{H}}_{rel}]=\frac{\widehat{\vec{P}^c}}{N_em^*}-\frac{e}{m^*}\vec{A}, \quad (8)$$

where $\hat{\rho}_{\vec{q}}=\Sigma_{\vec{k},\sigma}\hat{a}^\dagger_{\vec{k}+\vec{q}\sigma}\hat{a}_{\vec{k}\sigma}$ is the density-wave operator for electrons, $\vec{A}(t)=(\vec{E}_{op}/\Omega_f)\cos(\Omega_ft)$, and $\vec{E}_{op}(t)=\vec{E}_{op}\sin(\Omega_ft)$. Here, Ω_f is the infrared-field angular frequency and E_{op} is its amplitude. Applying a quantum-statistical average $\langle\langle\cdots\rangle\rangle$ to both Eqs. (7) and (8), we define the following quantities:

$$N_em^*\frac{d}{dt}\vec{u}_0\equiv\left\langle\left\langle\frac{d}{dt}\widehat{\vec{P}^c}\right\rangle\right\rangle, \quad (9)$$

$$\vec{u}_d\equiv\left\langle\left\langle\frac{d}{dt}\vec{R}^c\right\rangle\right\rangle=\vec{u}_0-\frac{e}{m^*\Omega_f}\vec{E}_{op}\cos(\Omega_ft), \quad (10)$$

where $\vec{u}_0=\langle\langle\widehat{\vec{P}^c}/N_em^*\rangle\rangle$ is the part of the drift velocity related to the center-of-mass canonical momentum.

The center-of-mass motion of electrons can be regarded as a *slow* motion in comparison with the *fast* relative scattering motion of electrons. Combining Eqs. (7)–(10) and using the Fermi (Bose) statistics for electron (phonon) operators $\hat{a}^\dagger_{\vec{k}\sigma},\hat{a}_{\vec{k}\sigma}$ ($\hat{b}^\dagger_{\vec{q}\lambda},\hat{b}_{\vec{q}\lambda}$), we get the force-balance equation[3]

$$N_e m^* \frac{d}{dt}\vec{u}_d = N_e m^* \frac{d}{dt}\vec{u}_0 + N_e e\vec{E}_{op}(t), \tag{11}$$

$$N_e m^* \frac{d}{dt}\vec{u}_0 = N_e e\vec{E}_{dc} + \vec{F}_i[\vec{u}_d] + \vec{F}_p[\vec{u}_d], \tag{12}$$

where $\vec{F}_i$ ($\vec{F}_p$) are the frictional forces due to impurity (phonon) scattering. For steady state or cases in which the time period of an optical field is much smaller than the momentum-relaxation time of electrons, the last term in Eq. (11) has no contribution. By considering the slow drifting motion of the center-of-mass in comparison with the ultrafast energy-relaxation process of electrons in the relative motion and keeping only the leading-order interaction of electrons with phonons and impurities, we can introduce a quantum-statistical average[11] to the frictional forces acting on all electrons which occur on the right-hand side of Eq. (12) due to impurity and phonon scattering. We know that the electron temperature should be determined by the ultrafast relative (internal) motion of electrons. In contrast, the drifting motion of electrons is very slow compared to the relative scattering process. As a result, when the frictional forces acting on all electrons are quantum-statistically averaged, one can use the Fermi-Dirac function as a first-order approximation with an effective temperature adiabatically evolving from $T^*(t=-\infty)=T$ to $T^*(t\geqslant 0)=T_e$. The anisotropic scattering in the relative motion of electrons produces nonzero frictional forces given by

$$\vec{F}_i[\vec{u}_d] = -2\pi n_i \mathcal{V}\sum_{\vec{k}}\sum_{\vec{q}} \vec{q}|U^{imp}(q)|^2(n_{\vec{k}+\vec{q}} - n_{\vec{k}}) \times \sum_{M=-\infty}^{\infty} J_{|M|}^2(|\mathcal{M}_{\vec{q}}|)\,\delta(\varepsilon_{k+q} - \varepsilon_k + \hbar\vec{q}\cdot\vec{u}_0 + M\hbar\Omega_f), \tag{13}$$

$$\vec{F}_p[\vec{u}_d] = -4\pi\sum_{\vec{k}}\sum_{\vec{q},\lambda}\vec{q}|C_{q\lambda}|^2(n_{\vec{k}+\vec{q}} - n_{\vec{k}})\sum_{M=-\infty}^{\infty} J_{|M|}^2(|\mathcal{M}_{\vec{q}}|) \times [N_0(\omega_{q\lambda}) - N_0'(\omega_{q\lambda} + \vec{q}\cdot\vec{u}_0 + M\Omega_f)] \times \delta(\varepsilon_{k+q} - \varepsilon_k + \hbar\omega_{q\lambda} + \hbar\vec{q}\cdot\vec{u}_0 + M\hbar\Omega_f). \tag{14}$$

Here, $M=0,\pm 1,\pm 2,\ldots,$ $J_M(x)$ is the Mth-order Bessel function, n_i is the impurity concentration, and $N_0(\omega_{q\lambda}) = [\exp(\hbar\omega_{q\lambda}/k_B T)-1]^{-1}$ is the Bose-Einstein function for equilibrium phonons with lattice temperature T. The electron-phonon coupling strength $|C_{q\lambda}|^2$ and electron-impurity coupling strength $|U^{imp}(q)|^2$ have essentially been modified by the multiplicative factor $J_{|M|}^2(|\mathcal{M}_{\vec{q}}|)$. Using Eq. (13), we get the same resistivity obtained from the Boltzmann transport equation to leading order for both nondegenerate and degenerate electron gases when impurity scattering is considered and $E_{op}=0$. At high temperatures ($T \geqslant 300$ K), it is known that the scattering from impurities becomes much less significant than that from phonons, and we will ignore it hereafter. $N_0'(\omega_{q\lambda}+\vec{q}\cdot\vec{u}_0+M\Omega_f)$ in Eq. (14) is obtained from $N_0(\omega_{q\lambda}+\vec{q}\cdot\vec{u}_0+M\Omega_f)$ by replacing the lattice temperature T with the nonequilibrium electron temperature T_e, which will be explicitly given[3] in Sec. III. The force-balance equation describes the momentum dissipation process for drifting electrons. It is easy to show from Eq. (14) that $\vec{F}_i[\vec{u}_d]=\vec{F}_p[\vec{u}_d]=0$ if $\vec{u}_0=\vec{0}$. Here, the center-of-mass drift causes a nonzero frictional force by producing a Doppler shift $\vec{q}\cdot\vec{u}_0$ relative to the phonon frequency $\omega_{q\lambda}$ and introducing anisotropic phonon scattering. On the other hand, the infrared field mainly modifies the electron-phonon coupling strength and drives T_e away from T by changing the average kinetic energy of the electrons. $n_{\vec{k}}$ in Eq. (14) is the occupation probability of electrons, and

$$\mathcal{M}_{\vec{q}} = \frac{e\vec{q}\cdot\vec{E}_{op}}{m^*\Omega_f^2}. \tag{15}$$

B. Boltzmann and Fokker-Planck equations

As we noted earlier, both uniform infrared and dc fields couple only to the center-of-mass motion of electrons but not directly to their relative motions. The drift of electrons, however, couples the center-of-mass and relative motions. At high temperatures, phonon scattering in the system will dominate impurity scattering. The first-order Coulomb effects can be included within the Hartree-Fock approximation. The Hartree energy shifts only the band edges and is irrelevant to our current work. The Fock energy between two electrons can be neglected compared to the average kinetic energy of each electron if $\sigma_{3D}^{1/3}a_B^* \gg 1$ with effective Bohr radius $a_B^* = 4\pi\epsilon_0\epsilon_r\hbar^2/m^*e^2$. For the concentration σ_{3D} chosen in this paper, $\sigma_{3D}^{1/3}a_B^* \gg 1$ is satisfied. The second-order Coulomb effects include screening and pair scattering. Screening has been considered in this paper within the static Thomas-Fermi approximation. A discussion about using a more sophisticated random-phase approximation can be found in a recent paper.[15] Pair scattering does not directly contribute to the drift velocity. For time scales much longer than the pair scattering time, this effect can be included phenomenologically by introducing a homogeneous lifetime $1/\Gamma_{ee}$ to electrons, where Γ_{ee} is the pair scattering rate on the order of 10 ps^{-1}. This will broaden the loss (δ) functions in both the force-balance and Boltzmann scattering equations. Obviously, this will not qualitatively change the features predicted in this paper. The force-balance equation[3] for the center-of-mass motion takes into account the momentum-dissipation effect. The energy-relaxation effect is included in the relative scattering motion. If we could assume an isotropic Fermi-Dirac function for hot electrons,[3] the electron temperature would be found from a first-order energy-balance equation for the relative motion of electrons. However, there is no justification for this assumption if the external field exists for all time. On the other hand, the Boltzmann scattering equation has been very successfully applied to explain laser damage of dielectric and semiconductor materials[10,12] under an intense laser field. We consider the situation in which the infrared photon energy is much smaller than the band gap of bulk GaAs and the amplitude of the infrared field is moderate. In this case, we can neglect the multiphoton interband excitation process. For the relative motion of

electrons under both dc and infrared fields, we use the Boltzmann scattering equation without the drift term to describe the relative scattering motion with phonons at high temperatures, given by

$$\frac{d}{dt}n_{\vec{k}}=\mathcal{W}_{\vec{k}}^{(\mathrm{in})}(1-n_{\vec{k}})-\mathcal{W}_{\vec{k}}^{(\mathrm{out})}n_{\vec{k}}, \tag{16}$$

where the scattering-in and scattering-out rates of electrons by phonons are[12]

$$\begin{aligned}\mathcal{W}_{\vec{k}}^{(\mathrm{in})}=&\frac{2\pi}{\hbar}\sum_{\vec{q},\lambda}|C_{q\lambda}|^2\sum_{M=-\infty}^{\infty}J_{|M|}^2(|\mathcal{M}_{\vec{q}}|)[n_{\vec{k}-\vec{q}}N_{\vec{q}\lambda}\delta(\varepsilon_k\\&-\varepsilon_{k-q}-\hbar\omega_{q\lambda}+\hbar\vec{q}\cdot\vec{u}_0-M\hbar\Omega_{\mathrm{f}})\\&+n_{\vec{k}+\vec{q}}(N_{\vec{q}\lambda}+1)\delta(\varepsilon_k-\varepsilon_{k+q}+\hbar\omega_{q\lambda}-\hbar\vec{q}\cdot\vec{u}_0\\&+M\hbar\Omega_{\mathrm{f}})],\end{aligned} \tag{17}$$

$$\begin{aligned}\mathcal{W}_{\vec{k}}^{(\mathrm{out})}=&\frac{2\pi}{\hbar}\sum_{\vec{q},\lambda}|C_{q\lambda}|^2\sum_{M=-\infty}^{\infty}J_{|M|}^2(|\mathcal{M}_{\vec{q}}|)[(1-n_{\vec{k}+\vec{q}})\\&\times N_{\vec{q}\lambda}\delta(\varepsilon_{k+q}-\varepsilon_k-\hbar\omega_{q\lambda}+\hbar\vec{q}\cdot\vec{u}_0-M\hbar\Omega_{\mathrm{f}})+(1\\&-n_{\vec{k}-\vec{q}})(N_{\vec{q}\lambda}+1)\delta(\varepsilon_{k-q}-\varepsilon_k+\hbar\omega_{q\lambda}-\hbar\vec{q}\cdot\vec{u}_0\\&+M\hbar\Omega_{\mathrm{f}})].\end{aligned} \tag{18}$$

The terms containing $N_{\vec{q}\lambda}+1$ represent the contributions from phonon emission, while the terms containing $N_{\vec{q}\lambda}$ are the contributions from phonon absorption. In the absence of both dc and infrared fields, the steady-state solution of Eq. (16) gives rise to a Fermi-Dirac function with the lattice temperature. The pair scattering effect can be phenomenologically included by changing the δ functions to Lorenzian functions with a homogeneous broadening $\hbar\Gamma_{\mathrm{ee}}$, where the scattering rate Γ_{ee} is on the order of 10 ps^{-1}. However, this does not qualitatively change the features predicted in this paper. The modification of electron-phonon scattering with $M\neq 0$ results from the drifting electrons interacting with the infrared field through phonon scattering. This effect is also historically known as *free-carrier absorption of photons*. The free-carrier absorption of photons occurs only close to the Fermi surface if the Fermi energy of electrons is much larger than (i.e., degenerate electrons) the electron temperature and phonon energies. In this case, by assuming small energies, e.g., $\hbar\omega_{q\lambda}$, $\hbar\Omega_{\mathrm{f}}$, $\hbar\vec{q}\cdot\vec{u}_0$ relative to the electron kinetic energy $\varepsilon_k\approx\varepsilon_{\mathrm{F}}$ with $\varepsilon_{\mathrm{F}}=\hbar^2(3\pi^2\sigma_{3\mathrm{D}})^{2/3}/2m^*$ at $T=0$ K for weak dc and moderate infrared fields, we can expand the Boltzmann scattering equation in Eq. (16) to second order with respect to these energies. As a result, the Fokker-Planck equation for drifting electrons under dc and infrared fields formally becomes[12]

$$\begin{aligned}&\frac{d}{dt}f(\varepsilon_{\vec{k}})+[V_{\mathrm{T}}(\varepsilon_{\vec{k}})+V_{\mathrm{F}}(\varepsilon_{\vec{k}})]\frac{d}{d\varepsilon_{\vec{k}}}f(\varepsilon_{\vec{k}})-[D_{\mathrm{T}}(\varepsilon_{\vec{k}})+D_{\mathrm{F}}(\varepsilon_{\vec{k}})]\\&\times\frac{d^2}{d\varepsilon_{\vec{k}}^2}f(\varepsilon_{\vec{k}})=[A_{\mathrm{T}}(\varepsilon_{\vec{k}})+A_{\mathrm{F}}(\varepsilon_{\vec{k}})]f(\varepsilon_{\vec{k}}).\end{aligned} \tag{19}$$

Here, $f(\varepsilon_{\vec{k}})=\rho(\varepsilon_k)n_{\vec{k}}$ is the continuous distribution function of electrons with density of states $\rho(\varepsilon_k)=\sqrt{(2m^*)^3\varepsilon_k}/2\pi^2\hbar^3$. We note that this equation is different from the standard Fokker-Planck equation[16] and cannot be written as a conservation law for particle number and current within the energy space even in the absence of source terms on the right-hand side of Eq. (19). But we will still call Eq. (19) the Fokker-Planck equation for simplicity. This equation is linear with respect to $f(\varepsilon_{\vec{k}})$ and, thus, greatly simplifies the calculation of the scattering dynamics of electrons. On the right-hand side of Eq. (19), we include two source terms. The coefficient for the thermal spontaneous phonon emission is

$$\begin{aligned}A_{\mathrm{T}}(\varepsilon_{\vec{k}})=&\frac{2\pi}{\hbar}\sum_{\vec{q},\lambda}|C_{q,\lambda}|^2[\delta(\varepsilon_k-\varepsilon_{k+q}+\hbar\omega_{q\lambda}-\hbar\vec{q}\cdot\vec{u}_0)\\&-\delta(\varepsilon_k-\varepsilon_{k-q}-\hbar\omega_{q\lambda}+\hbar\vec{q}\cdot\vec{u}_0)],\end{aligned} \tag{20}$$

and the coefficient for the field-induced spontaneous phonon emission is

$$\begin{aligned}A_{\mathrm{F}}(\varepsilon_{\vec{k}})=&\frac{\pi}{2\hbar}\sum_{\vec{q},\lambda}|C_{q,\lambda}|^2\mathcal{M}_{\vec{q}}^2[\delta(\varepsilon_k-\varepsilon_{k+q}+\hbar\omega_{q\lambda}-\hbar\vec{q}\cdot\vec{u}_0+\hbar\Omega_{\mathrm{f}})+\delta(\varepsilon_k-\varepsilon_{k+q}+\hbar\omega_{q\lambda}-\hbar\vec{q}\cdot\vec{u}_0-\hbar\Omega_{\mathrm{f}})\\&-\delta(\varepsilon_k-\varepsilon_{k-q}-\hbar\omega_{q\lambda}+\hbar\vec{q}\cdot\vec{u}_0+\hbar\Omega_{\mathrm{f}})-\delta(\varepsilon_k-\varepsilon_{k-q}-\hbar\omega_{q\lambda}+\hbar\vec{q}\cdot\vec{u}_0-\hbar\Omega_{\mathrm{f}})]\\&-\frac{\pi}{\hbar}\sum_{\vec{q},\lambda}|C_{q,\lambda}|^2\mathcal{M}_{\vec{q}}^2[\delta(\varepsilon_k-\varepsilon_{k+q}+\hbar\omega_{q\lambda}-\hbar\vec{q}\cdot\vec{u}_0)-\delta(\varepsilon_k-\varepsilon_{k-q}-\hbar\omega_{q\lambda}+\hbar\vec{q}\cdot\vec{u}_0)].\end{aligned} \tag{21}$$

In addition, we define in Eq. (19) the thermal energy transfer rate due to phonon scattering by

$$V_{\mathrm{T}}(\varepsilon_{\vec{k}})=\frac{2\pi}{\hbar}\sum_{\vec{q},\lambda}|C_{q,\lambda}|^2(\hbar\omega_{q\lambda}-\hbar\vec{q}\cdot\vec{u}_0)[N_{\vec{q}\lambda}\delta(\varepsilon_k-\varepsilon_{k-q}-\hbar\omega_{q\lambda}+\hbar\vec{q}\cdot\vec{u}_0)-(N_{\vec{q}\lambda}+1)\delta(\varepsilon_k-\varepsilon_{k+q}+\hbar\omega_{q\lambda}-\hbar\vec{q}\cdot\vec{u}_0)], \tag{22}$$

and the field-induced energy transfer rate by

$$V_{\mathrm{F}}(\varepsilon_{\vec{k}})=\frac{\pi}{2\hbar}\sum_{\vec{q},\lambda}|C_{q,\lambda}|^2\mathcal{M}_{\vec{q}}^2\{N_{\vec{q}\lambda}(\varepsilon_k-\varepsilon_{k-q})[\delta(\varepsilon_k-\varepsilon_{k-q}-\hbar\omega_{q\lambda}+\hbar\vec{q}\cdot\vec{u}_0+\hbar\Omega_{\mathrm{f}})+\delta(\varepsilon_k-\varepsilon_{k-q}-\hbar\omega_{q\lambda}+\hbar\vec{q}\cdot\vec{u}_0-\hbar\Omega_{\mathrm{f}})]$$
$$+(N_{\vec{q}\lambda}+1)(\varepsilon_k-\varepsilon_{k+q})[\delta(\varepsilon_k-\varepsilon_{k+q}+\hbar\omega_{q\lambda}-\hbar\vec{q}\cdot\vec{u}_0+\hbar\Omega_{\mathrm{f}})+\delta(\varepsilon_k-\varepsilon_{k+q}+\hbar\omega_{q\lambda}-\hbar\vec{q}\cdot\vec{u}_0-\hbar\Omega_{\mathrm{f}})]\}$$
$$-\frac{\pi}{\hbar}\sum_{\vec{q},\lambda}|C_{q,\lambda}|^2\mathcal{M}_{\vec{q}}^2(\hbar\omega_{q\lambda}-\hbar\vec{q}\cdot\vec{u}_0)[N_{\vec{q}\lambda}\delta(\varepsilon_k-\varepsilon_{k-q}-\hbar\omega_{q\lambda}+\hbar\vec{q}\cdot\vec{u}_0)-(N_{\vec{q}\lambda}+1)\delta(\varepsilon_k-\varepsilon_{k+q}+\hbar\omega_{q\lambda}-\hbar\vec{q}\cdot\vec{u}_0)]. \tag{23}$$

Finally, we define in Eq. (19) the thermal energy diffusion coefficient due to phonon scattering by

$$D_{\mathrm{T}}(\varepsilon_{\vec{k}})=\frac{\pi}{\hbar}\sum_{\vec{q},\lambda}|C_{q,\lambda}|^2(\hbar\omega_{q\lambda}-\hbar\vec{q}\cdot\vec{u}_0)^2[N_{\vec{q}\lambda}\delta(\varepsilon_k-\varepsilon_{k-q}-\hbar\omega_{q\lambda}+\hbar\vec{q}\cdot\vec{u}_0)+(N_{\vec{q}\lambda}+1)\delta(\varepsilon_k-\varepsilon_{k+q}+\hbar\omega_{q\lambda}-\hbar\vec{q}\cdot\vec{u}_0)], \tag{24}$$

and the field-induced energy diffusion coefficient by

$$D_{\mathrm{F}}(\varepsilon_{\vec{k}})=\frac{\pi}{4\hbar}\sum_{\vec{q},\lambda}|C_{q,\lambda}|^2\mathcal{M}_{\vec{q}}^2\{N_{\vec{q}\lambda}(\varepsilon_k-\varepsilon_{k-q})^2[\delta(\varepsilon_k-\varepsilon_{k-q}-\hbar\omega_{q\lambda}+\hbar\vec{q}\cdot\vec{u}_0+\hbar\Omega_{\mathrm{f}})+\delta(\varepsilon_k-\varepsilon_{k-q}-\hbar\omega_{q\lambda}+\hbar\vec{q}\cdot\vec{u}_0-\hbar\Omega_{\mathrm{f}})]$$
$$+(N_{\vec{q}\lambda}+1)(\varepsilon_k-\varepsilon_{k+q})^2[\delta(\varepsilon_k-\varepsilon_{k+q}+\hbar\omega_{q\lambda}-\hbar\vec{q}\cdot\vec{u}_0+\hbar\Omega_{\mathrm{f}})+\delta(\varepsilon_k-\varepsilon_{k+q}+\hbar\omega_{q\lambda}-\hbar\vec{q}\cdot\vec{u}_0-\hbar\Omega_{\mathrm{f}})]\}$$
$$-\frac{\pi}{2\hbar}\sum_{\vec{q},\lambda}|C_{q,\lambda}|^2\mathcal{M}_{\vec{q}}^2(\hbar\omega_{q\lambda}-\hbar\vec{q}\cdot\vec{u}_0)^2[N_{\vec{q}\lambda}\delta(\varepsilon_k-\varepsilon_{k-q}-\hbar\omega_{q\lambda}+\hbar\vec{q}\cdot\vec{u}_0)+(N_{\vec{q}\lambda}+1)\delta(\varepsilon_k-\varepsilon_{k+q}+\hbar\omega_{q\lambda}-\hbar\vec{q}\cdot\vec{u}_0)]. \tag{25}$$

The electron-phonon scattering depends not only on the electron distribution but also on the phonon distribution. The nonequilibrium phonon distribution can be computed from another scattering equation due to their coupling to electrons:[17]

$$\frac{d}{dt}N_{\vec{q}\lambda}=\Theta_{\vec{q}\lambda}^{em}(N_{\vec{q}\lambda}+1)-\Theta_{\vec{q}\lambda}^{abs}N_{\vec{q}\lambda}-\frac{N_{\vec{q}\lambda}-N_0(\omega_{q\lambda})}{\tau_{q\lambda}}, \tag{26}$$

where $\tau_{\vec{q}\lambda}$ is the relaxation time of phonons from the boundary scattering. In Eq. (26), the phonon-emission rate due to electron-phonon scattering to leading order is found to be[10]

$$\Theta_{\vec{q}\lambda}^{em}=\frac{4\pi}{\hbar}|C_{q,\lambda}|^2\sum_{\vec{k}}\frac{f(\varepsilon_{\vec{k}})}{\rho(\varepsilon_k)}\Bigg\{[\delta(\varepsilon_k-\varepsilon_{k+q}+\hbar\omega_{q\lambda}-\hbar\vec{q}\cdot\vec{u}_0)+\delta(\varepsilon_k-\varepsilon_{k-q}-\hbar\omega_{q\lambda}+\hbar\vec{q}\cdot\vec{u}_0)]\left(1-\frac{1}{2}\mathcal{M}_{\vec{q}}^2\right)+\frac{1}{4}\mathcal{M}_{\vec{q}}^2[\delta(\varepsilon_k$$
$$-\varepsilon_{k+q}+\hbar\omega_{q\lambda}-\hbar\vec{q}\cdot\vec{u}_0+\hbar\Omega_{\mathrm{f}})+\delta(\varepsilon_k-\varepsilon_{k-q}-\hbar\omega_{q\lambda}+\hbar\vec{q}\cdot\vec{u}_0-\hbar\Omega_{\mathrm{f}})+\delta(\varepsilon_k-\varepsilon_{k+q}+\hbar\omega_{q\lambda}-\hbar\vec{q}\cdot\vec{u}_0-\hbar\Omega_{\mathrm{f}})$$
$$+\delta(\varepsilon_k-\varepsilon_{k-q}-\hbar\omega_{q\lambda}+\hbar\vec{q}\cdot\vec{u}_0+\hbar\Omega_{\mathrm{f}})]\Bigg\}+\frac{4\pi}{\hbar}|C_{q,\lambda}|^2\sum_{\vec{k}}\frac{(\varepsilon_{k+q}-\varepsilon_k)}{\rho(\varepsilon_k)}\left[\frac{d}{d\varepsilon_{\vec{k}}}f(\varepsilon_{\vec{k}})\right]\Bigg\{\frac{1}{4}\mathcal{M}_{\vec{q}}^2[\delta(\varepsilon_k-\varepsilon_{k+q}+\hbar\omega_{q\lambda}$$
$$-\hbar\vec{q}\cdot\vec{u}_0+\hbar\Omega_{\mathrm{f}})+\delta(\varepsilon_k-\varepsilon_{k+q}+\hbar\omega_{q\lambda}-\hbar\vec{q}\cdot\vec{u}_0-\hbar\Omega_{\mathrm{f}})]+\delta(\varepsilon_k-\varepsilon_{k+q}+\hbar\omega_{q\lambda}-\hbar\vec{q}\cdot\vec{u}_0)\left(1-\frac{1}{2}\mathcal{M}_{\vec{q}}^2\right)\Bigg\}, \tag{27}$$

and the phonon-absorption rate is

$$\Theta_{\vec{q}\lambda}^{abs}=\frac{4\pi}{\hbar}|C_{q,\lambda}|^2\sum_{\vec{k}}\frac{f(\varepsilon_{\vec{k}})}{\rho(\varepsilon_k)}\Bigg\{[\delta(\varepsilon_k-\varepsilon_{k-q}-\hbar\omega_{q\lambda}+\hbar\vec{q}\cdot\vec{u}_0)+\delta(\varepsilon_k-\varepsilon_{k+q}+\hbar\omega_{q\lambda}-\hbar\vec{q}\cdot\vec{u}_0)]\left(1-\frac{1}{2}\mathcal{M}_{\vec{q}}^2\right)$$
$$+\frac{1}{4}\mathcal{M}_{\vec{q}}^2[\delta(\varepsilon_k-\varepsilon_{k-q}-\hbar\omega_{q\lambda}+\hbar\vec{q}\cdot\vec{u}_0+\hbar\Omega_{\mathrm{f}})+\delta(\varepsilon_k-\varepsilon_{k+q}+\hbar\omega_{q\lambda}-\hbar\vec{q}\cdot\vec{u}_0-\hbar\Omega_{\mathrm{f}})+\delta(\varepsilon_k-\varepsilon_{k-q}-\hbar\omega_{q\lambda}$$
$$+\hbar\vec{q}\cdot\vec{u}_0-\hbar\Omega_{\mathrm{f}})+\delta(\varepsilon_k-\varepsilon_{k+q}+\hbar\omega_{q\lambda}-\hbar\vec{q}\cdot\vec{u}_0+\hbar\Omega_{\mathrm{f}})]\Bigg\}+\frac{4\pi}{\hbar}|C_{q,\lambda}|^2\sum_{\vec{k}}\frac{(\varepsilon_{k-q}-\varepsilon_k)}{\rho(\varepsilon_k)}\left[\frac{d}{d\varepsilon_{\vec{k}}}f(\varepsilon_{\vec{k}})\right]\Bigg\{\frac{1}{4}\mathcal{M}_{\vec{q}}^2[\delta(\varepsilon_k-\varepsilon_{k-q}$$
$$-\hbar\omega_{q\lambda}+\hbar\vec{q}\cdot\vec{u}_0+\hbar\Omega_{\mathrm{f}})+\delta(\varepsilon_k-\varepsilon_{k-q}-\hbar\omega_{q\lambda}+\hbar\vec{q}\cdot\vec{u}_0-\hbar\Omega_{\mathrm{f}})]+\delta(\varepsilon_k-\varepsilon_{k-q}-\hbar\omega_{q\lambda}+\hbar\vec{q}\cdot\vec{u}_0)\left(1-\frac{1}{2}\mathcal{M}_{\vec{q}}^2\right)\Bigg\}. \tag{28}$$

Equations (11), (12), (16), and (26) can be combined together to study the electron-phonon drag effect.[7] The scattering interaction between drifting electrons and phonons constitutes a frictional force on the center-of-mass motion of electrons. The force-balance equation determines the center-of-mass drift velocity from Eq. (11), where upon ignoring the frictional force due to impurity scattering in Eq. (12) the remainder of the frictional force from phonons in Eq. (14) can now be written as

$$\vec{F}_p[\vec{u}_d]=-4\pi\sum_{\vec{k}}\frac{1}{\rho(\varepsilon_k)}\left[\frac{d}{d\varepsilon_{\vec{k}}}f(\varepsilon_{\vec{k}})\right]\sum_{\vec{q},\lambda}\vec{q}|C_{q,\lambda}|^2(\varepsilon_{k+q}-\varepsilon_k)\Bigg\{\left(1-\frac{1}{2}\mathcal{M}_{\vec{q}}^{\,2}\right)[N_0(\omega_{q\lambda})-N_0'(\omega_{q\lambda}+\vec{q}\cdot\vec{u}_0)]$$
$$\times\delta(\varepsilon_{k+q}-\varepsilon_k+\hbar\omega_{q\lambda}+\hbar\vec{q}\cdot\vec{u}_0)+\frac{1}{4}\mathcal{M}_{\vec{q}}^{\,2}\{[N_0(\omega_{q\lambda})-N_0'(\omega_{q\lambda}+\vec{q}\cdot\vec{u}_0+\Omega_f)]\,\delta(\varepsilon_{k+q}-\varepsilon_k+\hbar\omega_{q\lambda}+\hbar\vec{q}\cdot\vec{u}_0+\hbar\Omega_f)$$
$$+[N_0(\omega_{q\lambda})-N_0'(\omega_{q\lambda}+\vec{q}\cdot\vec{u}_0-\Omega_f)]\,\delta(\varepsilon_{k+q}-\varepsilon_k+\hbar\omega_{q\lambda}+\hbar\vec{q}\cdot\vec{u}_0-\hbar\Omega_f)\}\Bigg\}. \tag{29}$$

From Fröhlich electron-phonon coupling, we find the coupling matrix to be[12]

$$|C_q|^2=\left(\frac{\hbar\omega_{\rm LO}}{2\mathcal{V}}\right)\left(\frac{1}{\epsilon_\infty}-\frac{1}{\epsilon_s}\right)\frac{e^2}{\epsilon_0(q^2+Q_s^2)}, \tag{30}$$

where $\omega_{\rm LO}$ is the frequency of dominant longitudinal-optical (LO) phonon modes, ϵ_∞ and ϵ_s are the high-frequency and static dielectric constants of the host material, and $Q_s^2=(e^2/\epsilon_0\epsilon_r)(m^*/\pi^2\hbar^2)(3\pi^2\sigma_{3D})^{1/3}$ represents the Thomas-Fermi screening effect. For acoustic phonon scattering, on the other hand, we use the deformation-potential approximation.[17] This leads to

$$|C_{q\ell}|^2=\frac{\hbar\omega_{q\ell}}{2\rho c_\ell^2\mathcal{V}}\left[D^2+\frac{9}{32q^2}(eh_{14})^2\right]\left(\frac{q^2}{q^2+Q_s^2}\right)^2, \tag{31}$$

$$|C_{qt}|^2=\frac{\hbar\omega_{qt}}{2\rho c_t^2\mathcal{V}}\frac{13}{64q^2}(eh_{14})^2\left(\frac{q^2}{q^2+Q_s^2}\right)^2, \tag{32}$$

where $\lambda=\ell,t$ corresponds to one longitudinal and two transverse acoustic-phonon modes, c_ℓ and c_t are the sound velocities for these modes, ρ is the ion mass density, D is the deformation-potential coefficient, and h_{14} is the piezoelectric constant. Applying the Debye model to low-energy acoustic phonons, we get $\omega_{q\lambda}=c_\lambda q$ with $\lambda=\ell,t$.

III. RADIATIVE EFFECT AND ELECTRON TRANSPORT UNDER DC AND INFRARED FIELDS

It is known that the scattering between LO phonons and electrons becomes dominant at high temperatures in polar semiconductors, such as GaAs. In this section, we discuss the radiative effect and electron transport under both an infrared field and a dc electric field by including the optical-phonon scattering at high temperatures in GaAs. We consider the indirect absorption of the infrared field by electrons in the relative motion and the resulting change in the nonequilibrium electron temperature for an anisotropic distribution of electrons. We assume an outside heat bath connected to the GaAs system to keep a constant lattice temperature. Under this assumption we further assume $N_{\vec{q}\lambda}\approx N_0(\omega_{q\lambda})$ for phonons under a weak electron-phonon coupling in the relative scattering motion of electrons (neglecting the electron-phonon drag effect). By using these approximations, analytical expressions for the expansion coefficients in the Fokker-Planck equation can be obtained.

For the case with both dc and infrared fields, the thermal quantities $A_T(\varepsilon_{\vec{k}})$, $V_T(\varepsilon_{\vec{k}})$, and $D_T(\varepsilon_{\vec{k}})$ can be obtained from the functions defined in Eqs. (A1)–(A3) (in the appendix)

$$A_T(\varepsilon_{k_\perp},\varepsilon_{k_\parallel})=\frac{4\alpha}{\pi(\hbar\omega_{\rm LO})^{3/2}}[G_0^+(k_\perp,k_\parallel)-G_0^-(k_\perp,k_\parallel)], \tag{33}$$

$$V_T(\varepsilon_{k_\perp},\varepsilon_{k_\parallel})=-\frac{4\alpha}{\pi\sqrt{\hbar\omega_{\rm LO}}}[G_1^+(k_\perp,k_\parallel)-G_1^-(k_\perp,k_\parallel)], \tag{34}$$

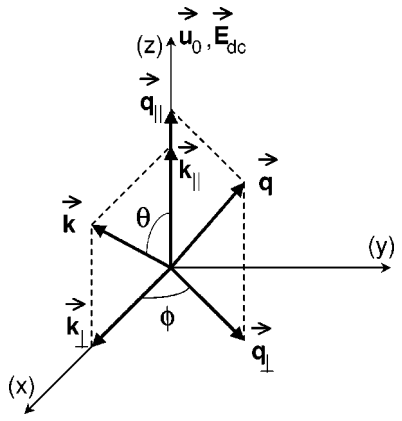

FIG. 1. Illustration for orientations of electron ($\vec{k}$) and phonon ($\vec{q}$) wave vectors. $\vec{E}_{\rm dc}$ and $\vec{u}_0$ lie in the (z) direction. The electron wave vector $\vec{k}$ can be decomposed as a parallel component ($\vec{k}_\parallel$) (in the z direction) and a perpendicular component $\vec{k}_\perp$ (in the x direction). The phonon wave vector $\vec{q}$ can also be decomposed in the same way. The angle between $\vec{q}_\perp$ and the x direction within the x-y plane is denoted as ϕ, and the angle between $\vec{k}$ and the z direction within the x-z plane is represented by θ.

$$D_{\mathrm{T}}(\varepsilon_{k_\perp},\varepsilon_{k_\parallel})=\frac{2\alpha\sqrt{\hbar\omega_{\mathrm{LO}}}}{\pi}[G_2^{+}(k_\perp,k_\parallel)+G_2^{-}(k_\perp,k_\parallel)], \tag{35}$$

where $\alpha=(e^2/16\pi\epsilon_0)\sqrt{2m^*}\omega_{\mathrm{LO}}^2(1/\epsilon_\infty-1/\epsilon_s)$, $k_\perp=\sqrt{2m^*\varepsilon_{k_\perp}}/\hbar$, and $k_\parallel=\sqrt{2m^*\varepsilon_{k_\parallel}}/\hbar$. The anisotropic dependence on $\varepsilon_{k_\perp}$ and $\varepsilon_{k_\parallel}$ in Eqs. (33)–(35) is a consequence of the applied dc electric field, as shown in Fig. 1.

The field-induced quantities $A_{\mathrm{F}}(\varepsilon_{\vec{k}})$, $V_{\mathrm{F}}(\varepsilon_{\vec{k}})$, and $D_{\mathrm{F}}(\varepsilon_{\vec{k}})$, proportional to E_{op}^2, can be obtained from the functions defined in Eqs. (A4)–(A6):

$$\begin{aligned}A_{\mathrm{F}}(\varepsilon_{k_\perp},\varepsilon_{k_\parallel})=&\frac{\alpha}{\pi(\hbar\omega_{\mathrm{LO}})^{3/2}}[R_{+}^{(+1)}(k_\perp,k_\parallel)+R_{+}^{(-1)}(k_\perp,k_\parallel)\\&-R_{-}^{(+1)}(k_\perp,k_\parallel)-R_{-}^{(-1)}(k_\perp,k_\parallel)]\\&-\frac{2\alpha}{\pi(\hbar\omega_{\mathrm{LO}})^{3/2}}[R_{+}^{(0)}(k_\perp,k_\parallel)-R_{-}^{(0)}(k_\perp,k_\parallel)],\end{aligned} \tag{36}$$

$$\begin{aligned}V_{\mathrm{FT}}(\varepsilon_{k_\perp},\varepsilon_{k_\parallel})=&-\frac{\alpha}{\pi\sqrt{\hbar\omega_{\mathrm{LO}}}}[S_{+}^{(+1)}(k_\perp,k_\parallel)+S_{+}^{(-1)}(k_\perp,k_\parallel)\\&-S_{-}^{(+1)}(k_\perp,k_\parallel)-S_{-}^{(-1)}(k_\perp,k_\parallel)],\end{aligned} \tag{37}$$

$$V_{\mathrm{J}}(\varepsilon_{k_\perp},\varepsilon_{k_\parallel})=\frac{2\alpha}{\pi\sqrt{\hbar\omega_{\mathrm{LO}}}}[S_{+}^{(0)}(k_\perp,k_\parallel)-S_{-}^{(0)}(k_\perp,k_\parallel)], \tag{38}$$

$$\begin{aligned}D_{\mathrm{FT}}(\varepsilon_{k_\perp},\varepsilon_{k_\parallel})=&\frac{\alpha\sqrt{\hbar\omega_{\mathrm{LO}}}}{2\pi}[T_{+}^{(+1)}(k_\perp,k_\parallel)+T_{+}^{(-1)}(k_\perp,k_\parallel)\\&+T_{-}^{(+1)}(k_\perp,k_\parallel)+T_{-}^{(-1)}(k_\perp,k_\parallel)],\end{aligned} \tag{39}$$

$$D_{\mathrm{A}}(\varepsilon_{k_\perp},\varepsilon_{k_\parallel})=-\frac{\alpha\sqrt{\hbar\omega_{\mathrm{LO}}}}{\pi}[T_{+}^{(0)}(k_\perp,k_\parallel)+T_{-}^{(0)}(k_\perp,k_\parallel)], \tag{40}$$

$V_{\mathrm{F}}(\varepsilon_{\vec{k}})=V_{\mathrm{FT}}(\varepsilon_{\vec{k}})+V_{\mathrm{J}}(\varepsilon_{\vec{k}})$, and $D_{\mathrm{F}}(\varepsilon_{\vec{k}})=D_{\mathrm{FT}}(\varepsilon_{\vec{k}})+D_{\mathrm{A}}(\varepsilon_{\vec{k}})$. $V_{\mathrm{FT}}(\varepsilon_{\vec{k}})$ and $D_{\mathrm{FT}}(\varepsilon_{\vec{k}})$ with $M=\pm1$ represent the infrared-field-induced corrections to relevant thermal quantities $V_{\mathrm{T}}(\varepsilon_{\vec{k}})$ and $D_{\mathrm{T}}(\varepsilon_{\vec{k}})$, respectively. $V_{\mathrm{J}}(\varepsilon_{\vec{k}})>0$ for $M=0$ comes from the effect of Joule heating due to power absorption from the infrared field by electrons, and $D_{\mathrm{A}}(\varepsilon_{\vec{k}})<0$ for $M=0$ is a result of the antidiffusion of electrons in energy space due to a correction to spontaneous phonon emission by Joule heating.[12] The anisotropic dependence on $\varepsilon_{k_\perp}$ and $\varepsilon_{k_\parallel}$ in Eqs. (36)–(40) reflects different radiative effects for two polarizations (parallel and perpendicular to $\vec{u}_0$) of the incident infrared field in addition to the existence of a dc field (parallel to $\vec{u}_0$), as shown in Fig. 1.

With the help of functions defined in Eqs. (A7)–(A9), the frictional force from phonon scattering can be expressed as

$$\begin{aligned}\frac{F_p}{N_{\mathrm{e}}m^*}=&\frac{4\alpha}{\pi^3\sigma_{3\mathrm{D}}}\sqrt{\frac{1}{2m^*}}\int_{-\infty}^{+\infty}dk_\parallel\int_0^{+\infty}k_\perp dk_\perp\frac{1}{\rho(\varepsilon_{k_\perp}+\varepsilon_{k_\parallel})}\\&\times\left[\frac{\partial f(\varepsilon_{k_\perp},\varepsilon_{k_\parallel})}{\partial\varepsilon_{k_\perp}}+\frac{\partial f(\varepsilon_{k_\perp},\varepsilon_{k_\parallel})}{\partial\varepsilon_{k_\parallel}}\right][H(k_\perp,k_\parallel)+W_1(k_\perp,k_\parallel)\\&+W_2^{(+1)}(k_\perp,k_\parallel)+W_2^{(-1)}(k_\perp,k_\parallel)].\end{aligned} \tag{41}$$

The anisotropic scattering contributes to the frictional force in Eq. (41) in two respects: (i) through the functions H, W_1, and $W_2^{\pm1}$ defined in Eqs. (A7)–(A9) and (ii) through the distribution function $f(\varepsilon_{k_\perp},\varepsilon_{k_\parallel})$ obtained from the Fokker-Planck equation for degenerate electrons. Having calculated these expansion coefficients in Eqs. (33)–(40) and the frictional force in Eq. (41), the Fokker-Planck equation in Eq. (19) can be solved simultaneously with the force-balance equation in Eq. (11) along with the boundary conditions $f(0,0)=\partial f(0,\varepsilon_{k_\parallel})/\partial\varepsilon_{k_\perp}=\partial f(\varepsilon_{k_\perp},0)/\partial\varepsilon_{k_\parallel}=0$.

When electrons stay in a nonequilibrium state, we cannot define an electron temperature through the Fermi-Dirac distribution.[3] However, we can still define a general nonequilibrium electron temperature through the average energy of each electron in the relative motion. By calculating $\mu(T_{\mathrm{e}})$ for fixed T_{e} and $\sigma_{3\mathrm{D}}$, T_{e} can be found from the following equation:[2]

$$\begin{aligned}&\frac{2\pi\hbar^2}{m^*\sigma_{3\mathrm{D}}}\int_0^{+\infty}\frac{k^4dk}{1+\exp\{[\varepsilon_k-\mu(T_{\mathrm{e}})]/k_{\mathrm{B}}T_{\mathrm{e}}\}}\\&\quad=\left\{e\int_0^{\tau_{\mathrm{p}}}[\vec{E}_{\mathrm{dc}}+\vec{E}_{\mathrm{op}}(t)]\cdot\vec{u}_{\mathrm{d}}(t)dt-\frac{1}{2}m^*|\vec{u}_{\mathrm{d}}(t)|^2\right\}\\&\qquad+\frac{1}{\sigma_{3\mathrm{D}}}\int_0^{+\infty}f(\varepsilon_{\vec{k}})\varepsilon_k d\varepsilon_{\vec{k}},\end{aligned} \tag{42}$$

where $\mu(T_{\mathrm{e}})$ is the chemical potential for free electrons at the temperature T_{e}. The combination of the first and second terms on the right-hand side of Eq. (42) represents the net increase of internal energy of electrons from frictional forces, and the last term comes from the change in the average kinetic energy of electrons in the relative motion mainly due to indirect photon absorption. This T_{e} depends on both external and system parameters, such as $\sigma_{3\mathrm{D}}$, $\hbar\omega_{\mathrm{LO}}$, m^*, E_{dc}, E_{op}, and Ω_{f}, but not on the electron kinetic energy, and it is different from the lattice temperature T. For very high lattice temperatures at which the quantum degeneracy can be neglected, the left side of Eq. (42) can simply be replaced by $\frac{3}{2}k_{\mathrm{B}}T_{\mathrm{e}}$ In Eq. (42), the momentum-relaxation time τ_{p} is given by

$$\frac{1}{\tau_{\mathrm{p}}}=\frac{1}{\sigma_{3\mathrm{D}}}\int_0^{+\infty}\left[\frac{1}{\tau_{\mathrm{T}}(\varepsilon_{\vec{k}})}+\frac{1}{\tau_{\mathrm{F}}(\varepsilon_{\vec{k}})}\right]f(\varepsilon_{\vec{k}})d\varepsilon_{\vec{k}}, \tag{43}$$

where the state-dependent nonlocal quantities can be expressed through

$$\frac{1}{\tau_{\mathrm{T}}(\varepsilon_{\vec{k}})}=\frac{2\pi}{\hbar}\sum_{\vec{q}}|C_q|^2[\mathcal{R}_{k,k+q}\delta(\varepsilon_k-\varepsilon_{k+q}+\hbar\omega_{\mathrm{LO}}-\hbar q_{\parallel}u_0)+\mathcal{R}_{k,k-q}\delta(\varepsilon_k-\varepsilon_{k-q}-\hbar\omega_{\mathrm{LO}}+\hbar q_{\parallel}u_0)], \tag{44}$$

$$\begin{aligned}\frac{1}{\tau_{\mathrm{R}}(\varepsilon_{\vec{k}})}=&\frac{\pi}{2\hbar}\sum_{\vec{q}}|C_q|^2\mathcal{M}_q^2\{\mathcal{R}_{k,k+q}[\delta(\varepsilon_k-\varepsilon_{k+q}+\hbar\omega_{\mathrm{LO}}-\hbar q_{\parallel}u_0+\hbar\Omega_{\mathrm{f}})+\delta(\varepsilon_k-\varepsilon_{k+q}+\hbar\omega_{\mathrm{LO}}-\hbar q_{\parallel}u_0-\hbar\Omega_{\mathrm{f}})]\\&+\mathcal{R}_{k,k-q}[\delta(\varepsilon_k-\varepsilon_{k-q}-\hbar\omega_{\mathrm{LO}}+\hbar q_{\parallel}u_0+\hbar\Omega_{\mathrm{f}})+\delta(\varepsilon_k-\varepsilon_{k-q}-\hbar\omega_{\mathrm{LO}}+\hbar q_{\parallel}u_0-\hbar\Omega_{\mathrm{f}})]\}\\&-\frac{\pi}{\hbar}\sum_{\vec{q}}|C_q|^2\mathcal{M}_q^2[\mathcal{R}_{k,k+q}\delta(\varepsilon_k-\varepsilon_{k+q}+\hbar\omega_{\mathrm{LO}}-\hbar q_{\parallel}u_0)+\mathcal{R}_{k,k-q}\delta(\varepsilon_k-\varepsilon_{k-q}-\hbar\omega_{\mathrm{LO}}+\hbar q_{\parallel}u_0)].\end{aligned} \tag{45}$$

The magnitude of $1/\tau_{\mathrm{p}}$ reflects the strength of the frictional force. In the above two equations, we have defined the angular-distribution factor for electron momentum dissipation[18]

$$\mathcal{R}_{k,k\pm q}=1-\frac{\vec{k}\cdot(\vec{k}\pm\vec{q})}{|\vec{k}\cdot(\vec{k}\pm\vec{q})|}. \tag{46}$$

IV. NUMERICAL RESULTS AND DISCUSSION

In our numerical calculations, we have chosen GaAs as the host material. $\vec{E}_{\mathrm{dc}}$ is assumed in the z direction, as shown in Fig. 1, and $\vec{E}_{\mathrm{op}}$ is chosen to be either parallel or perpendicular to $\vec{E}_{\mathrm{dc}}$. For GaAs we have taken parameters as follows: $m^*=0.067m_0$ with free-electron mass m_0, $\sigma_{3\mathrm{D}}=1\times10^{18}\ \mathrm{cm}^{-3}$, $\hbar\omega_{\mathrm{LO}}=36$ meV, $\epsilon_{\mathrm{r}}=12$, $\epsilon_{\mathrm{s}}=11$, $\epsilon_{\infty}=13$, $\hbar\Omega_{\mathrm{f}}=25$ meV, and $T=300$ K. Other parameters, such as E_{dc} and E_{op}, will be given directly in the figure captions. In the numerical calculations below, we will show results only for steady-state cases.

In Fig. 2 the scaled distribution function $\overline{f}(\varepsilon_{k_\perp},\varepsilon_{k_\parallel})$ (in units of $\sigma_{3\mathrm{D}}/\varepsilon_{\mathrm{F}}$) is exhibited as a function of $\varepsilon_{k_\perp}$ (dashed curve with $\varepsilon_{k_\parallel}=0$) and of $\varepsilon_{k_\parallel}$ (solid curve with $\varepsilon_{k_\perp}=0$) for $\vec{E}_{\mathrm{op}}\|\vec{E}_{\mathrm{dc}}$ with $E_{\mathrm{dc}}=0.58$ kV/cm and $E_{\mathrm{op}}=20$ kV/cm. A substantial difference between our model and the energy-balance approach[11] can be seen clearly here; energy-balance equation give the dashed curve. It is evident from this figure that the distribution function of electrons exhibits an anisotropic dependence on $\varepsilon_{k_\parallel}$ and $\varepsilon_{k_\perp}$, which is absent in the energy-balance approach. This anisotropy is attributed to the existence of $\vec{E}_{\mathrm{dc}}$ and $\vec{E}_{\mathrm{op}}$ applied to the GaAs system.

We display in Fig. 3 the scaled distribution function $\overline{f}(\varepsilon_{k_\perp},\varepsilon_{k_\parallel})$ as a function of parallel kinetic energy $\varepsilon_{k_\parallel}$ of electrons with $\varepsilon_{k_\perp}=0$ for $\vec{E}_{\mathrm{op}}\|\vec{E}_{\mathrm{dc}}$ (solid curve) and $\vec{E}_{\mathrm{op}}\perp\vec{E}_{\mathrm{dc}}$ (dashed curve) with $E_{\mathrm{dc}}=0.58$ kV/cm and $E_{\mathrm{op}}=20$ kV/cm. From this figure we find that the distribution function of electrons evidently depends on the polarization of the incident infrared field. This is a consequence of the different modifications to electron-phonon scattering in the presence of an incident infrared field with parallel and perpendicular polarization relative to $\vec{E}_{\mathrm{dc}}$. This anisotropic coupling to the infrared field will contribute differently to u_0 (compare the solid curves in Figs. 4 and 5).

In Fig. 4 we show the calculated drift velocity u_0 (solid curve and left axis) and the nonequilibrium electron temperature T_{e} (dashed curve and right axis) as functions of the amplitude of the incident infrared field E_{op} for $\vec{E}_{\mathrm{op}}\|\vec{E}_{\mathrm{dc}}$ with $E_{\mathrm{dc}}=0.58$ kV/cm. From the figure we find that u_0 increases sublinearly with E_{op} in the range of E_{op} shown, similar to results reported by Lei at high E_{op}, $T=10$ K, and terahertz frequencies.[11] However, the higher temperature $T=300$ K used in our calculation brings u_0 down below 10^7 cm/s. In addition, the increase of T_{e} is greatly limited at $T=300$ K and the range for the ratio T_{e}/T is significantly reduced.

In addition, we present the calculated drift velocity u_0 (solid curve and left axis) and nonequilibrium electron temperature T_{e} (dashed curve and right axis) as functions of E_{op}

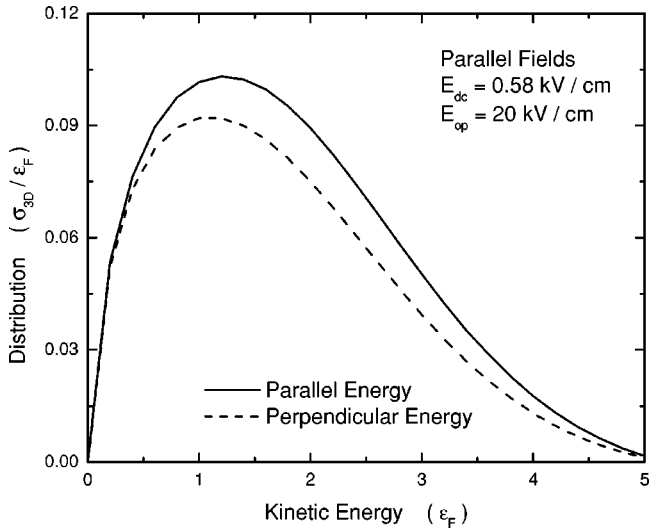

FIG. 2. Scaled distribution function $\overline{f}(\varepsilon_{k_\perp},\varepsilon_{k_\parallel})$ in units of $\sigma_{3\mathrm{D}}/\varepsilon_{\mathrm{F}}$ as a function of parallel kinetic energy $\varepsilon_{k_\parallel}/\varepsilon_{\mathrm{F}}$ (solid curve with $\varepsilon_{k_\perp}=0$) and of perpendicular kinetic energy $\varepsilon_{k_\perp}/\varepsilon_{\mathrm{F}}$ (dashed curve with $\varepsilon_{k_\parallel}=0$) for $\vec{E}_{\mathrm{op}}\|\vec{E}_{\mathrm{dc}}$. Here, $\varepsilon_{k_\parallel}=\hbar^2k_\parallel^2/2m^*$, $\varepsilon_{k_\perp}=\hbar^2k_\perp^2/2m^*$, $\varepsilon_{\mathrm{F}}=\hbar^2(3\pi^2\sigma_{3\mathrm{D}})^{2/3}/2m^*$ at $T=0$ K, $\vec{E}_{\mathrm{dc}}=0.58$ kV/cm, and $\vec{E}_{\mathrm{op}}=20$ kV/cm.

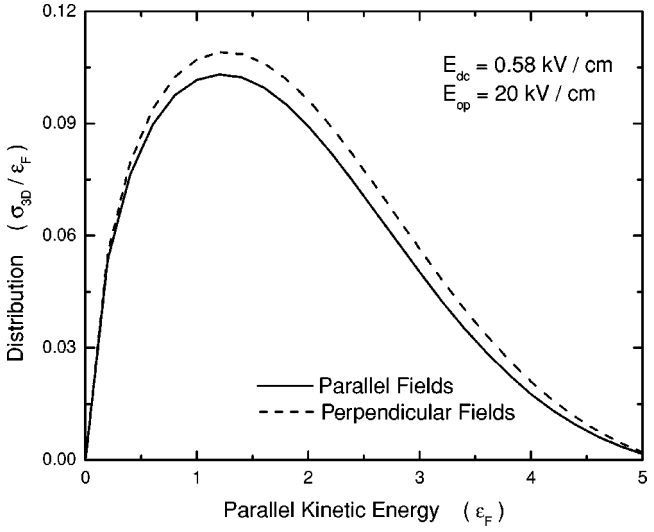

FIG. 3. Scaled distribution function $\overline{f}(\varepsilon_{k_\perp}, \varepsilon_{k_\parallel})$ in units of $\sigma_{3D}/\varepsilon_F$ as a function of parallel kinetic energy $\varepsilon_{k_\parallel}/\varepsilon_F$ of electrons with $\varepsilon_{k_\perp}=0$ for parallel polarization $\vec{E}_{op}\|\vec{E}_{dc}$ (solid curve) and perpendicular polarization $\vec{E}_{op}\perp\vec{E}_{dc}$ (dashed curve). Here, $\vec{E}_{dc}=0.58$ kV/cm and $\vec{E}_{op}=20$ kV/cm.

in Fig. 5 for $\vec{E}_{op}\perp\vec{E}_{dc}$ with $E_{dc}=0.58$ kV/cm. By comparing this figure with Fig. 4 we find that u_0 decreases with E_{op} instead of increasing with E_{op} in this case. Although u_0 decreases with E_{op}, T_e still increases with E_{op} due to the increased average kinetic energy of electrons by power absorption from the applied infrared field.

The effect of the polarization of the incident infrared field can be further visualized by looking at the momentum-relaxation rate $1/\tau_p$ in Fig. 6 as a function of E_{op} for $\vec{E}_{op}\perp\vec{E}_{dc}$ (solid curve) and $\vec{E}_{op}\|\vec{E}_{dc}$ (dashed curve) with $E_{dc}=0.58$ kV/cm. From this figure we find that the momentum-relaxation process is suppressed by the incident infrared field for parallel polarization but enhanced for perpendicular polarization. This result provides an explanation for Figs. 4 and 5, where u_0 for parallel polarization in Fig. 4 increases with E_{op} due to the suppression of momentum relaxation or frictional force, while u_0 for perpendicular polarization in Fig. 5 decreases with E_{op} due to the enhancement of momentum relaxation.

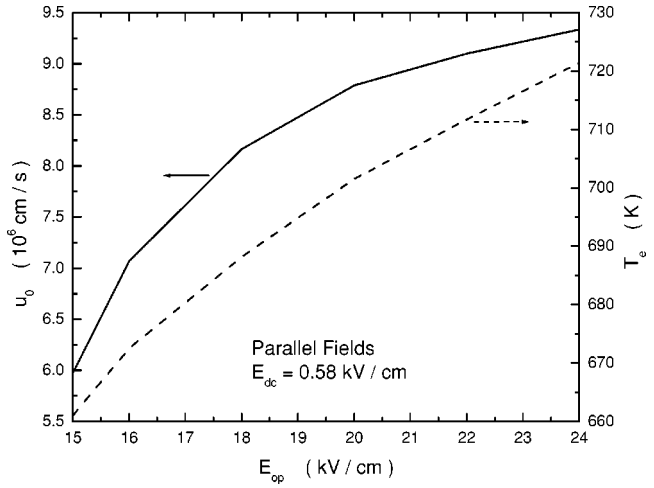

FIG. 4. Calculated drift velocity u_0 in units of 10^6 cm/s (solid curve and left axis) and nonequilibrium electron temperature T_e in units of K (dashed curve and right axis) as functions of the amplitude of the incident infrared field E_{op}. Here, E_{dc} is set to be 0.58 kV/cm and $\vec{E}_{op}$ is assumed to be parallel to $\vec{E}_{dc}$.

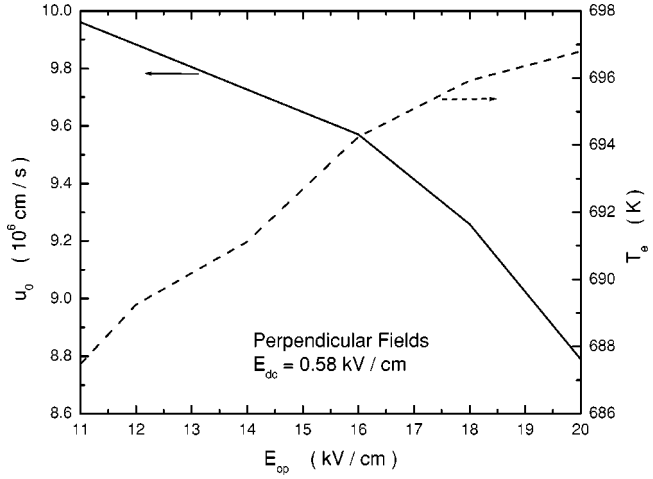

FIG. 5. Calculated drift velocity u_0 in units of 10^6 cm/s (solid curve and left axis) and nonequilibrium electron temperature T_e in units of K (dashed curve and right axis) as functions of the amplitude of the incident infrared field E_{op}. Here, E_{dc} is set to be 0.58 kV/cm and $\vec{E}_{op}$ is assumed to be perpendicular to $\vec{E}_{dc}$.

V. CONCLUSIONS

In conclusion, we have developed a model, in which electrons are interacting with an infrared field in a bulk semicon-

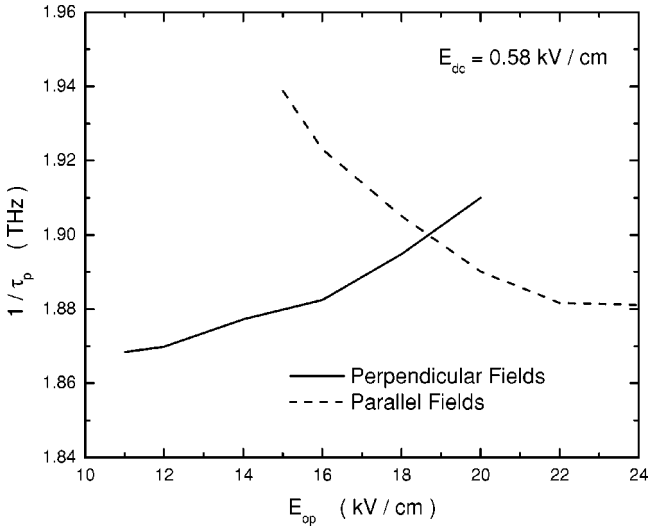

FIG. 6. Calculated momentum-relaxation rate $1/\tau_p$ in units of THz as a function of the amplitude of the incident infrared field E_{op} for perpendicular polarization $\vec{E}_{op}\perp\vec{E}_{dc}$ (solid curve) and parallel polarization $\vec{E}_{op}\|\vec{E}_{dc}$ (dashed curve). Here, E_{dc} is set to be 0.58 kV/cm.

ductor while they are undergoing transport due to an applied strong dc electric field, by coupling the force-balance equation for the slow center-of-mass motion of electrons and the Fokker-Planck equation for the ultrafast relative scattering motion of degenerate electrons. This allows us to fully include in our model the anisotropic energy-relaxation process, which was neglected in the past when the first-order energy-balance equation was used. At the same time, it also allows us to include the anisotropic coupling of electrons to the incident infrared field which gives rise to different frictional forces for polarizations of the infrared field perpendicular or parallel to the applied dc field. Based on this model, we can study high-field transport of electrons under strong dc and infrared fields beyond the relaxation-time approximation.

From our numerical study, we have discovered the anisotropic dependence of the electron distribution function on parallel and perpendicular kinetic energies of electrons. We have also quantified the effect of anisotropic coupling to the incident infrared field with polarizations parallel and perpendicular to the applied dc electric field. We have found that the drift velocity of electrons increases with the amplitude of the infrared field for the parallel polarization due to a suppression of momentum relaxation but decreases with the infrared-field amplitude for the perpendicular polarization due to an enhancement of momentum relaxation. The heating of electrons has been precisely described beyond the first-order energy-balance equation in addition to the inclusion of anisotropic coupling to an incident infrared field having different polarizations.

The first requirement for the Fokker-Planck equation to be a good approximation to the Boltzmann scattering equation in the relative scattering motion of electrons is that the Fermi energy ε_F must be greater than $k_B T$ for equilibrium electrons. The other requirements include $\varepsilon_F \gg \hbar\omega_{q\lambda}$, $\hbar\Omega_f$, and $\hbar q_\parallel u_0$. When a dipole coupling of electrons to an external optical field exists, i.e., using a grating to produce a nonuniform field or with photon energy greater than the band gap of the host materials, the incoherent Boltzmann scattering equation should be replaced by full coherent density-matrix equations with off-diagonal matrix elements.

ACKNOWLEDGMENTS

The authors are grateful to Professor S. W. Koch and Professor J. V. Moloney for their helpful discussions and comments on the relative scattering motion of electrons.

APPENDIX

We can decompose the electron and phonon wave vectors along the direction of $\vec{E}_{dc}$ and perpendicular to it, as shown in Fig. 1. Here, for optical-phonon scattering of electrons at high temperatures in polar semiconductors we first define three functions that determine $A_T(\varepsilon_{\vec{k}})$, $V_T(\varepsilon_{\vec{k}})$, and $D_T(\varepsilon_{\vec{k}})$ in the case of $\vec{E}_{dc} \neq 0$ but $\vec{E}_{op} = 0$ [see Eqs. (33)–(35)]:

$$G_0^{\pm}(k_\perp, k_\parallel) = \frac{\sqrt{2m^*\hbar\omega_{LO}}}{\hbar} \int_{-\infty}^{+\infty} dq_\parallel \int_0^{+\infty} dq_\perp \chi_{\pm}(\vec{k},\vec{q},0) \times \frac{q_\perp}{q_\parallel^2 + q_\perp^2 + Q_s^2}, \tag{A1}$$

$$G_1^{\pm}(k_\perp, k_\parallel) = \frac{\sqrt{2m^*\hbar\omega_{LO}}}{\hbar} \int_{-\infty}^{+\infty} dq_\parallel \int_0^{+\infty} dq_\perp \chi_{\pm}(\vec{k},\vec{q},0) \times \frac{q_\perp}{q_\parallel^2 + q_\perp^2 + Q_s^2} \left(1 - \frac{q_\parallel u_0}{\omega_{LO}}\right) \left[N_0(\omega_{LO}) + \frac{1}{2} \pm \frac{1}{2}\right], \tag{A2}$$

$$G_2^{\pm}(k_\perp, k_\parallel) = \frac{\sqrt{2m^*\hbar\omega_{LO}}}{\hbar} \int_{-\infty}^{+\infty} dq_\parallel \int_0^{+\infty} dq_\perp \chi_{\pm}(\vec{k},\vec{q},0) \times \frac{q_\perp}{q_\parallel^2 + q_\perp^2 + Q_s^2} \left(1 - \frac{q_\parallel u_0}{\omega_{LO}}\right)^2 \left[N_0(\omega_{LO}) + \frac{1}{2} \pm \frac{1}{2}\right]. \tag{A3}$$

In the above three equations, we have introduced the following function for $k_\perp q_\perp \neq 0$:

$$\chi_{\pm}(\vec{k},\vec{q},M) = \frac{\theta(2k_\perp q_\perp - |q_\perp^2 + q_\parallel^2 \pm 2k_\parallel q_\parallel \mp 2m^*(\omega_{LO} - q_\parallel u_0 + M\Omega_f)/\hbar|)}{\sqrt{4k_\perp^2 q_\perp^2 - [q_\perp^2 + q_\parallel^2 \pm 2k_\parallel q_\parallel \mp 2m^*(\omega_{LO} - q_\parallel u_0 + M\Omega_f)/\hbar]^2}},$$

where $\theta(x)$ is a unit step function. On the other hand, for $k_\perp q_\perp = 0$ we define

$$\chi_{\pm}(\vec{k},\vec{q},M) = \pi\delta\left[q_\perp^2 + q_\parallel^2 \pm 2k_\parallel q_\parallel \mp \frac{2m^*}{\hbar}(\omega_{LO} - q_\parallel u_0 + M\Omega_f)\right].$$

Moreover, we define another three functions that will determine $A_F(\varepsilon_{\vec{k}})$, $V_F(\varepsilon_{\vec{k}})$, and $D_F(\varepsilon_{\vec{k}})$ in the case of $\vec{E}_{op} \neq 0$ with $M = 0, \pm 1$ [see Eqs. (36)–(40)]:

$$R_{\pm}^{(M)}(k_\perp, k_\parallel) = \frac{\gamma\sqrt{2m^*\hbar\omega_{LO}}}{\hbar} \int_{-\infty}^{+\infty} dq_\parallel \int_0^{+\infty} dq_\perp \chi_{\pm}(\vec{k},\vec{q},M) \times \frac{q_\perp(\alpha_\perp q_\perp^2 + \alpha_\parallel q_\parallel^2)}{q_\parallel^2 + q_\perp^2 + Q_s^2}, \tag{A4}$$

$$S_{\pm}^{(M)}(k_\perp,k_\parallel)=\frac{\gamma\sqrt{2m^*\hbar\omega_{\rm LO}}}{\hbar}\int_{-\infty}^{+\infty}dq_\parallel\int_0^{+\infty}dq_\perp\chi_\pm(\vec{k},\vec{q},M)\times\frac{q_\perp(\alpha_\perp q_\perp^2+\alpha_\parallel q_\parallel^2)}{q_\parallel^2+q_\perp^2+Q_{\rm s}^2}\left(1-\frac{q_\parallel u_0}{\omega_{\rm LO}}+\frac{M\Omega_{\rm f}}{\omega_{\rm LO}}\right)\times\left[N_0(\omega_{\rm LO})+\frac{1}{2}\pm\frac{1}{2}\right],\tag{A5}$$

$$T_{\pm}^{(M)}(k_\perp,k_\parallel)=\frac{\gamma\sqrt{2m^*\hbar\omega_{\rm LO}}}{\hbar}\int_{-\infty}^{+\infty}dq_\parallel\int_0^{+\infty}dq_\perp\chi_\pm(\vec{k},\vec{q},M)\times\frac{q_\perp(\alpha_\perp q_\perp^2+\alpha_\parallel q_\parallel^2)}{q_\parallel^2+q_\perp^2+Q_{\rm s}^2}\left(1-\frac{q_\parallel u_0}{\omega_{\rm LO}}+\frac{M\Omega_{\rm f}}{\omega_{\rm LO}}\right)^2\times\left[N_0(\omega_{\rm LO})+\frac{1}{2}\pm\frac{1}{2}\right],\tag{A6}$$

where $\gamma=(eE_{\rm op}/m^*\Omega_{\rm f}^2)^2$. We take $\alpha_\perp=0$ and $\alpha_\parallel=1$ for $\vec{E}_{\rm op}\|\vec{E}_{\rm dc}$, but we set $\alpha_\parallel=0$ and $\alpha_\perp=1/2$ for $\vec{E}_{\rm op}\perp\vec{E}_{\rm dc}$ (unpolarized infrared field). Finally, for the frictional force in Eq. (41) we define the following functions with $M=0,\pm1$:

$$H(k_\perp,k_\parallel)=\int_{-\infty}^{+\infty}dq_\parallel\int_0^{+\infty}dq_\perp\xi(\vec{k},\vec{q},0)\frac{q_\perp q_\parallel}{q_\parallel^2+q_\perp^2+Q_{\rm s}^2}\times[N_0(\omega_{\rm LO})-N_0'(\omega_{\rm LO}+q_\parallel u_0)]\left(1+\frac{q_\parallel u_0}{\omega_{\rm LO}}\right),\tag{A7}$$

$$W_1(k_\perp,k_\parallel)=-\frac{\gamma}{2}\int_{-\infty}^{+\infty}dq_\parallel\int_0^{+\infty}dq_\perp\xi(\vec{k},\vec{q},0)\times\frac{q_\perp q_\parallel(\alpha_\perp q_\perp^2+\alpha_\parallel q_\parallel^2)}{q_\parallel^2+q_\perp^2+Q_{\rm s}^2}[N_0(\omega_{\rm LO})-N_0'(\omega_{\rm LO}+q_\parallel u_0)]\left(1+\frac{q_\parallel u_0}{\omega_{\rm LO}}\right),\tag{A8}$$

$$W_2^{(M)}(k_\perp,k_\parallel)=\frac{\gamma}{4}\int_{-\infty}^{+\infty}dq_\parallel\int_0^{+\infty}dq_\perp\xi(\vec{k},\vec{q},M)\times\frac{q_\perp q_\parallel(\alpha_\perp q_\perp^2+\alpha_\parallel q_\parallel^2)}{q_\parallel^2+q_\perp^2+Q_{\rm s}^2}[N_0(\omega_{\rm LO})-N_0'(\omega_{\rm LO}+q_\parallel u_0+M\Omega_{\rm f})]\left(1+\frac{q_\parallel u_0+M\Omega_{\rm f}}{\omega_{\rm LO}}\right).\tag{A9}$$

In the above three equations, we have introduced the following function for $k_\perp q_\perp\neq0$:

$$\xi(\vec{k},\vec{q},M)=\frac{\theta(2k_\perp q_\perp-|q_\perp^2+q_\parallel^2+2k_\parallel q_\parallel+2m^*(q_\parallel u_0+\omega_{\rm LO}+M\Omega_{\rm f})/\hbar|)}{\sqrt{4k_\perp^2q_\perp^2-[q_\perp^2+q_\parallel^2+2k_\parallel q_\parallel+2m^*(q_\parallel u_0+\omega_{\rm LO}+M\Omega_{\rm f})/\hbar]^2}}.$$

On the other hand, for $k_\perp q_\perp=0$ we define

$$\xi(\vec{k},\vec{q},M)=\pi\delta\left[q_\perp^2+q_\parallel^2+2k_\parallel q_\parallel+\frac{2m^*}{\hbar}(q_\parallel u_0+\omega_{\rm LO}+M\Omega_{\rm f})\right].$$

[1] H. Fröhlich and B.V. Paranjape, Proc. Phys. Soc. London, Sect. B **69**, 21 (1956).
[2] M.R. Arai, Appl. Phys. Lett. **42**, 906 (1983).
[3] X.L. Lei and C.S. Ting, Phys. Rev. B **32**, 1112 (1985).
[4] J. M. Ziman, *Principles of the Theory of Solids*, 1st ed. (Cambridge University Press, Cambridge, England, 1964), pp. 179–186.
[5] X.-G. Zhao, G.A. Georgakis, and Q. Niu, Phys. Rev. B **56**, 3976 (1997).
[6] R. Kubo, J. Phys. Soc. Jpn. **12**, 570 (1957).
[7] F.G. Bass and Yu.G. Gurevich, Sov. Phys. Usp. **14**, 113 (1971); Yu.G. Gurevich and O.L. Mashkevich, Phys. Rep. **181**, 327 (1989).
[8] C.S. Ting, S.C. Ying, and J.J. Quinn, Phys. Rev. B **14**, 4439 (1976).
[9] T. Apostolova, D.H. Huang, and D.A. Cardimona, Phys. Rev. B **67**, 205323 (2003).
[10] A. Kaiser, B. Rethfeld, H. Vicanek, and G. Simon, Phys. Rev. B **61**, 11 437 (2000).
[11] X.L. Lei, J. Appl. Phys. **84**, 1396 (1998).
[12] T. Apostolova, D.H. Huang, P.M. Alsing, J. McIver, and D.A. Cardimona, Phys. Rev. B **66**, 075208 (2002).
[13] M. Lindberg and S.W. Koch, Phys. Rev. B **38**, 3342 (1988); J.V. Moloney, R.A. Indik, J. Hader, and S.W. Koch, J. Opt. Soc. Am. B **11**, 2023 (1999).
[14] L. D. Landau and E. M. Lifshitz, *Quantum Mechanics*, 3rd ed. (Pergamon Press, Oxford, 1976), pp. 1–45.
[15] X.L. Lei and C.S. Ting, J. Phys. C **18**, 77 (1985).
[16] L.H. Holway and D.W. Fradin, J. Appl. Phys. **46**, 279 (1975).
[17] S.K. Lyo and D.H. Huang, Phys. Rev. B **66**, 155307 (2002).
[18] M. Sparks, D.L. Mills, R. Warren, T. Holstein, A.A. Maradudin, L.J. Sham, E. Loh, Jr., and D.F. King, Phys. Rev. B **24**, 3519 (1981).