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Period and phase of microwave-induced resistance oscillations and zero-resistance states in two-dimensional electron systems

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We present phenomenological analysis of the period and the phase of microwave-induced resistance oscillations (MIRO) and "zero-resistance" states (ZRS) recently observed in two-dimensonal electron systems. While MIRO are found to be well described by a "1/4-cycle shift," we find that as MIRO evolve into ZRS with increasing magnetic field B, the phase becomes progressively smaller, decreasing roughly as $1/B$. As a result, both maxima and minima in the ZRS regime form periodic in $1/B$ sequences, without a 1/4-cycle shift. A simple model based on oscillatory density of states is found to provide a unified description of the period/phase of MIRO/ZRS and leads to an interpretation of previously reported fine structures in terms of multiphoton processes.

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Using microwave (MW) photoconductivity spectroscopy of two-dimensional electron systems (2DES)[1] originally employed in experiments on microwave-induced resistance oscillations (MIRO),[2] two research groups have recently reported on a series of "zero-resistance" states[3,4] (ZRS) emerging from the MIRO minima in ultrahigh quality samples. Manifesting a novel dissipationless regime, such states appear when the MW frequency $\omega = 2\pi f$ somewhat exceeds an integer multiple of the cyclotron frequency $\omega_c = eB/m$ of the 2DES (m is the effective electron mass) and are characterized by an exponentially small low-temperature resistance and a nearly classical Hall resistance. More recently, experiments have been extended to probe dc conductivity in Corbino rings, revealing "zero-conductance" states[5] (ZCS) in agreement with standard dc magnetotransport tensor relation. Discovery of ZRS has triggered a surge of theoretical interest[6] and has been confirmed in independent experiments.[7,8] As it was realized that even the origin of MIRO lacks understanding, the first step forward was made by Durst *et al.*[9] who related the phenomenon to radiation-induced impurity-assisted scattering (in fact, similar ideas were proposed decades ago by Ryzhii[10]). Regardless of the microscopic nature of the MIRO, it was established experimentally[1] that high sample mobility favors MIRO amplitude, therefore, one could intuitively expect that advances in the sample quality would eventually lead to zero or negative resistance at the MIRO minima. As the later scenario is not realized,[3,4] Andreev *et al.*[11] presented strong arguments showing that a negative resistance (conductance) state, regardless of its origin, is intrinsically unstable to formation of current (Hall field) domains[11–13] which give rise to ZRS (ZCS). On the other hand, as these and other theories remain untested, current understanding of the phenomenon appears far from complete. Furthermore, while there seems to be a consensus about the period of the ZRS, the value of the "phase" remains one of the puzzles surrounding experimental reports.[3,4] Since the majority of the proposed models account for or even rely on a specific value of the phase, it is important to address this issue.

In this paper we present detailed analysis of the period and the phase of the MIRO and ZRS, which complements original experimental findings.[3,4] More specifically, we find that while MIRO are well described by a "1/4-cycle shift," in the ZRS regime the phase decreases roughly as $1/B$. Such dependence virtually eliminates a 1/4-cycle shift[3] and appears, under reasonable assumptions, consistent with the model[9] based on the oscillatory density of states (DOS). We also find that reconciling experiment with theory[9] requires a somewhat ($\approx 5\%$) reduced value of the effective mass, as compared to the generally accepted value of $0.067m_0$. Finally, our analysis explains the fine structures first reported in Ref. 4 in terms of multiphoton processes.

Despite a great deal of similarity between the data presented in Ref. 3 and that of Ref. 4, conclusions regarding the phase of the ZRS differ greatly. Mani *et al.*[3] have found the positions of the maxima/minima ($\pm$, respectively) of MIRO and ZRS to be universally described by

$$\varepsilon_j^{\pm} = j \mp 1/4, \tag{1}$$

where $\varepsilon \equiv \omega/\omega_c$ and j is a positive integer. According to Eq. (1) resistance extrema could be obtained assuming that photoresistance is described by a single-harmonic function, e.g., $\cos[2\pi(\varepsilon+1/4)] = -\sin(2\pi\varepsilon)$.

On the other hand, Zudov *et al.*[4] have reported that *major* ($j \lesssim 4$) maxima can be roughly fitted to $\varepsilon_j^{+} \approx j$. The positions of the major minima, contrary to the maxima, are not defined since ZRS span a wide B range. Naively one could take the ZRS center as its position but the high-temperature data[4] and asymmetry of the major maxima[4] rule against such assumption. Instead, it was proposed[4] that ZRS minima could also be viewed as a roughly periodic (in $1/B$) sequence, with no apparent phase, although with an enhanced period (cf. Fig. 3 in Ref. 4). Observations of Ref. 4 are summarized as follows:

$$\varepsilon_j^{\pm} = \alpha^{\pm} j, \quad j \lesssim 4, \tag{2}$$

where $\alpha^{\pm}$ is a constant close to unity. Higher-order ($j \gtrsim 4$) MIRO were found to conform to Eq. (1) although such approach required a somewhat low value of the effective mass ($m_{\mathrm{MIRO}} = 0.064m_0$), as opposed to $m_{\mathrm{ZRS}} = 0.068$ obtained using Eq. (2) (with $\alpha^{+} = 1$) for $j \lesssim 4$.[14]

It immediately follows that, experimentally, the boundary of applicability of Eqs. (1) and (2), i.e., $j \approx 4$, also seems to

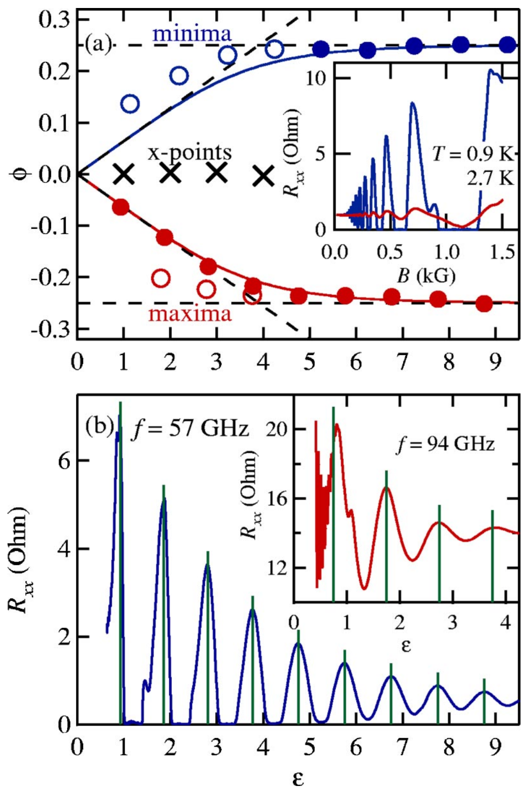

FIG. 1. (a) Solid (open) circles are experimental values of the phase $\phi^{\pm}=\varepsilon_j^{\pm}-j$ extracted from the data (Ref. 4) (see inset) taken at $T=0.9$ K (2.7 K) and $f=57$ GHz; ×-symbols mark the crossing points of the two traces and, according to the theory (Ref. 9) represent CR harmonics. Solid (dashed) lines are exact (asymptotic) solutions of Eq. (8) for $\Gamma=0.3$ K. (b) Magnetoresistance[4] as a function of ε under MW illumination of $f=57$ GHz taken at $T=0.9$ K. Abscissas of vertical lines are calculated using Eqs. (3) and (8) for the maxima. Inset shows magnetoresistance (Ref. 2) from lower-mobility ($\mu=3.0\times10^6$ cm^2/V s) 2DES for $f=94$ GHz (for sample details see Ref. 2) and vertical lines are given by Eq. (1).

separate MIRO and ZRS regimes; being described by different equations (and different experimental values of m), MIRO occurs at $j\gtrsim4$, while ZRS appear at $j\lesssim4$. To reconcile Eqs. (1) and (2) we propose the following expression:

$$\varepsilon_j^{\pm}=j+\phi^{\pm}(\varepsilon), \tag{3}$$

where $\phi^{\pm}$ is now allowed to vary with ε, approaching ∓0.25 as ε increases. In what follows, we analyze the phase $\phi^{\pm}\equiv\varepsilon_j^{\pm}-j$ extracted from our experimental data,[4,15] as a function of ε.[16]

In Fig. 1(a) we present the phase $\phi^{\pm}$ extracted from magnetoresistance traces[4] (cf. inset) taken at $T=0.9$ K (solid circles) and $T=2.7$ K (open circles) for $f=57$ GHz. Horizontal dashed lines mark a "1/4-cycle" shift, which is readily observed in experiment for $\varepsilon\gtrsim4$. At $\varepsilon\lesssim4$, positions of the minima at this T cannot be accurately determined, but for the maxima we observe a dramatic reduction of $|\phi^{+}|$, roughly linear with ε, i.e.,

$$\phi^{+}=-\beta^{+}\varepsilon, \tag{4}$$

with $\beta^{+}\approx6.4\times10^{-2}\ll1$. We note that, in agreement with earlier observations,[4] such a dependence does not produce *any* phase but affects the period. Indeed, substituting Eq. (4) into Eq. (3) one obtains $\varepsilon_j^{+}=(1+\beta^{+})^{-1}j$, which is just Eq. (2) with $\alpha^{+}=(1+\beta^{+})^{-1}$. Since we previously assumed $\alpha^{+}=1$ the mass was overestimated by a factor of $1+\beta^{+}$. After correction one obtains $m=m_{\rm ZRS}/(1+\beta^{+})\approx0.064m_0=m_{\rm MIRO}$, so both MIRO and ZRS are now described by a single, albeit somewhat low, effective mass. At elevated temperatures, e.g., $T=2.7$ K, ZRS are destroyed and the minima positions at $\varepsilon\lesssim4$ could be accessed. While somewhat enhanced at this T, the extracted phase (open circles) reveals the symmetry, i.e., $\phi^{-}(\varepsilon)\approx-\phi^{+}(\varepsilon)$, which, combined with obvious symmetry in the MIRO regime at any T hints on the symmetry of the ZRS. Therefore, we expect the maxima and the minima to be symmetrically detuned from the cyclotron resonance (CR) harmonics ($\varepsilon_j=j$), which, according to theory,[9] are indifferent to MW radiation. Experimentally, CR harmonics correspond to the higher-B sequence of the crossing points [points of intersection of the low- and high-T traces in the inset in Fig. 1(a)], which are represented by × symbols. These crossing points are characterized by zero phase and the same value of the effective mass, i.e., $m=0.064m_0$, over the whole range of ε.

It is interesting to examine our experimental observations in terms of a "toy model" proposed by Durst *et al.*[9] who suggested that MW photoresistance is expected roughly to follow the derivative of the DOS taken at $E=\hbar\omega$, so the extrema are described by

$$\left.\frac{d^2N(E)}{dE^2}\right|_{E=\hbar\omega}=0. \tag{5}$$

Since in weak magnetic field the DOS oscillates as $\cos(2\pi E/\hbar\omega_c)$, Eq. (1) is easily recovered. However, with increasing magnetic field cyclotron energy eventually exceeds the Landau level (LL) width and DOS can no longer be described by a single-harmonic function. As a result, in regular magnetotransport, Shubnikov–de Haas (SdH) effect evolves into a quantum Hall effect with increasing magnetic field (or sample mobility). Intuitively, one could think that similar magnetic-field-driven transition might be responsible for the evolution of the MIRO (Ref. 2) into ZRS.[3,4]

For quantitative analysis we assume that LL's are Lorentzians[17] with a field-independent width Γ and the DOS can be written as

$$N(E)=\frac{1}{\pi^2\ell_0^2}\sum_n\frac{\Gamma}{(E-n\hbar\omega_c)^2+\Gamma^2}, \tag{6}$$

where n denotes the LL index and $\ell_0=\sqrt{\hbar/eB}$ is the magnetic length. As we are constrained to very high LL's ($\hbar\omega_c\lesssim\hbar\omega\ll E_F$, E_F is the Fermi energy), the summation can be taken over infinite limits. After introducing dimensionless variables [$\varepsilon=E/\hbar\omega_c$, $\gamma=\Gamma/\hbar\omega_c$, and $n(\varepsilon)=\hbar\omega_c\pi\ell_0^2N(E)$], one obtains

$$n(\varepsilon)=[\cos^2(\pi\varepsilon)\tanh(\pi\gamma)+\sin^2(\pi\varepsilon)\coth(\pi\gamma)]^{-1}. \tag{7}$$

Substituting Eq. (7) in Eq. (5) [i.e., $d^2n(\varepsilon)/d\varepsilon^2|_{\varepsilon=\omega/\omega_c}$] and solving for ε we arrive at Eq. (3) with the phase of a symmetric form:

$$\phi^{\pm}=\mp\frac{1}{2\pi}\arccos\psi, \tag{8}$$

where $\psi=1/2-y+\sqrt{y^2-y+9/4}$ and $y=\cosh^2(\pi\gamma)$.

At lower magnetic fields ($\gamma\gg1$), $y\gg1$, $\psi\sim y^{-1}\ll1$, so $\phi^{\pm}\approx\mp1/4$, and Eq. (3) reduces to Eq. (1). More interesting results emerge at higher magnetic fields ($\gamma\ll1$), when LL's are well separated. In this limit, $y\approx1+\pi^2\gamma^2/2$, $\psi\approx1+2\pi^2\gamma^2/3$, which leads to $\phi^{\pm}\approx\mp\gamma/\sqrt{3}$ (the same result can be obtained by considering an isolated Lorentzian line). We immediately notice that the phase is linear with ε, in agreement with experiment [cf. Fig. 1(a)]:

$$\phi^{\pm}=\mp\frac{1}{\sqrt{3}}\frac{\Gamma}{\hbar\omega}\varepsilon. \tag{9}$$

A few comments are appropriate. First, direct comparison of Eqs. (9) and (4) yields $\Gamma=\sqrt{3}|\beta^{+}|\hbar\omega\approx0.3$ K$\ll\hbar\omega\approx2.7$ K. Second, since MIRO ($j\gtrsim4$) and ZRS ($j\lesssim4$) conform to limiting cases of Eqs. (3) and (8) [Eq. (1) and Eq. (2), respectively] the model suggests that ZRS develop from the MIRO minima as a result of a magnetic-field-driven transition taking place around $\hbar\omega_c/(2\Gamma)\sim1$. Such a transition would roughly occur when $\phi^{\pm}=\mp\gamma/\sqrt{3}$ reaches its low-field limit of $\mp1/4$, which, indeed, happens when $\hbar\omega_c/(2\Gamma)=\sqrt{3}/2\sim1$. The number of resolved ZRS can be then estimated as $\hbar\omega/(2\Gamma)\approx4$, in agreement with experiment. Third, it is interesting to mention that Eq. (9) suggests a direct method to probe Γ, which is not directly accessible in standard magnetotransport. Finally, we mention that detailed microscopic calculations using a self-consistent Born approximation predict similar reduction of the phase with increasing magnetic field.[18]

In Fig. 1(a) we now present $\phi^{\pm}$ (solid lines), calculated using Eq. (8) for $\Gamma=0.3$ K, and observe good agreement with experimental data.[4] Dashed lines crossing about $\varepsilon\approx4$ represent asymptotes of Eq. (8), i.e., $\phi^{\pm}=\mp\gamma/\sqrt{3}$ ($\gamma\ll1$) and $\phi^{\pm}=\mp0.25$ ($\gamma\gg1$). Experimentally, this intersection roughly marks a transition from MIRO to ZRS [cf. Fig. 1(b)] and an enhancement of the phase at higher T can now be related to the thermal broadening of LL's. Using Eqs. (3) and (8) we can also compute the maxima positions for the whole range of ε. In Fig. 1(b) we present the results of such calculations shown by vertical lines along with experimental trace[4] for $f=57$ GHz replotted as a function of ε. While it was shown before[14] that Eq. (1) works well only for $\varepsilon\gtrsim4$ and Eq. (2) for $\varepsilon\lesssim4$, Eqs. (3) and (8) provide excellent agreement over the whole range of ε. Since the model suggests larger phase in lower-mobility samples, it seems worthwhile to reanalyze the data of Ref. 2, where the phase shift was not self-evident and the maxima were associated with CR harmonics. Replotting the data[2] (cf. Fig. 2 from Ref. 2) as a function of ε in the inset of Fig. 1(b) we observe a fairly good agreement by fitting the maxima positions with Eq. (1); a somewhat reduced phase for the $j=1$ peak explains why the data, especially when plotted as a function of B, could be roughly described[2] without a phase shift. It is obvious that such approach also leads to a reduced value of the effective mass ($\sim0.063m_0$), than assumed before.[2] While the low-mass value cannot be explained at this point, it hints on a possibility that MW photoresistance does not reflect a conventional cyclotron mass, i.e., the mass, which would appear in standard MW absorption measurements.

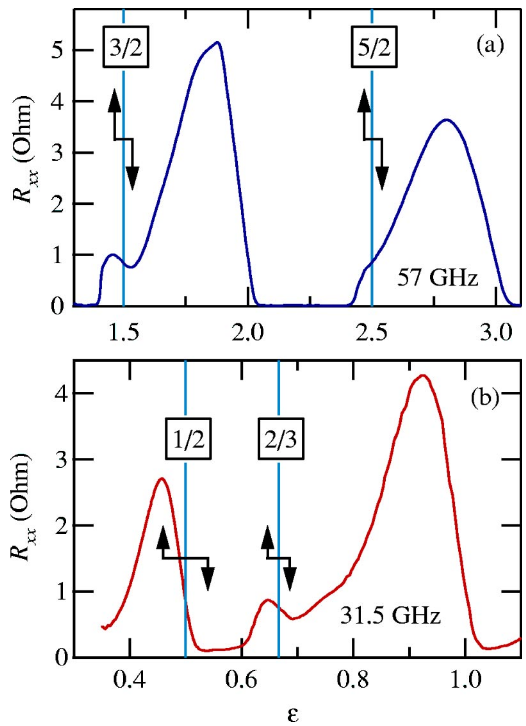

FIG. 2. Magnetoresistance under MW illumination of frequency (a) 57 GHz and (b) 31.5 GHz. Vertical lines are drawn at $\varepsilon=j/m=3/2$, 5/2 (a) and 1/2, 2/3 (b). Vertical arrows, placed symmetrically about j/m, mark maximum-minimum pairs at $\varepsilon^{\pm}_{jm}=j/m\mp\phi_{jm}$.

Once we have established that photoresistance extrema can be viewed as maximum-minimum pairs positioned symmetrically about $\varepsilon_j=j$, we can generalize Eq. (3) for the processes involving multiple photons. While it might seem unlikely that such higher-order processes be readily resolved experimentally, our data[4] suggest that such a scenario deserves close examination, especially in light of recent theoretical comments.[9,18,19] For the case of $\gamma\ll1$, m-photon processes are to produce maximum-minimum pairs positioned roughly symmetrically about *fractional* ε and Eq. (3) is modified as follows:

$$\varepsilon^{\pm}_{jm}=\frac{j}{m}\mp\phi_{jm}, \tag{10}$$

where $m=2,3,4,\ldots$ (here, we do not attempt to calculate ϕ_{jm} and explicitly include its sign). For instance, two-photon ($m=2$) processes would reveal themselves at half-integer values of ε, e.g., 1/2, 3/2, 5/2, ... To prove the feasibility of such scenario, we present in Fig. 2(a) the magnetoresistance

tance data[4] taken under illumination with MW radiation of $f=57$ GHz. A structure centered at $\varepsilon=3/2$ and marked by vertical arrows is clearly observed and similar structure seems to develop around $\varepsilon=5/2$. While such secondary peaks may have other origins, structures emerging at $\varepsilon<1$ present stronger support for multiphoton transitions as these naturally allow to enter the region of $\varepsilon<1$. In Fig. 2(b) we show magnetoresistance data[20] for $f=31.5$ GHz and focus on the region of $\varepsilon<1$. The structure positioned about $\varepsilon=1/2$ is comparable in amplitude to the primary single-photon structure around $\varepsilon=1$. In addition, there appears yet another maximum-minimum pair close to $\varepsilon=2/3$ suggesting a three-photon process. As has been noticed before,[4] such features are best observed at low MW frequencies, i.e., $f\lesssim 40$ GHz. Based on a good agreement, we believe that secondary peaks first reported in Ref. 4 are due to multiphoton processes as prescribed by Eq. (10) for $m=2$. While systematic power-dependence experiments are deferred for future studies,[20] here we mention that observation of secondary features requires high MW intensity, further supporting the multiphoton scenario.

In summary, we have studied the period and the phase of the MW photoresistance over the wide range of ε, covering both MIRO and ZRS regimes. As MIRO evolves into ZRS with increasing magnetic field, the phase reduces dramatically, decreasing as $1/B$. Maxima and minima occur roughly symmetrically about CR harmonics and can be described by different periods and zero phase, in agreement with our earlier report.[4] Within a simplified framework of the DOS model,[9] ZRS and MIRO can be viewed as two different experimental regimes separated by the condition $\hbar\omega_c/(2\Gamma)\sim 1$. Despite obvious oversimplification, the model seems to capture the behavior of the period and the phase quite well, allowing for universal (single equation, single effective mass) description of the MIRO and ZRS. In addition, it points out on multiphoton processes as the origin of fine structures first reported in Ref. 4. The physics underlying a ($\sim 5\%$) reduction of the effective mass, which is required to consistently explain the data, is unclear at this point and presents an interesting problem calling for further investigations, e.g., magnetoabsorption experiments.

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[16] We calculate ε using $m=0.064m_0$. This value is obtained from the period in two regimes characterized by a constant phase: MIRO maxima and minima ($\phi^{\pm}=\mp 0.25$) for $\varepsilon\gtrsim 4$ and crossing points ($\phi=0$) for all ε.
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