Zeros of the order parameter of $d_{x^2-y^2}$ superconducting film in the presence of uniform current

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I show that additional p-wave component is generated in the pure d-wave superconductor in the presence of the uniform current. When the current flows in the antinodal direction the spectrum has a gap over all Fermi surface. If the current flows in the nodal direction, the gap opens only in the direction parallel to the current. The correction to the current due to the p-wave component of the order parameter is linear in vector potential. I show that the spin-orbit coupling is responsible for the additional component of the order parameter.

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Recently it was proposed that classification of the order parameter in the ferromagnetic superconductor may be performed in accordance with corepresentations of nonunitary magnetic group.[1–3] Similar approach was used in Ref. 4 to consider the effect of broken time-reversal symmetry at T^* on the superconducting state in high-T_c superconductors. Here I would like to extend this type of analysis for the case of a d-wave superconductor with uniform current.

There is a common belief that the high-temperature superconductors have nontrivial order parameter transforming as $B_{1g}(x^2-y^2)$ representation of D_{4h} point group. When external magnetic field is applied the symmetry of the order parameter is reduced.[5,6] As a result, for magnetic field directed along the c axis the new order parameter has the form $d_{x^2-y^2}+id_{xy}$. Furthermore, additional component of the order parameter may be induced by the surface. Since any surface breaks inversion symmetry, additional p-wave component is generated near the surface due to the surface induced spin-orbit coupling.[7] In the following I show that a constant superconducting current reduces the symmetry of the order parameter to $d_{x^2-y^2}+ip_x$ when the current is directed along the x axis. Such form of the order parameter is consistent with the group-theory analysis and may be generated by the spin-orbit coupling. I will construct the corresponding invariant terms in the free energy and show how p wave is generated from the microscopic BCS theory.

Since a constant superfluid current is a real vector that changes sign under time-reversal transformation, the total D_{4h} group will be reduced to nonunitary (magnetic) group $D_{2h}(C_{2v})$. I consider for simplicity that superfluid current is directed along the x axis. Unitary subgroup C_{2v} has four elements E, C_{2x}, σ_h, σ_y and four representations A_1, A_2, B_1, B_2. In accordance with the general theory of corepresentations[8] all four irreducible representations belong to the class "a." This means that each irreducible representation generates one nonequivalent corepresentation of the nonunitary group. All corepresentations are one dimensional and are listed in the Table I together with the basis functions. Here a and b are real numbers and $\epsilon=\exp(2i\phi)$ where ϕ is real. Following Ref. 2 I introduced factor 2 in the exponent. To clarify the derivation of basis functions in Table I I construct the basis function for the A_1 corepresentation. It is easy to see that $\exp(-i\phi)(k_x^2-k_y^2)$ transforms as the A_1 representation of the unitary subgroup as well as corresponding corepresentation of the nonunitary group. Since the group in the presence of uniform current does not have the center of inversion, a mixture of singlet and triplet order parameter is possible. The general expression for the triplet part of the order parameter has the following form:

$$\boldsymbol{\Psi}=\hat{x}f_x(k_x,k_y)+\hat{y}f_y(k_x,k_y)+\hat{z}f_z(k_x,k_y), \tag{1}$$

where $\hat{x}$, $\hat{y}$, $\hat{z}$ are unit axial vectors in the **a**, **b**, **c** axis directions, respectively. The functions $f_{x,y,z}(k_x,k_y)$ are odd with respect to the $\mathbf{k}\rightarrow-\mathbf{k}$ transformation. Applying the σ_h operation to $f_{x,y}(k_x,k_y)$ I obtain that $f_{x,y}(k_x,k_y)=-f_{x,y}(k_x,k_y)$ for A_1 and B_1 representations. Therefore $f_{x,y}(k_x,k_y)=0$ for A_1 and B_1 cases. Applying all operations from Table I to the remaining function $f_z(k_x,k_y)$ I obtain the following set of equations:

$$\begin{aligned} f_z(k_x,-k_y)&=-f_z(k_x,k_y),\\ f_z(-k_x,-k_y)^*&=\epsilon f_z(k_x,k_y),\\ f_z(-k_x,k_y)^*&=-\epsilon f_z(k_x,k_y). \end{aligned} \tag{2}$$

It is easy to see that the function

$$\Psi(k_x,k_y)=\exp(-i\phi)(d_{x^2-y^2}\oplus i\eta\hat{z}k_y), \tag{3}$$

where η is a real constant, transforms as A_1 corepresentation the nonunitary group $D_{2h}(C_{2v})$ in Table I. This means that the uniform current in the x direction generates the p_x wave contribution to the gap function. The gap function in that case is defined as $\hat{\Delta}(k_x,k_y)=i(d_{x^2-y^2}+i\eta\sigma_3k_y)\sigma_2$. Since $\hat{\Delta}\hat{\Delta}^\dagger\propto\hat{\sigma}_0$ the corresponding phase is unitary and the gap in the excitation spectrum is determined by $|d_{x^2-y^2}|^2+\eta^2k_y^2$.[9] As a result, the gap is fully developed over all Fermi surface.

Let us show that the term in the free energy,

$$i\eta'(v_xp_x-v_yp_y)\psi_d^*+\text{c.c.}, \tag{4}$$

is an invariant of the group D_{4h}. Here η' is a real constant, (v_x,v_y) are components of the superfluid velocity, and ψ_d, p_x, and p_y are components of the order parameter transforming as $d_{x^2-y^2}$, $\hat{z}k_y$, and $\hat{z}k_x$, respectively. Indeed, (v_x,v_y) and (p_x,p_y) transform as the E_u representations of the group D_{4h}. Since the direct product $E_u\times E_u=A_{1g}+A_{2g}+B_{1g}+B_{2g}$ contains $B_{1g}(d_{x^2-y^2})$ representation, Eq. (4) repre-

TABLE I. Corepresentations of the nonunitary group, when current flows in the x direction.

	E	C_{2x}	σ_h	σ_y	RI	$R\sigma_x$	RC_{2z}	RC_{2y}	Basis functions
A_1	1	1	1	1	ϵ	ϵ	ϵ	ϵ	$k_x^2-k_y^2+ia\sigma_3k_y$
A_2	1	1	-1	-1	ϵ	ϵ	$-\epsilon$	$-\epsilon$	$k_yk_z+ia\sigma_1k_x+ib\sigma_2k_y$
B_1	1	-1	1	-1	ϵ	$-\epsilon$	ϵ	$-\epsilon$	$k_xk_y+ia\sigma_3k_x$
B_2	1	-1	-1	1	ϵ	$-\epsilon$	$-\epsilon$	ϵ	$k_xk_z+ia\sigma_1k_y+ib\sigma_2k_x$

sents the true scalar. The presence of i in Eq. (4) is important since the superfluid current is antisymmetric with respect to the time-reversal symmetry.

The existence of the p component of the order parameter in the presence of the supercurrent becomes apparent if I write relevant Lifshitz invariant in the free energy:[10]

$$F=F_d+i\eta''[\psi_d^*(D_xp_x-D_yp_y)-\psi_d(D_x^*p_x^*-D_y^*p_y^*)]+\alpha_p(|p_x|^2+|p_y|^2), \tag{5}$$

where $D_l=-i\nabla_l-2eA_l$ $(l=x,y,z)$, η'' is a real constant, $\mathbf{A}$ is a vector potential, and F_d is the free energy for the d-wave superconductor without current. In Eq. (5) the additional quadratic in p_x, p_y term is written to secure $p_x=p_y=0$ solution if $\eta''=0$. Assuming that $\psi_d=\psi_{d0}\exp(imvx)$ and $(p_x,p_y)=(p_x,0)\exp(imvx)$ I obtain a contribution to the free energy, which is similar to Eq. (4). By minimizing Eq. (5) with respect to p_x I obtain the amplitude of the p component of the order parameter: $p_x=i\eta''mv\psi_d/\alpha_p$.

When the current is flowing in the nodal direction, $B_{1g}(x^2-y^2)$ representation of the D_{4h} group is no longer A_1 corepresentation of the reduced nonunitary group. Group-theoretical analysis for this case is presented in Table II. It should be pointed out that this case is similar to the case II considered in Ref. 4. Similarly to the case when the current flows in the x direction, the corresponding phase is unitary, $\hat{\Delta}\hat{\Delta}^\dagger\propto\hat{\sigma}_0$, and the gap function has the form $|d_{x^2-y^2}|^2+b^2(k_x+k_y)^2$. Contrary to the previous case, however, gap opens only in the direction parallel to the current, and the spectrum remains gapless in the direction perpendicular to the current.[11]

Let us now discuss the correction to the current when the additional component of the order parameter is generated. It is easy to see from Eq. (5) that the additional contribution to the current is given by

$$j_x=j_{dx}+2ie\eta''(\psi_d^*p_x-\psi_dp_x^*). \tag{6}$$

Substituting $p_x=i\eta''mv\psi_d/\alpha_p$ to the last equation and taking into account that in the quasiclassical limit $mv=-2eA$, correction to the supercurrent has the form

$$\mathbf{j}=\mathbf{j}_d+8e^2\eta''^2\mathbf{A}|\psi_d|^2/\alpha_p. \tag{7}$$

At this point I should point out that usually the nonlinear London equation is discussed in the case of the d-wave superconductors with the nodes in the spectrum.[11] Nonlinear corrections to the London equation appear due to the Doppler shift of the spectrum in the presence of the superflow. As a result, in some regions of the Fermi surface the excitation energy becomes negative leading to the finite quasiparticle current. However, as I have shown here, the generation of the p-wave component of the order parameter in the presence of the superflow leads to the correction to the supercurrent which is *linear* in the vector potential. Opening of the p-wave component of the order parameter leads to the correction to the penetration depth and could be detected experimentally.

At the end I suggest one possible microscopic mechanism, which causes the appearance of the p component of the order parameter in the presence of uniform current. I assume that current flows along the x axis. Spin-orbit coupling in that case could be written as

$$H_{so}=i\gamma\sum_k\psi_{\mathbf{k}}^\dagger[\mathbf{v}\times\partial_{\mathbf{k}}]_z\sigma_3\psi_{\mathbf{k}},$$

where v is the superfluid velocity and γ is the spin-orbit coupling constant. Following Balatsky[5] I can calculate the correction to the anomalous Green function $\hat{F}(\mathbf{k},\omega)=\hat{\Delta}_0(\mathbf{k})/D(\mathbf{k},\omega)$, where $D(\mathbf{k},\omega)=\omega^2+\xi(\mathbf{k})^2+|\Delta_0(\mathbf{k})|^2$, using perturbation theory due to the interaction H_{so}. The first correction to $\hat{F}^0(\mathbf{k},\omega)$ is

TABLE II. Corepresentations of the nonunitrary group, when current flows in the nodal direction.

	E	U_{xy}	σ_h	σ_{xy}^-	RI	$R\sigma_{xy}$	RC_{2z}	RU_{xy}^-	Basis functions
A_1	1	1	1	1	ϵ	ϵ	ϵ	ϵ	$k_x^2+k_y^2+ak_xk_y+ib\sigma_3(k_x-k_y)$
A_2	1	1	-1	-1	ϵ	ϵ	$-\epsilon$	$-\epsilon$	$(k_x-k_y)k_z+ia(\sigma_1k_y+\sigma_2k_x)$
$+$									$ib(\sigma_1k_x+\sigma_2k_y)$
B_1	1	-1	1	-1	ϵ	$-\epsilon$	ϵ	$-\epsilon$	$k_x^2-k_y^2+ia\sigma_3(k_x+k_y)$
B_2	1	-1	-1	1	ϵ	$-\epsilon$	$-\epsilon$	ϵ	$(k_x+k_y)k_z+ia(\sigma_1k_x-\sigma_1k_y)$
									$\times ib(\sigma_1k_y-\sigma_2k_x)$

$$\delta\hat{F}(\mathbf{k},\omega)=i\gamma v\widehat{G^0}(\mathbf{k},\omega)\partial_{k_y}\sigma_3\hat{F}^0(\mathbf{k},\omega)$$
$$=i\gamma v\frac{[i\omega-\xi(\mathbf{k})]\sigma_3\Delta_0}{D(\mathbf{k},\omega)^2}\sin(k_y)i\sigma_2$$
$$=-i\gamma v\frac{[-i\omega+\xi(\mathbf{k})]\sigma_1\Delta_0}{D(\mathbf{k},\omega)^2}\sin(k_y). \qquad (8)$$

Here I take into account that $\hat{\Delta}_0(\mathbf{k})=\Delta_0(\cos(k_x)-\cos(k_y))i\sigma_2$, and $\widehat{G^0}(\mathbf{k},\omega)=[i\omega-\xi(\mathbf{k})]\sigma_0/D(\mathbf{k},\omega)$. To estimate the p-wave correction to the order parameter I assume a repulsive separable interaction in the following form: $V_y(\mathbf{k},\mathbf{k}')=V_y\sin(k_y)\sin(k_y')$. As a result, the correction to $\Delta_0(\mathbf{k})$ is given by

$$\Delta_1(\mathbf{k})=T\sum_{\omega,\mathbf{k}}V_y\sin(k_y)\sin(k_y')\delta\hat{F}(\mathbf{k}',\omega)$$
$$=-iC\gamma v\frac{\Delta_0}{E_F}N_0V_y\sin(k_y)\sigma_1. \qquad (9)$$

Here C is a real constant of the order of 1, and N_0 is the density of states at the Fermi surface. To estimate the effect in absolute units I assume that spin-orbit coupling constant $\gamma v_f\approx 0.05$–0.1 eV. This estimate is supported by the recent measurement of the band splitting on the surface[12] and is of the order of the spin-orbit coupling in Ni. Direct estimate of Eq. (9) shows that $\Delta_1/\Delta_0\sim\gamma v_f\Delta_0/E_f^2$. Here I assume that superfluid velocity is $v\sim\Delta_0/k_f$. Using typical for cuprates values for the Fermi energy $E_f\sim 0.1$–0.2 eV and $\Delta_0\sim 0.05$ eV I obtain that $\Delta_1/\Delta_0\sim 0.05$–$0.25$. These estimates show that predicted correction to the order parameter is relatively large and could be observed experimentally. As it was suggested above the effect could be detected by measuring penetration depth in the presence of supercurrent. According to Eq. (7) penetration depth should be reduced in the presence of current. As a result penetration depth measured at the constant magnetic field should be different from penetration depth measured at the constant current. Moreover, temperature dependence of the penetration depth at low temperature should satisfy Nernst principle[13] leading to $\partial\lambda/\partial T\rightarrow 0$ as $T\rightarrow 0$. Another possibility to see the effect was proposed in Ref. 7. Generation of p-wave component leads to anisotropy of the Knight shift at low temperatures.

In conclusion, I have shown, that additional p-wave component appears in the case of d-wave superconductor in the presence of uniform supercurrent. This effect leads to the additional contribution to the current linear in the vector potential. It is shown that spin-orbit coupling is responsible for this effect.

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