Surface plasmon polariton scattering by a small particle placed near a metal surface: An analytical study

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(Received 12 May 2003; revised manuscript received 29 August 2003; published 30 January 2004)

Scattering of light by a small particle placed near a metal surface is studied via analytical calculations of the particle extinction cross section. Considering a small spherical particle, we express the extinction cross section via the total electric-field Green's tensor of a metal-dielectric interface structure. Analytic expressions are derived for the parts of Green's tensor that govern the excitation of *p*- and *s*-polarized waves propagating away from the interface, and waves propagating along and being localized at the interface, viz., surface plasmon polaritons (SPP's). This allows us in turn to divide the extinction cross section into parts associated with scattering of light into different types of electromagnetic waves. The scattering cross sections related to SPP-to-SPP scattering, and scattering of SPP's into waves propagating away from the interface, are studied with respect to the dielectric constant of the metal and the height of the scatterer above the interface. In the case where the light wavelength is close to the SPP resonance, the SPP-to-SPP scattering cross section can be orders of magnitude larger compared to the extinction cross section of a particle in free space, whereas in the case of a nearly perfect conductor, the SPP-to-SPP cross section tends to 0. The efficiency of SPP-to-SPP scattering is calculated and, e.g., for the metal dielectric constant -100 (order of magnitude for gold at the light wavelength 1500 nm) it is found to be above 60% for the optimum scatterer-surface distance.

DOI: 10.1103/PhysRevB.69.045422 PACS number(s): 78.68.+m, 71.36.+c, 02.70.−c, 03.65.Nk

I. INTRODUCTION

Surface plasmon polaritons (SPP's) represent a type of electromagnetic excitation which is bound to and propagating along metal-dielectric interfaces.[1] Scattering of SPP's by surface random roughness has been extensively studied over the last 30 years with main emphasis on the far-field distribution of light scattered out of the surface plane. In recent years there has been a rapidly growing interest in the possibility of SPP control and manipulation in the surface plane by artificially created surface structures. Theoretical models[2–4] considering two-dimensional SPP scattering configurations and corresponding experiments[5] with SPP's being incident on one-dimensional (linelike) surface structures have been reported. SPP guiding along thin metal stripes (including those embedded in dielectric) (Refs. 6–10) has been investigated. Local SPP excitation and scattering by nanoparticles placed randomly[11] or intentionally (to form micro-optical elements)[12] as well as SPP scattering by artificial surface scatterers[13,14] has also revealed interesting avenues worth further exploration. Quite recently, metallic microstructures consisting of periodically arranged surface particles have been shown to exhibit a band gap for SPP's and to allow SPP guiding (at wavelengths in the band gap) along narrow channels free from particles.[15–17] In general, the SPP band gap phenomenon[18,19] is similar to the photonic band gap effect, i.e., the inhibition of light propagation in (quasi-two-dimensional) photonic crystals.[20]

Scattering of SPP's by surface features constitutes a core of the aforementioned scattering configurations. Low-loss SPP manipulation requires that the radiation is kept in the surface plane, i.e., that the SPP-to-SPP scattering is considerably stronger than the SPP scattering into waves propagating away from the metal surface. When considering the efficiency of SPP-to-SPP scattering, it would be convenient to divide the total scattered field into propagating SPP field components, components related to waves propagating away from the metal-dielectric interface, and a remaining quasistatic near-field term, and thereby to evaluate how large a fraction of the power of the scattered field is related to SPP's. The total field scattered by a small particle can be related to Green's tensor for a metal-dielectric interface structure (see, e.g., Ref. 11). The existing formulations of the Green's tensor are suitable for calculating the total scattered field but not for extracting the part of the scattered field which is related to SPP's and the parts related to other types of fields. Another possibility for calculating the total field is to make a numerical calculation based on expanding the field in vector spherical harmonics around the center of the particle, see, e.g., Refs. 21,22. Although this method is also not suitable for extracting the part of the field related to SPP's, it is nevertheless possible to evaluate the power scattered into SPP's from the total field by integrating the Poynting vector flux over an infinite plane.[21]

In this paper we study analytically the scattering of a SPP by a small particle placed near a metal surface and evaluate the efficiency of the SPP-to-SPP scattering. The approach is that we first provide a relation between the extinction cross section of the particle and the total electric-field Green's tensor for the metal-dielectric interface structure. We then present a new formulation of Green's tensor for a metal-dielectric interface structure, where the Green's tensor is divided into parts that govern the excitation of SPP's, *s*- and *p*-polarized waves propagating away from the interface, and a quasistatic (near) field, respectively. Analytic expressions are derived for the parts of the Green's tensor related to propagating waves. The extinction cross section is thereafter divided into terms related to scattering into SPP's and scattering into other types of waves. The analytic method is restricted to the case of a small scattering particle which is not located directly at the metal surface, and material absorption is not considered. A method based on field expansion into vector spherical harmonics would have less restrictions, but then we would not have the advantages of being able to extract the part of the scattered field related to excitation of SPP's, and being able to obtain (relatively simple) analytic expressions for the particle scattering cross section.

It should be noted that the component of Green's tensor that we are mainly interested in is often referred to as the density of states. Some similarities exist between the type of calculations presented in this paper and calculations for the emission of light by a dipole source, and spontaneous emission from a two-level atom treated in a semiclassical approach as a dipole source, see, e.g., Refs. 23–37 The similarities reflect the fact that the electromagnetic field related to scattering by a small particle is equivalent (within the framework of the electric dipole approximation) to the radiation by a dipole source driven by the incident field.

The paper is organized as follows. In Sec. II we provide a relation between the scattering cross section for a small particle and the retarded Green's tensor of the structure in which the particle is placed. In Sec. III Green's tensor for a metal-dielectric interface structure is constructed through an eigenmode expansion which allows decomposition of Green's tensor into parts that govern the excitation of SPP's, *s*- and *p*-polarized waves propagating away from the interface, and a quasistatic field. In Sec. IV we apply the results of the preceding sections and study scattering of a SPP by a small particle placed near a metal surface. We study SPP-to-SPP scattering as well as scattering from SPP's to waves propagating away from the metal-dielectric interface, and finally we evaluate the SPP scattering efficiency. The conclusions are offered in Sec. V.

II. SCATTERING THEORY

A measure of how much light is scattered out of a beam of light incident upon a particle is the extinction cross section[38] of the particle given by

$$C_{ext}=\mathrm{Im}\left(\frac{k_0}{|\boldsymbol{E}_0|^2}\int [\boldsymbol{E}_0(\boldsymbol{r})]^*\cdot[\varepsilon(\boldsymbol{r})-\varepsilon_{ref}(\boldsymbol{r})]\boldsymbol{E}(\boldsymbol{r})d^3r\right), \tag{1}$$

where $\boldsymbol{E}_0$ is the electric field of the incident beam (the field that we would have if the scattering particle was not there), $\boldsymbol{E}$ is the total field being the incident beam plus the scattered field, $k_0=2\pi/\lambda$, with λ being the free-space wavelength, $\boldsymbol{r}$ is the position coordinate, $\varepsilon(\boldsymbol{r})$ is the dielectric constant of the structure under consideration including the scattering particle, whereas $\varepsilon_{ref}(\boldsymbol{r})$ represents the dielectric constant of the reference structure without the scattering particle. In general the extinction is the power removed from the incident beam due to scattering and absorption. However, for our choice of materials there is no absorption and in the following the extinction is equivalent to scattered light. The extinction is proportional to the product of the extinction cross section and the amplitude of the electric field squared at the site of the particle.

The total field can, e.g., be obtained by solving the Lippmann-Schwinger integral equation

$$\boldsymbol{E}(\boldsymbol{r})=\boldsymbol{E}_0(\boldsymbol{r})-\int \boldsymbol{G}(\boldsymbol{r},\boldsymbol{r}')k_0^2\cdot[\varepsilon(\boldsymbol{r}')-\varepsilon_{ref}(\boldsymbol{r}')]\boldsymbol{E}(\boldsymbol{r}')d^3r', \tag{2}$$

where $\boldsymbol{G}$ is the retarded Green's tensor for the reference structure. Green's tensor is the solution to the following equation:

$$-\boldsymbol{\nabla}\times\boldsymbol{\nabla}\times\boldsymbol{G}(\boldsymbol{r},\boldsymbol{r}')+k_0^2\varepsilon_{ref}(\boldsymbol{r})\boldsymbol{G}(\boldsymbol{r},\boldsymbol{r}')=\boldsymbol{I}\delta(\boldsymbol{r}-\boldsymbol{r}'), \tag{3}$$

where $\boldsymbol{I}$ is the unit tensor and δ is the Dirac delta function. The reference structure that we consider here consists of a half space $(z>0)$ of dielectric with dielectric constant $\varepsilon_1 >0$, and a half space $(z<0)$ of metal with dielectric constant $\varepsilon_2<0$. Because we are interested in metal-dielectric interfaces that support SPP's we furthermore have to require $\varepsilon_2<-\varepsilon_1$. We consider a spherical particle located in the upper half space with dielectric constant ε_p, and radius a, and with the position of the center of the particle $z>0$.

For the case of a spherical particle which is small with respect to the wavelength, which is not located directly on the metal surface $(z/a\gg 1)$, and when the incident field $\boldsymbol{E}_0(\boldsymbol{r})$ is approximately constant across the particle, we can assume that the field $\boldsymbol{E}(\boldsymbol{r})$ is constant inside the particle. The assumption can be justified by considering Eq. (2). In the case where $\boldsymbol{E}_0(\boldsymbol{r})$ can be considered constant over the region of the particle the assumption that $\boldsymbol{E}(\boldsymbol{r})$ is also constant requires that $\int\boldsymbol{G}(\boldsymbol{r},\boldsymbol{r}')k_0^2[\varepsilon(\boldsymbol{r}')-\varepsilon_{ref}(\boldsymbol{r}')]d^3r'$ is independent of $\boldsymbol{r}$ for $\boldsymbol{r}$ being a position inside the particle. In the electrostatic limit this requirement is satisfied to a good approximation when $z/a\gg 1$. The requirement would be perfectly satisfied for a particle in a homogeneous medium, but the presence of the metal surface adds a contribution to $\boldsymbol{G}(\boldsymbol{r},\boldsymbol{r}')$ relative to the homogeneous medium case which, for particles close to the interface, decays as $(z+z')^{-3}$, and for particles located directly on the surface the result will be the excitation of higher-order multipole components in the scattered field.[39–41]

For $z/a\gg 1$ the real part of $\int\boldsymbol{G}(\boldsymbol{r},\boldsymbol{r}')k_0^2[\varepsilon(\boldsymbol{r}') -\varepsilon_{ref}(\boldsymbol{r}')]d^3r'$ will be dominated by the integral over the singularity of $\boldsymbol{G}(\boldsymbol{r},\boldsymbol{r}')$ for $\boldsymbol{r}\approx\boldsymbol{r}'$. The singularity is similar to the singularity of Green's tensor for a homogeneous medium in the long-wavelength or electrostatic limit, in which case we have[42]

$$\mathrm{Re}\left(\int_V \boldsymbol{G}(\boldsymbol{r},\boldsymbol{r}')k_0^2[\varepsilon(\boldsymbol{r}')-\varepsilon_{ref}(\boldsymbol{r}')]d^3r'\right)\approx-\frac{1}{3}\,\frac{\varepsilon_p-\varepsilon_1}{\varepsilon_1}\boldsymbol{I} \tag{4}$$

for all $\boldsymbol{r}$ inside the small particle. For the imaginary part of $\boldsymbol{G}$ no electrostatic approximation applies $[\mathrm{Im}(\boldsymbol{G})=\boldsymbol{0}$ in the electrostatic limit]. In this case, however, we may note that $\mathrm{Im}[\boldsymbol{G}(\boldsymbol{r},\boldsymbol{r}')]$ is nonsingular and can be considered constant for $\boldsymbol{r}$, $\boldsymbol{r}'$ inside the small particle if $a\ll\lambda/(2\pi\sqrt{\varepsilon_1})$, i.e.,

$$\mathrm{Im}\left(\int_V \boldsymbol{G}(\boldsymbol{r},\boldsymbol{r}')k_0^2[\varepsilon(\boldsymbol{r}')-\varepsilon_{ref}(\boldsymbol{r}')]d^3r'\right)\approx \mathrm{Im}[\boldsymbol{G}(\boldsymbol{r},\boldsymbol{r})]k_0^2(\varepsilon_p-\varepsilon_1)V. \tag{5}$$

As it has been justified that both real and imaginary parts of the integral are approximately independent of $\boldsymbol{r}$ inside the particle when certain restrictions are imposed on z/a and a/λ, the field inside the particle is given by

$$\boldsymbol{E}=\left(\boldsymbol{I}+\int_V \boldsymbol{G}(\boldsymbol{r},\boldsymbol{r}')k_0^2[\varepsilon(\boldsymbol{r}')-\varepsilon_{ref}(\boldsymbol{r}')]d^3r'\right)^{-1}\cdot\boldsymbol{E}_0. \tag{6}$$

The extinction cross section thereby reduces to

$$C_{ext}=k_0^3\left(\frac{\varepsilon_p-\varepsilon_1}{\varepsilon_p+2\varepsilon_1}4\pi a^3\right)^2\varepsilon_1^2\boldsymbol{\mu}^*\cdot\{-\mathrm{Im}[\boldsymbol{G}(\boldsymbol{r},\boldsymbol{r})]\}\cdot\boldsymbol{\mu}, \tag{7}$$

where $\boldsymbol{\mu}=\boldsymbol{E}_0/|\boldsymbol{E}_0|$. It is known that the radiation from a dipole antenna with dipole moment $\boldsymbol{\mu}$ is proportional to $\boldsymbol{\mu}^*\cdot[-\mathrm{Im}(\boldsymbol{G}(\boldsymbol{r},\boldsymbol{r}))]\cdot\boldsymbol{\mu}$, and thus Eq. (7) can be interpreted in the way that the light scattered out of the incident beam corresponds to the light emitted from a dipole antenna (driven by the incident beam) located at the position of the particle.

In this paper we will, on the basis of Eq. (7) and an eigenmode expansion of $\boldsymbol{G}$, make a further interpretation, where C_{ext} is divided into three parts,

$$C_{ext}=C_{ext}^{s-pol.}+C_{ext}^{p-pol.}+C_{ext}^{SPP}, \tag{8}$$

where the three terms correspond to scattering into s-polarized and p-polarized waves propagating away from the interface and surface plasmon polaritons, respectively. By dividing C_{ext} in this way it will become possible to address the question of how large a fraction of the scattered light is scattered into surface plasmon polariton waves.

III. CONSTRUCTION OF GREEN'S TENSOR THROUGH AN EIGENMODE EXPANSION

In this section we will construct Green's tensor $\boldsymbol{G}$. A detailed description of the method for constructing Green's tensor through an eigenmode expansion is given in Ref. 36 In this paper we will use the method of Ref. 36 to construct the Green's tensor for our case of a metal-dielectric interface that supports SPP's. The eigenmode expansion method allows decomposition of the metal-dielectric interface Green's tensor into parts that govern scattering of light into s- and p-polarized modes propagating away from the interface, surface plasmon polaritons, and a part related to a quasistatic near-field.

Green's tensor can be constructed through an eigenmode expansion of the form

$$\boldsymbol{G}(\boldsymbol{r},\boldsymbol{r}')=\boldsymbol{G}_{GT}(\boldsymbol{r},\boldsymbol{r}')+\boldsymbol{G}_L(\boldsymbol{r},\boldsymbol{r}')=\sum_n\frac{\boldsymbol{E}_n(\boldsymbol{r})\boldsymbol{E}_n^*(\boldsymbol{r}')}{N_n\lambda_n}+\sum_n\frac{\nabla\phi_n(\boldsymbol{r})[\nabla\phi_n(\boldsymbol{r}')]^*}{M_nk_0^2}, \tag{9}$$

where the generalized transverse part of Green's tensor $\boldsymbol{G}_{GT}$ is constructed from the complete set of transverse eigenmodes $\boldsymbol{E}_n(\boldsymbol{r})$ given by

$$-\nabla\times\nabla\times\boldsymbol{E}_n(\boldsymbol{r})+k_0^2\varepsilon(\boldsymbol{r})\boldsymbol{E}_n(\boldsymbol{r})=\lambda_n\varepsilon(\boldsymbol{r})\boldsymbol{E}_n(\boldsymbol{r}), \tag{10}$$

$$\nabla\cdot[\varepsilon(\boldsymbol{r})\boldsymbol{E}_n(\boldsymbol{r})]=0, \tag{11}$$

with the orthogonality relation

$$\int\varepsilon(\boldsymbol{r})\boldsymbol{E}_n(\boldsymbol{r})\cdot[\boldsymbol{E}_m(\boldsymbol{r})]^*d^3r=\delta_{nm}N_n. \tag{12}$$

The longitudinal or quasistatic part $\boldsymbol{G}_L$ is constructed from longitudinal eigenmodes that can be found from a complete set of scalar eigenmodes $\phi_n(\boldsymbol{r})$ satisfying

$$\nabla\cdot[\varepsilon(\boldsymbol{r})\nabla\phi_n]=\sigma_n\phi_n(\boldsymbol{r}), \tag{13}$$

with the orthogonality relation

$$\int\varepsilon(\boldsymbol{r})\nabla\phi_n(\boldsymbol{r})\cdot[\nabla\phi_m(\boldsymbol{r})]^*d^3r=\delta_{nm}M_n. \tag{14}$$

The term $\boldsymbol{G}_L$ does not contribute to the extinction cross section Eq. (7) since $\boldsymbol{G}_L(\boldsymbol{r},\boldsymbol{r})$ does not have an imaginary part for our choice of dielectric constants. In the following we will therefore concentrate on deriving $\boldsymbol{G}_{GT}$.

A. SURFACE PLASMON POLARITON CONTRIBUTION

The excitation of surface plasmon polaritons by a dipole source near the metal surface is governed by the part of Green's dyadic which is related to surface plasmon polariton eigenmodes. The surface plasmon polariton eigenmodes may be written as

$$\boldsymbol{E}_{\boldsymbol{\kappa}_\rho}^{SPP}(\boldsymbol{r})=\left(\hat{z}-i\sqrt{\frac{\varepsilon_1}{-\varepsilon_2}}\hat{\kappa}_\rho\right)e^{i\boldsymbol{\kappa}_\rho\cdot\boldsymbol{\rho}}e^{-\sqrt{\varepsilon_1/(-\varepsilon_2)}\kappa_\rho z},\quad z>0, \tag{15}$$

$$\boldsymbol{E}_{\boldsymbol{\kappa}_\rho}^{SPP}(\boldsymbol{r})=\left(\frac{\varepsilon_1}{\varepsilon_2}\hat{z}-i\sqrt{\frac{\varepsilon_1}{-\varepsilon_2}}\hat{\kappa}_\rho\right)e^{i\boldsymbol{\kappa}_\rho\cdot\boldsymbol{\rho}}e^{\sqrt{-\varepsilon_2/\varepsilon_1}\kappa_\rho z},\quad z<0, \tag{16}$$

where $\hat{z}$ is a unit vector normal to the surface, $\boldsymbol{\kappa}_\rho$ is an in-plane wave vector, $\hat{\kappa}_\rho=\boldsymbol{\kappa}_\rho/\kappa_\rho$ $(\kappa_\rho=|\boldsymbol{\kappa}_\rho|)$, and $\boldsymbol{\rho}$ is the in-plane position.

The eigenvalue $\lambda_{\boldsymbol{\kappa}_\rho}$, orthogonality relation, and normalization factor $N_{\boldsymbol{\kappa}_\rho}$ are given by

$$\lambda_{\boldsymbol{\kappa}_\rho}=k_0^2-\kappa_\rho^2\frac{\varepsilon_1+\varepsilon_2}{\varepsilon_1\varepsilon_2}, \tag{17}$$

$$\int\varepsilon(\boldsymbol{r})\boldsymbol{E}_{\boldsymbol{\kappa}_\rho}^{SPP}(\boldsymbol{r})\cdot[\boldsymbol{E}_{\boldsymbol{\kappa}_\rho'}^{SPP}(\boldsymbol{r})]^*d^3r=\delta(\boldsymbol{\kappa}_\rho-\boldsymbol{\kappa}_\rho')N_{\boldsymbol{\kappa}_\rho}, \tag{18}$$

$$N_{\boldsymbol{\kappa}_\rho}=(2\pi)^2\frac{1}{2}\frac{\sqrt{\varepsilon_1(-\varepsilon_2)}}{\kappa_\rho}\left(1-\frac{\varepsilon_1^2}{\varepsilon_2^2}\right). \tag{19}$$

By inserting in Eq. (9) we find the surface plasmon polariton contribution to Green's tensor

$$\boldsymbol{G}_{SPP}(\boldsymbol{r},\boldsymbol{r}')=\int_{\kappa_\rho=0}^{\infty}\kappa_\rho^2 d\kappa_\rho\int_{\phi_{\boldsymbol{\kappa}_\rho}=0}^{2\pi}d\phi_{\boldsymbol{\kappa}_\rho}\frac{\left(\hat{z}-i\sqrt{\frac{\varepsilon_1}{-\varepsilon_2}}\hat{\kappa}_\rho\right)\left(\hat{z}+i\sqrt{\frac{\varepsilon_1}{-\varepsilon_2}}\hat{\kappa}_\rho\right)e^{i\boldsymbol{\kappa}_\rho\cdot(\boldsymbol{\rho}-\boldsymbol{\rho}')}e^{-\sqrt{\varepsilon_1/(-\varepsilon_2)}\kappa_\rho(z+z')}}{(2\pi)^2\frac{1}{2}\sqrt{\varepsilon_1(-\varepsilon_2)}\left(1-\frac{\varepsilon_1^2}{\varepsilon_2^2}\right)\left(k_0^2\frac{\varepsilon_1\varepsilon_2}{\varepsilon_1+\varepsilon_2}-\kappa_\rho^2+i\epsilon\right)\frac{\varepsilon_1+\varepsilon_2}{\varepsilon_1\varepsilon_2}}, \tag{20}$$

where $\phi_{\boldsymbol{\kappa}_\rho}$ is an angle that defines the direction of $\hat{\kappa}_\rho$ and the infinitesimal positive number ϵ is necessary for obtaining the retarded Green's tensor. If we set $\boldsymbol{\rho}'=\boldsymbol{0}$ the integration over $\phi_{\boldsymbol{\kappa}_\rho}$ may be carried out analytically by using the formulas given in Appendix, which leads to the simplified expression

$$\boldsymbol{G}_{SPP}(\boldsymbol{r},\boldsymbol{r}')=\int_{\kappa_\rho=0}^{\infty}\kappa_\rho^2 d\kappa_\rho e^{-\sqrt{\varepsilon_1/(-\varepsilon_2)}\kappa_\rho(z+z')}\frac{\left[\hat{z}\hat{z}J_0(\kappa_\rho\rho)+\frac{\varepsilon_1}{\varepsilon_2}\left(\hat{\rho}\hat{\rho}J_0''(\kappa_\rho\rho)+\hat{\phi}\hat{\phi}\frac{J_0'(\kappa_\rho\rho)}{\kappa_\rho\rho}\right)+(\hat{z}\hat{\rho}-\hat{\rho}\hat{z})\sqrt{\frac{\varepsilon_1}{-\varepsilon_2}}J_0'(\kappa_\rho\rho)\right]}{2\pi\frac{1}{2}\sqrt{\varepsilon_1(-\varepsilon_2)}\left(1-\frac{\varepsilon_1^2}{\varepsilon_2^2}\right)\left(k_0^2\frac{\varepsilon_1\varepsilon_2}{\varepsilon_1+\varepsilon_2}-\kappa_\rho^2+i\epsilon\right)\frac{\varepsilon_1+\varepsilon_2}{\varepsilon_1\varepsilon_2}}, \tag{21}$$

where $\hat{\rho}$ and $\hat{\phi}$ ($\hat{\rho}=\boldsymbol{\rho}/\rho$, $\rho=|\boldsymbol{\rho}|$) are cylindrical in-plane coordinate unit vectors, J_0 is the Bessel function of the first kind of order 0, and the prime refers to the derivative with respect to the argument. Although in this paper we have restricted ourselves to the case of real dielectric constants, a procedure similar to that in Ref. 36 based on constructing Green's tensor using a biorthogonal set of eigenmodes for the case of complex dielectric constants also leads to Eq. (21). The expression (21) may therefore also be used for the case of metals with a complex dielectric constant.

By using the identity

$$\frac{1}{x+i\epsilon}=P\frac{1}{x}-i\pi\delta(x), \tag{22}$$

where P refers to the principal value, it is possible to split $\boldsymbol{G}_{SPP}$ into two parts,

$$\boldsymbol{G}_{SPP}^{\delta}(\boldsymbol{r},\boldsymbol{r}')=-i\frac{k_{SPP}}{2}\frac{e^{-\sqrt{\varepsilon_1/(-\varepsilon_2)}k_{SPP}(z+z')}}{\sqrt{\varepsilon_1(-\varepsilon_2)}\left(1-\frac{\varepsilon_1^2}{\varepsilon_2^2}\right)\frac{\varepsilon_1+\varepsilon_2}{\varepsilon_1\varepsilon_2}}\left[\hat{z}\hat{z}J_0(k_{SPP}\rho)+\frac{\varepsilon_1}{\varepsilon_2}\left(\hat{\rho}\hat{\rho}J_0''(k_{SPP}\rho)+\hat{\phi}\hat{\phi}\frac{J_0'(k_{SPP}\rho)}{k_{SPP}\rho}\right)+(\hat{z}\hat{\rho}-\hat{\rho}\hat{z})\sqrt{\frac{\varepsilon_1}{-\varepsilon_2}}J_0'(k_{SPP}\rho)\right], \tag{23}$$

$$\boldsymbol{G}_{SPP}^{P}(\boldsymbol{r},\boldsymbol{r}')=P\int_{\kappa_\rho=0}^{\infty}\kappa_\rho^2 d\kappa_\rho e^{-\sqrt{\varepsilon_1/(-\varepsilon_2)}\kappa_\rho(z+z')}\frac{\left[\hat{z}\hat{z}J_0(\kappa_\rho\rho)+\frac{\varepsilon_1}{\varepsilon_2}\left(\hat{\rho}\hat{\rho}J_0''(\kappa_\rho\rho)+\hat{\phi}\hat{\phi}\frac{J_0'(\kappa_\rho\rho)}{\kappa_\rho\rho}\right)+(\hat{z}\hat{\rho}-\hat{\rho}\hat{z})\sqrt{\frac{\varepsilon_1}{-\varepsilon_2}}J_0'(\kappa_\rho\rho)\right]}{2\pi\frac{1}{2}\sqrt{\varepsilon_1(-\varepsilon_2)}\left(1-\frac{\varepsilon_1^2}{\varepsilon_2^2}\right)\left(k_0^2\frac{\varepsilon_1\varepsilon_2}{\varepsilon_1+\varepsilon_2}-\kappa_\rho^2\right)\frac{\varepsilon_1+\varepsilon_2}{\varepsilon_1\varepsilon_2}}, \tag{24}$$

where $k_{SPP}=k_0\sqrt{\varepsilon_1\varepsilon_2/(\varepsilon_1+\varepsilon_2)}$ is the surface plasmon polariton wave number. The term $\boldsymbol{G}_{SPP}^{\delta}$ can in principle be obtained from the total Green's tensor by extracting the contribution to a Sommerfeld integral related to the pole of the reflection coefficient for p-polarized waves incident on the metal interface.[43] For the case of real dielectric constants we have a simple exact analytic expression for the imaginary part of $\boldsymbol{G}_{SPP}$. However, for the real part of $\boldsymbol{G}_{SPP}$ it is in general necessary to carry out a principal value integral. In the case where ρ is large, and $z+z'$ is small, in both cases on a wavelength scale, it is possible to give an analytic approximation to $\boldsymbol{G}_{SPP}^{P}$ which is asymptotically correct as ρ increases towards infinity, resulting in

$$\boldsymbol{G}_{SPP}(\boldsymbol{r},\boldsymbol{r}')\approx -i\frac{k_{SPP}}{2}\frac{e^{-\sqrt{\varepsilon_1/(-\varepsilon_2)}k_{SPP}(z+z')}}{\sqrt{\varepsilon_1(-\varepsilon_2)}\left(1-\frac{\varepsilon_1^2}{\varepsilon_2^2}\right)\frac{\varepsilon_1+\varepsilon_2}{\varepsilon_1\varepsilon_2}}H_0^{(1)}(k_{SPP}\rho)\left(\hat{z}\hat{z}-\frac{\varepsilon_1}{\varepsilon_2}\hat{\rho}\hat{\rho}+(\hat{z}\hat{\rho}-\hat{\rho}\hat{z})i\sqrt{\frac{\varepsilon_1}{-\varepsilon_2}}\right), \tag{25}$$

where $H_0^{(1)}$ is the Hankel function of first kind and order 0. The approximation (25) can be very useful for evaluating scattering between particles located close to a metal surface, when the scatterers are separated by a few wavelengths.[17]

For the case of real dielectric constants it is relatively simple to evaluate scattering into SPP's, since the principal value integral does not contribute to the imaginary part of Green's tensor, i.e.,

$$-\mathrm{Im}[\boldsymbol{G}_{SPP}(\boldsymbol{r},\boldsymbol{r})]=\frac{k_{SPP}}{2\sqrt{\varepsilon_1|\varepsilon_2|}\left(1-\frac{\varepsilon_1^2}{\varepsilon_2^2}\right)\frac{\varepsilon_1+\varepsilon_2}{\varepsilon_1\varepsilon_2}}\left(\hat{z}\hat{z}+\frac{\varepsilon_1}{|\varepsilon_2|}\frac{1}{2}(\hat{\rho}\hat{\rho}+\hat{\phi}\hat{\phi})\right)e^{-\sqrt{\varepsilon_1/|\varepsilon_2|}k_{SPP}2z}. \tag{26}$$

B. *s*-POLARIZED WAVE CONTRIBUTION

The *s*-polarized eigenmodes may be written as

$$\boldsymbol{E}^{s-pol.}_{\boldsymbol{\kappa}_\rho,\kappa_{z1}}(\boldsymbol{r})=\hat{\phi}_{\boldsymbol{\kappa}_\rho}[e^{-i\kappa_{z1}z}+r^s(\kappa_\rho,\kappa_{z1})e^{i\kappa_{z1}z}]e^{i\boldsymbol{\kappa}_\rho\cdot\boldsymbol{\rho}},\quad z>0, \tag{27}$$

$$\boldsymbol{E}^{s-pol.}_{\boldsymbol{\kappa}_\rho,\kappa_{z1}}(\boldsymbol{r})=\hat{\phi}_{\boldsymbol{\kappa}_\rho}[1+r^s(\kappa_\rho,\kappa_{z1})]e^{-i\kappa_{z2}z}e^{i\boldsymbol{\kappa}_\rho\cdot\boldsymbol{\rho}},\quad z<0, \tag{28}$$

where $\hat{\phi}_{\boldsymbol{\kappa}_\rho}$ is an in-plane unit vector perpendicular to $\boldsymbol{\kappa}_\rho$, κ_{z1} is the out-of-plane wave-vector component, and

$$\lambda_{\boldsymbol{\kappa}_\rho,\kappa_{z1}}=k_0^2-(\kappa_\rho^2+\kappa_{z1}^2)/\varepsilon_1, \tag{29}$$

$$\kappa_{z2}=\sqrt{\kappa_\rho^2\left(\frac{\varepsilon_2}{\varepsilon_1}-1\right)+\frac{\varepsilon_2}{\varepsilon_1}\kappa_{z1}^2}. \tag{30}$$

The Fresnel reflection coefficient for *s*-polarized waves is given by

$$r^s(\kappa_\rho,\kappa_{z1})=\frac{\kappa_{z1}-\kappa_{z2}}{\kappa_{z1}+\kappa_{z2}}. \tag{31}$$

Taking advantage of the fact that the magnitude of the Fresnel reflection coefficient equals 1 in our case of using real dielectric constants the orthogonality relation and normalization factor are given by

$$\int \varepsilon(\boldsymbol{r})\boldsymbol{E}_{\boldsymbol{\kappa}_\rho,\kappa_{z1}}(\boldsymbol{r})\cdot[\boldsymbol{E}_{\boldsymbol{\kappa}'_\rho,\kappa'_{z1}}(\boldsymbol{r})]^*d^3r = \delta(\boldsymbol{\kappa}_\rho-\boldsymbol{\kappa}_\rho{}')\,\delta(\kappa_{z1}-\kappa'_{z1})N_{\boldsymbol{\kappa}_\rho,\kappa_{z1}} + \text{non}-\text{singular terms}, \tag{32}$$

$$N_{\boldsymbol{\kappa}_\rho,\kappa_{z1}}=(2\pi)^3\varepsilon_1. \tag{33}$$

Construction of the *s*-polarized wave contribution to Green's dyadic by inserting in Eq. (9) leads to

$$\boldsymbol{G}_{s-pol}(\boldsymbol{r},\boldsymbol{r}')=\int_{\kappa_{z1}=0}^{\infty}d\kappa_{z1}\int_{\kappa_\rho=0}^{\infty}\kappa_\rho d\kappa_\rho\int_{\phi_{\boldsymbol{\kappa}_\rho}=0}^{2\pi}d\phi_{\boldsymbol{\kappa}_\rho}\hat{\phi}_{\boldsymbol{\kappa}_\rho}\hat{\phi}_{\boldsymbol{\kappa}_\rho}e^{i\boldsymbol{\kappa}_\rho\cdot(\boldsymbol{\rho}-\boldsymbol{\rho}')} \times\frac{e^{-i\kappa_{z1}(z-z')}+e^{i\kappa_{z1}(z-z')}+e^{i\kappa_{z1}(z+z')}r^s(\kappa_\rho,\kappa_{z1})+e^{-i\kappa_{z1}(z+z')}[r^s(\kappa_\rho,\kappa_{z1})]^*}{(2\pi)^3(k_0^2\varepsilon_1-\kappa_\rho^2-\kappa_{z1}^2+i\epsilon)}. \tag{34}$$

It is convenient to make the substitution $\kappa_{z1}=k\cos\theta$, $\kappa_\rho=k\sin\theta$, and make use of the formulas in Appendix ($\boldsymbol{\rho}'$ ($\boldsymbol{\rho}'=\boldsymbol{0}$) to cast the integral into a form suitable for evaluation of the scattering, i.e.,

$$\boldsymbol{G}_{s-pol.}(\boldsymbol{r},\boldsymbol{r}')=\int_{\theta=0}^{\pi/2}d\theta\int_{k=0}^{\infty}k^2\sin\theta dk\left(\hat{\rho}\hat{\rho}\frac{-J_0'(\kappa_\rho\rho)}{\kappa_\rho\rho}+\hat{\phi}\hat{\phi}[-J_0''(\kappa_\rho\rho)]\right)\frac{1}{(2\pi)^2(k_0^2\varepsilon_1-k^2+i\epsilon)}\{2\cos[k\cos\theta(z-z')] +2\mathrm{Re}[e^{ik\cos\theta(z+z')}r^s(\theta)]\}, \tag{35}$$

where

$$r^s(\theta)=\frac{\cos\theta-i\sqrt{\sin^2\theta-\varepsilon_2/\varepsilon_1}}{\cos\theta+i\sqrt{\sin^2\theta-\varepsilon_2/\varepsilon_1}}. \tag{36}$$

In Eq. (35) the angle θ may be interpreted as an angle of propagation with respect to the surface normal vector. By using Eq. (22) the scattering of light into s-polarized modes can be written as an integral over an angular emission spectrum $\boldsymbol{I}_{s-pol}(\theta)$ for waves propagating away from the interface, i.e.,

$$-\mathrm{Im}[\boldsymbol{G}_{s-pol}(\boldsymbol{r},\boldsymbol{r})]=\int_{\theta=0}^{\pi/2} I_{s\text{-}pol}(\theta)\sin\theta d\theta, \tag{37}$$

where

$$\boldsymbol{I}_{s\text{-}pol}(\theta)=\frac{k_0\sqrt{\varepsilon_1}}{4\pi}\frac{1}{2}\{1+\mathrm{Re}[e^{ik_0\sqrt{\varepsilon_1}\cos\theta 2z}r^s(\theta)]\}(\hat{\rho}\hat{\rho}+\hat{\phi}\hat{\phi}). \tag{38}$$

The Green's tensor contribution $\boldsymbol{G}_{s\text{-}pol}$ may also be cast in the form of a Sommerfeld integral by applying the residue theorem to Eq. (34), whereby

$$\boldsymbol{G}_{s\text{-}pol}(\boldsymbol{r},\boldsymbol{r}')=\frac{i}{4\pi k_1^2}\int_{\kappa_\rho=0}^{\infty} d\kappa_\rho[e^{i\kappa_{z1}|z-z'|}+e^{i\kappa_{z1}(z+z')}r^s(\kappa_\rho,\kappa_{z1})]\left(\hat{\phi}\hat{\phi}\kappa_\rho\frac{k_1^2}{\kappa_{z1}}J_0''(\kappa_\rho\rho)+\hat{\rho}\hat{\rho}\frac{J_0'(\kappa_\rho\rho)}{\rho}\frac{k_1^2}{\kappa_{z1}}\right), \tag{39}$$

where in this expression $\kappa_{z1}=\sqrt{k_0^2\varepsilon_1-\kappa_\rho^2}$ and $k_1^2=k_0^2\varepsilon_1$.

C. *P*-POLARIZED WAVE CONTRIBUTION

The p-polarized eigenmodes may be written

$$\boldsymbol{E}^{p\text{-}pol}_{\boldsymbol{\kappa}_\rho,\kappa_{z1}}(\boldsymbol{r})=\left(\frac{\hat{z}\kappa_\rho+\hat{\kappa}_\rho\kappa_{z1}}{\sqrt{\kappa_\rho^2+\kappa_{z1}^2}}e^{-i\kappa_{z1}z}+r^p(\kappa_\rho,\kappa_{z1})\times\frac{\hat{z}\kappa_\rho-\hat{\kappa}_\rho\kappa_{z1}}{\sqrt{\kappa_\rho^2+\kappa_{z1}^2}}e^{+i\kappa_{z1}z}\right)e^{i\boldsymbol{\kappa}_\rho\cdot\boldsymbol{\rho}},\quad z>0 \tag{40}$$

where the Fresnel reflection coefficient for p-polarized waves is given by

$$r^p(\kappa_\rho,\kappa_{z1})=\frac{\varepsilon_2\kappa_{z1}-\varepsilon_1\kappa_{z2}}{\varepsilon_2\kappa_{z1}+\varepsilon_1\kappa_{z2}}. \tag{41}$$

The expressions for eigenvalue, κ_{z2}, and normalization factor, are similar to the case of s-polarized waves, and the contribution to Green's tensor from p-polarized waves can be constructed in a similar way, i.e.,

$$\boldsymbol{G}_{p\text{-}pol}(\boldsymbol{r},\boldsymbol{r}')=\int_{\kappa_{z1}=0}^{\infty}d\kappa_{z1}\int_{\kappa_\rho=0}^{\infty}\kappa_\rho d\kappa_\rho\int_{\phi_{\kappa_\rho}=0}^{2\pi}d\phi_{\kappa_\rho}\frac{\boldsymbol{E}^{p\text{-}pol}_{\boldsymbol{\kappa}_\rho,\kappa_{z1}}(\boldsymbol{r})[\boldsymbol{E}^{p\text{-}pol}_{\boldsymbol{\kappa}_\rho,\kappa_{z1}}(\boldsymbol{r}')]^*}{(2\pi)^3(k_0^2\varepsilon_1-\kappa_\rho^2-\kappa_{z1}^2+i\epsilon)}. \tag{42}$$

Following the scheme used for s-polarized waves we can make the substitution $\kappa_{z1}=k\cos\theta$, $\kappa_\rho=k\sin\theta$, make use of the formulas in Appendix, and use Eq. (22) to evaluate the imaginary part of Green's tensor for $\boldsymbol{r}=\boldsymbol{r}'$, resulting in

$$-\mathrm{Im}[\boldsymbol{G}_{p\text{-}pol}(\boldsymbol{r},\boldsymbol{r})]=\int_{\theta=0}^{\pi/2}I_{p\text{-}pol}(\theta)\sin\theta d\theta, \tag{43}$$

where

$$\boldsymbol{I}_{p\text{-}pol}(\theta)=\frac{k_0\sqrt{\varepsilon_1}}{4\pi}(\hat{z}\hat{z}\sin^2\theta\{1+\mathrm{Re}[e^{ik_0\sqrt{\varepsilon_1}\cos\theta(2z)}r^p(\theta)]\}+\tfrac{1}{2}(\hat{\phi}\hat{\phi}+\hat{\rho}\hat{\rho})\cos^2\theta\{1-\mathrm{Re}[e^{ik_0\sqrt{\varepsilon_1}\cos\theta(2z)}r^p(\theta)]\}), \tag{44}$$

$$r^p(\theta)=\frac{\varepsilon_2\cos\theta-\varepsilon_1 i\sqrt{\sin^2\theta-\varepsilon_2/\varepsilon_1}}{\varepsilon_2\cos\theta+\varepsilon_1 i\sqrt{\sin^2\theta-\varepsilon_2/\varepsilon_1}}. \tag{45}$$

The contribution for p-polarized waves can also be cast in the form of a Sommerfeld integral by use of the residue theorem. As a preparation to this we will transform the integral for κ_{z1} into an integral over the interval from $-\infty$ to $+\infty$, i.e.,

$$G_{p\text{-}pol}(\mathbf{r},\mathbf{r}')=\int_{\kappa_{z1}=-\infty}^{\infty}d\kappa_{z1}\int_{\kappa_\rho=0}^{\infty}\kappa_\rho d\kappa_\rho\int_{\phi_{\kappa_\rho}=0}^{2\pi}d\phi_{\kappa_\rho}\frac{e^{i\boldsymbol{\kappa}_\rho\cdot(\boldsymbol{\rho}-\boldsymbol{\rho}')}}{(2\pi)^3(k_0^2\varepsilon_1-\kappa_\rho^2-\kappa_{z1}^2+i\epsilon)}\Bigg\{\hat{z}\hat{z}\frac{\kappa_\rho^2}{\kappa_\rho^2+\kappa_{z1}^2}[e^{i\kappa_{z1}(z-z')}+e^{i\kappa_{z1}(z+z')}r^p(\kappa_\rho,\kappa_{z1})]+\hat{\kappa}_\rho\hat{\kappa}_\rho\frac{\kappa_{z1}^2}{\kappa_\rho^2+\kappa_{z1}^2}[e^{i\kappa_{z1}(z-z')}-e^{i\kappa_{z1}(z+z')}r^p(\kappa_\rho,\kappa_{z1})]+(\hat{z}\hat{\kappa}_\rho+\hat{\kappa}_\rho\hat{z})\frac{\kappa_{z1}\kappa_\rho}{\kappa_\rho^2+\kappa_{z1}^2}(-e^{i\kappa_{z1}(z-z')})-(\hat{z}\hat{\kappa}_\rho-\hat{\kappa}_\rho\hat{z})\frac{\kappa_{z1}\kappa_\rho}{\kappa_\rho^2+\kappa_{z1}^2}[-e^{i\kappa_{z1}(z+z')}r^p(\kappa_\rho,\kappa_{z1})]\Bigg\}. \quad (46)$$

The integration over κ_{z1} can be carried out by means of residue calculus. Here one should observe the poles for $1/(\kappa_{z1}^2+\kappa_\rho^2)$, r^p, and for $1/(k_0^2\varepsilon_1-\kappa_{z1}^2-\kappa_\rho^2+i\epsilon)$. The residue related to the poles $\kappa_{z1}=\pm i\kappa_\rho$ actually results in a term that except for opposite sign is equal to $G_L(\mathbf{r},\mathbf{r}')$. The residue related to the pole of r^p results in a term that except for opposite sign is equal to $G_{SPP}(\mathbf{r},\mathbf{r}')$.

By calculating the residue related to the poles of $1/(k_0^2\varepsilon_1-\kappa_{z1}^2-\kappa_\rho^2+i\epsilon)$, and further calculating the integral over ϕ_{κ_ρ} by using the formulas in Appendix ($\boldsymbol{\rho}'=\mathbf{0}$), we obtain

$$G_{p\text{-}pol}(\mathbf{r},\mathbf{r}')+G_{SPP}(\mathbf{r},\mathbf{r}')+G_L(\mathbf{r},\mathbf{r}')=\frac{-i}{4\pi k_0^2\varepsilon_1}\int_{\kappa_\rho=0}^{\infty}d\kappa_\rho\Bigg(\hat{z}\hat{z}J_0(\kappa_\rho\rho)\frac{\kappa_\rho^3}{\kappa_{z1}}(e^{i\kappa_{z1}|z-z'|}+r^pe^{i\kappa_{z1}(z+z')})+\left[\hat{\phi}\hat{\phi}\left(\frac{-J_0'(\kappa_\rho\rho)}{\kappa_\rho\rho}\right)+\hat{\rho}\hat{\rho}[-J_0''(\kappa_\rho\rho)]\right]\kappa_\rho\kappa_{z1}(e^{i\kappa_{z1}|z-z'|}-r^pe^{i\kappa_{z1}(z+z')})+i(\hat{z}\hat{\rho}+\hat{\rho}\hat{z})\kappa_\rho^2J_0'(\kappa_\rho\rho)e^{i\kappa_{z1}|z-z'|}-i(\hat{z}\hat{\rho}-\hat{\rho}\hat{z})\kappa_\rho^2J_0'(\kappa_\rho\rho)r^pe^{i\kappa_{z1}(z+z')}\Bigg). \quad (47)$$

The total Green's tensor is obtained as the sum of Eqs. (47) and (39).

IV. MODAL DISTRIBUTION OF SCATTERED LIGHT

The results of the preceding section, in particular Eqs. (43), (37), and (26), now give us the possibility to divide the extinction cross section into three terms, namely,

$$C_{ext}^{SPP}=k_0^3\left(\frac{\varepsilon_p-\varepsilon_1}{\varepsilon_p+2\varepsilon_1}4\pi a^3\right)^2\varepsilon_1^2\boldsymbol{\mu}^*\cdot\{-\mathrm{Im}[G_{SPP}(\mathbf{r},\mathbf{r})]\}\cdot\boldsymbol{\mu}, \quad (48)$$

$$C_{ext}^{s\text{-}pol}=k_0^3\left(\frac{\varepsilon_p-\varepsilon_1}{\varepsilon_p+2\varepsilon_1}4\pi a^3\right)^2\varepsilon_1^2\boldsymbol{\mu}^*\cdot\{-\mathrm{Im}[G_{s\text{-}pol}(\mathbf{r},\mathbf{r})]\}\cdot\boldsymbol{\mu}, \quad (49)$$

$$C_{ext}^{p-pol.}=k_0^3\left(\frac{\varepsilon_p-\varepsilon_1}{\varepsilon_p+2\varepsilon_1}4\pi a^3\right)^2\varepsilon_1^2\boldsymbol{\mu}^*\cdot\{-\mathrm{Im}[G_{p\text{-}pol}(\mathbf{r},\mathbf{r})]\}\cdot\boldsymbol{\mu}, \quad (50)$$

which govern scattering into SPP's and *s*- and *p*-polarized modes propagating away from the metal-dielectric interface, respectively. It is convenient to normalize these cross sections with respect to the scattering cross section of a particle located in a homogeneous dielectric with dielectric constant ε_1, i.e.,

$$C_{ext}^{hom}=k_0^3\left(\frac{\varepsilon_p-\varepsilon_1}{\varepsilon_p+2\varepsilon_1}4\pi a^3\right)^2\varepsilon_1^2\left(\frac{k_0\sqrt{\varepsilon_1}}{4\pi}\frac{2}{3}\right). \quad (51)$$

In the following, we shall specifically consider the case of the incident wave being a SPP with a plane phase front [cf. Eq. (15)]. The part of the extinction cross section, which is related to SPP-to-SPP scattering, is shown in Fig. 1 as a function of the dielectric constant of the metal (ε_2) for different heights of the scatterer z above the air-metal interface ($\varepsilon_1=1$). Here λ refers to the free-space wavelength of light. Note that in the case of the light wavelength being close to the SPP resonance, in which case ε_2 is close to $-\varepsilon_1$, the SPP-to-SPP scattering cross section may become orders of magnitudes larger than the free-space scattering cross section. This enhancement can be understood from Eq. (26) as a

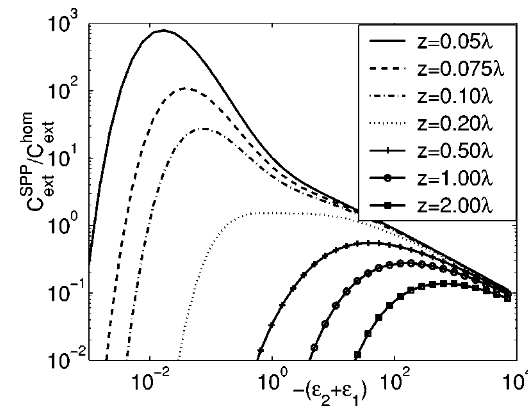

FIG. 1. Normalized extinction cross section related to scattering from a surface plasmon polariton (SPP) to SPP's for a particle located in air ($\varepsilon_1=1$) at the height z above a metal surface with metal dielectric constant ε_2. λ is the free-space wavelength of light.

result of the denominator approaching zero for $\varepsilon_2 = -\varepsilon_1$. The cross section will, however, not become infinitely large for $z > 0$, since the term $e^{-\sqrt{\varepsilon_1/(-\varepsilon_2)}k_{SPP}2z}$ in Eq. (26) also goes to zero at the SPP resonance, and exactly at resonance there is no scattering into SPP's. Note that as the height of the scatterer above the surface increases, the maximum in the SPP-to-SPP extinction cross section dependence moves away from the SPP resonance. For a scatterer-to-surface distance $z = \lambda$, the optimization of SPP-to-SPP scattering requires the dielectric constant to be close to -100. In the perfect conductor limit ($\varepsilon_2 = -\infty$), the SPP-to-SPP extinction cross section tends to zero, which is quite natural because of the absence of SPP's in this limit. However, the magnitude of ε_2 has to be very large before scattering into SPP's can be considered negligibly small.

Note that especially in the vicinity of the SPP resonance the requirement for the incident SPP to not vary appreciably across the particle leads to a restriction on the sphere radii for which our theory is valid, namely, $a \ll \sqrt{(\varepsilon_1 + \varepsilon_2)/\varepsilon_1\varepsilon_2}\lambda/2\pi$. For the case of, e.g., $\varepsilon_1 = 1$, $\varepsilon_2 = -1.1$ the requirement would be $a \ll 0.05\lambda$.

The calculations in this paper have been restricted to the case of real dielectric constants for two reasons. The first reason is that the evaluation of the various parts of the extinction cross section becomes particularly simple. When we, e.g., calculate $\mathrm{Im}[\boldsymbol{G}_{SPP}(\boldsymbol{r},\boldsymbol{r})]$ we do not have to be concerned with the principal value integral Eq. (24). The second reason is that for the case of metals with absorption the amount of light scattered into SPP's can only be evaluated in an approximate sense, which fundamentally is related to the fact that for lossy materials a complete set of orthogonal modes no longer exists. However, our expression for the part of Green's tensor related to excitation of SPP's Eq. (21) and the approximation Eq. (25) are also valid for metals with absorption. From Eq. (25) we observe that the SPP excitation is relatively unaffected by the presence of a small metal loss $[\mathrm{Im}(\varepsilon_2) \ll \mathrm{Re}(\varepsilon_2)]$ as long as $\mathrm{Im}(\varepsilon_1 + \varepsilon_2) \ll \mathrm{Re}(\varepsilon_1 + \varepsilon_2)$, which requires the loss to be very small as we approach the SPP resonance.

The scattering from SPP's into *s*- and *p*-polarized waves propagating away from the air-metal interface is presented in Fig. 2. Here we note that for large magnitudes of ε_2, the SPP-to-*s*-polarization extinction cross section rapidly decreases, because the in-plane field component of the incident SPP field tends to zero in this case (and *s*-polarized waves can only be excited by an in-plane field component at the site of the scatterer). For the SPP-to-*p*-polarization scattering, the extinction cross section on the other hand increases for large magnitudes of ε_2, becoming eventually larger than the total free-space extinction cross section. Optimum SPP-to-SPP scattering efficiency does not necessarily imply that the scatterer height must be very small, because the different parts of the extinction cross section do not have the same dependence on the height of the scatterer. The fraction of scattered light that goes into SPP's relative to the total scattered light is shown in Fig. 3. It is seen that for small heights z of the scatterer, the efficiency of SPP-to-SPP scattering can be close to 100% in the vicinity of the SPP resonance. As the height z increases, the optimum efficiency of this scattering

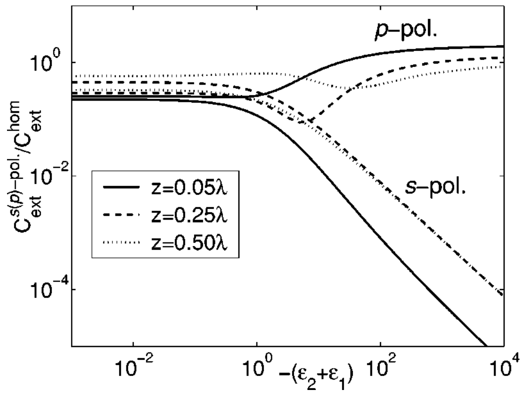

FIG. 2. Normalized extinction cross section related to scattering from a surface plasmon polariton to *s*-polarized and *p*-polarized waves that propagate away from an air-metal interface by a particle located in air ($\varepsilon_1 = 1$) at the height z above the metal surface with metal dielectric constant ε_2. λ is the free-space wavelength of light.

requires progressively larger magnitudes of ε_2. Thus, for example, for a height $z \approx 0.7\lambda$, the optimum dielectric constant of the metal is found at $\varepsilon_2 \approx -100$. Note though that this height is not optimum for this particular dielectric constant. Indeed, for scatterers very close to the surface, the SPP-to-SPP scattering efficiency is below 40%, whereas if the height is increased to 0.4λ the efficiency can be increased above 60%. If the height is further increased to 0.7λ, the efficiency is again close to 40% and becomes negligible when the scatterer moves further away from the surface.

V. CONCLUSION

In conclusion, we have presented an analytical study of scattering of light by a small spherical particle located near a metal-dielectric interface, focusing on the SPP-to-SPP scattering. The extinction cross section for a small spherical particle has been expressed in terms of the electric-field Green's tensor for the structure in which the particle is placed. A formulation has been developed for Green's tensor of a

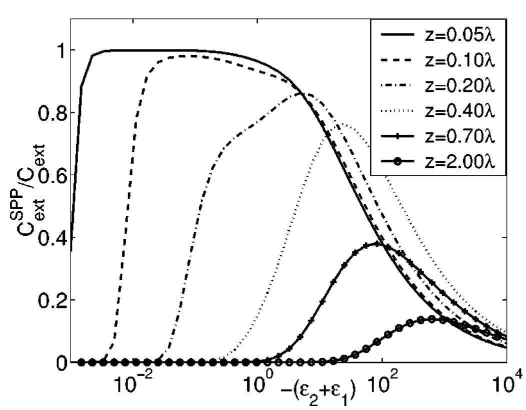

FIG. 3. Fraction of scattered light which is scattered into surface plasmon polaritons by a particle located in air ($\varepsilon_1 = 1$) at the height z above a metal surface with metal dielectric constant ε_2. λ is the free-space wavelength of light.

metal-dielectric interface structure, in which Green's tensor is divided into parts that govern SPP excitation, excitation of *s*- and *p*-polarized waves propagating away from the metal-dielectric interface, and a part responsible for a quasistatic (near) field. This formulation of Green's tensor allowed us to divide the extinction cross section into parts accounting for scattering into SPP's and waves propagating away from the interface, respectively. The approach developed has been applied to the case of SPP scattering by a particle placed in air above a metal surface. It has been shown that for the dielectric constant of metal being close to, but not exactly at, the SPP resonance, the extinction cross section for the SPP-to-SPP scattering can be orders of magnitude larger compared to the free-space particle extinction cross section. We have further found that as the height of the scatterer above the surface increases, the metal dielectric constant maximizing the SPP-to-SPP extinction cross section moves away from that corresponding to the SPP resonance. It was also seen that as the metal dielectric constant approaches the perfect conductor limit the SPP-to-SPP extinction cross section tends to zero. Finally, we have evaluated the efficiency of the SPP-to-SPP scattering, i.e., the fraction of the scattered power which is related to scattering into SPP's. For the case of particles placed close to the metal surface and metal dielectric constants corresponding to being close to the SPP resonance, the efficiency of the SPP-to-SPP scattering was found to be very close to 100%. As the height of the scatterer increases, the optimum efficiency of SPP-to-SPP scattering requires progressively larger magnitudes of the metal dielectric constant, i.e., that we are further and further away from the SPP resonance. For a specific metal dielectric constant, it is in turn possible to optimize the height of the particle with respect to the efficiency of the SPP-to-SPP scattering. We believe that the results obtained can be used as practical guidelines in experimental investigations of SPP scattering phenomena, e.g., when dealing with SPP micro-optical elements.

APPENDIX: FOMULAS INVOLVING BESSEL FUNCTIONS

$$\frac{1}{2\pi}\int_{\phi_{\boldsymbol{\kappa}_\rho}=0}^{2\pi} d\phi_{\boldsymbol{\kappa}_\rho} e^{i\boldsymbol{\kappa}_\rho\cdot\boldsymbol{\rho}} = J_0(\kappa_\rho\rho), \tag{A1}$$

$$\frac{1}{2\pi}\int_{\phi_{\boldsymbol{\kappa}_\rho}=0}^{2\pi} d\phi_{\boldsymbol{\kappa}_\rho} \hat{\phi}_{\boldsymbol{\kappa}_\rho}\hat{\phi}_{\boldsymbol{\kappa}_\rho} e^{i\boldsymbol{\kappa}_\rho\cdot\boldsymbol{\rho}} = \hat{\rho}\hat{\rho}\frac{-J_0'(\kappa_\rho\rho)}{\kappa_\rho\rho} + \hat{\phi}\hat{\phi}[-J_0''(\kappa_\rho\rho)], \tag{A2}$$

$$\frac{1}{2\pi}\int_{\phi_{\boldsymbol{\kappa}_\rho}=0}^{2\pi} d\phi_{\boldsymbol{\kappa}_\rho} \hat{\kappa}_\rho\hat{\kappa}_\rho e^{i\boldsymbol{\kappa}_\rho\cdot\boldsymbol{\rho}} = \hat{\phi}\hat{\phi}\left(\frac{-J_0'(\kappa_\rho\rho)}{\kappa_\rho\rho}\right) + \hat{\rho}\hat{\rho}[-J_0''(\kappa_\rho\rho)], \tag{A3}$$

$$\frac{1}{2\pi}\int_{\phi_{\boldsymbol{\kappa}_\rho}=0}^{2\pi} d\phi_{\boldsymbol{\kappa}_\rho} \hat{z}\hat{\kappa}_\rho e^{i\boldsymbol{\kappa}_\rho\cdot\boldsymbol{\rho}} = \hat{z}\hat{\rho}[-iJ_0'(\kappa_\rho\rho)]. \tag{A4}$$

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