Persistent currents in the presence of nonclassical electromagnetic fields

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Persistent currents in mesoscopic rings and cylinders threaded by a magnetostatic flux and also by monochromatic nonclassical electromagnetic fields are considered. The results depend on the quantum state of the nonclassical electromagnetic fields. It is shown that quantum and thermal noise in the field reduces the current and can change its character from diamagnetic to paramegnetic, and vice versa. Four different examples of nonclassical electromagnetic fields are considered (number eigenstates, coherent states with randomized phase, coherent states, and thermal states) and the corresponding currents are calculated. Two-mode entangled electromagnetic fields are also considered, and the effect of entanglement on the currents is studied.

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I. INTRODUCTION

It is already well established, both theoretically[1] and experimentally,[2] that in mesososcopic multiply connected nonsuperconductive samples, such as cylinders or rings (carbon nanotubes are also an example), threaded by static magnetic flux, persistent currents are developed. They are direct currents in the equilibrium state[3,4] in a normal metal or semiconducting sample with a size-induced energy gap at the Fermi surface. For a recent review, see Ref. 4. There is still disagreement between the theory and experiment concerning the actual amplitude of the persistent currents (the experimentally observed currents are much larger than the theoretically predicted ones).

In this paper we explore a different regime, namely, persistent currents in the case where in addition to the magnetostatic flux we also have electromagnetic fields. Related experimental work has been reported in Ref. 5. However, in this paper we go much further by considering nonclassical electromagnetic fields.[6]

Nonclassical electromagnetic fields are carefully prepared in a particular quantum state described mathematically by a density matrix ρ. In this case we know not only the average values $\langle E\rangle$ and $\langle B\rangle$ of the electric and magnetic fields, but also the standard deviations ΔE and ΔB, which describe both the quantum the classical noise. Classical noise can be eliminated at least in principle, but the quantum noise will be present. Nonclassical electromagnetic fields have been used to manipulate and control the quantum noise (subject to the uncertainty principle constraint). For example, squeezed electromagnetic fields have very small ΔE at the expense of large ΔB so that the product $\Delta E \Delta B$ obeys the uncertainty relation.

An alternative way of describing nonclassical electromagnetic fields is by knowing the statistics of photons $p_N = \langle N|\rho|N\rangle$ threading the ring. The distribution p_N is also a way to describe the noise. In nonclassical electromagnetic fields it can be narrower than a Poisson distribution, in contrast to classical ones, where it is a Poisson or a wider than Poisson distribution. Preparation of these fields in the laboratory is certainly nontrivial, but several states (squeezed states, number eigenstates, Schrödinger catstates, etc.) have been produced at both microwave[7] and optical frequencies[6] in the last 15 years. The latest effort in this area is to consider two-mode fields and produce entangled states.

The interaction of mesoscopic devices with nonclassical electromagnetic fields is an interdisciplinary area that studies how the quantum phenomena in these carefully prepared electromagnetic fields will affect the currents in the device. More specifically, what is the effect of the quantum noise ΔE and ΔB, of the photon statistics, or of the entanglement (in the case of two-mode fields) on the currents?

In our paper we discuss persistent currents in mesoscopic thin rings threaded by both classical and nonclassical electromagnetic fields. However, we want to stress that the considerations presented are also valid for a set of rings stacked along a certain axis and for a thin cylinder made of a material with a flat Fermi surface.

We limit our discussion to microwave radiation since its energy can be smaller than the energy gap at the Fermi surface in the typical ring. In that sense the presence of quantum light does not move the system far from the equilibrium and it can be treated as a perturbation of our system. In this way considerations concerning various nonequilibrium effects[8] can be omitted. The resulting system remains in equilibrium, and the application of equilibrium formulas for the current is justified. We first consider monochromatic fields produced in a cavity and discuss the influence of the quantum noise on the amplitude of the currents. We also consider two-mode fields with frequencies ω_1 and ω_2. Here we consider both separable (classically correlated) fields described with density matrices of the type

$$\rho = \sum_i p_i \rho_{1i} \otimes \rho_{2i}, \tag{1}$$

where p_i are probabilities and ρ_{1i}, ρ_{2i} density matrices describing the two modes; and also entangled (quantum mechanically correlated) fields. We study their effect on the electric currents in the rings and compare and contrast the results.

The work belongs in the general context of studying fully quantum mechanical devices comprised of mesoscopic devices interacting with nonclassical electromagnetic fields.

Such devices operate with few electrons and few photons deeply into the quantum regime and are potentially useful for quantum technologies. Related work in the context of Josephson devices has been presented in Ref. 9 and in the context of Aharonov-Bohm electron interference in Ref. 10.

II. PERSISTENT CURRENTS

We consider a thin metallic or semiconducting ring threaded by the classical magnetic flux ϕ. We limit our discussion to thin rings since it allows us to neglect the self-inductance effects in the system.

For our model calculations let us consider first a ring of circumference l_x with an even number M_e of electrons. The persistent current running at temperature T in the ring is given by[3]

$$I_e(\phi/\phi_0,T)=I_0\sum_{n=1}^{\infty} A_n(T)\sin\left(\frac{2\pi n\phi}{\phi_0}\right) \tag{2}$$

with

$$A_n(T)=\frac{4T}{\pi T^*}\,\frac{\exp(-nT/T^*)}{1-\exp(-2nT/T^*)}\cos(nk_F l_x), \tag{3}$$

where the flux quantum $\phi_0 := 2\pi/e$ (in units $\hbar=c=k_B=1$). The amplitude I_0 of the current depends crucially on the material and the band filling and is given by

$$I_0 := heM_e/(2l_x^2 m_e), \tag{4}$$

where m_e is the electron mass. The characteristic temperature is given by the relation $k_B T^*=\Delta_F/2\pi^2$ where k_B is the Boltzmann constant, Δ_F is the energy gap at the Fermi surface, and k_F is the Fermi wave vector. The current Eq. (2) is a periodic function of ϕ with period ϕ_0.

The characteristic of a current flowing in a ring with an odd number M_e of electrons can be obtained by the shift $\phi \to \phi+\phi_0/2$ in Eq. (2). The slope of the current characteristic (2) at $\phi=0$ (i.e., the sign of the derivative with respect to ϕ) allows one to distinguish the parity of M_e. A current that at $\phi=0$ has a positive slope we *call aparamagnetic current*, whereas a current with a negative slope at $\phi=0$ we *call a diamagnetic current*. Rings with an even (odd) number of electrons M_e exhibit a paramagnetic (diamagnetic) persistent current.

The formulas (2) and (3) are valid not only for quasi-one-dimensional rings but also, if the amplitude I_0 is replaced by $\tilde{M}I_0$, where $\tilde{M}$ is the number of current channels in the system,[11] for a set of rings stacked along a certain axis or a mesoscopic cylinder made of a material with a flat Fermi surface. Currents that are similar to Eq. (2) and (3) have also been found for carbon nanotubes.[12] In the particular case of zigzag nanotubes [of rolling vector $(m_1,0)$] with a lowered Fermi surface, the currents obtained are paramagnetic for even m_1 and diamagnetic for odd m_1. The following analysis can therefore be extended to them.

III. MONOCHROMATIC MICROWAVES

It is well known that the vector potential $\mathbf{A}$ and the electric field $\mathbf{E}$ are dual canonical variables of the quantized electromagnetic field. Since both the magnetic flux $\phi=\oint_C \mathbf{A}\, dl$ and the electromotive force $V_{EM}=\oint_C \mathbf{E}\, dl$ are related to the canonical variables via an integration performed over the circumference of the ring, the flux ϕ and the electromotive force $V_{EM}=-i\omega\partial_\phi$ form an equivalent pair of dual variables[9,10] satisfying

$$[\phi,\omega^{-1}V_{EM}]=i. \tag{5}$$

We introduce the annihilation operator

$$a=\frac{1}{\sqrt{2}}(\phi+i\omega^{-1}V_{EM}). \tag{6}$$

The Hamiltonian of the monochromatic electromagnetic field reads

$$H=\omega\left(a^\dagger a+\frac{1}{2}\right). \tag{7}$$

Assuming that the back reaction is negligible (i.e., the electromagnetic field created by the electrons in the ring is negligible), we find[9,10] that the flux operator evolves in time as

$$\phi=\frac{1}{\sqrt{2}}[\exp(i\omega t)a^\dagger+\text{H.c.}]. \tag{8}$$

Renormalization of the flux ϕ will be required for various ring sizes. The ring size should be such that the energy gap at the Fermi surface is much greater than the energy of the microwaves.

In the following we introduce the dimensionless variable $x:=\phi/\phi_0$ and the current $I(x,T):=I_e(x,T)/I_0$. If M_e is odd, the current $I(x,t)=I_o(x,T)/I_0=I_e(x+1/2,T)/I_0$. The most general total current is $I(x,T)=pI_e+(1-p)I_o$, where p is the probability of occurrence of even M_e.[13]

We assume that the magnetic flux threading the ring has a classical component λ (magnetostatic flux or low frequency electromagnetic field) and a quantum component x_q (high frequency electromagnetic field with $\hbar\omega \gg k_B T$):

$$x=\lambda+x_q. \tag{9}$$

We define the complex current operator as

$$\begin{aligned} I_c(x,T) &:= \sum_{n=1}^{\infty} A_n(T)\exp(i2\pi nx) \\ &= \sum_{n=1}^{\infty} A_n(T)\exp(i2\pi n\lambda)\exp(i2\pi nx_q). \end{aligned} \tag{10}$$

The expectation value is calculated by taking the trace $\langle I_c\rangle=\mathrm{Tr}(\rho I_c)$ with respect to the density operator ρ of the nonclassical electromagnetic field. The imaginary part of this expectation value is equal to the observed current, i.e.,

$$I(x,T)=\Im\langle I_c\rangle. \tag{11}$$

Changing the time origin $t \to t - \pi/2\omega$ and introducing $\xi := \sqrt{2}\pi/\phi_0$ allows us to rewrite the current operator in the form

$$I_c(x,T) := \sum_{n=1}^{\infty} A_n(T)\exp(i2\pi n\lambda)D(n\xi e^{i\omega t}), \qquad (12)$$

where $D(A) := \exp(Aa^{\dagger} - A^{*}a)$ is the displacement operator. The calculation of the expectation value of the current reduces to the calculation of the so-called Weyl function

$$W(\zeta_n) = \mathrm{Tr}\{\rho D[n\xi \exp(i\omega t)]\}, \qquad \zeta_n = n\xi \exp(i\omega t). \qquad (13)$$

The current is given by

$$\langle I_c \rangle = \sum_{n=1}^{\infty} A_n(T)\exp(i2\pi n\lambda)W(\zeta_n). \qquad (14)$$

The result is in general time dependent, but in special examples it might be time independent.

In order to see the effect of the nonclassical nature of the electromagnetic field, we compare the various terms in the sum of Eq. (10) with the corresponding terms in the sum of Eq. (7) for the case where x_q is a classical number. It is seen that the phase factor $\exp(i2\pi n x_q)$, which has absolute value equal to 1, is replaced by the Weyl function, which has $|W(\zeta_n)| < 1$. We interpret this as reduction of the current due to the noise (classical and quantum) in the electromagnetic field.

IV. EXAMPLES

In this section we consider various types of nonclassical electromagnetic fields and calculate the corresponding currents. We first work in the zero temperature limit where the current $I(x,0) = I(x)$ with $A_n(0) = 2\cos(k_F l_x)/(\pi n)$. This limit is convenient for our model calculations since both the phase coherence of the electrons in the ring and the quantum properties of the flux are most visible. In Sec. IV (D) we also consider thermal electromagnetic fields and calculate the corresponding currents. The Weyl functions needed for these calculations were given in Ref. 9.

A. Number eigenstates

Here the nonclassical component of the electromagnetic field is assumed to be in a number eigenstate. In this case $\rho = |N\rangle\langle N|$, and the total current is:

$$\langle I_c \rangle = \sum_{n=1}^{\infty} A_n(0)\exp(i2\pi, n\lambda)B_n(N),$$

$$B_n(N) = \exp(-n^2\xi^2/2)L_N(n^2\xi^2), \qquad (15)$$

where L_N are Laguerre polynomials. In this particular example the result is time independent. This is related to the fact that the phase in number eigenstates is random.

A plot of the current I as a function of the classical flux λ for several values of N is given in Fig. 1. We see that the presence of the number eigenstates not only modifies the amplitude of the current but also leads to qualitatively different results. Let us consider the response of the system for an externally applied infinitesimally small classical flux. The response can be either paramagnetic [for the slope $\partial I(\lambda = 0)/\partial\lambda > 0$] or diamagnetic [for the slope $\partial I(\lambda = 0)/\partial\lambda < 0$]. In the following we consider first a paramagnetic current flowing in a ring with an even number M_e of electrons. The current is a periodic function of λ with period $\lambda = 1$ and when it is driven by the classical flux only it is paramagnetic at small λ since $\partial I(\lambda = 0)/\partial\lambda = +\infty$. If we switch on the nonclassical flux the paramagnetic current decreases, and there exists some critical $N_c^{para} \approx 20$ for which the current becomes diamagnetic for small λ (Fig. 1). The behavior of a ring with odd M_e can be deduced from Fig. 1 when we move the origin in Fig. 1 by $\lambda = 1/2$. The current when driven by the classical flux only is then diamagnetic for small λ. The appearance of the nonclassical flux decreases the current's amplitude, and for $N_c^{dia} \approx 45$ the slope of the characteristic at $\lambda = 0$ changes sign. It follows from the considerations presented that for $N_c^{para} < N < N_c^{dia}$ the system always has diamagnetic reaction to small λ, no matter what is the parity of M_e. We also notice the unexpected result that, although the slope of the current at $\lambda \to 0$ for the diamagnetic current is smaller than for the paramagnetic one, it requires bigger N to change the sign ($N_c^{dia} > N_c^{para}$). This phenomenon can be understood from Eqs. (11) and (15), where the first two nonvanishing terms of the series determine the qualitative character of the current. In the case of an initially paramagnetic current all harmonics $\sin(2\pi n\lambda)$ are paramagnetic, whereas in the case of a diamagnetic current the odd harmonics $\sin[2\pi n(\lambda + 1/2)]$ are diamagnetic while the even harmonics are paramagnetic. This together with a close inspection of the magnitude and sign of $B_1(N)$ and $B_2(N)$ shows that it requires larger N to change the sign of the slope of the current at $\lambda = 0$ in the initially diamagnetic case since both the first nonvanishing terms support a diamagnetic current for $N < N_c^{dia}$.

This mechanism explains all the differences appearing in the paper in the critical behavior between para- and diamagnetic currents with respect to an externally applied small flux.

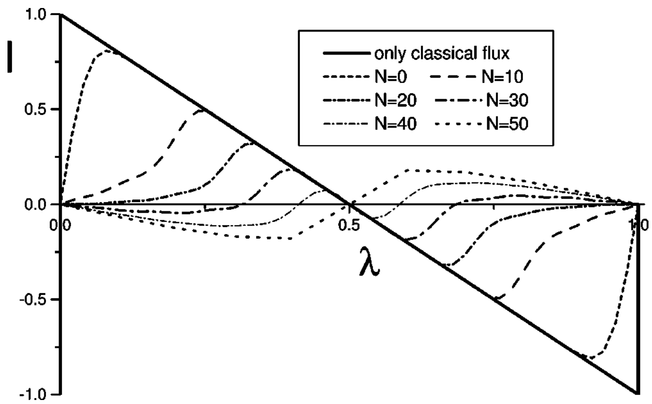

FIG. 1. The current Eq. (11) in the presence of the number eigenstate vs applied classical flux calculated by use of Eq. (15).

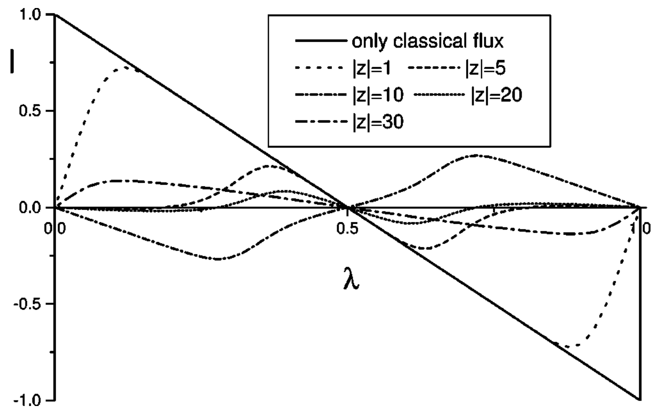

FIG. 2. The current Eq. (11) vs external classical flux in the presence of a coherent state with randomized phase for several values of $|z|$ calculated by use of Eq. (18).

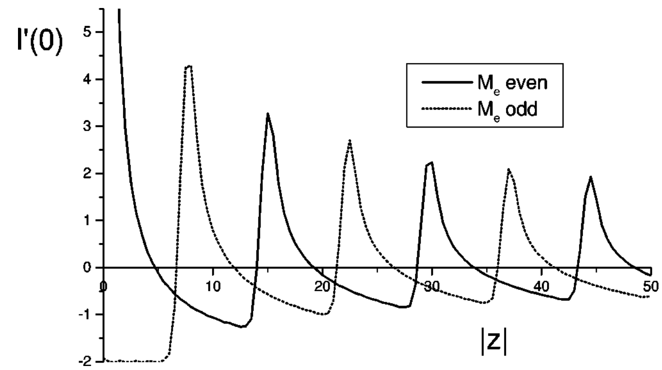

FIG. 3. The slope $I'(0)$ in the presence of a coherent state with randomized phase for *initially* para- and diamagnetic currents.

The current for $N=0$ (vacuum) is different from the current in the absence of a quantum flux

$$\langle I_c\rangle=\sum_{n=1}^{\infty} A_n(0)\exp(i2\pi,n\lambda)B_n^{vac},$$

$$B_n^{vac}:=B_n(0)=\exp(-n^2\xi^2/2). \tag{16}$$

It is a nontrivial example of the role played by the quantum vacuum fluctuations at the mesoscopic level. The vacuum $|0\rangle\langle 0|$ is a pure state, and thus the only noise present in the system is quantum noise caused by the finite fluctuations of the flux operator. The terms B_n^{vac} in the series given in Eq. (16) are Weyl functions. We see that fluctuations of the monochromatic vacuum modify the current characteristics, removing the nondifferentiability (infinite slope) of the current.

B. Coherent states with randomized phase

In this and the next section we consider electromagnetic fields in coherent states. We start with the more realistic (and experimentally easier) case where the phase of the coherent state[14] $|z\rangle:=D(z)|0\rangle$ is unknown. We assume *coherent states with fully randomized phase* described by the density matrix

$$\rho=\int\frac{d\Theta_z}{2\pi}\| z|e^{i\Theta_z}\rangle\langle |z|e^{i\Theta_z}|. \tag{17}$$

The current in the ring is in this case given by the formula

$$\langle I_c\rangle=\sum_{n=1}^{\infty} A_n(0)\exp(i2\pi n\lambda)B_n(|z|),$$

$$B_n(|z|)=\exp(-n^2\xi^2/2)J_0(2\xi|z|n), \tag{18}$$

where J_0 are Bessel functions.[15] In this particular example the result is time independent. This is related to the fact that the phase is random.

A plot of the current vs the externally applied classical flux in the ring with an even M_e is given in Fig. 2. The presence of the quantum flux results in lowering of the current amplitude (Fig. 2). As a result, the paramagnetic current changes into a diamagnetic one when $|z|$ attains some critical value $|z|_c^{para}\approx 5$. With further increase of $|z|$ we obtain the paramagnetic current again. A shift of the origin in Fig. 2 ($\lambda\to\lambda+1/2$) gives information about a system with an initially diamagnetic current (i.e., with odd M_e). We see that the current changes to paramagnetic (Fig. 3) for $|z|_c^{dia}\approx 7$.

Since the character of the current is fully characterized by the slope $I'(0):=\partial I(\lambda=0)/\partial\lambda$, we plot the slope as a function of $|z|$ in Fig. 3. Again, one can recognize a range of the parameter $|z|_c^{para}<|z|<|z|_c^{dia}$ where the only reaction of the system to a small classical flux is always diamagnetic, with no regard to the parity of M_e. This happens when both curves in Fig. 3 are below zero. It means that the quantum flux can change a qualitatively persistent current from paramagnetic to diamagnetic, and vice versa. The amplitude of the current flowing in the presence of a coherent state with randomized phase is sensitive to the value of $|z|$. With increasing $|z|$ the maximal value of the current for $0<\lambda<1/2$ is an oscillating function with decreasing amplitude. First the current is damped and then changes its character from para- to diamagnetic. Further, its amplitude increases, passes through a maximum, and later decreases, causing the appearance of a paramagnetic current. This scenario is periodic in $|z|$, but the next maximal value of the amplitude is always smaller than the previous one.

The behavior of the current flowing in a mesoscopic ring threaded by a flux with a nonclassical component in a coherent state with randomized phase can by explained by similar arguments as applied for the current flowing in the presence of flux in the number eigenstate [with the modification $B_n(N)\to B_n(|z|)$].

The influence of the nonclassical flux on persistent currents in both the number eigenstate with small N and the coherent state with randomized phase with small $|z|$ is, to some extent, similar to the influence of temperature or impurities. They all result in decreasing the amplitude of the persistent current. This similarity goes even deeper if one realizes that the slope of an initially diamagnetic current flowing in the ring with an odd number M_e of electrons is more ''stable'' compared to a paramagnetic current, with respect to the nonclassical flux, as can be seen from Figs. 1 and 3. If either $|z|$ or N is small, the slope is almost constant, as in the case of a current at finite temperature in the absence of a quantum flux. With increasing $|z|$ or N the similarity is no longer present, since neither the temperature nor the impurities can change the current Eq. (2) from para- to diamagnetic.

C. Coherent states

In this section we study the properties of the current in a mesoscopic ring in the presence of the standard coherent states $\rho=|z\rangle\langle z|$ where $|z\rangle:=D(z)|0\rangle$.[14] The current is in this case given by the formula

$$\langle I_c\rangle=\sum_{n=1}^{\infty} A_n(0)\exp(i2\pi n\lambda)B_n(|z|,\Theta_z),$$

$$B_n(|z|,\Theta_z)=\exp(-n^2\xi^2/2)\sum_{k=-\infty}^{\infty} J_k(2\xi|z|n)\times\exp[ik(\omega t-\Theta_z)], \tag{19}$$

where $\Theta_z:=\mathrm{Arg}z$ (Ref. 9) and J_k are Bessel functions.[15] It is seen that the current is time dependent. This is related to the fact that coherent states have a phase (which we called Θ_z) that evolves in time into $\Theta_z-\omega t$. In contrast, the previous example where the phase of the coherent states is randomized gave a time-independent current.

We note that the current not only has frequency ω but also all the higher harmonics $N\omega$. This is due to the highly nonlinear nature of the device.

It might be difficult to observe all these high frequencies experimentally in a direct way. However, we can effectively convert them into dc currents, if we add a time-dependent component to the classical part of the flux. This shifts the frequency of a particular component to zero, and we can observe it as dc current. Let $\lambda=\lambda_0+\Omega t$ where Ω is such that

$$2\pi n\Omega=k\omega, \tag{20}$$

where k is an integer. In other words, the ratio $2\pi\Omega/\omega=q=k/n$ is a rational number. The current possesses two types of direct component corresponding either to $\Omega<0$ (upper signs) or to $\Omega>0$ (lower signs) and is given by

$$I_{dc}=\sum_{n=1}^{\infty} A_n(0)J_{\pm qn}(2\xi|z|n)\sin(\Lambda_n^{\mp}), \tag{21}$$

where $\Lambda_n^{\mp}=n(2\pi\lambda_0\mp q\Theta_z)$. Such direct currents parametrized by the rational numbers q are analogous to the *Shapiro steps* in a current flowing in a superconducting ring with a Josephson junction,[9] but in the case of the nonsuperconducting rings they are more "dense," since they are labeled by rational numbers and not by integers as in the case of Shapiro steps. We limit our discussion to the case $q=1$ since the amplitude of the current for $q>1$ is small. The current corresponding to $\Omega<0$ is obtained if the classical flux is decreased from λ_0 through $\lambda=0$ and further to negative values of the flux. If $\Omega>0$ the flux is growing linearly.

In the presence of a coherent state one needs to take into account two parameters describing the state of the quantum flux. The first is the phase Θ_z and the second is $|z|$. A plot of the current Eq. (21) for several values of the phase and $|z|=1$ vs λ_0 is given in Fig. 4. The phase Θ_z of the coherent state causes a horizontal shift of the current characteristics. We see that for $\Theta_z\neq 0$ we get a finite persistent current even in the absence of static magnetic flux ($\lambda_0=0$).

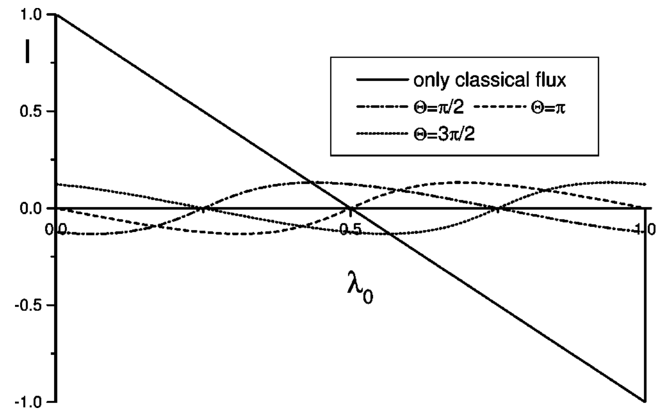

FIG. 4. The current Eq. (11) in the presence of a coherent state with $|z|=1$ for several values of the phase Θ_z vs external static classical flux λ_0 and $\Omega<0$ calculated by use of Eq. (21).

The influence of the amplitude $|z|$ (Fig. 5) modifies both the amplitude and the character of the current (Fig. 6) in a periodic way. It is similar to the behavior observed in a system subject to a coherent state with randomized phase. The maximal value of the current is an oscillating function of $|z|$ with decreasing amplitude. The main difference appears if one investigates a coherent state with small $|z|$. In the case of a coherent state with randomized phase, the current smoothly approaches the vacuum characteristics. For standard coherent states, a small value of $|z|$ results in a vanishingly small amplitude of the current. In the limit $|z|=0$, the current for $\Omega\neq 0$ is no longer direct. If the quantum flux is in the vacuum state the current is given by the formula Eq. (16) with $B_n^{vac}=B_n(|z|=0,\Theta_z=0)=B_n(N=0)$. For very small values of $|z|$ the dc current flowing in the ring with even M_e ($\Omega<0$) is paramagnetic, which can be deduced from Fig. 5, where we show the current vs λ_0 for fixed $\Theta_z=0$ for differ-

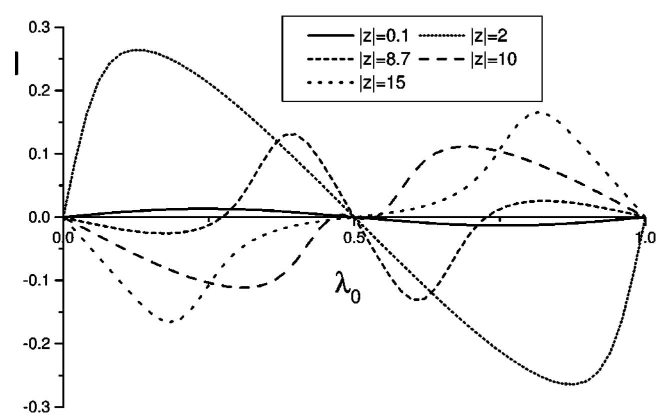

FIG. 5. The current Eq. (11) in the presence of a coherent state vs λ_0 for $\Theta_z=0$ and several values of $|z|$ calculated by use of Eq. (21).

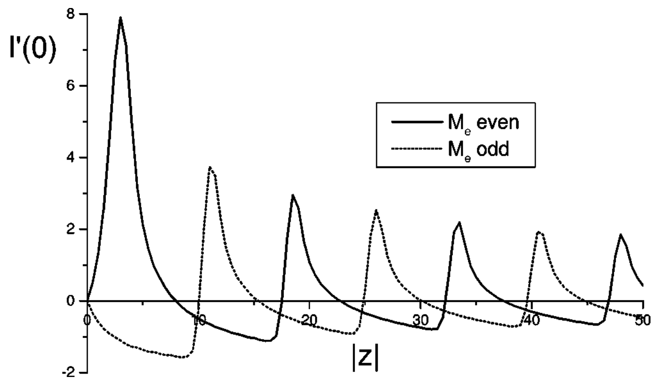

FIG. 6. The slope $I'(0)$ vs $|z|$ in the presence of the coherent state $\| z|\rangle\langle|z|\|$ for a system exhibiting initially para- or diamagnetic reaction for infinitesimal flux.

ent values of $|z|$. Increasing $|z|$ results in increase of the amplitude of the current to some maximal value near $|z| \approx 10$ (Fig. 6). Further increasing $|z|$ leads to a decrease of the amplitude and to a system with a diamagnetic current (negative slope at $\lambda_0 = 0$). This behavior is periodic, but the maximal values of the current attainable in each ''period'' decrease with increasing $|z|$. The range of $|z|$ where both curves in Fig. 6 are below the $|z|$- axis, indicating a negative slope $I'(0)$, is again the union of intervals. If $\Omega > 0$ the current behaves as if it were running in the ring with odd M_e. Both currents corresponding to positive and negative Ω are identical only for $\Theta_z = \pi/2$ and $\Theta_z = 3\pi/2$. The dependence of the current on the phase of the coherent state leads to a finite value $I(\lambda_0 = 0) \neq 0$ of the current in the absence of a static magnetic flux. The value $I(\lambda_0 = 0)$ vs Θ_z is plotted in Fig. 7. We see that for almost all values of the phase the current $I(\lambda_0 = 0) \neq 0$ with no regard to other parameters. This means that under certain conditions it is possible to obtain a persistent current driven by *a time-dependent flux*.

There is a range of parameters of the coherent state of the flux driving a persistent current when, again to some extent, the influence of the nonclassical light is similar to the influence of temperature or impurities, i.e., the amplitude of the current is slightly lowered. In contrast to the case of a coherent state with randomized phase, this range does *not* appear for small $|z|$ but rather somewhere near $|z| \approx 3$, where the amplitude of the current is maximal. Inspection of the gradient of the slope $I'(0)$ in Fig. 6 leads to the conclusion that the diamagnetic current is more ''stable'' with respect to the coherent state than the paramagnetic current. This ''stability'' is not so strong as in the case of the flux in the number eigenstate or in a coherent state with randomized phase.

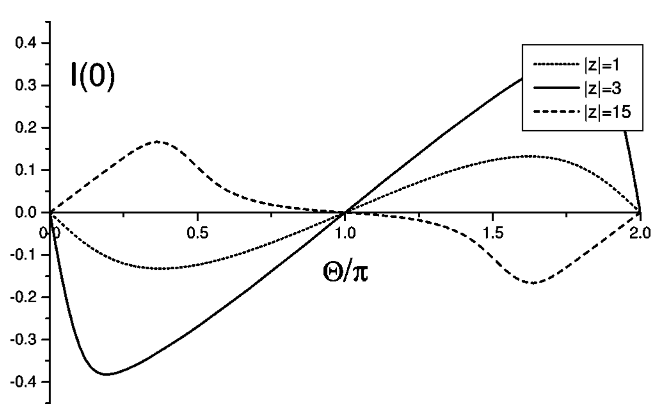

FIG. 7. The current $I(\lambda_0 = 0)$ vs Θ_z for $|z| = 1$.

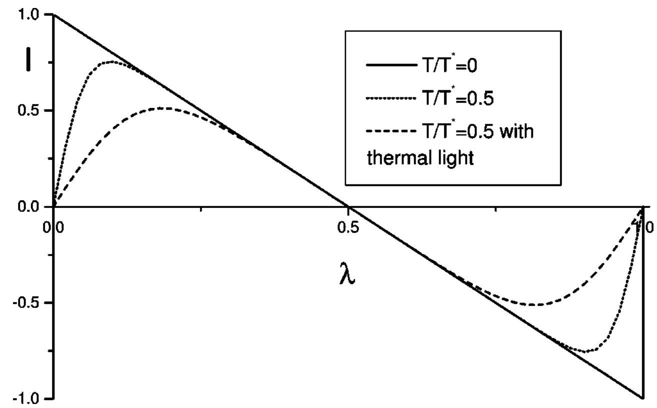

FIG. 8. The current vs λ for flux with and without thermal noise in a ring with $\beta\omega \sim 0.05T^*/T$.

D. Thermal fields

Here we consider thermal fields described by the density matrix

$$\rho = (1 - e^{-\beta\omega}) \sum_N e^{-N\beta\omega} |N\rangle\langle N|. \tag{22}$$

In this case the current is

$$\langle I_c \rangle = \sum_{n=1}^{\infty} A_n(T) \exp(i2\pi n\lambda) B_n^{th}(T),$$

$$B_n^{th}(T) = \exp[-n^2 \xi^2 \coth(\beta\omega/2)/2], \tag{23}$$

where $\beta = 1/k_B T$. The result in this case is time dependent. As one might expect, the equilibrium fluctuations whether classical[13] or quantum (Fig. 8) do not destroy completely the persistent currents which survive in realistic experiments. As an example, let us consider $\omega \sim 1\,\mathrm{cm}^{-1}$ and a ring with $T^* \sim 100\mathrm{K}$; then $\beta\omega \sim 0.05T^*/T$. Thus the effective decrease of the amplitude is not destructive for current in the region $T \leqslant T^*$.

V. TWO-MODE MICROWAVES

We now consider two-mode microwaves. In this case Eq. (8) becomes

$$x_q = \frac{1}{\phi_0\sqrt{2}} [\exp(i\omega_1 t) a_1^\dagger + \exp(-i\omega_1 t) a_1] + \frac{1}{\phi_0\sqrt{2}} [\exp(i\omega_2 t) a_2^\dagger + \exp(-i\omega_2 t) a_2], \tag{24}$$

where the indices 1, 2 refer to the two modes. Consequently the expectation value of the current is given by

$$\langle I_c\rangle = \sum_{n=1}^{\infty} A_n(T)\exp(i2\pi n\lambda)W(\zeta_{1n},\zeta_{2n}), \tag{25}$$

where the two-mode Weyl function is

$$W(\zeta_{1n},\zeta_{2n}) = \mathrm{Tr}\{\rho D_1[n\xi\exp(i\omega_1 t)]D_2[n\xi\exp(i\omega_2 t)]\}, \tag{26}$$

with $\zeta_{in}=n\xi\exp(i\omega_i t)$. If the two microwave modes are uncorrelated the density matrix is factorizable, $\rho=\rho_1\otimes\rho_2$ (where ρ_1,ρ_2 are the density matrices of the two modes), and the result for the corresponding Weyl function is

$$W(\zeta_{1n},\zeta_{2n}) = W_1(\zeta_{1n})W_2(\zeta_{2n}), \tag{27}$$

where

$$W_i(\zeta_{in}) = \mathrm{Tr}[\rho_i D_i(\zeta_{in})], \tag{28}$$

which we insert in Eq. (25) to find the current.

Separable systems are those that are correlated classically and are described by density matrices of the form

$$\rho = \sum_N p_N \rho_{1N}\otimes\rho_{2N}, \qquad N=1,\dots,M, \tag{29}$$

where ρ_{1N} and ρ_{2N} are two sets of density matrices describing the first and second modes, respectively, and p_N are probabilities. Entangled systems are those that are correlated quantum mechanically and whose density matrices cannot be written in the form of Eq. (24). For separable systems,

$$W(\zeta_{1n},\zeta_{2n}) = \sum_N p_N W_{1N}(\zeta_{1n})W_{2N}(\zeta_{2n}), \tag{30}$$

which we insert in Eq. (25) to find the current.

We discuss the effect of entanglement between the two microwave modes on the current, with an example. We consider the entangled state $|s\rangle = 2^{-1/2}[|01\rangle + |10\rangle]$ where $|01\rangle,|10\rangle$ are two mode number eigenstates. For comparison we also consider the corresponding separable state

$$\rho_{sep} = \frac{1}{2}[|01\rangle\langle 01| + |10\rangle\langle 10|]. \tag{31}$$

Clearly, the density matrix of the entangled state $\rho_{ent} = |s\rangle\langle s|$ can be written as

$$\rho_{ent} = \rho_{sep} + \frac{1}{2}[|01\rangle\langle 10| + |10\rangle\langle 01|]. \tag{32}$$

The corresponding Weyl function for the separable state is

$$W_{sep}(\zeta_{1n},\zeta_{2n}) = \left[1-\frac{n^2\xi^2e^2}{2}\right]\exp\left(-\frac{1}{2}n^2\xi^2e^2\right) \tag{33}$$

and for the entangled state

$$W_{ent}(\zeta_{1n},\zeta_{2n}) = W_{sep}(\zeta_{1n},\zeta_{2n}) + W_{cross}(\zeta_{1n},\zeta_{2n})$$

$$W_{cross}(\zeta_{1n},\zeta_{2n}) = -\frac{n^2\xi^2e^2}{2}\cos[(\omega_1-\omega_2)t] \times \exp\left(-\frac{1}{2}n^2\xi^2e^2\right). \tag{34}$$

These calculations have been presented and used in a different context in Ref. 10. Inserting them in Eq. (25) to find the current, it is seen that, in the particular example that we have considered, the current is time independent for classically correlated (separable) microwaves and has an extra time-dependent component in the case of entangled microwaves.

The results are an example of how purely quantum phenomena (which have no classical analog) in the electromagnetic fields can affect mesoscopic devices.

VI. SUMMARY

Mesoscopic rings and cylinders are very sensitive devices that can ''feel'' the quantum mechanical nature of the electromagnetic field. In this paper we discussed mesoscopic rings or cylinders with quasi-one-dimensional conductance (flat Fermi surface). However, that kind of analysis can also be applied to cylinders with an arbitrary Fermi surface. In such systems the formulas (2) and (3) for the persistent current are slightly different[11,16] and the amplitude of the current is reduced, but the qualitative results to not change.

We investigated persistent currents in mesoscopic rings and cylinders in the presence of both the classical and nonclassical components of the electromagnetic flux. We showed that the nonclassical light does not destroy persistent currents but decreases their amplitude and can lead to a change of character from para- to diamagnetic, and vice versa. The current flux characteristics depend strongly on the state of the nonclassical electromagnetic field.

Persistent currents can flow for nonclassical flux in a number eigenstate and in a coherent state with randomized phase in the presence of *static* classical flux. We showed that for a certain range of parameters describing the state of the quantum light a mesoscopic ring reacts with a diamagnetic current for a small classical magnetic flux, with no regard to the parity of M_e.

We also found that in the case of nonclassical flux in the standard coherent state and in the presence of a *time-dependent* classical flux satisfying the condition given by Eq. (20), we can also get persistent currents. Such currents bear a resemblance to Shapiro steps in a current flowing in the Josephson junction.

We showed how the vacuum noise whose source is the uncertainties of the quantum operators modifies the current characteristics. In the last section we discussed the influence of the equilibrium (thermal) noise on the properties of the current. We showed that it does not qualitatively change the behavior of the currents. This, together with our earlier studies,[13] leads to the conclusion that persistent currents survive in the presence of equilibrium fluctuations provided that the parameters of the system are far from critical (e.g., the energy of the thermal excitation is not comparable with the gap at the Fermi surface).

The work presented in the paper shows the interplay between quantum phenomena in electromagnetic fields and quantum phenomena in mesoscopic devices. It shows that mesoscopic rings and cylinders can serve as detectors of nonclassical light. It shows a possible application of, e.g., carbon nanotubes, which are an example of mesoscopic cylinders exhibiting coherent motion of electrons. On the other hand, there is the possibility of *controlling* persistent current amplitude in mesoscopic devices by applying a suitably chosen nonclassical flux.

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