Inverse flux quantum periodicity in the amplitudes of commensurability oscillations in two-dimensional lateral surface superlattices

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We report strong, amplitude modulated, commensurability oscillations in the magnetoresistance of short period, square, two-dimensional, lateral surface superlattices with symmetric potentials. The amplitude of the oscillations is strongly enhanced when one magnetic-flux quantum (h/e) passes through an integral number of cells of the superlattice. The temperature dependence of the strong oscillations agrees with the theory for commensurability oscillations in one-dimensional superlattices, but the smaller oscillations between these are more rapidly attenuated by increasing temperature. Although the structure we observe has the same flux periodicity as expected for the Landau-level substructure known as the Hofstadter butterfly, such substructure will not be resolved at the temperatures of measurement (1–10 K). We compare our data instead to a recent theoretical model which treats exactly this case, and find significant points of agreement.

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Lateral surface superlattices (LSSL's), in which a periodic potential is applied to a two-dimensional electron system (2DES), have been studied for many years. The most significant feature in their magnetoresistance is generally the series of oscillations, periodic in reciprocal magnetic field, which are called commensurability or Weiss oscillations (CO's).[1] These can be strong if the modulating potential varies only in one dimension, but the oscillations are much weaker if the same modulation is applied in two orthogonal directions, and can have the opposite phase.[2]

Theoretically, both semiclassical and quantum-mechanical perturbation theories describe the one-dimensional (1D) data successfully. However when these techniques are extended to two dimensions, both the quantum-mechanical approach considering only the diffusional term in the conductivity,[3] and the early semiclassical calculations[4] predict that the CO's for a 2D pattern with the same modulation in both principal directions should be of the same amplitude as in the 1D case. This contradicts the experimental evidence. The suppression of the 2D CO's was ascribed in a comprehensive development of the methods introduced in Ref. 2 to the effect of the subband splitting resulting from the 2D nature of the potential.[5] A more direct, semiclassical picture of 2D LSSL's based on the motion of the guiding center in real space[6] shows that *asymmetry* in the potential landscape is important in defining the magnitudes of the CO's that are observed. If the two principal Fourier components of the periodic potential are unequal in magnitude, then strong CO's are expected for current flowing in the modulation direction of the larger component, and CO's should be absent for current flow in the orthogonal direction. No CO's should be seen at all if the Fourier components are equal, provided that the mobility is sufficiently high. This picture has been developed in a more comprehensive calculation[7] and experimental studies of a wide range of symmetric and asymmetric 2D LSSL's[8,9] have confirmed its main features.

In this paper we describe large, amplitude modulated CO's in short period LSSL's with *symmetric* modulation. The amplitude is enhanced when a quantum of magnetic flux (h/e) passes through an integral number of unit cells of the LSSL. The fields at which this modulation occurs point to a quantum-mechanical origin. We shall argue that the modulation is a manifestation of structure in the collisional or scattering term in the conductivity, which is predicted by a current theoretical approach[10] at these magnetic fields, even though the internal structure of the Landau levels due to the Hofstadter butterfly is not resolved.

The LSSL's considered here were fabricated by patterning the surfaces of $GaAs/Al_{0.3}Ga_{0.7}As$ heterostructures using electron beam lithography and shallow wet etching, to produce either 50 nm diameter pillars on a 100 nm period square LSSL[8] or 40 nm diameter holes in a 80 nm structure.[9] Figure 1 shows normal and inverted AFM images of an 80 nm period LSSL of holes. The heterostructures contained a 6 nm strained layer of $In_{0.2}Ga_{0.8}As$, 10 nm below the surface, and the depth of the holes (20 nm) was sufficient to cut through this layer and produce a periodic stress.

The potential in such samples arises from two main effects, depletion due to removal of material and a piezoelectric field due to the stress. Interference between these two effects usually breaks the symmetry of the potential,[8,9] which is undesirable in the experiments reported here. We therefore forward biased the samples using an overlying gate to eliminate the depletion effect, leaving only the piezoelectric field. The mobilities of the samples under these conditions were typically 70 $m^2V^{-1}\ s^{-1}$. Results from different Hall bars, aligned in the $[011]$ and $[01\bar{1}]$ directions on a (100) substrate and fabricated in close proximity on the same wafer are reported. For these directions the piezoelectric potentials are equal in magnitude but opposite in sign,[11,12] producing a symmetric square potential displaced by half a superlattice period in one axial direction with respect to the pattern on

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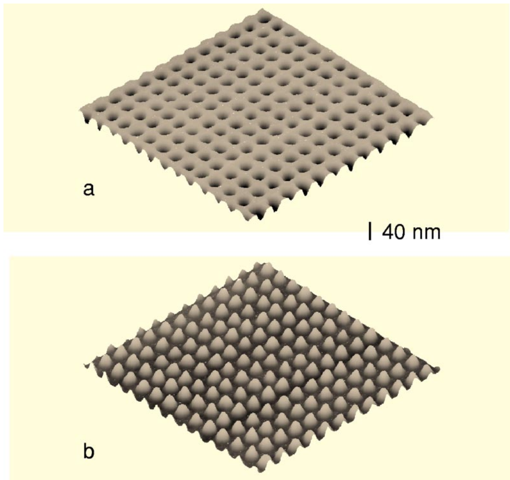

FIG. 1. AFM images of an etched 80 nm period square LSSL of holes without a gate but after removal of the resist; (a) normal view, (b) inverted view. The depth of the holes is approximately 20 nm.

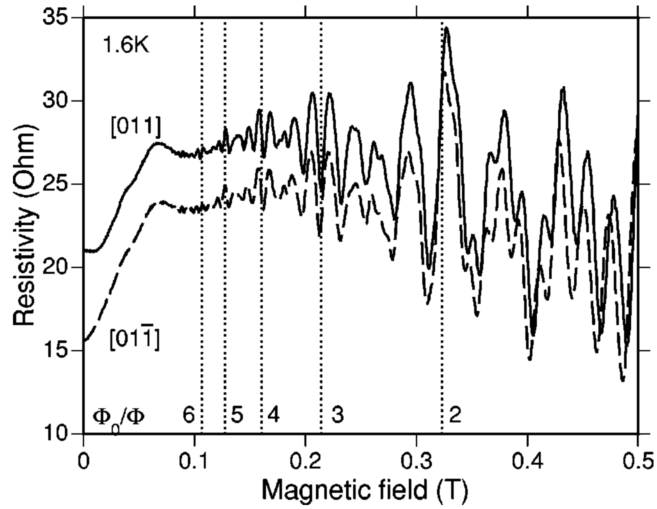

FIG. 3. Magnetoresistance at 1.6 K with +0.4 V gate bias for the same LSSL's as in Fig. 2. The dotted vertical lines indicate the magnetic-field values at which one flux quantum (h/e) passes through 2, 3, 4, 5, and 6 unit cells of the superlattice, where the magnetoresistance oscillations are enhanced. Shubnikov–de Haas oscillations can be seen above 0.3 T.

the surface. The magnetoresistance of the samples was measured at liquid helium temperatures using standard alternating current (a.c.) techniques.

Magnetoresistance data for the 80 nm period superlattices are shown in Figs. 2 and 3; similar characteristics were measured on two further pairs of devices. The traces for the two orthogonal directions show almost identical structures. At 5.5 K (Fig. 2) the Shubnikov–de Haas oscillations die away around 0.8 T leaving the CO's at lower fields. The minima of the CO's are in the expected flat-band positions given by the commensurability condition $2R_c/a=k-\frac{1}{4}$, where R_c is the cyclotron radius at the Fermi energy, a is the period of the superlattice, and k is an integer index. This is shown by the vertical bars in Fig. 2, which are plotted for k up to 12. We do not observe the "antiphase" oscillations seen previously for much longer periods of modulation,[2] with maxima at these positions. The absence of any other minima shows that only the principal period of the LSSL is involved in the structure; there is no evidence for any diagonal Fourier components, as are seen in similar samples aligned with the cube axes.[8,9]

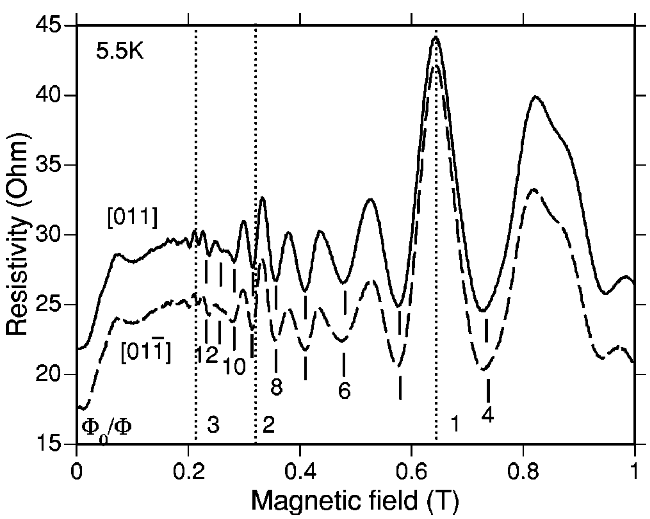

FIG. 2. Magnetoresistance at 5.5 K with +0.4 V gate bias from 80 nm period 2D LSSL's aligned with the [011] and [01$\bar{1}$] directions and prepared at the same time. The dotted vertical lines indicate the magnetic-field values at which one flux quantum (h/e) passes through 1, 2, and 3 unit cells of the superlattice, where the magnetoresistance oscillations are enhanced. The vertical bars indicate the expected locations of the CO minima with the correct 80 nm period for the indices shown.

A striking feature of the data is the *amplitude modulation*: the CO's are large close to fields of 0.64, 0.32, 0.21 . . . T. At these fields the flux Φ through one unit cell of the superlattice obeys $\Phi_0/\Phi=\alpha$ where the flux quantum $\Phi_0=h/e$ and $\alpha=1,2,3,\ldots$. In other words, one quantum of magnetic flux passes through an integral number of unit cells at these fields. The strong oscillations can be seen up to $\alpha=6$ at 1.6 K in Fig. 3 and for $\alpha=1$ and 2 the modulation was visible up to 9 K.

To confirm that the modulation is associated with magnetic flux, and therefore the *area* of the unit cell, we also studied LSSL's with 100 nm period (Fig. 4). Similar structure was seen but this time at fields of 0.41 T, 0.21 T, . . . (millikelvin temperatures were required to see the structures at $\alpha=3$ and 4). Samples containing a 100 nm square superlattices of holes also showed similar amplitude modulation at the same fields. The magnetic fields at which the strong oscillations were seen in LSSL's of these two periods differ by the ratio of the *areas* of the unit cell, 16/25. In contrast, the magnetic field for CO's depends on the *linear* dimension of the superlattice, which gives a ratio of 4/5. This shows that the modulation cannot arise from interference between CO's associated with different Fourier components of the potential and supports a relation with inverse flux quantization. Although the modulation is most pronounced for a symmetric potential, it persists to some extent with mild asymmetry

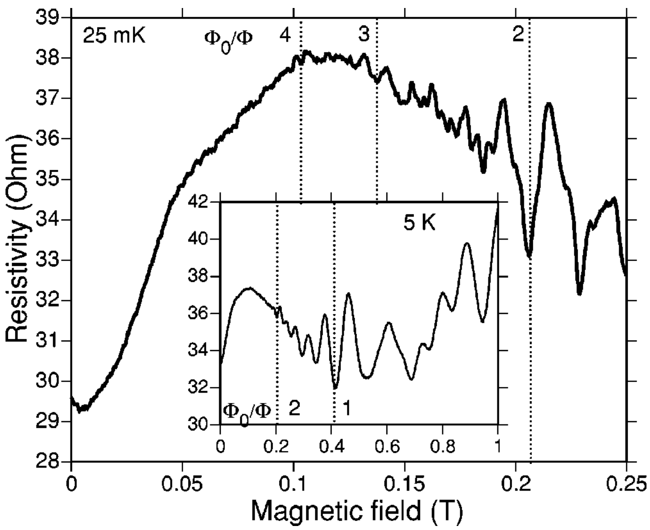

FIG. 4. Magnetoresistance at 25 mK and $+0.3$ V gate bias for a sample aligned with the [011] direction of 50 nm diameter pillars on a 100 nm period square superlattice. The dotted vertical lines indicate the magnetic-field values at which one flux quantum (h/e) passes through 2, 3, and 4 cells of the superlattice, where enhanced oscillations are observed. Inset: the same sample measured at 5 K, showing the enhanced oscillations at $\Phi_0/\Phi=1$ and 2.

[Fig. 2(b) of Ref. 8], especially at $\Phi_0/\Phi=1$, but disappears completely when the asymmetry is large [see Fig. 2(a) of Ref. 13)].

We have studied the temperature dependence of the amplitudes of the CO's for the 80 nm period samples. Representative data are given in Fig. 5. For all the large amplitude CO's near integer values of the index α, Fig. 5(a) shows that the temperature dependence was in excellent agreement with the standard models for 1D CO's.[14,15] Note that the fitting function used $T/T_a \sinh(T/T_a)$, where the characteristic temperature $T_a=\hbar\omega_c a/2\pi k_B\lambda_F$, ω_c is the cyclotron frequency, k_B is Boltzmann's constant, and λ_F is the Fermi wavelength, depends, for a constant period a, only on the ratio of the temperature and the magnetic field, which are both known. Thus the only disposable parameter used in any of the fits is the zero-temperature amplitude. Apart from the points at low temperature and high field, where the Shubnikov–de Haas oscillations have a significant systematic effect, the fit is consistent with the random errors for all this data. This indicates that the strong oscillations and 1D CO's depend on the same energy scale, which is larger than the cyclotron energy by a factor of $\frac{1}{2}k_F a$ $(k_F=2\pi/\lambda_F)$.[14]

The standard model also fits the CO's between $\alpha=1$ and $\alpha=2$ well [see the data for the $k=6$ peak in Fig. 5(a)]. However, Fig. 5(b) shows that the CO's die away much more rapidly with temperature than predicted between the next pairs of large amplitude regions ($\alpha=2,3$ and $\alpha=3,4$). This indicates that a smaller energy scale is involved. In the next low amplitude region (between $\alpha=4,5$), the minimum corresponding to $k=20$ is entirely absent (see Fig. 3). The zero-temperature amplitudes obtained from this fitting procedure are plotted versus the CO index k in Fig. 5(c). The enhanced amplitudes near integer values of α are clear.

These observations show that strong CO's are seen in symmetric 2D LSSL's with short periods. The enhancement when one flux quantum passes through an integral number of cells of the superlattice points to a quantum-mechanical origin. Superficially it appears to be in agreement with early analyses of the problem,[3,5] which predict that CO's of equal amplitude will occur for transport in the two principal directions in a symmetric 2D periodic potential; their amplitude is reduced when the substructure of the Landau levels is resolved[5] but this does not apply to our samples. However, these early quantum-mechanical models give no modulation of the CO's other than the usual monotonic decay with inverse field and therefore do not explain key features of our observations.

The quantum mechanics of a 2DES subject to a symmetric 2D periodic potential and a perpendicular magnetic field have been widely studied since Hofstadter[16] predicted a remarkable self-similar energy spectrum for the limit of an isolated tight-binding band due to a strong periodic potential, the Hofstadter butterfly. This spectrum is plotted over the range $0<\Phi/\Phi_0<1$ and repeats with period Φ_0 at higher fields. Within the lowest repeat, the band breaks into α subbands at $\Phi/\Phi_0=1/\alpha$, the same fields as the structure observed in our data. However there is no sign in our data of the reflected structures predicted at $\Phi/\Phi_0=(\alpha-1)/\alpha$. Therefore we do not accept this crude application of the Hofstadter prediction to our data. A full quantum-mechanical calculation[17] predicts magnetoresistance structures at $\Phi/\Phi_0=\alpha$ but does not show inverse flux quantum structure or commensurability oscillations and is therefore not directly relevant to our work.

Our system lies closer to the opposite limit of isolated Landau levels in a weak 2D periodic potential. This has two main features (Ref. 5 and earlier references therein). (1) Each Landau level broadens into a band whose overall width follows the same commensurability relation as in a 1D LSSL, with minima at the flat-band fields given by $2R_c/a=k-\frac{1}{4}$.[18–20] (2) Within this varying width each Landau band shows a Hofstadter-like spectrum as a periodic function of the *inverse* flux quantum Φ_0/Φ.

In current LSSL's the overall width varies more rapidly with magnetic field than the internal structure. Some indications of the internal structure have been seen in high-mobility samples at millikelvin temperatures in the field range $\Phi_0/\Phi<1$.[21,22]

In our samples the scattering is too strong for us to resolve substructure in the Landau levels directly. Indeed in the higher-temperature experiments, the Landau levels themselves are not resolved in the field range of interest. In the previous paper Vasilopoulos, Wang, and Peeters[10] have analyzed the conductivity of the 2D modulated system in just this limit. As in their previous analysis of the 1D problem,[15] they use an evaluation of the conductivity tensor in which the diagonal part is divided into diffusive and collisional contributions. The diffusive or band-conduction component is dominated by net current carrying states and leads in this case to commensurability oscillations in the usual way. It is however the collisional or scattering contribution due to hopping between localized states, which generates conductivity

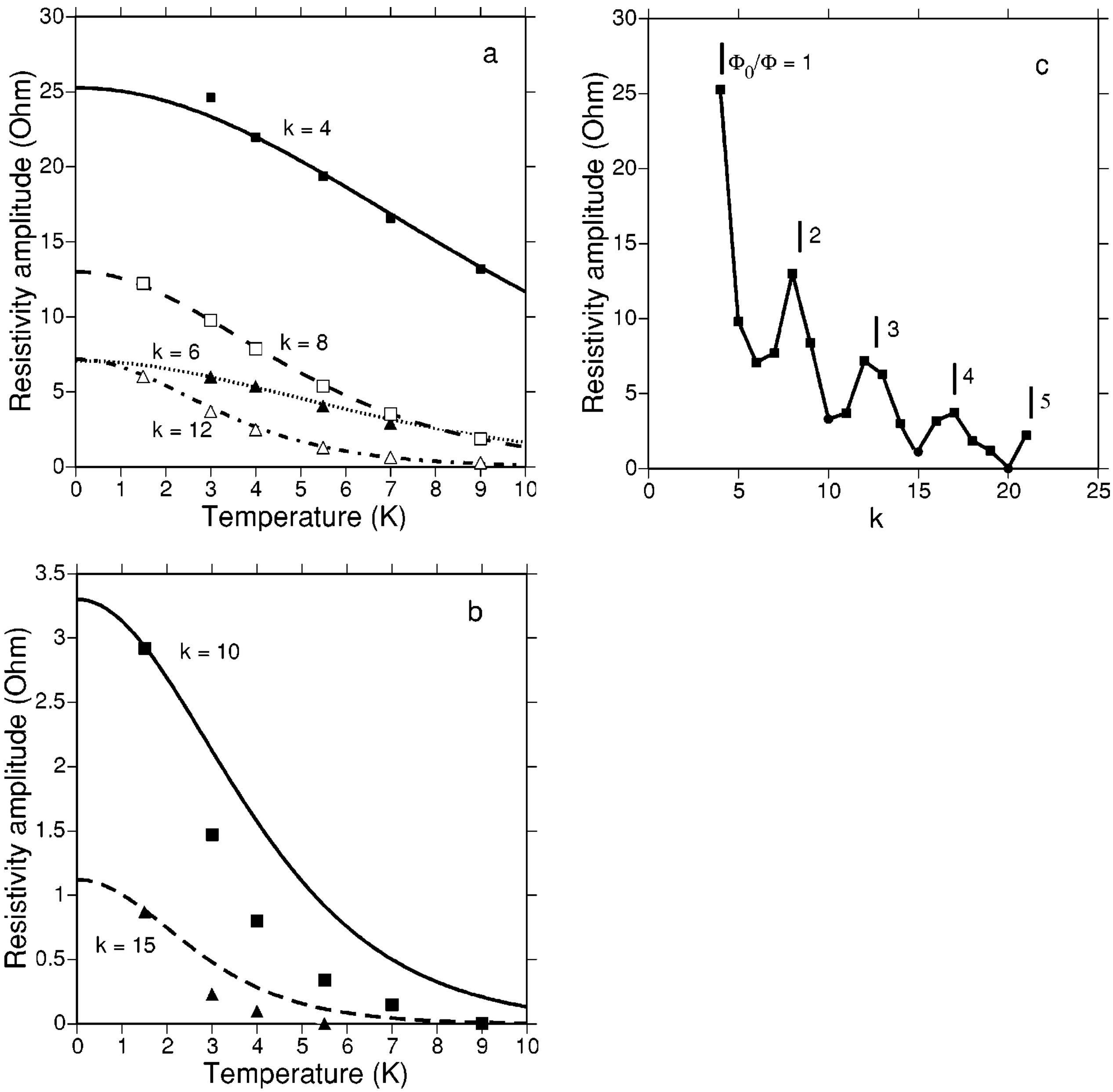

FIG. 5. Temperature dependence of the amplitudes of the CO's for a 80 nm period, [011] sample at the index numbers k indicated. (a) Oscillations that obey the standard theory for 1D. The random measurement error associated with each point is typically $\pm 0.3\,\Omega$. The point at 3 K for $k=4$ is seriously affected by Shubnikov–de Haas oscillations. (b) Oscillations between integer values of Φ_0/Φ that are more strongly attenuated than the standard theory predicts. (c) Extrapolated zero-temperature amplitudes for the CO's (errors estimated at $\pm 0.5\,\Omega$). The large amplitudes of the oscillations close to integer values of Φ_0/Φ are clearly visible. Circles mark points for which the extrapolation is less reliable.

peaks where one flux quantum intersects an integral number of cells. These translate to resistivity structures at the same magnetic fields, in agreement with experiment.

In Fig. 6 we compare the data of Fig. 2 with the computations of Vasilopoulos *et al.* for the same sample parameters, assuming a field independent relaxation time (taking a $B^{-1/2}$ dependence does not greatly affect the comparison). The agreement between theory and experiment around the strong peak at $\alpha=1$ is excellent. Between $\alpha=1$ and 2, weaker CO structures resulting in the main from the diffusion term, although reduced in amplitude by the antiphase collisional contribution, are also in good agreement with experiment. At $\alpha=2$, another strong collisional peak is predicted and observed, although the agreement between the peak shapes is not as good as at $\alpha=1$, in that the minimum observed at the $k=9$ flat-band position at a slightly lower field is not resolved in the theory. This is not a complete surprise as the theoretical estimates for the diffusion contribution use a constant or $B^{-1/2}$ dependence for the relaxation time, whereas in practice it may oscillate.[2] At lower fields strong peaks are predicted at $\alpha=3$,4, and 5, but in the experimental data, these structures are much weaker than pre-

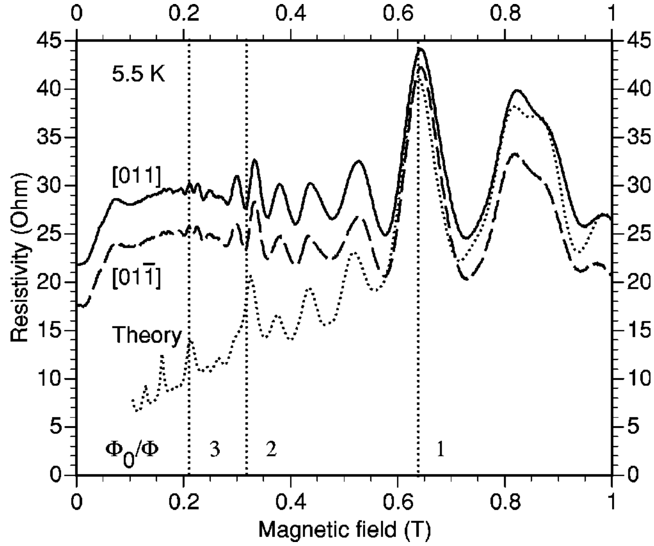

FIG. 6. The data of Fig. 2 replotted together with the theoretical trace calculated in Ref. 10 for the same parameters, assuming a magnetic-field independent scattering rate. The dotted vertical lines are at fields where one flux quantum (h/e) passes through 1, 2, and 3 cells of the superlattice, and where strong magnetoresistance structures are calculated and observed.

dicted. This is confirmed if the experimental temperature dependences (Fig. 5) are compared with the theoretical curves (Fig. 10 of Ref. 10); the peaks at integer α are predicted to be much more robust than is observed.

Finally, we consider the question of why the enhanced amplitudes at integer values of α have not been reported before. We believe that there are two reasons for this, both associated with the short period of our devices. First, the small areas of the cells of the LSSL described here mean that the conditions $\alpha=1$, 2, etc. occur at relatively high fields and the higher integer structures are not smeared out by scattering, giving the characteristic series of enhanced oscillations we observe. Second, the accompanying theoretical work demonstrates that the structures at integer α are intrinsically stronger in short period devices. With the benefit of hindsight, one can see enhanced oscillation amplitudes at $\alpha=1$ in many published results.

In summary, we have observed strong commensurability oscillations in short period, square, lateral surface superlattices with symmetric modulation. No oscillations should be seen in high mobility samples under these conditions according to semiclassical theory. We see an amplitude modulation of the oscillations, which are particularly strong when a quantum of magnetic flux (h/e) passes through an integral number of unit cells. The enhanced commensurability oscillations are thermally robust and obey the existing model developed to describe the amplitudes of 1D commensurability oscillations. A current theoretical analysis predicts similar enhanced resistivity peaks occurring at the same fields. Theory and experiment are generally in good agreement though detailed differences do remain.

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